T.I.P. :- for Distinct & Positive Roots

$D > 0$

$S > 0$

$P > 0$

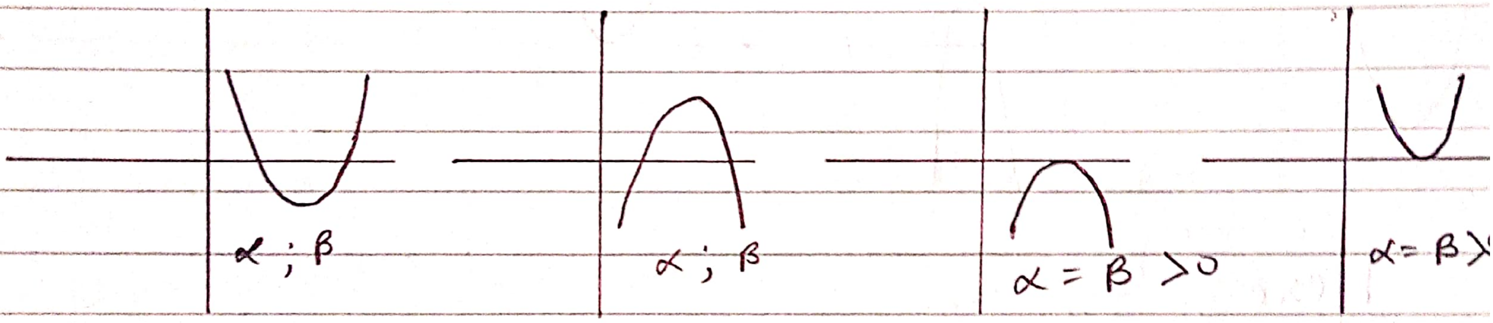
① & ②'s intersectⁿ; we get :-

$\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

$\therefore$ $\boxed{A}$ & $\boxed{D}$ are the correct answers

• Location of Roots :-

Case - I). When both roots are positive



Here,

① $D \geq 0$ ② Sum > 0 ③ $P. > 0$

For Example :-

If $x^2 - kx + 4 = 0$. Then, find 'k' such that both roots are positive.

Solⁿ. $D \geq 0 \Rightarrow k^2 - 4(4) \geq 0$

$\therefore k \in (-\infty, -4] \cup [4, \infty)$ — ①

$S > 0 \Rightarrow -(-k)/1 > 0 \Rightarrow \boxed{k > 0}$ — ②

$P > 0 \Rightarrow \frac{4}{1} > 0 \Rightarrow \boxed{k \in R}$ — ③

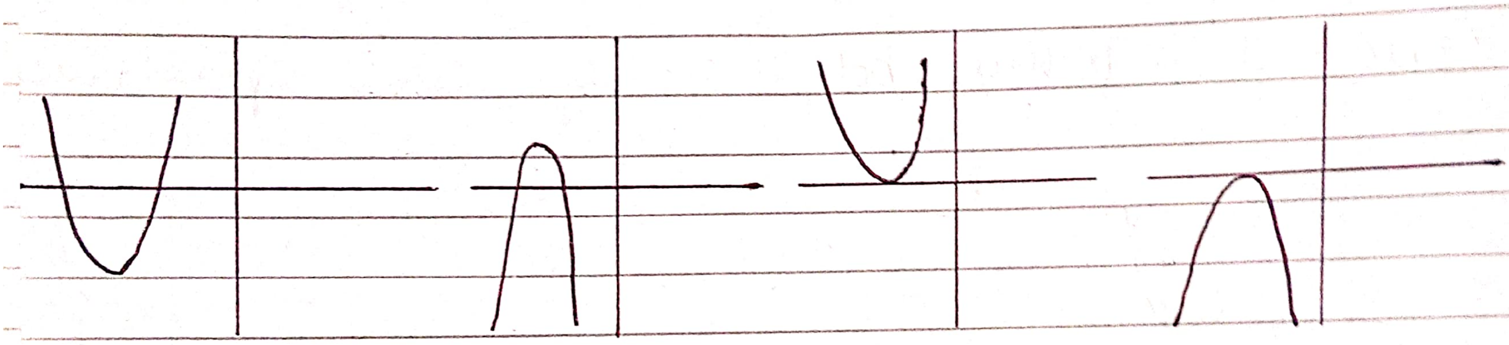
Now,

① $\cap$ ② $\cap$ ③

$\downarrow$

$K \in [4, \infty)$

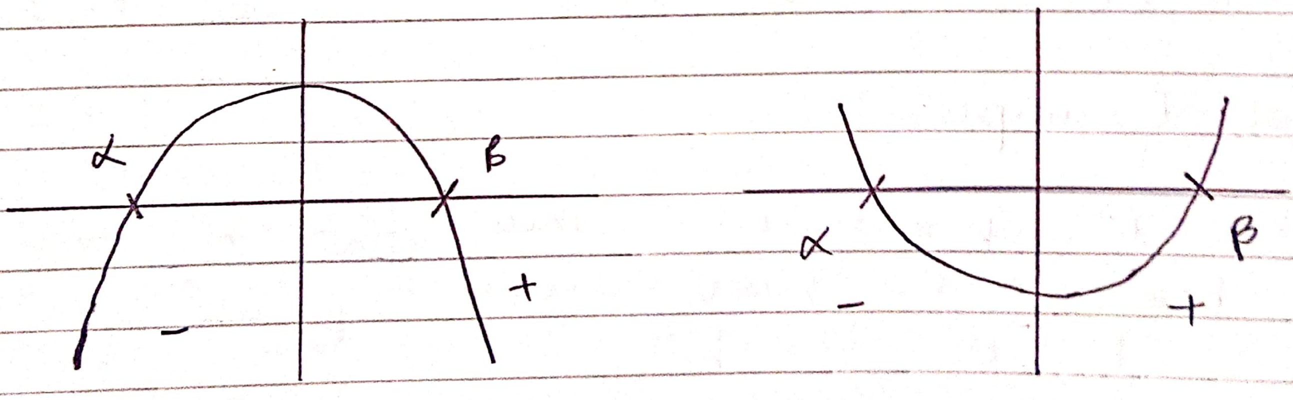
Case – II). When both roots are –ve.



Here,

① $D \geq 0$ ② $S < 0$ ③ $P > 0$

Case – III). When roots are of opp. sign.

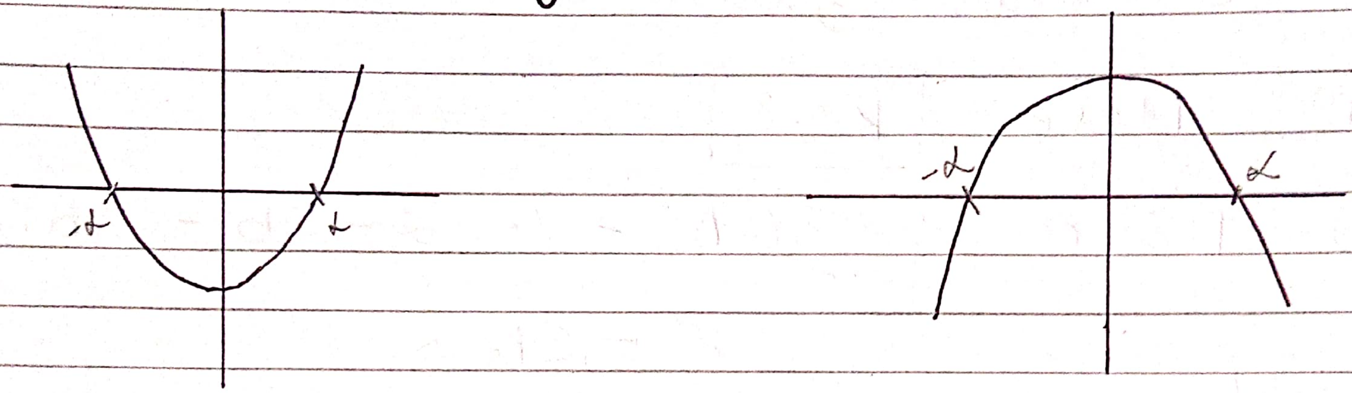


Here,

① $D > 0$ ② $P < 0$

Not imp. to check this condition

Case – IV). When Roots are equal in magnitude but Opposite in Sign.



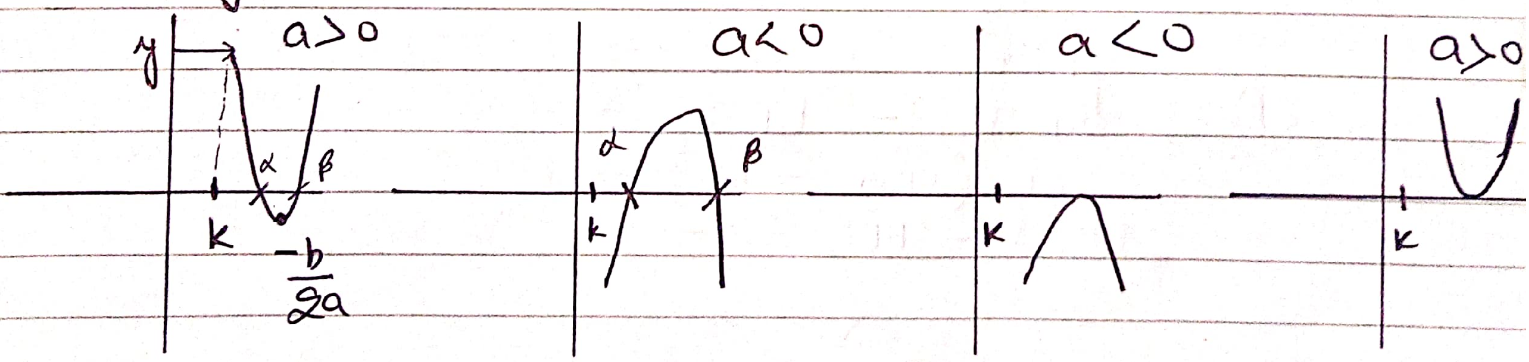
Here,

(1) $D > 0$ Can be Skipped

(2) $S = 0$

(3) $P < 0$

Case – V). When both roots are greater than a real no. 'K'.



Here,

(1) $D \geq 0$

(2) $a \cdot f(K) > 0$

(3) $-\frac{b}{2a} > K$

Q. For which value of 'b' both the roots of an equation $x^2 - 6bx + 2 - 2a + 9a^2 = 0$ exceed greater than 3?

Soln Here, $\boxed{k = 3}$

① $D \geq 0 \Rightarrow \cancel{36a^2} - 4(2 - 2b + \cancel{9b^2}) \geq 0$

$$2 - 2b \leq 0$$

$$\boxed{b \geq 1} \quad — ①$$

② $a \cdot f(x) > 0$

$\Rightarrow 1 \times f(3) > 0$

$\Rightarrow 9 - 6a(3)(2 - 2a + 9a^2) > 0$

$\Rightarrow 9a^2 - 20a + 11 > 0$

$\Rightarrow 9a^2 - 11a - 9a + 11 > 0$

$\Rightarrow a(9a - 11) - 1(9a - 11) > 0$

$\Rightarrow (9a - 11)(a - 1) > 0$

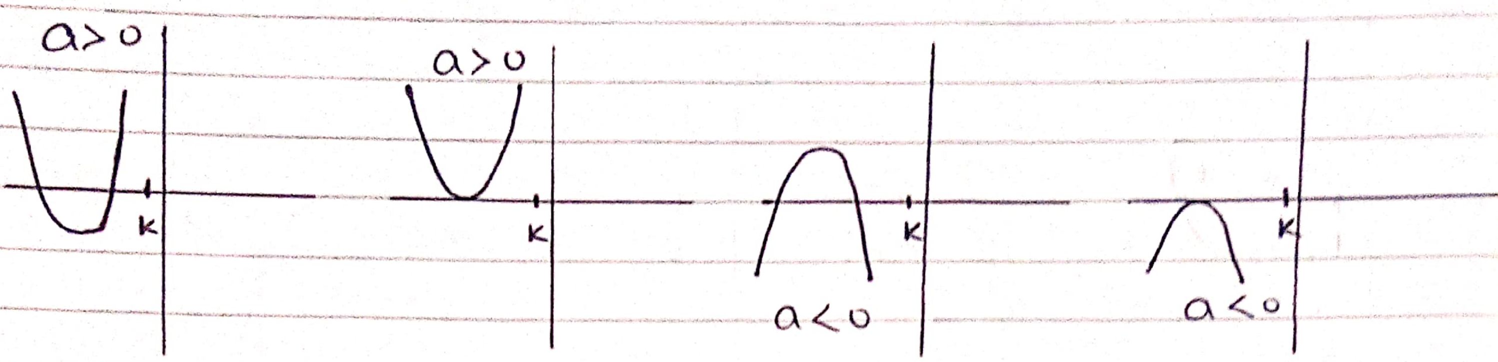
$$\begin{array}{ccccc} + & & - & & + \\ & 1 & & \frac{11}{9} & \end{array}$$

$\therefore a \in (-\infty, 1) \cup (\frac{11}{9}, \infty)$

③ $k < -\frac{b}{2a} \Rightarrow 3 < -\frac{(-6a)}{2} \Rightarrow a > 1$

$\therefore a \in (\frac{11}{9}, \infty)$

Case - VI). When both roots are less than a real no. K.



Here,

(1) $D \geq 0$ (2) $a \cdot f(K) > 0$

(3) $K > -\frac{b}{2a}$

Case - VII). When one root is less than K and other root is greater than K.

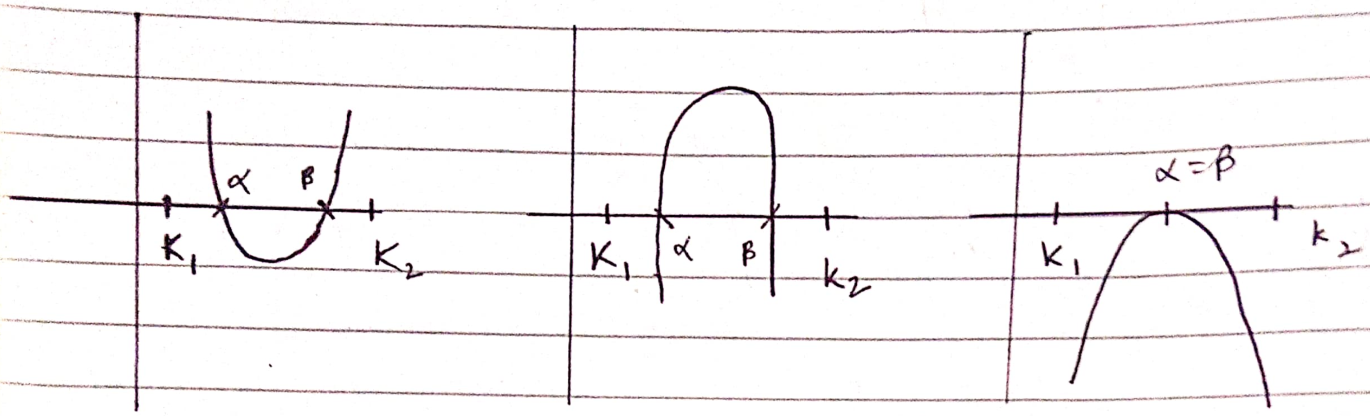
OR

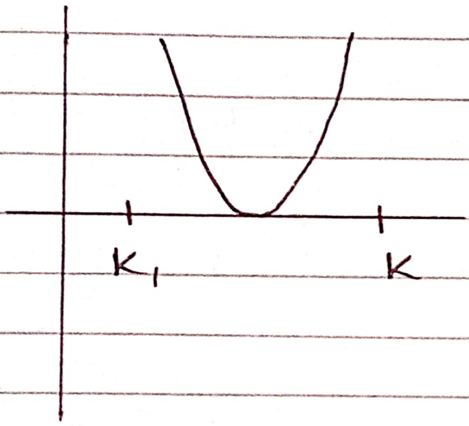
When 'K' lies b/w both the roots.

Here:-

(1) $D > 0$ (2) $a \cdot f(x) < 0$

Case-VIII). When both roots are lying b/w K_1 and K_2.





Here;

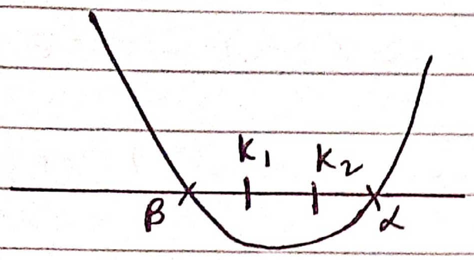
① $a \cdot f(k_1) > 0$

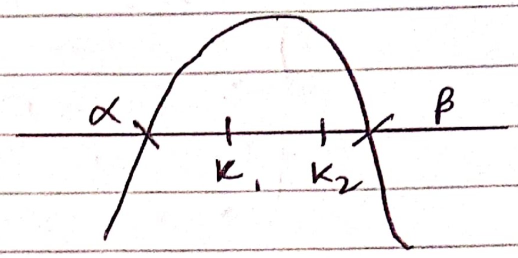
② $a \cdot f(k_2) > 0$

③ $k_1 < \frac{-b}{2a} < k_2$

④ $D \geq 0$

Case - IX). When one root is less than K_1 and other root is greater than K_2.





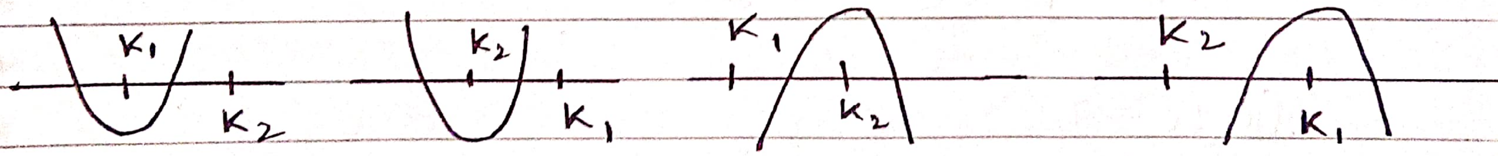
Here;

① $D > 0$ ② $a \cdot f(K_1) < 0$

③ $a \cdot f(K_2) < 0$

V. Imp.

Case - X). When exactly one root lies b/w K_1 & K_2.



$f(K_1) < 0$, $f(K_2) > 0$ | $f(K_1) > 0$, $f(K_2) < 0$ | $f(K_1) < 0$, $f(K_2) > 0$ | $f(K_1) > 0$, $f(K_2) < 0$

Here,

$$\boxed{f(K_1) f(K_2) < 0}$$

Q. If $x^2 + 2(k-1)x + k + 5 = 0$. Then, find 'k' such that equation has atleast one positive root.

Sol^n:

Case-I) When both roots are +ve

→ $D \geq 0 \rightarrow k \in (-\infty, -1] \cup [4, \infty)$

→ $S > 0 \rightarrow k < 1$

→ $P > 0 \rightarrow k > -5$

$\therefore k \in (-5, -1]$

Case-II) When one root +ve & other root is -ve.

→ $D > 0 \rightarrow \propto$

→ $P > 0 \rightarrow k < -5$

$\therefore k \in (-\infty, -5)$

Case-III) When one root is zero and other root is +ve.

→ $D \geq 0 \rightarrow k \in (-\infty, -1] \cup [4, \infty)$

→ $a \cdot f(0) > 0 \rightarrow k > -5$

→ $-\frac{b}{2a} > 0 \rightarrow k < 1$

$\boxed{\therefore k \in (-\infty, -1)}$