- Pseudo Quadratic Eqn. :-

for Example :-

$\sin^4\theta + 3\sin^2\theta + 4 = 0$

Let $\sin^2\theta = t \Rightarrow \therefore \boxed{t^2 + 3t + 4 = 0}$

Q. $x^4 + (1-2k)x^2 + (k^2-1) = 0$ → Bi-Quad. / Pseudo quad. Eqn.

Find the values of 'k' for which the equation has :-

Solⁿ. Let $x^2 = t$. Then,

$t^2 + (1-2k)t + (k^2-1) = 0 \; < \alpha, \beta$

where,

$t = \alpha$	$t = \beta$
$x^2 = \alpha$	$x^2 = \beta$
$x = \pm\sqrt{\alpha}$	$x = \pm\sqrt{\beta}$

(A) 4 Real Solⁿ (Real & Distinct) :-

For 4 Real Solⁿ, the roots of 't' quad → $\alpha ; \beta$ both have to be +ve or greater than 0.

Case-I). Greater than a real no. 0.

(1) $D > 0 \rightarrow (1-2k)^2 - 4(k^2-1) > 0$

$1 + 4k^2 - 4k - 4k^2 + 4 > 0$

$\therefore \boxed{k < 5/4}$

(2) $-\frac{b}{2a} > 0 \Rightarrow \boxed{k > \frac{1}{2}}$

(3) $a\,f(0) > 0 \Rightarrow (k^2 - 1) > 0$

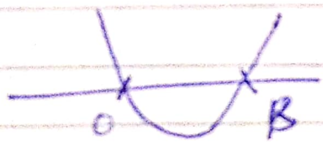
$\therefore\ k \in (-\infty, -1) \cup (1, \infty)$

Final $\Rightarrow$ (1) $\cap$ (2) $\cap$ (3)

$k \in \left(1, \frac{5}{4}\right)$

(B) Three Sol^n :-

$\rightarrow$ For 3 solutions, let one of the root be 0 and other be β



$\rightarrow$ Here,

$P = 0$

$k^2 - 1 = 0$

$\boxed{k = \pm 1}$

Also,

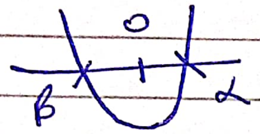
(1) $D > 0 \Rightarrow \boxed{k < \frac{5}{4}}$

(2) $-\frac{b}{2a} > 0 \Rightarrow \boxed{k > \frac{1}{2}}$

$\therefore$ Finally $\boxed{k = \pm 1}$

(C) Two Solⁿ :-

$\alpha > 0 \; ; \; \beta < 0$

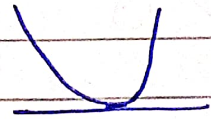


$\Rightarrow a f(0) < 0$

$\Rightarrow k^2 - 1 < 0$

$\therefore k \in (-1, 1)$

$\alpha = \beta > 0$



$\Rightarrow D = 0$

$\boxed{k = 5/4}$

$\Rightarrow \frac{-b}{2a} > 0 \Rightarrow \boxed{k = \frac{1}{2}}$

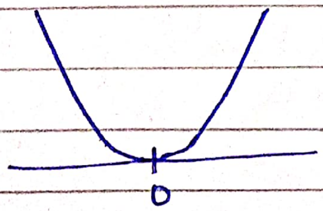
$\Rightarrow a \cdot f(0) > 0$

$\therefore k \in (-\infty, -1) \cup (1, \infty)$

$\therefore$ finally, $k \in (-1, 1) \cup \left\{\frac{5}{4}\right\}$

(D) One Solⁿ :-

when $\alpha = \beta = 0$



$\therefore \boxed{k \in \phi}$

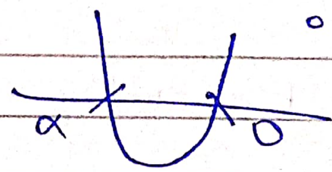
$\Rightarrow D = 0 \Rightarrow \boxed{k = 5/4}$

$\Rightarrow \frac{-b}{2a} = 0 \Rightarrow \boxed{k = \frac{1}{2}}$

$\Rightarrow f(0) = 0 \Rightarrow \boxed{k^2 - 1 = 0}$

when $0 > \alpha$ and $\beta = 0$

$\therefore \boxed{k = -1}$



$\Rightarrow D > 0 \Rightarrow \boxed{k < 5/4}$

$\Rightarrow \frac{-b}{2a} < 0 \Rightarrow \boxed{k < \frac{1}{2}}$

$\Rightarrow a \cdot f(0) = 0 \Rightarrow k = \pm$

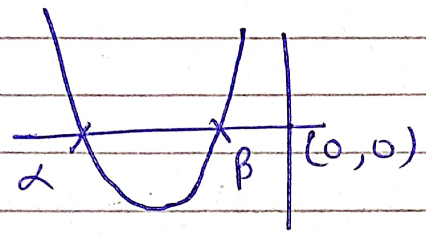
(E) NO Solⁿ :-

When α & $\beta \Rightarrow$ Imaginary

$\Rightarrow D < 0 \Rightarrow \boxed{K > \frac{5}{4}}$

or

$\alpha ; \beta < 0$



Here,

$\Rightarrow D \geq 0 \Rightarrow \boxed{K \leq 5/4}$

$\Rightarrow \frac{-b}{2a} < 0 \Rightarrow \boxed{K < \frac{1}{2}}$

$\Rightarrow a \cdot f(0) > 0$

$\therefore K \in (-\infty, -1) \cup (1, \infty)$

$\therefore$ Finally $K \in (-\infty, -1) \cup (\frac{5}{4}, \infty)$

Important Questions from Rank Booster Course

QUADRATIC EQUATIONS

Q1. Let (x, y, z) be points with integer coordinates satifying the system of homogenous equat^n:-

$3x - y - z = 0$ —— 1

$-3x + z = 0$ —— 2

$-3x + 2y + z = 0$ —— 3

Then, the no. of such points for which $x^2 + y^2 + z^2 \leq 100$ is

Sol^n: From, equat^n 2, $\boxed{z = 3x}$, & putting it in eq. 1; we get, $\boxed{y = 0}$. Thus,

$x^2 + (0)^2 + (3x)^2 \leq 100 \Rightarrow \boxed{x^2 \leq 10}$

Thus, $x \in \{0, \pm 1, \pm 2, \pm 3\}$

Q2. Let α & β be two roots of equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:-

Sol^n: By Quadratic formula, we get

$x = -(i+1)$ & $(-1+i)$

So, $\alpha^2 = 2i$; $\beta^2 = -2i$

So, $\alpha^{15} + \beta^{15} = (\alpha^2)^7 \alpha + (\beta^2)^7 \beta = 2^7(-i) = -256$

Q. If $(1-p)$ is a root of quad. eq. $x^2 + px + (1-p) = 0$, then, its roots are :-

Sol^n Let the roots be α & $(1-p)$.

Thus, $\alpha + (1-p) = \frac{-p}{1} \Rightarrow \boxed{\alpha = -1}$

Also, $(\alpha)(1-p) = (1-p) \Rightarrow \boxed{p = 1} \Rightarrow (1-p) = \boxed{0}$

$\therefore$ Roots are 0 & -1

Q. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :-

(a) Infinite number of real roots.

(b) No real roots.

(c) exactly one real root.

(d) exactly four real root.

Sol^n Let $t = e^{\sin x}$; then,

$$t - \frac{1}{t} - 4 = 0 \Rightarrow t^2 - 4t - 1 = 0$$

$$\therefore t = \frac{4 \pm \sqrt{16+4}}{2} = \boxed{2 \pm \sqrt{5}}$$

$\therefore e^{\sin x} = 2 - \sqrt{5}$ which is not possible as $2-\sqrt{5}$ is $-ve$ but 'e' cannot be $-ve$

And. $e^{\sin x} = 2 + \sqrt{5}$ is possible.

Thus, The eqn. $e^{\sin x} - e^{-\sin x} - 4 = 0$ has no. real roots.

Q. The no. of real roots of eqn. $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$

(a) 3 (b) 2 (c) 1 (d) 0

Q. If $y = \frac{x^2 + 2x + C}{x^2 + 4x + 3C}$, then what are the possible values of c for which $y \in R$?

Sol^n. $yx^2 + 4xy + 3cy = x^2 + 2x + C$

$\Rightarrow (y-1)x^2 + x(4y-2) + (3cy - c) = 0$

$\Rightarrow$ Since, $x \in R \rightarrow D \geq 0$, Thus,

$\Rightarrow (4y-2)^2 - 4(3cy-c)(y-1) \geq 0$

$\Rightarrow (2y-1)^2 - c(y-1)(3y-1) \geq 0$

$\Rightarrow 4y^2 + 1 - 4y - c(3y^2 - 4y + 1) \geq 0$

Since, Quad. ≥ 0, then, $\boxed{D < 0}$

$\Rightarrow (4c-4)^2 - 4(4-3c)(1-c) < 0$

$\Rightarrow c^2 - c < 0$

$\Rightarrow \boxed{c \in (0, 1]}$

Q. Find the roots of the equatⁿ $9x^2 - 18|x| + 5 = 0$ which belong to the Domain of $f(x) = \log(x^2 - x - 2)$.

Solⁿ. Let $|x| = t$; then, $9t^2 - 18t + 5 = 0$

$\Rightarrow 9t^2 - 18t + 5 = 0 \Rightarrow [3t-1][3t-5] = 0$

$\therefore t = |x| = 1/3 \Rightarrow x = \pm 1/3$

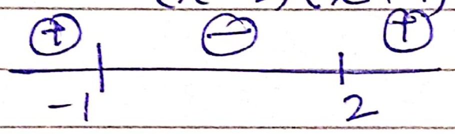
$\&\ t = |x| = 5/3 \Rightarrow x = \pm 5/3$

Now, Domain of $\log(x^2 - x - 2)$ is

$x^2 - x - 2 > 0$

$x^2 - 2x + x - 2 > 0$

$(x-2)(x+1) > 0$



$\rightarrow x \in (-\infty, -1) \cup (2, \infty)$

Thus, $-5/3$ is the only root in the domain.

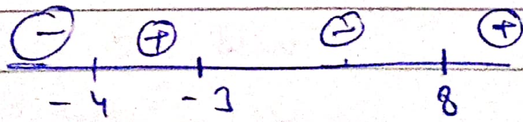
Q. Solve for x :-

$$\log_{0.6}\left(\log_6\left(\frac{x^2+x}{x+4}\right)\right) < 0$$

Solⁿ. $\log_6\left(\frac{x^2+x}{x+4}\right) < (0.6)^0 = 1 \Rightarrow \log_6\left(\frac{x^2+x}{x+4}\right) > 1$

$\Rightarrow \left(\frac{x^2+x}{x+4}\right) > 6^1 \Rightarrow \frac{x^2 + x - 6(x+4)}{x+4} > 0$

$\Rightarrow \frac{x^2 + x - 6x - 24}{x+4} > 0 \Rightarrow \frac{(x-8)(x+3)}{(x+4)} > 0$



$\rightarrow x \in (-4, -3) \cup (8, \infty)$

Q. Find the minimum value of $f(x) = x^2 + \frac{1}{x^2} - 6x - 6/x + 2$

Sol^n. On factorising, we get :- $\left(x+\frac{1}{x}\right)^2 - 6\left(x+\frac{1}{x}\right)$

Let $\left(x+\frac{1}{x}\right) = t$; then, $f(x) = t^2 - 6t$
$= (t-3)^2 - 9$

Since, $x + \frac{1}{x} = t = y$

$\Rightarrow yx = x^2 + 1 \Rightarrow \boxed{x^2 - yx + 1 = 0}$

Here, $D \geq 0 \Rightarrow y^2 - 4 \geq 0$

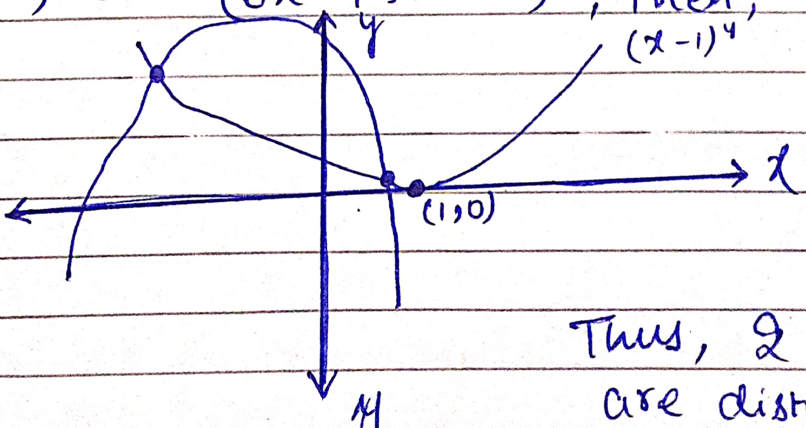
$\Rightarrow (y-2)(y+2) \geq 0$
$\Rightarrow y \in (-\infty, -2] \cup [2, \infty)$

On putting $\boxed{y = t = 3}$, we get -9 as the minimum value.

Q. The number of Distinct Real Roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Sol^n $x^4 - 4x^3 + 12x^2 + x - 1$ can be written as,

$\Rightarrow (x-1)^4 + 6x^2 + 5x - 2 = 0$
$\Rightarrow (x-1)^4 = -(6x^2 + 5x - 2)$, Then,



Thus, 2 roots which are distinct & Real are found.

Q. If $\alpha ; \beta$ are roots of the eq. $2x^2 + 6x + b =$ ($b < 0$) then $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is less than,

(A) 2 (B) -2

(C) 18 (D) None of Above

Q. If $f(x,y) = x^2 - 4x + y^2 + 6y$ and $0 \leq x \leq 1$ & $0 \leq y \leq 1$. Then, find min. value

Sol: Through hit and trial method, we got :-

$y^2 + 6y = 0$ on putting $\boxed{y=0}$

Also, we got, $x^2 - 4x = \boxed{-3}$ on putting $\boxed{x=1}$ as the minimum value.

Thus, minimum value of expression is $\boxed{-3}$.