Basics Of Inequalities

(1) Equalities :-

$x^2 = 16$

$x = \pm\sqrt{16}$

$\boxed{x = \pm 4}$

$x^2 = 4$

$x = \sqrt{4}$

$\boxed{x = 2}$ ✓ & $\boxed{x = \pm 2}$ ✗

$\boxed{\sqrt{x} = +ve}$ and $\boxed{-\sqrt{x} \Rightarrow -ve}$

(2) Inequalities :-

→ Signs :-

$>$ → Greater than

$<$ → Less than

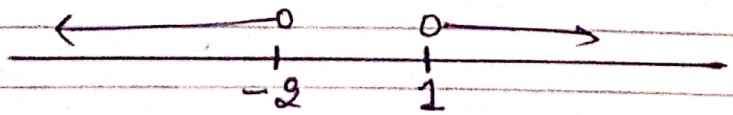
$\geq$ → Greater than or equal to

$\leq$ → Less than or equal to

(3) Intervals :-

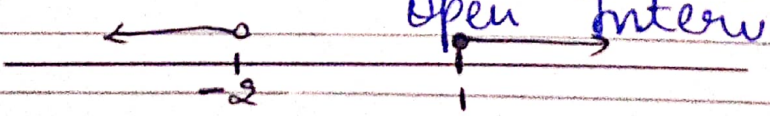
→ (,) → Open Interval

For Example :-



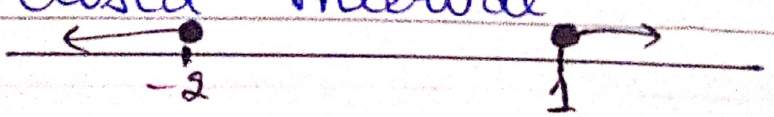
→ (,] / [,) → Semi-Closed / Semi-Open Interval

For Example :-



→ [,] → Closed Interval

For Example :-



→ ○ → Open and We are not considering the value on which these are put.

● → Closed We are considering the values on which these are put.

→ Curly Brackets :- { , }
Only discrete values are accepted.
Ex :- {1, 2, 3, 4, 5 ... }

→ ∞ and −∞ are always on / in round / open brackets.

→ Always we go Left → to Right in a number line to define its intervals.

→ If $a > b$
then we can do → $a + c > b + c$.

→ If $a > b$ and $c > 0$
then, $ac > bc$

→ If $a > b$ and $c < 0$
then $ac < bc$

✱ Sign Of Inequality changes if its is multiplied by a negative quantity.

→ If $a > b$
Case - I → $c > 0$ → $a/c > b/c$

Case - II → $c < 0$ → $a/c < b/c$

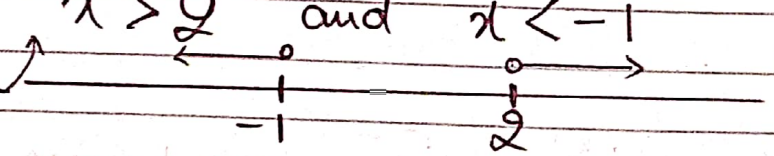
→ $a \times b > 0$

Case - I → $a > 0$ and $b > 0$
Case - II → $a < 0$ and $b < 0$

→ $a \times b < 0$
Case - I → $a > 0$ and $b < 0$
Case - II → $a < 0$ and $b > 0$

(4) Union in Intervals :-
↓
Combine
↓
OR

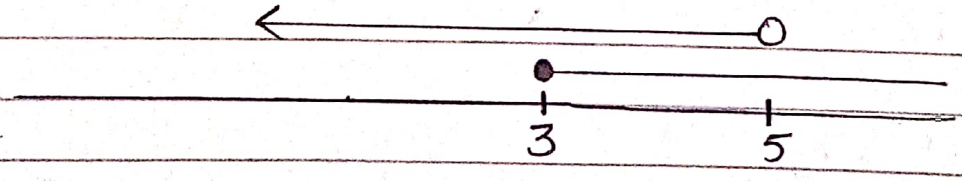
for Example :- $x > 2$ and $x < -1$



$\therefore x \in (-\infty, -1) \cup (2, \infty)$

(5) Intersection in Intervals :-
↓
Common → $\cap$
↓
And

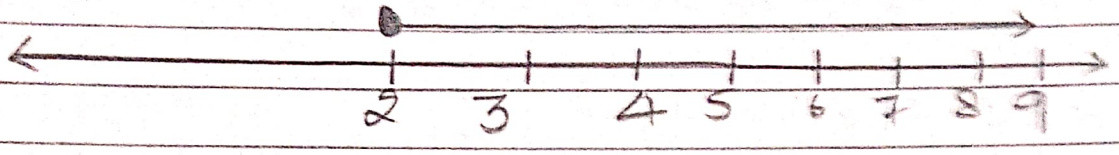
for Example :- $x \geq 3$ and < 5



$x \in [3, 5)$

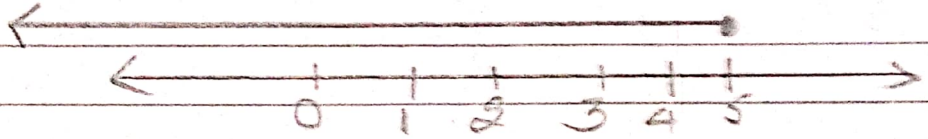
Some Miscellaneous Examples on Inequalities

Q. $x \geq 2$



$x \in [2, \infty)$

Q $x \leq +5$

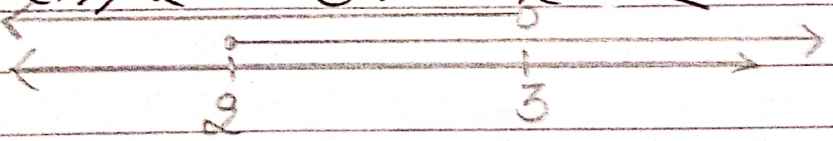


$x \in (-\infty, 5]$

Q. $2 < x < 5$

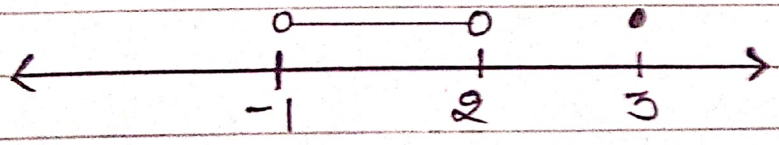
$\rightarrow x \in (2, 5)$

Q. $x \geq 2$ Or $x < 3$



$\rightarrow x \in (-\infty, \infty)$

Q



$\rightarrow x \in (-1, 2) \cup \{3\}$

(6) Wavy Curve Method :-

Rule-1 :- R.H.S → zero

Rule-2 :- Factorize → L.H.S. in the form of $(x-a)$.

Rule-3 :- Check for coefficient of $x = +1$

Rule-4 :- $(\ )^{\text{o}} \rightarrow$ P-odd → Sign change
$(\ )^{\text{e}} \rightarrow$ P-even → No Sign change

Rule-5 :- Opt. for +ve value → $>$
Opt. for -ve value → $<$

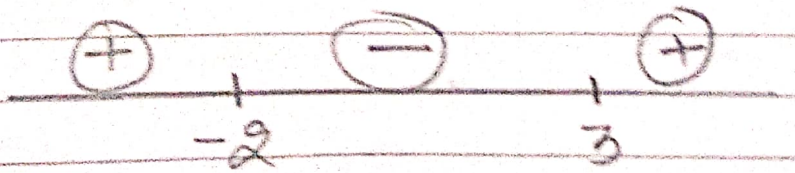
Rule-6 :- $>$ and $<$ → Open Bracket
$\geq$ and $\leq$ → closed Bracket

→ Some Questions on Wavy Curve Method :-

Q. $(x-1)(x+2) > 0$

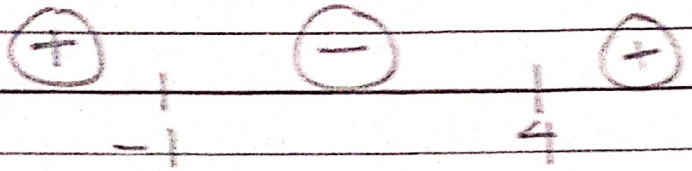
$x \in (-\infty, -2) \cup (1, \infty)$

Q. $(x+2)(x-3) > 0$



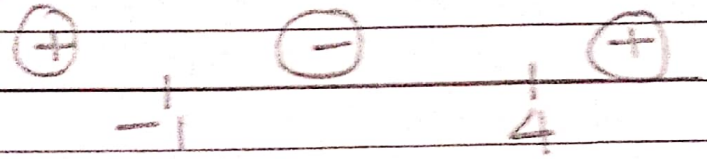
$x \in (-\infty, -2) \cup (3, \infty)$

Q. $(x+1)(x-4) \geq 0$



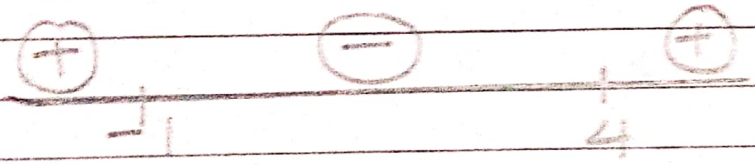
$x \in (-\infty, -1] \cup [4, \infty)$

Q. $(x+1)(x-4) < 0$



$x \in (-1, 4)$

Q. $(x+1)(x-4) \leq 0$



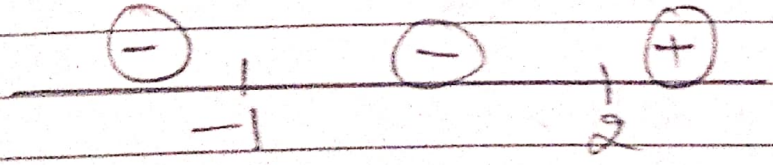
$\rightarrow \boxed{x \in [-1, 4]}$

Q. $(x+1)(x-4)(x+10) > 0$

$x \in (-10, -1) \cup (4, \infty)$

Imp.

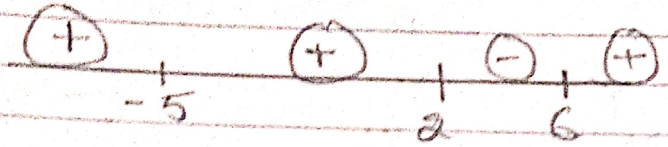
Q. $(x+1)^2 (x-2)^3 \leq 0$



$x \in (-\infty, -1] \cup [-1, 2]$

or $x \in (-\infty, 2]$

Q. $(x-2)^3 (x+5)^4 (x-6) \geq 0$



$x \in (-\infty, 2] \cup [6, \infty)$

Q. $(x+3)^3 (x-1)^4 (x-4)^5 (x+6)^6 \leq 0$

$x \in (-3, 1) \cup (1, 4)$

Q. $(1-x)(2+x)(x+3) \geq 0$

$\Rightarrow (-1)(x-1)(x+2)(x+3) \geq 0$ [Coefficient of $x = 1$]

$\Rightarrow (x-1)(x+2)(x+3) \leq 0$

$x \in (-\infty, -3] \cup [-2, 1]$

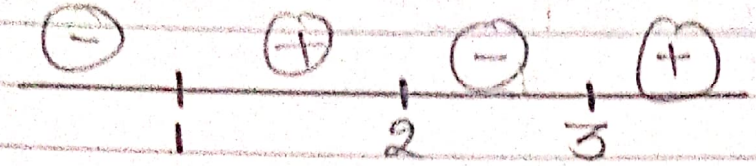
Q. $(2-x)(2+x)(3-x) > 0$

$\Rightarrow (-1)(-1)(x-2)(x+2)(x-3) > 0$

$\Rightarrow (x-2)(x+2)(x-3) > 0$

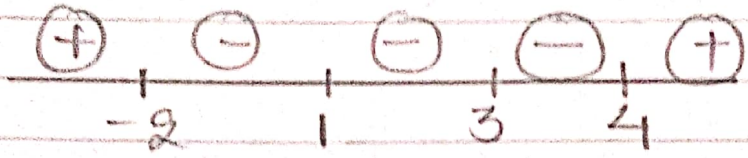
$x \in (-2, 2) \cup (3, \infty)$

Q. $\frac{(x-1)(x-2)}{(x-3)} > 0$



$x \in (1, 2) \cup (3, \infty)$

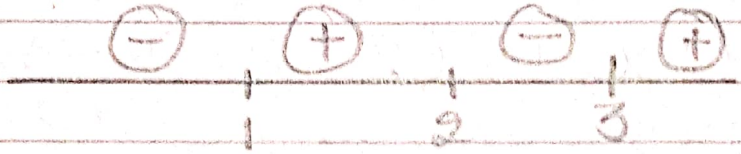
Q. $\frac{(x-1)^2 (x+2)}{(x-3)^4 (x-4)^5} > 0$



$x \in (-2, 1) \cup (1, 3) \cup (3, 4)$

Q. $\frac{(x-1)(x-2)}{(x-3)} \geq 0$

Denominator → Open

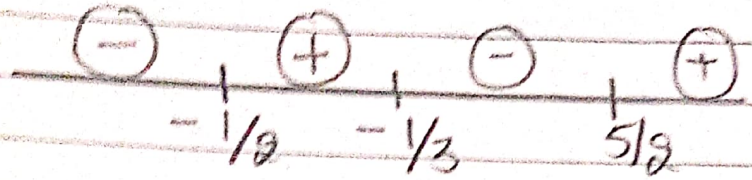


$x \in [1, 2] \cup (3, \infty)$

Q. $(x+1)(x-3)^2(x+4) \leq 0$

$x \in [-4, -1] \cup \{3\}$

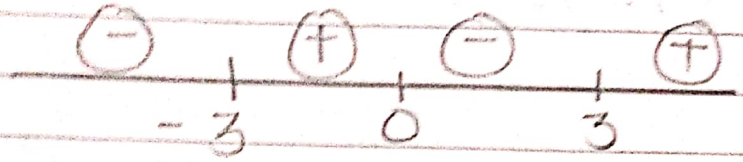
Q. $(3x+1)(1+2x)(2x-5) < 0$



$x \in (-\infty, -1/2) \cup (-1/3, 5/2)$

Q. $x(x^2-9) < 0$

$\Rightarrow (x-0)(x+3)(x-3) < 0$

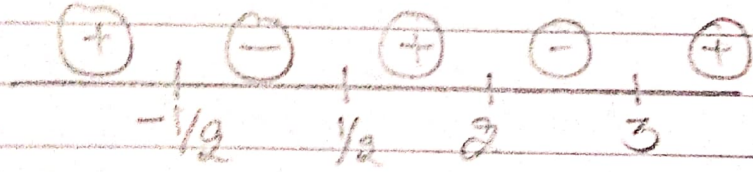


$x \in (-\infty, -3) \cup (0, 3)$

Q. $(1-4x^2)(x^2-5x+6) < 0$

$\Rightarrow (-1)(4x^2-1)(x-2)(x-3) < 0$

$\Rightarrow (2x+1)(2x-1)(x-2)(x-3) > 0$



$x \in (-\infty, -1/2) \cup (1/2, 2) \cup (3, \infty)$

$\longrightarrow \dfrac{(x-1)(x-2)}{(x^2+1)} > 0$

All +ve ← (x^2+1) → non-factorisable

Then, $\longrightarrow (x-1)(x-2) > 0$