T.I.P. → Output of Exponential functions are always +ve.

- Exponential functions :-

→ Conditions :- $a>0$ & $a \neq 1$

→ Representations :- $f(x) = (a)^x$

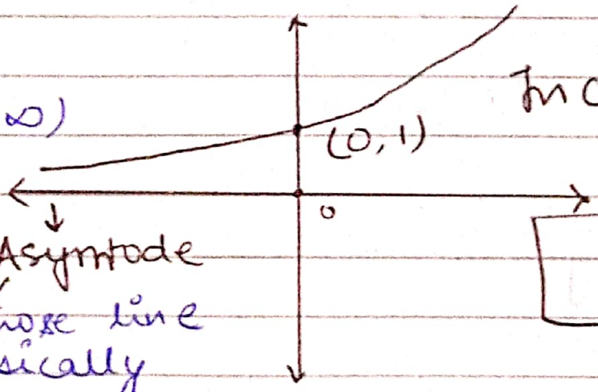
↳ base of the exponential function

For Example :- $(2)^x$, $(1/2)^x$, $(3)^x$, $(e)^x$.

→ Graph :-

Domain :- R

Range :- $(0, \infty)$



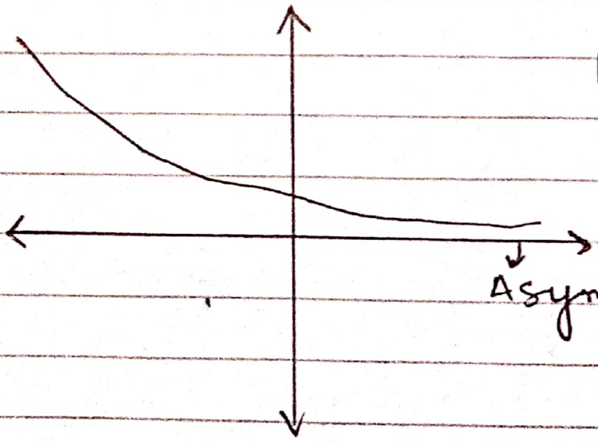
$\boxed{a>1}$

Increasing Nature

$\boxed{\text{For } f(x) = a^x}$

↳ $\boxed{y \propto x}$

Asymtodes are those line which are basically tangent to the curve at infinity.



Decreasing Nature

$\boxed{\text{for } 0<a<1}$

↳ $\boxed{y \propto \frac{1}{x}}$

- Signum function :-

If Sgm (x)
→ $x>0$ → 1
→ $x=0$ → 0
→ $x<0$ → -1

- Logarithmic functions :-

→ $\log_b^a = c \iff \boxed{a = b^c}$

→ $f(x) = \log_b^x$.

V. Imp.

Here, $x>0$, $b>0$ & $b \neq 1$

Q. $f(x) = \log_4^{x-2}$; find the domain.

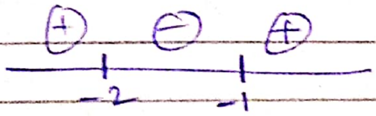
$$\boxed{x - 2 > 0}$$

$$\therefore \boxed{x > 02}$$

Q. $f(x) = \log_4^{(x^2+3x+2)}$

$$x^2 + 3x + 2 > 0$$

$$(x+2)(x+1) > 0$$



$$\therefore x \in (-\infty, -2) \cup (-1, \infty)$$

Q. $f(x) = \log_4^{\sqrt{x^2+3x+2}}$

⟶ Same as above.

Q. If $f(x) = (\log_{10}(\log_{10}(\log_{10}(\log_{10}^{x})$, then, find $D_f = ??$

Soln. Here, $x > 0$ & $\log_{10} \log_{10} \log_{10}^{x} > 0$

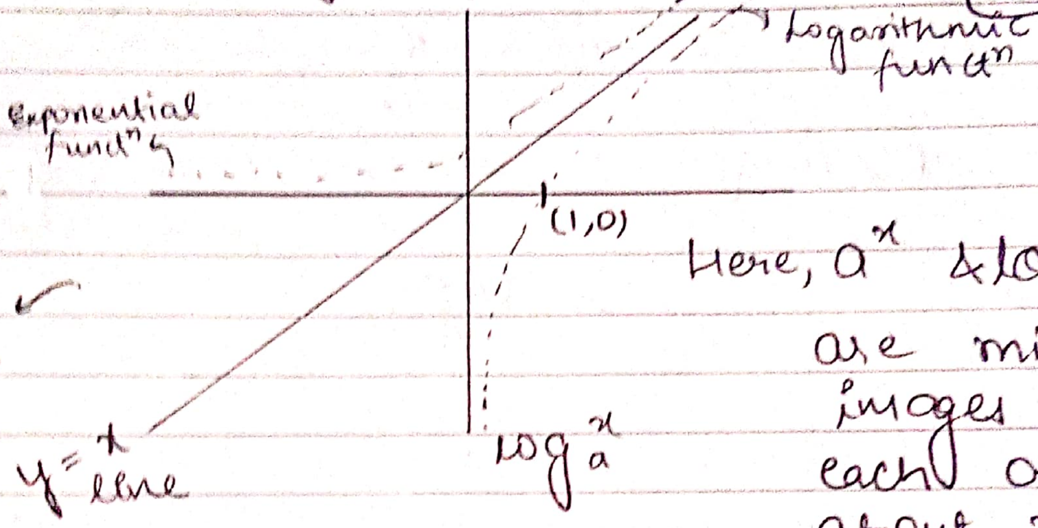
So, on solving, we get, $\boxed{x > 1000}$

$\therefore D_f \in (100, \infty)$

V.V.V. Imp

* Graphs of logarithmic functions :-

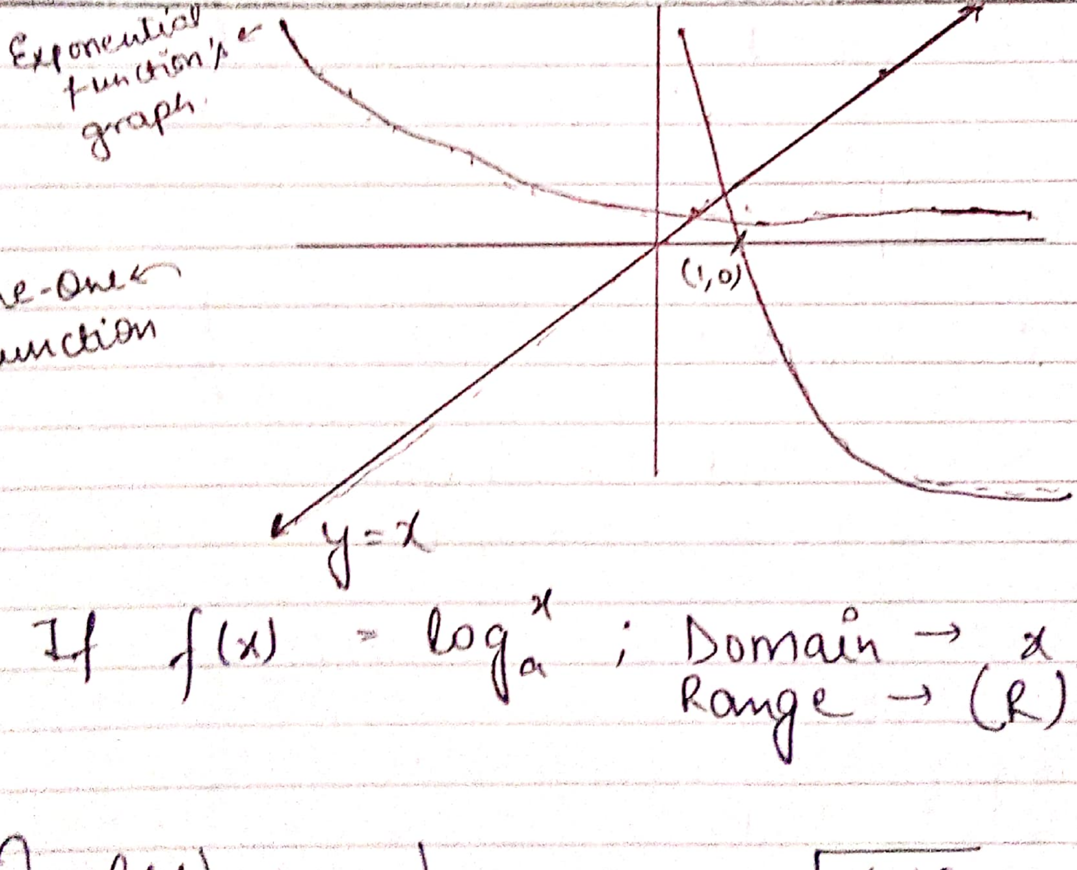
$\boxed{a > 1}$



One-One function. ✓

Here, a^x & $\log_a^x$ are mirror images of each other about $y = x$

$\boxed{0 < a < 1}$



One-One function

Both are Mirror images of each other.

If $f(x) = \log_a^x$; Domain → $x > 0$
Range → (R) → $(-\infty, \infty)$

Q. $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

Sol^n

$1 - x > 0$	$x + 2 \geq 0$
$\boxed{x < 1}$	$\boxed{x \geq -2}$

∴ $x \in [-2, 1) - \{0\}$

Trick T.I.P. :- $\log_a^{x} > b \rightarrow \boxed{x > a^b}$

⑬

Q. $\log_2^{2x-1} > \log_2^{3x-5}$

Soln $2x - 1 > 3x - 5$ $\Big\{ \begin{matrix} 2x - 1 > 0 \\ x > \frac{1}{2} \end{matrix} \Big\{ \begin{matrix} 3x - 5 > 0 \\ x > \frac{5}{3} \end{matrix} \Big\}$ $x \in \left(\frac{5}{3},\right.$

$\Rightarrow \boxed{x < 4}$

Q. $\log_{1/2}^{x^2+2x+1} > \log_{1/2}^{(-x-1)}$

$x^2 + 2x + 1 < -x - 1$
$x^2 + 3x + 2 < 0$
$(x+2)(x+2) < 0$

$\Big\{ \begin{matrix} x^2 + 2x + 1 > 0 \\ (x+1)^2 > 0 \\ x \in R - \{-1\} \end{matrix}$

& $-x - 1 > 0$
$\boxed{x < -1}$

$\therefore$ final $x \in (-2, -1)$

V. Imp

Q. $\log_x^{0.5} > 2$, find the Domain :-

Soln. for $0 < x < 1$

$0.5 < x^2$
$x^2 - 1/2 > 0$
$\Rightarrow \left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right) > 0$

$\Rightarrow$ 

$x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, \infty\right)$

$\hookrightarrow x \in (\sqrt{1/2}, 1)$

for $x > 1$

$0.5 > x^2$
$x^2 - 1/2 > 0$
$\Rightarrow \left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right) > 0$

$\Rightarrow \boxed{x \in \phi}$

Q. $f(x) = \sqrt{\log_8^x}$; find the Domain ?

Sol^n $\log_8^x \geqslant 0$

⇒ $x \geqslant (8)^0$

⇒ $x \geqslant 1$

∴ $x \in [1,\infty)$

$x > 0$

∴ $x \in (0,\infty)$

⇒ $\boxed{x \in [1,\infty)}$

Q. $f(x) = \sqrt{\log_{1/10}^{x+5}}$; find the domain ?

Sol^n. $x+5 > 0$ ⇒ $\boxed{x > -5}$ ——①

and $\log_{1/10}^{(x+5)} \geqslant 0$ ⇒ $x+5 \leq (1/10)^0$ ⇒ $x+5 \leq 1$ ——②

∴ from ① & ②;

$x \in (-5,-4]$

Q. $f(x) = \log_2(\log_{1/2} x)$

Sol^n $\boxed{x > 0}$ and $\log_{1/2}^x > 0$ ⇒ $\boxed{x < (1/2)^0}$

∴ $\boxed{x \in (0,1)}$

Q. $f(x) = \log_{x^2+3x+2}(x+3)$; find the Domain ?

Sol^n. $x+3 > 0$ ⇒ $\boxed{x > -3}$

and $x^2+3x+2 \neq 0$ ⇒ $(x+1)(x+2) \neq 0$

∴ $x \in (-3,\infty) - \{-1,-2\}$

Q $f(x) = \log_e (x^2 - 4x + 7)$; find $R_f = ??$

Sol^n. $f(x) = \log_e (x^2 - 4x + 7)$

$\hookrightarrow [3\ \infty)$

$\therefore\ y = [\log 3, \infty)$

Q. $f(x) = \log_e (-x^2 + 5x - 6)$

Sol^n. $D = (-x^2 + 5x - 6) \Rightarrow (-\infty, 1/4]$

$\therefore\ D_{f(x)} = (-\infty, \ln 1/4]$

Q. $f(x) = \log_e (\sqrt{-x^2 + 5x - 6})$

Sol^n $D = (-x^2 + 5x - 6)^{1/2} = [0, 1/2]$

$\therefore\ D_{f(x)} = (-\infty, \ln 1/2]$