Basics Of Logarithm

① Defination Of logarithm :-

$$\boxed{\log_b^{N} = a} \leftrightarrows \boxed{N = b^{a}}$$

Q. $\log_8^{16}$

$(8)^{4/3} = 16$

Q. $\log_{81}^{27}$

$(81)^{3/4} = 27$

② Properties Of logarithm :-

(A) $\log_b (m)^n = n \log_b m$

For Example :-

$$\log_2^{8} = \log_2^{(2)^3} = 3 \log_2^{2} = 3$$

(B) $\log_a^{a} = 1$

(C) $\log_{b^n}^{(x)^m} = \dfrac{m}{n} \log_b^{x}$

For Example :- $\log_{16}^{32}$

$$\Rightarrow \log_{(2)^4}^{(2)^5} \Rightarrow \frac{5}{4} \log_2^{2} = 5/4$$

imp

☆ $\log_b^{N} = a \Rightarrow N = (b)^{a}$

$\rightarrow N > 0$ $\rightarrow b > 0$

$\rightarrow b \neq 1$ $\rightarrow 0 < b < 1$

Q. $\log_2^{(\cos 45^\circ)} = \log_2^{(1/\sqrt{2})} = \log_2^{(2)^{-1/2}}$

$\Rightarrow \frac{-1}{2} \log_2^2 = \underline{\underline{-1/2}}$

(D) $\log_b^1 = 0 \leftrightharpoons b^0 = 1$

Q. $\log_{1/3}^{9\sqrt{3}} \Rightarrow \log_{3^{-1}}^{(3)^{5/2}}$

$\Rightarrow \frac{5/2}{-1} \log_3^3 \Rightarrow -5/2$

Q. $\log_{2\sqrt{3}}^{1728}$

$\Rightarrow \log_{2\sqrt{3}}^{(12)^3} \Rightarrow 3 \log_{2\sqrt{3}}^{12} \Rightarrow 3 \log_{2\sqrt{3}}^{2^2 \cdot 3}$

$\Rightarrow 3 \cdot \log_{2\sqrt{3}}^{2^2 \cdot (\sqrt{3})^2} \Rightarrow 6 \log_{2\sqrt{3}}^{2\sqrt{3}}$

$\boxed{\Rightarrow 6}$

Q. $\log_2 (\log_2^4) = ??$

$\log_2^2 = 1$

Q. $\log_5 \sqrt{5 \cdot \sqrt{5 \cdot \sqrt{5 \ldots \infty}}}$

$\rightarrow \sqrt{5 \cdot \sqrt{5 \cdot \sqrt{5 \cdot \ldots \infty}}} = x$

$\rightarrow \sqrt{5x} = x \rightarrow 5x = x^2$

$x^2 - 5x = 0$

$\Downarrow$

$x(x-5) = 0$

$\underbrace{x}_{0 \ (\times)} \quad \underbrace{(x-5)}_{5 \ (\checkmark)}$

$\boxed{\therefore x = 5}$

$\Rightarrow \log_5^x \Rightarrow \log_5^5 = \underline{\underline{1}}$

Q. $\log_b^{\tan 1^\circ} \cdot \log_b^{\tan 2^\circ} \ldots \ldots \log_b^{\tan 89^\circ}$

$\downarrow$

$\log_b^{\tan 45^\circ} = \log_b^1 = 0$

$\boxed{\therefore 0 = \text{Answer}}$

Q. $\log \sin 1^\circ \cdot \log \sin 2^\circ \ldots \ldots \log_{\sin 90^\circ}$

Here, ~~$\log_{\sin} 90$~~ $\log_b^{\sin 90^\circ} = \log_b^1 = 0$

(E) $\log(mn) = \log m + \log n$

(F) $\log\left(\frac{m}{n}\right) = \log m - \log n$

For Example $\Rightarrow \log \frac{2}{3} = \log 2 - \log 3$

(G) Natural Logarithm :-

$e \rightarrow$ euler's Constant $\rightarrow 2.713$

Irrational No.

Written as :- $\log_e N = \ln N$

Q. $\log_2(\log_2(\log_3(\log_3 27^3))) = ??$

$\Rightarrow \log_2(\log_2(\log_3(3\log_3 27)))$

$\Rightarrow \log_2(\log_2(\log_3 3\times 3))$

$\Rightarrow \log_2(\log_2 2)$

$\Rightarrow \log_2 1 = 0$

Q. $\log_{\sec\alpha}(\cos^3\alpha)$; where $\alpha \in (0, 90^\circ)$

$\Rightarrow 3\log_{\sec\alpha}\cos\alpha \Rightarrow 3\log_{\sec\alpha}(\sec\alpha)^{-1}$

$\Rightarrow -3\log_{\sec\alpha}\sec\alpha \Rightarrow -3$

(H) $\log_b a = -1$ if $\boxed{ab = 1}$

for Example :-

$\log_{1/3} 3 = -1$ | $\log_{\cot\alpha}\tan\alpha = -1$

(I) $\log_b^{a} = \dfrac{\log_c^{a}}{\log_c^{b}}$], Base Changing Theorem

For Example :-

$$\log_2^{5} = \frac{\log_e^{5}}{\log_e^{2}}$$

Q. $\log_2^{3} \cdot \log_3^{4} \cdot \log_4^{5} \cdots\cdots \log_n^{(n+1)} = 10$

$$\Rightarrow \frac{\log_e^{3}}{\log_e^{2}} \cdot \frac{\cancel{\log_e^{4}}}{\cancel{\log_e^{3}}} \cdot \frac{\cancel{\log_e^{5}}}{\cancel{\log_e^{4}}} \cdots\cdots \frac{\log_e^{(n+1)}}{\cancel{\log_e^{n}}}$$

$$\Rightarrow \frac{\log_e^{(n+1)}}{\log_e^{(2)}} \Rightarrow \log_2^{(n+1)} = 10$$

$$\Rightarrow \log^{(n+1)} = 2^{10} \Rightarrow n+1 = 1024$$

$$\boxed{\Rightarrow n = 1023}$$

Q. $\log_2^{10} - \log_8^{125} = ??$

$$\Rightarrow \log_2^{10} - \log_{2^3}^{5^3} \Rightarrow \log_2^{10} - \frac{\cancel{3}}{\cancel{3}} \log_2^{5}$$

$$\Rightarrow \log_2^{10} - \log_2^{5} \Rightarrow \log_2^{10/5} \Rightarrow \log_2^{2} = 1$$

(II) $(a)^{\log_a^{x}} = x$

For Example :-

$$(2)^{\log_2^{9}} = 9$$

(K) $a^{\log_b c} = c^{\log_b a}$

For Example :-

$(7)^{\log_3 5} = (5)^{\log_3 7}$

Q. $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3} = ??$

$\Rightarrow \cancel{7^{\log_3 5}} + \cancel{3^{\log_5 7}} - \cancel{7^{\log_3 5}} - \cancel{3^{\log_5 7}}$

V. Imp. $\Rightarrow 0$

(L) $\dfrac{1}{\log_a N} \Rightarrow \log_N a$