Sol^n Let $\sqrt{3+4i} = z = x+iy$, then,

$3+4i = x^2 + 2xyi - y^2$ → $x^2-y^2 = 3$; $2xy = 4 \Rightarrow \boxed{y = 2/x}$

$\therefore x^4 - 4 = 3x^2 \Longrightarrow$ Let $x^2 = t$, then,

$\Rightarrow t^2 - 3t - 4 = 0$

$\Rightarrow (t-4)(t+1) = 0$

$\Rightarrow x^2 = 4 \Longrightarrow x = 2$ & $x = -2$

$y = 1$ & $y = -1$

$\therefore z = 2+i$ or $z = -(2+i)$

So, $\sqrt{3+4i} = \pm(2+i)$

(15) Logarithm of a Complex Number:-

Let for a Complex No. 'z',

$\Rightarrow \log_e(x+iy) = \log_e z$

$\Rightarrow \log_e z = \log_e(re^{i\theta}) = \log_e r + \log_e e^{i\theta}$

$\boxed{\log z = \log_e r + i\theta}$

Q. If $z = i^i$, then, find its arg. & state whether it is purely real or purely imaginary.

Sol^n $\log z = i \log i = i\log(1)e^{i\pi/2} = \frac{i^2\pi}{2}$

$\therefore z = e^{-\pi/2}$ → Argument = 0 ; Purely Real

Q. For a complex no. Z, if $Z = \frac{\pi}{4}(i+1)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i} + \frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right)$ then the value of $\left(\frac{|Z|}{amp.(Z)}\right)$ is equal to $\underline{4.00}$

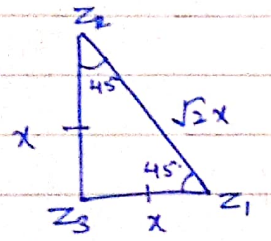
Sol^n Here, $Z = \frac{\pi}{\cancel{4}}(\cancel{2}i)^2\left(\frac{1+\pi+\pi+1}{\cancel{\sqrt{\pi}}+i+\pi i-\cancel{\sqrt{\pi}}}\right) = -i^2\pi\left(\frac{2(\pi+\cancel{1})}{\cancel{i}(\cancel{\pi}+1)}\right)$

$\therefore \boxed{Z = 2\pi i}$

So, $|Z| = 2\pi$ and $amp(Z) = \pi/2 \Rightarrow \left(\frac{|Z|}{amp(Z)}\right) = \underline{4}$

Q. Let the complex nos. $Z_1, Z_2,$ and Z_3, are the vertices A, B, and C respectively, of an isosceles right-angled ΔABC, with right-angle at C, then the value of $\frac{(Z_1-Z_2)^2}{(Z_1-Z_3)(Z_3-Z_2)} = \underline{2.00}$.

Sol^n



Here, $\frac{Z_2-Z_1}{Z_3-Z_1} = \left|\frac{Z_3-Z_1}{Z_2-Z_1}\right| e^{i\pi/4} = \sqrt{2}.e^{i\pi/4}$

$\Rightarrow \boxed{\frac{Z_1-Z_2}{Z_1-Z_3} = \frac{1}{\sqrt{2}}e^{i\pi/4}}$ — (1)

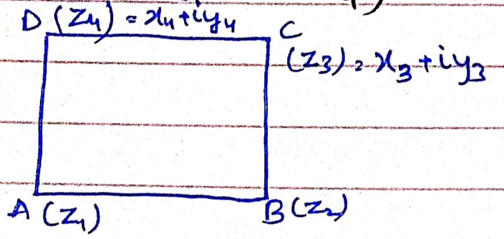
Also, $\frac{Z_1-Z_2}{Z_3-Z_2} = \left|\frac{Z_1-Z_2}{Z_3-Z_2}\right| e^{i\pi/4} = \sqrt{2}\,e^{i\pi/4}$

$\Rightarrow \boxed{\frac{Z_1-Z_2}{Z_3-Z_2} = \sqrt{2}\,e^{i\pi/4}}$ — (2)

$\therefore \left(\frac{Z_1-Z_2}{Z_1-Z_3} \times \frac{Z_1-Z_2}{Z_3-Z_2}\right) = \underline{2}$

Q. If A $(2+3i)$ and B $(3+4i)$ are two vertices of a sq. ABCD (taken in A.C.W. order) in a complex plane, then the value of $|z_3|^2 - |z_4|^2$ (where C is z_3 and D is z_4) is equal to $\underline{12.00}$.

Solⁿ



$$\frac{z_4 - z_1}{z_2 - z_1} = (1)\, e^{i\pi/2} = i$$

$$\Rightarrow z_4 - z_1 = i(z_2 - z_1) = i(1+i) = \underline{-1+i}$$

$$\boxed{z_4 = 1+4i}$$

Also, $\frac{z_3 - z_2}{z_1 - z_2} = e^{i(-\pi/2)} = -i$

$$\Rightarrow z_3 - z_2 = -i(-i-1) = i - 1 \Rightarrow \boxed{z_3 = 2+5i}$$

$$\therefore |z_3|^2 - |z_4|^2 = \underline{12}$$

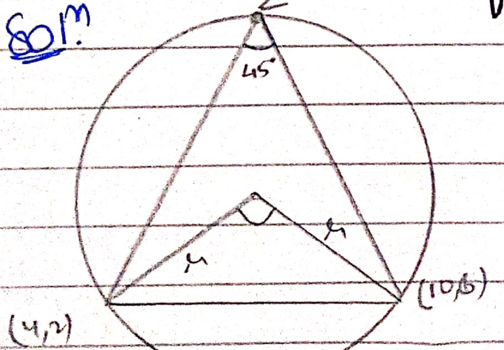
Q. For a complex no. z, if $|z-1+i| + |z+i| = 1$, then, the range of principle arg.(z) is :-

(A) $[-\pi/4, \pi/4]$ (B) $[\pi/4, \pi/2]$

(C) $[-\pi/2, -\pi/4]$ (D) $[-\frac{\pi}{2}, \pi/2]$

Q. If $\arg\left(\frac{z-(10+6i)}{z-(4+2i)}\right) = \frac{\pi}{4}$, then, the perimeter of the locus of z is ______.

Solⁿ



Here,

$$r^2 + r^2 = 36 + 16$$

$$\boxed{r = \sqrt{26}}$$

Now, for perimeter,

$$\frac{\cancel{270}^{3}}{\cancel{360}_{4}} \times 2\pi(\sqrt{26}) = \boxed{\frac{3\pi\sqrt{26}}{2} \text{ unit}}$$

Q. The value of $\left(\sum_{K=1}^{4}\left(\sin\frac{2\pi K}{5} - i\cos\frac{2\pi K}{5}\right)\right)^4$ is $\underline{1.00}$

Sol^n. Here, $\left(\sum_{K=1}^{4} -i\left(\cos\frac{2\pi K}{5} + i\sin\frac{2\pi K}{5}\right)\right)^4 = \left(\sum_{K=1}^{4} -i\, e^{\frac{i2\pi K}{5}}\right)^4$

Sum of first 4 roots of 5th Root of unity

5th Root of unity

Since, $1+\omega+\omega^2+\omega^3+\omega^4 = 0$

$\Rightarrow \boxed{1+\omega^2+\omega^3+\omega^4 = -\omega^5}$

V. Imp. $\Rightarrow (-i)^4\left(e^{\frac{i2\pi(5)}{5}}\right)^4 = (1)(-1)^4 = \underline{1}$

Q. Let $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, and $z_1 + z_2 + z_3 = 3+\sqrt{5}i$,
then, the value of $Re(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1)$ is $\underline{0.00}$

Sol^n. Since, $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2 + \bar{z}_1 z_2)$

Similarly,

$|z_1+z_2+z_3|^2 = (z_1+z_2+z_3)(\bar{z}_1+\bar{z}_2+\bar{z}_3)$

$= |z_1|^2 + |z_2|^2 + |z_3|^2 + z_1\bar{z}_2 + z_1\bar{z}_3 + z_2\bar{z}_1 + z_2\bar{z}_3 + z_3\bar{z}_1 + z_3\bar{z}_2$

$z + \bar{z}$

$(|z_1+z_2+z_3|)^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + 2Re(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1)$

$\Rightarrow 14 = 1+4+9+2X \Rightarrow \boxed{X = 0}$

Q. If z_1, z_2, z_3, are 3 distinct complex no. such that $\frac{3}{|z_1-z_2|} = \frac{5}{|z_2-z_3|} = \frac{7}{|z_3-z_1|}$, then, the value of

$\frac{9}{z_1-z_2} + \frac{25}{z_2-z_3} + \frac{49}{z_3-z_1} = \underline{0.00}$

Sol^n. Let $\frac{3}{|z_1-z_2|} = \frac{5}{|z_2-z_3|} = \frac{7}{|z_3-z_1|} = K$

$\Rightarrow \frac{9}{|z_1-z_2|^2} = \frac{25}{|z_2-z_3|^2} = \frac{49}{|z_3-z_1|^2} = K^2$

$$\therefore \frac{9}{|z_1-z_2|^2} = \frac{9}{(z_1-z_2)(\bar{z}_1-\bar{z}_2)} = k^2 \Rightarrow \frac{9}{(z_1-z_2)} = k^2(\bar{z}_1-\bar{z}_2)$$

Similarly, $\frac{9}{z_1-z_2} + \frac{25}{z_2-z_1} + \frac{49}{z_2-z_1} = k^2(\bar{z}_1-\bar{z}_2) + k^2(\bar{z}_2-\bar{z}_1) + k^2(\bar{z}_3-\bar{z}_1) = 0$

Q. If $Im\left(\frac{iz-2}{z-i}\right) = -1$, represents part of a circle, the the radius of the circle is <u>0.75</u>.

Soln Let $z = x+iy$, then,

$$\left(\frac{i(x+iy)-2}{(x+iy)-i}\right) \Rightarrow \frac{ix-(y+2)}{x+i(y-1)} \times \underbrace{\frac{x-i(y-1)i}{x-(y-1)i}}_{\text{Imaginary} = -1}$$

$$\Rightarrow Im\left(\frac{iz-2}{z-i}\right) \Rightarrow \frac{x^2+(y+2)(y-1)}{x^2+(y-1)^2} = -1$$

$$\rightarrow x^2+y^2-\tfrac{1}{2}y-\tfrac{1}{2}=0 \longrightarrow \text{Eqn. of circle.}$$

$$\therefore \boxed{r = 3/4}$$

Imp

Q. If $\lambda \in R$, such that the origin and the non-real roots of the equation $2z^2+2z+\lambda=0$ form the vertices of an equilateral $\triangle$ in the argand plane, then $1/\lambda$ is equal to <u>1.50</u>.

Soln If z_1 & z_2 are the roots of $2z^2+2z+\lambda=0$, and along with origin, z_1 & z_2 are forming equilateral $\triangle$, then,

$$z_1^2+z_2^2 = z_1z_2$$

$$\rightarrow (z_1+z_2)^2-2z_1z_2 = z_1z_2$$

$$\rightarrow \left(\frac{-2}{2}\right)^2 = 3\left(\frac{\lambda}{2}\right) \Rightarrow \boxed{\frac{1}{\lambda}=\frac{3}{2}}$$

Q. Let $O = (0,0)$, $A = (3,0)$, $B = (0,-1)$ and $C = (3,2)$, then, the minimum value of $|z| + |z-3| + |z+i| + |z-3-2i|$ occurs at the

(A) point of intersection of AB and CO
(B) point of intersection of AC and BO
(C) point of intersection of CB and AO
(D) Mean of O, A, B, C.

Soln. Here, min $(|z| + |z-3|) \rightarrow$ Lies on line joining OA.
min. $(|z-i| + |z-3-2i|) \rightarrow$ lies on line BC

$\therefore$ min. $(|z| + |z-3| + |z+i| + |z-3-2i|) \Rightarrow$ P.o.i. of AO & BC.

Q. The no. of solutions of the equation $z^3 + \frac{3(\bar{z})^2}{|z|} = 0$ are. 5.00 .

Q. The minimum value of the expression $|3z-3| + |2z-4|$ is equal to 2.00 .

Soln. $|3z-3| + |2z-4| \Rightarrow 3|z-1| + 2|z-2|$

$= |z-1| + 2(|z-1| + |z-2|)$ → min. at line joining (1,0) & (2,0), specifically at (1,0), then,

$= 0 + 2(1) = 2$

Q. The solution of the equation $|z| - z = 1 + 2i$ is

Sol^n Let $z = x + iy$, then,

$$\sqrt{x^2+y^2} = x + iy + 1 + 2i \;\Big\langle\; \begin{matrix} \sqrt{x^2+y^2} - x = 1 \Rightarrow \boxed{x = 3/2} \\ -y = 2 \Rightarrow \boxed{y = -2} \end{matrix}$$

$\therefore \boxed{z = 3/2 - 2i}$

Q. Let 'n' be a positive integer and a complex no. with unit modulus is a solution of the equation $z^n + z + 1 = 0$, then, the value of 'n' can be

(A) 87 (B) 97

(C) 222 (D) 104

Sol^n:

Q. If $P(z)$ is a variable point in the complex plane such that $Im\left(-\frac{1}{z}\right) = \frac{1}{4}$, then, the value of the perimeter of the locus of $P(z)$ is 12.56

Sol^n: Let $z = x + iy$, then, $\frac{-1}{z} = \frac{-(x-iy)}{x^2+y^2}$

So, $\frac{y}{x^2+y^2} = \frac{1}{4} \Rightarrow x^2+y^2-4y = 0$

$\boxed{\text{radius} = 2}$

$\therefore$ Perimeter $= 2\pi(2) = \underline{12.56}$

Q. A complex no. z is said to be unimodular if $|z|=1$. Let z_1 and z_2 are complex no. such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is a unimodular and z_2 is not unimodular, then, the point z_1 lies on a

(A) circle of radius $\sqrt{2}$.
(B) st. line parallel to x-axis
(C) st. line parallel to y-axis
(D) circle of radius 2

Q. If equat^n $(z-1)^n = z^n = 1$, has solut^n, then n can be

(A) 4 (B) 15 (C) 12 (D) 21

Sol^n.

Q. If $P_1 = 1 - \frac{\omega}{2} + \frac{\omega^2}{4} - \frac{\omega^3}{8} + \dots \infty$ and $P_2 = \frac{1-\omega^2}{2}$, then,

$P_1 P_2 =$ 1.00

Sol^n. $P_1 = \frac{2}{2+\omega}$; $P_2 = \frac{1-\omega^2}{2}$ $\Rightarrow$ $P_1 P_2 = \frac{1-\omega^2}{2+\omega} = \frac{1-\omega^2}{1+1+\omega^2}$

$= \frac{1-\omega^2}{1-\omega^2}$ $\underline{=1}$

Q. Let $z = re^{i\theta}$ ($r > 0$ and $\pi < \theta \leq 3\pi$) is a root of the equation $z^8 - z^7 + z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0$. If the sum of all the values of θ is $k\pi$, then k is 16.00 .

Q. If $z = 3 - 4i$, then the value of expression $z^4 - 3z^3 + 3z^2 + 99z - 95 =$ <u>5.00</u>.

Sol^n: