- Modulus function:-

Imp

$\boxed{|\text{Input}| = +\text{ve Output}}$ → Property of modulus funct^n

For Ex :-

$|-5| = 5$

$|5| = 5$

$|-\sqrt{2}| = \sqrt{2}$

$|-\sqrt{3}\pi| = \sqrt{3}\pi$

→ Removal of Modulus function :-

$$|\text{Input}| = \begin{cases} \text{Input} \geq 0 \rightarrow +(\text{Input}) \\ \text{Input} < 0 \rightarrow -(\text{Input}) \end{cases}$$

For Ex:-

(A) $f(x) = |x| \Rightarrow \begin{cases} +x & (x \geq 0) \\ -x & (x < 0) \end{cases}$

+ve Output

(B) $f(x) = |2x-3| \Rightarrow \begin{cases} +(2x-3) & (2x-3 \geq 0) \\ -(2x-3) & (2x-3 < 0) \end{cases}$

$\boxed{+\text{ve Output}}$

→ Modulus Equalities :-

For Ex :-

(A) $f(x) = |x-3| = 5$

If $x - 3 \geq 0 \longrightarrow x - 3 = 5$

$= \boxed{x = 8}$

If $x - 3 < 0 \longrightarrow -(x-3) = 5$

$= \boxed{x = -2}$

②

(B) $f(x) = \left|\frac{3}{2} - 2x\right| = 4$

$\rightarrow$ If $3-4x \geq 0 \rightarrow 3-4x = 8$

$\boxed{x = -5/4}$

$\rightarrow$ If $3/2 - 2x < 0 \rightarrow \boxed{x = 11/4}$

(C) $f(x) = |x^2 - 7| = 2$

$\rightarrow x^2 - 7 = \pm 2$

$x^2 = 9$

$\boxed{x = \pm 3}$

$x^2 = -2 + 7$

$\boxed{x = \pm\sqrt{5}}$

(D) $f(x) = |2x-4| = -2$

*

$\rightarrow x \in \phi$

(E) $f(x) = |x|^2 - 5|x| + 6 = 0$

*

Let $|x| = t$

Then,

$t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t = 2$ or $t = 3$

$|x| = 2$ or $|x| = 3$

$x = \pm 2$ or $x = \pm 3$

(f) $x^2 + 5|x| + 6 = 0$

$|x|^2 + 5|x| + 6 = 0$

Let $|x| = t$

Then,

$t^2 + 5t + 6 = 0$

$(t+2)(t+3) = 0$

$t = -2$ | $t = -3$

$|x| = -2$ | $|x| = -3$

$x \in \phi$

→ Modulus Inequalities :-

(A) $|x| \geq 2$

imp. → $x \in (-\infty, -2] \cup [2, \infty)$

$$\boxed{|\text{Input}| \geq k \ (+ve)}$$

$\text{Input} \in (-\infty, -k] \cup [k, \infty)$

(B) $|x| \geq -2$

→ $\boxed{x \in R}$

(C) $|x| > 3$

→ $x \in (-\infty, -3) \cup (3, \infty)$

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D) $|x| \leq 2$

$x \in [-2, 2]$

Q. find the value of x in the following functions :-

Imp

A) $|x-3| \geq 5$

Let $\boxed{x-3 = t}$

$\therefore \quad |t| \geq 5$

$t \leq -5 \ \cup \ t \geq 5$

$x-3 \leq -5 \ \cup \ x-3 \geq 5$

$\boxed{x \leq -2} \ \cup \ \boxed{x \geq 8}$

$x \in (-\infty, -2] \cup [8, \infty)$

or

$x \in R - \{(-2, 8)\}$

B) $|x-4| \leq 5$

Let $|x-4| = |t|$, then,

$|t| \leq 5$

$\therefore \quad -5 \leq t \leq 5$

$-5 \leq x-4 \leq 5$

$-1 \leq x \leq 9$

$x \in [-1, 9]$

(C) $|x+2| > 0$

$\boxed{x \neq -2}$

$\boxed{\therefore x \in \mathbb{R} - \{-2\}}$

(D) $|2x-3| \geq 7$

Let $|t| \geq 7$, then,

$t \geq 7 \quad \cup \quad t \leq -7$

$2x-3 \geq 7 \quad \cup \quad 2x-3 \leq -7$

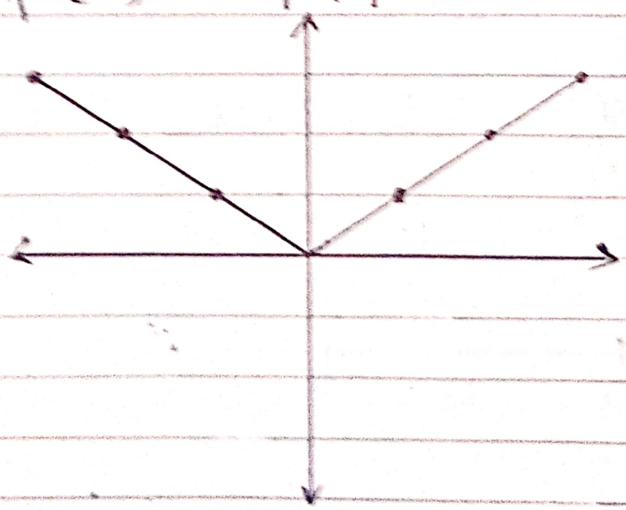
$2x \geq 10 \quad \cup \quad 2x \leq -4$

$\boxed{x \geq 5} \quad \cup \quad \boxed{x \leq -2}$

$\therefore x \in (-\infty, -2] \cup [5, \infty)$

→ Graph of Modulus function:-

$f(x) = |x|$



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Q. Find the Domain :-

(A) $f(x) = \sqrt{9 - |x|}$

$9 - |x| \geq 0$

$|x| \leq 9$

$\therefore x \in [-9, 9]$

(B) $f(x) = \dfrac{1}{\sqrt{3 - |x|}}$

$3 - |x| > 0$

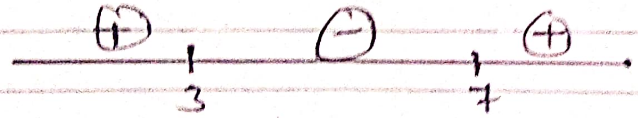
$|x| < 3$

$\therefore x \in (-3, 3)$

(C) $f(x) = \dfrac{1}{\sqrt{(|x| - 3)(|x| - 7)}}$

Let $|x| = t$. Then,

$\therefore (t - 3)(t - 7) > 0$



$t < 3 \cup t > 7$

$|x| < 3 \cup |x| > 7$

$x \in (-3, 3) \cup x \in (-\infty, -7) \cup (7, \infty)$

①

Imp.

② $|x-1| + |x-2| = 5$

$x < 1$	$1 \leq x < 2$	$x \geq 2$
$-(x-1)-(x-2) = 5$	$+(x-1)-(x-2) = 5$	$+(x-1)+(x-2) = 5$
$\boxed{x = -1}$	$-1+2 = 5$	$2x - 3 = 5$
	$1 = 5$	$2x = 8$
-1 Ans.	$\boxed{\text{False}}$	$\boxed{x = 4}$
	$x \in \Phi$	Ans.

$$\therefore x \in \{-1, 4\}$$

Greatest Integer function :-

→ Representation :- $f(x) = [x]$ g.i.f.

→ It is the greatest integer less than or equal to x.

For Example :-

i). $[5.4] = 5$ | ii). $[7.8] = 7$

iii). $[-5.7] = -6$ | iv). $[-9] = -9$

v). $[2.9999....] = 2$ | vi). $[-100.6] = -101$

vii). $x \in [0,1) = 0$ | viii). $[x] = 5$

$\therefore x \in [5,6)$

ix). $[x] = -2$

$\therefore x \in [-2,-1)$

x). $[x] = 2.3$

$\therefore x \in \phi$

xi). $[x-3] = 3$

Let $x - 3 = t$,

then $[t] = 3$

$\therefore 3 \leq t < 4$

$3 \leq x-3 < 4$

$\boxed{6 \leq x < 7}$

xii). $[2x-5] = 4$

Let $2x-5 = t$

then $[t] = 4$

$4 \leq t < 5$

$4 \leq 2x-5 < 10$

$9/2 \leq x < 5$

$\therefore x \in [4.5, 5)$

xiii). $[3x-2] = -7$

Let $3x-2 = t$

then, $-7 \leq [t] < -6$

$-7 \leq 3x-2 < -6$

$-5/3 \leq x < -4/3$

xiv. $[x] > 4$, find x

$x \in [5, \infty)$

xv). $[x] \geq 4$, find x.

$x \in [4, \infty)$

xvi). $[x] \leq 4$, find x.

$\therefore x \in (-\infty, 5)$

xvii) $[x] < 4$, find x.

$\therefore x \in (-\infty, 4)$

xviii). $[x] > -6$ find x.

$\therefore x \in [-5, \infty)$

$[x] \geq 1$, find x

$\therefore x \in [1, \infty)$

$[x] \leq 5$, find x.

$\therefore x \in (-\infty, 6)$

$[x] \leq 1/2$, find x.

$\therefore x \in (-\infty, 1)$

$[3x-2] > 4$, find x.

$\therefore 3x - 2 \geq 5$

$3x \geq 7$

$x \geq 7/3$

$\therefore x \in \left[\frac{7}{3}, \infty\right)$

$f(x) = [|x|]$, find D. & R.

Domain $\in$ R

Range $\in$ Whole Numbers.

$[x] < 1/3$

$\therefore x \in (-\infty, 1)$

$[x] \geq 3/2$, find x.

$\therefore x \in [2, \infty)$

$f(x) = \frac{|x-3|}{x-3}$, find d. & R.

Domain $\Rightarrow \in R - \{3\}$

Range $\Rightarrow$

$$\frac{+(x-3)}{x-3} \rightarrow +1 \qquad \frac{-(x-3)}{x-3} \rightarrow -1$$

$\therefore$ Range $\in \{-1, 1\}$

$f(x) = [|x|]$ and D $\in [-3, 2]$

Now, find Range.

Range $\in \{0, 1, 2, 3\}$

②

$f(x) = \sqrt{5-[x]}$; find D.

$5 - [x] \geq 0$

$[x] \leq 5$

$\therefore x \in (-\infty, 6)$

$f(x) = \dfrac{1}{\sqrt{2[x]-5}}$

$2[x] - 5 > 0$

$[x] > 5/2$

$\therefore x \in [3, \infty)$

$f(x) = \dfrac{1}{\sqrt{[2x-5]+7}}$

$\Rightarrow [2x-5]+7 > 0$

$\Rightarrow [2x-5] > -7$

$\Rightarrow 2x-5 \geq -6$

$\Rightarrow 2x \geq -1$

$\Rightarrow \boxed{x \geq -1/2}$

$[x] + [-x] = \begin{cases} 0 ; & x \in I \\ -1 ; & x \notin I \end{cases}$

- Fractional Part:-

→ Represented by :- $f(x) = \{\ \}$

Fractional Part $= [x] + \{x\}$

For Ex:-

→ $\{5.7\} = 5.7 - 5 = 0.7$

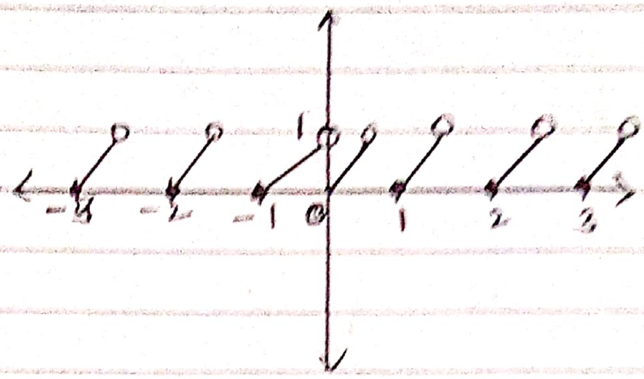
→ $\{7.9\} = 0.9$.

→ $\{8.2\} = 0.2$

→ $\{-5.7\} = 1 - \{5.7\} = 1 - 0.7$
$= \boxed{0.3}$

→ $\{-6.6\} = 1 - \{+6.6\} = 1 - 0.6$
$= \boxed{0.4}$

• Graph of fractional Part :-



Q. $f(x) = \sqrt{\{x\}}$, (find the domain) :-

Sol^n $\boxed{x \in R}$

Reason :- As all the values of R in $\{x\}$ will range from $[0, 1)$, therefore $\boxed{x \in R}$.

Q. $f(x) = \frac{1}{\sqrt{\{x\}}}$, find the domain.

Sol^n $x \in R - \{I\}$

Q. $f(x) = \sqrt{\{x\} - 2}$, find the domain.

$0 \leq \{x\} < 1$ and $\{x\} - 2 \geq 0$

$\Rightarrow \{x\} \geq 2$

$\boxed{\therefore x \in \phi}$

Q. $f(x) = [\{x\}] = 0$, find the domain

Sol^n $D_f \Rightarrow R$

Q. $f(x) = \{[x]\} = 0$, find the domain

Sol^n $D_f \Rightarrow R$

Imp.

Q. Solve $2[x] = \{x\} + 3$

Soln

Case-I) when $x \in I$,

$\therefore \; 2[I] = \{I\} + 3$

$\Rightarrow \; 2I = 0 + 3$

$\Rightarrow \; \boxed{I = 3/2}$

$\boxed{\therefore \; x \in \phi}$

Case-II) when $x \notin I$,

when $x \notin I$, then $x \in I + f$

$\therefore \; 2[I + f] = \{I + f\} + 3$

$\Rightarrow \; 2I = f + 3$

$f = 2I - 3$

Now, Always $\boxed{0 \leq f < 1} \Rightarrow$ Case-wise. $\boxed{0 < f < 1}$

$\therefore \; 0 < 2I - 3 < 1$

$\Rightarrow \; 3 < 2I < 4$

$\boxed{3/2 < I < 2} \rightarrow I \in \phi$

$\boxed{\text{Finally, } x \in \phi}$

Imp.

Q. Solve $2[x] = 2\{x\} + 3$

* Case-I) when $x \in I$

$\therefore 2I = 2(0) + 3$

$\Rightarrow I = 3/2$

$\boxed{\therefore x \in \phi}$

Case-II). when $x \notin I$

$\therefore x \in I + f$

$2I = 2f + 3$

$2f = 2I - 3$

$\therefore f = \frac{2I - 3}{2}$

Here, $0 < f < 1$

$\Rightarrow 0 < \frac{2I - 3}{2} < 1$

$\Rightarrow 0 < 2I - 3 < 2$

$\Rightarrow 03 < 2I < 2 + 3$

$\Rightarrow 3/2 < I < 5/2 \Rightarrow$

$\boxed{\therefore I \in 2}$

Now, $f = \frac{2I - 3}{2} = \frac{2 \times 2 - 3}{2} = 0.5$

Now, $I + f = x = 2 + 0.5 = \boxed{2.5}$

x

Q. Solve $2x + [x] = 5$

Case - I)

When $x \in I$,

$\Rightarrow 2I + [I] = 5$

$\Rightarrow 3I = 5 \Rightarrow \boxed{I = 5/3}$ $\times$

$\therefore x \in \phi$

Case - II)

When $x \notin I = I + f$

$\Rightarrow 2(I + f) + [I + f] = 5$

$\Rightarrow 2I + 2f + I = 5$

$\Rightarrow 3I + 2f = 5$

$\Rightarrow \boxed{f = \frac{5 - 3I}{2}}$

Here, $0 < f < 1$

$\Rightarrow 0 < \frac{5 - 3I}{2} < 1$

$\Rightarrow 0 < 5 - 3I < 2$

$\Rightarrow -05 < -3I < -3$

$\Rightarrow \frac{5}{3} > I > 1 \rightarrow \boxed{x = \phi}$

$\boxed{I \in \phi}$