#-5

- Some other Important Questions :-

Q. If $f(x) = \sin(\log x)$; then,

$f(x/y) + f(xy) - 2f(x)\cos(\log y) = ??$

(A) 0
(B) $\sin(\log x)$
(C) $\cos(\log x)$
(D) $\cos(\log y)$

Sol^n $\sin\left(\log\left(\frac{x}{y}\right)\right) + \sin(\log(xy)) - 2\sin(\log x)\cos(\log y)$

$\Rightarrow \sin(\log x)\cos(\log y) - \sin(\log y)\cos(\log x) + \sin(\log x) + \sin(\log y) - 2\sin(\log x)(\cos(\log y))$

$\Rightarrow$ **0**

Q. The Domain of the funct^n $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ = the greatest integer less than or equal to x is :-

(A) R
(B) $[0, \infty)$
(C) $(-\infty, 0]$
(D) None of these

Sol^n. Actual Domain :- $[0, \infty) \cup -Z$

Q. The Domain of $\sqrt{\left(\frac{1}{2}\log[3x - x^2]\right)}$ is

(A) $\left(\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$
(B) $\left[\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right]$

(C) $[3-\sqrt{5}, 3+\sqrt{5}]$ (D) None of these,

Sol^{n}. $\Rightarrow \frac{1}{2}\log_e(3x-x^2) \geq 0$

$\Rightarrow \log_e(3x-x^2) \geq 0$

$\Rightarrow (3x-x^2) \geq e^0$

$\Rightarrow 3x-x^2 \geq 1 \Rightarrow -x^2+3x-1 \geq 0$

$\Rightarrow \boxed{x^2-3x+1 \leq 0}$

$\therefore x \in \left[\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right]$

Q. If $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

for $x > 2$, then $f(11) = ??$

(A) 7/6 (B) 5/6

(C) 6/7 (D) 5/7

Sol^{n}: $\frac{1}{\sqrt{11+2\sqrt{18}}} + \frac{1}{\sqrt{11+2\sqrt{18}}}$

$\Rightarrow \frac{1}{\sqrt{9+2\cdot3\cdot\sqrt{2}+2}} + \frac{1}{\sqrt{9-2\cdot3\cdot\sqrt{2}+2}}$

$\Rightarrow \frac{1}{\sqrt{(3+\sqrt{2})^2}} \quad \frac{1}{\sqrt{(3-\sqrt{2})^2}}$

$\Rightarrow \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}} \Rightarrow \boxed{\frac{6}{7}}$

Q. For a real no. x, $[x]$ denotes g.i.f The value of :-

$$\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \left[\frac{1}{2} + \frac{2}{100}\right] + \cdots + \left[\frac{1}{2} + \frac{99}{100}\right]$$

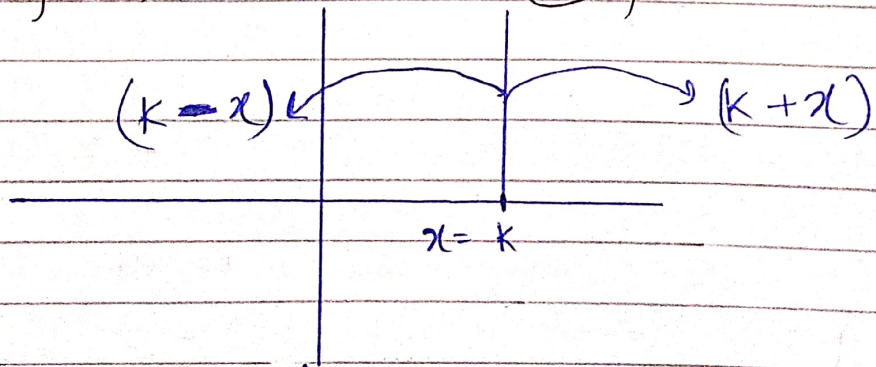
(A) 49 (B) 50
(C) 48 (D) 51

Sol^{n}: $\left[\frac{1}{2}\right] + \left[\frac{1}{2} + \frac{1}{100}\right] + \cdots \left[\frac{1}{2} + \frac{49}{100}\right] = 0$

$\Rightarrow \left[\frac{1}{2} + \frac{50}{100}\right] + \left[\frac{1}{2} + \frac{51}{100}\right] + \cdots \left[\frac{1}{2} + \frac{99}{100}\right] = 50$

Q. The graph of the function $y = f(x)$ is symmetrical about the line $x = k$ then,

(A) $f(x+k) = f(x-k)$ (B) $f(x) = -f(-x)$
(C) $f(x) = f(-x)$ (D) $f(k+x) = f(k-x)$



$\therefore f(k+x) = f(k-x)$

Q. Find the Domain of $f(x) = \log_7(\log_5 \log_3 \log_2 (2x^2+5x^2-14x))$

Sol^n.

$\log_5 \log_3 \log_2 (2x^3+5x^2-14x) > 0$

$\Rightarrow \log_5 \log_3 \log_2 (2x^3+5x^2-14x) > 5^0 \cdot 1$

$\Rightarrow \log_2 (2x^3+5x^2-14x) > 3^1$

$\Rightarrow (2x^3+5x^2-14x) > 2^3$

$\Rightarrow 2x^3+5x^2-14x-8 > 0$

$\Rightarrow (x-2)(2x+1)(x+4) > 0$

$\ominus \; -4 \; \oplus \; -\frac{1}{2} \; \ominus \; 2 \; \oplus \quad \rightarrow x \in (-4, -\frac{1}{2}) \cup (2, \infty)$

Q. Find the Domain & Range of

$y = \sqrt{1+x} + \sqrt{2-x}$

Sol^n $\Rightarrow$ Domain $\rightarrow$ $x \in [-1, 2]$

$\Rightarrow$ Range $\Rightarrow$ $y = \sqrt{2-x} + \sqrt{1+x}$

$y^2 = 2-x+1+x+2\sqrt{(2-x)(1+x)}$

$y^2 = 3 + 2\sqrt{2-x^2+x}$

$= \boxed{3 + 2\sqrt{\frac{9}{4} - \left(x-\frac{1}{2}\right)^2}}$

$(x = \frac{1}{2})$ Max. $\rightarrow \sqrt{6}$

Min. $(x = 2) \rightarrow \sqrt{3}$

$\therefore \boxed{\text{Range} :- [\sqrt{3}, \sqrt{6}]}$

Q. Find Range of $f(x) = \frac{x^2-3x+2}{x^2+x-6}$.

Sol^n) Range :- $y = \frac{(x-2)(x-1)}{(x-2)(x+3)}$ $[x \neq 2]$

$\Rightarrow y = \frac{(x-1)}{(x+3)}$ $[x \neq 2]$

$\Rightarrow \boxed{\frac{3y+1}{1-y} = x}$ $\begin{matrix} \nearrow y \neq 1 \\ \searrow y \neq 1/5 \ [x \neq 2] \end{matrix}$

$\therefore$ Range :- $y \in R - \left\{1, \frac{1}{5}\right\}$

Q. Find the Domain of $f(x) = \log_{[x+\frac{1}{x}]} |x^2-x-6|$

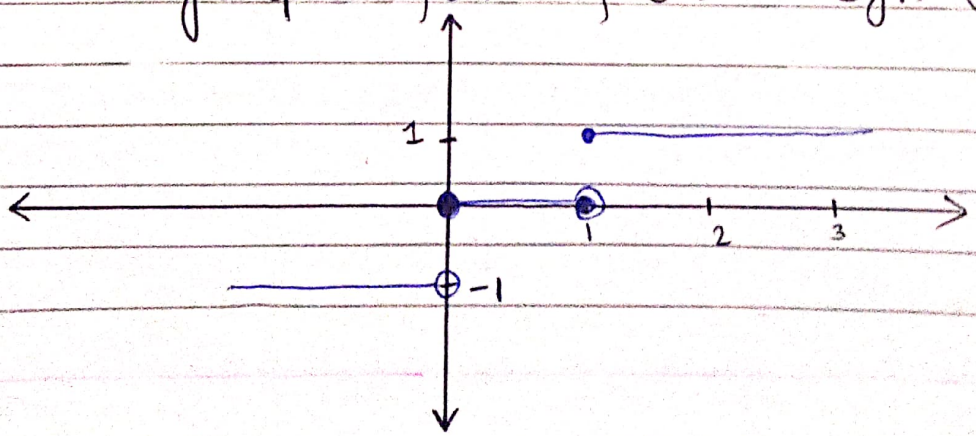
Sol^n Here, $\left[x+\frac{1}{x}\right] \geq 2$ [By AM $\geq$ GM]

$\therefore \boxed{x \in (0, \infty)}$

Also, $x^2-x-6 \longrightarrow (x \neq -2)$ & $(x \neq 3)$

$\therefore \boxed{D_f \in (0, \infty) - \{3\}}$

Q. Form the graph for $f(x) = \text{Sgn}([x])$



Q. Find Domain of $f(x) = \sqrt{(5x-6-x^2)}\,[\ln\{x\}] + \sqrt{7x-5-2x^2} + \left(\ln\left(\frac{7}{2}-x\right)\right)^{-1}$

Sol^n

$[\ln\{x\}]$ (when $\{x\} \neq 1$, i.e. $= 0$)

And, $7x-5-2x^2 \geq 0$

$\Rightarrow (x-1)(2x-5) \leq 0$

$\Rightarrow x \in \left[1, \frac{5}{2}\right]$

Also, $\frac{1}{\ln\left(\frac{7}{2}-x\right)}$ $\Rightarrow \frac{7}{2}-x > 0$

$\Rightarrow \boxed{x < 7/2}$

$\frac{7}{2}-x \neq 1 \rightarrow \boxed{x \neq 5/2}$

$\therefore D_f = (1,2) \cup (2, 5/2)$

Q. Find Domain of the funct^n,

$y = \log_{10}(1 - \log_{10}(x^2-5x+16))$

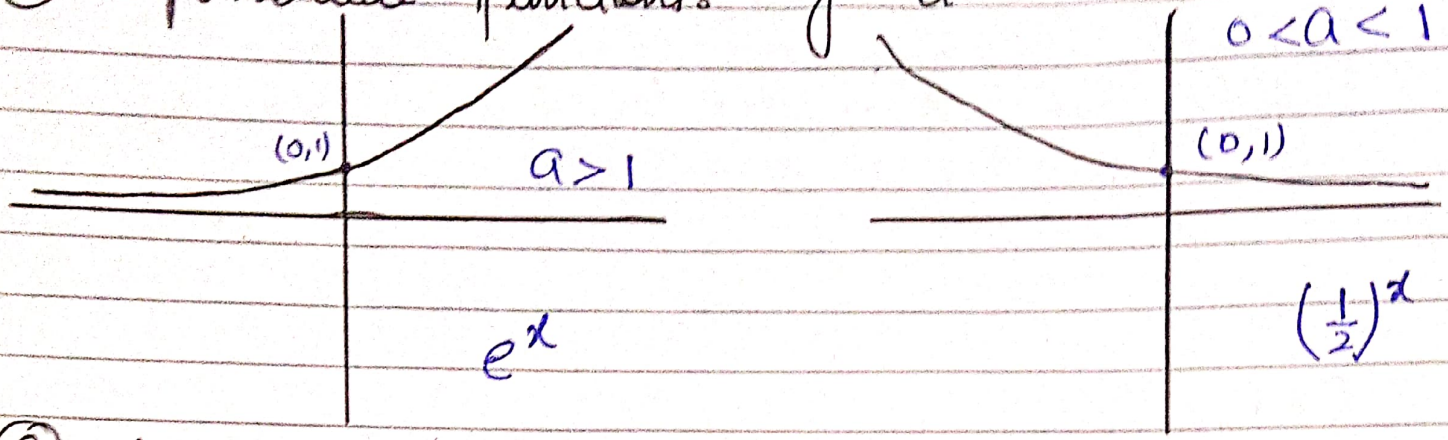
Sol^n $\Rightarrow 1 - \log_{10}(x^2-5x+16) > 0$

$\Rightarrow \log_{10}(x^2-5x+16) < 1$

$\Rightarrow (x^2-5x+16) < 10 \Rightarrow x^2-5x+6 < 0$

$\Rightarrow \boxed{x \in (2,3)}$

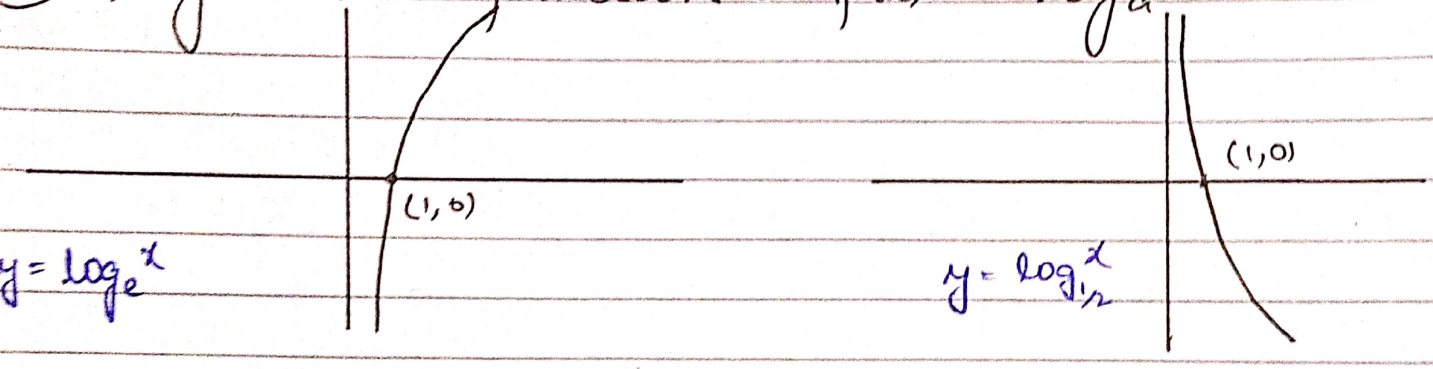
① Exponential functions:- $y = a^x$



e^x

$\left(\frac{1}{2}\right)^x$

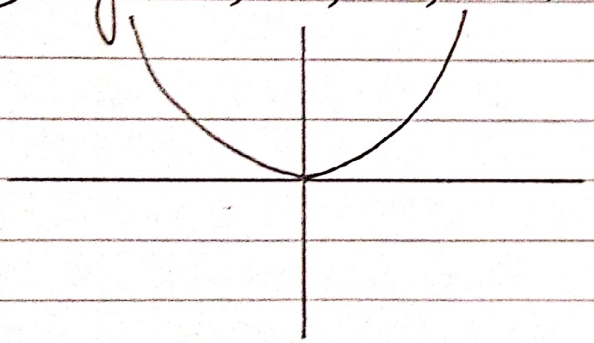
② Logarithmic functions :- $f(x) = \log_a x$



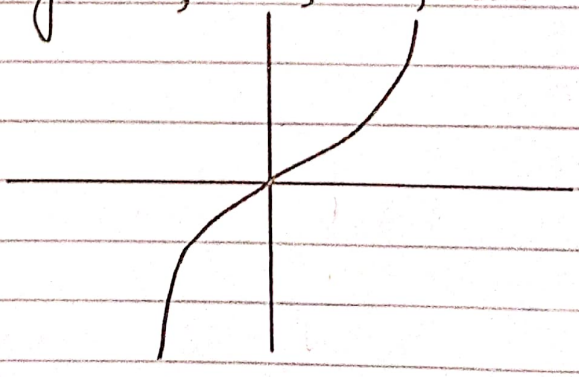
$y = \log_e x$

$y = \log_{1/2} x$

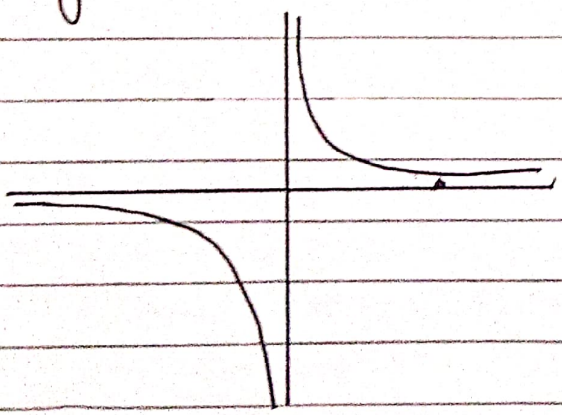
③ $y = x^2$; x^4 ; x^6 ; x^8 , ...



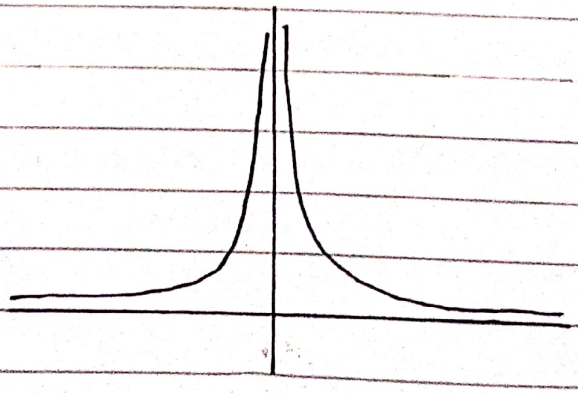
④ $y = x^3$; x^5 ; x^7 ; ....



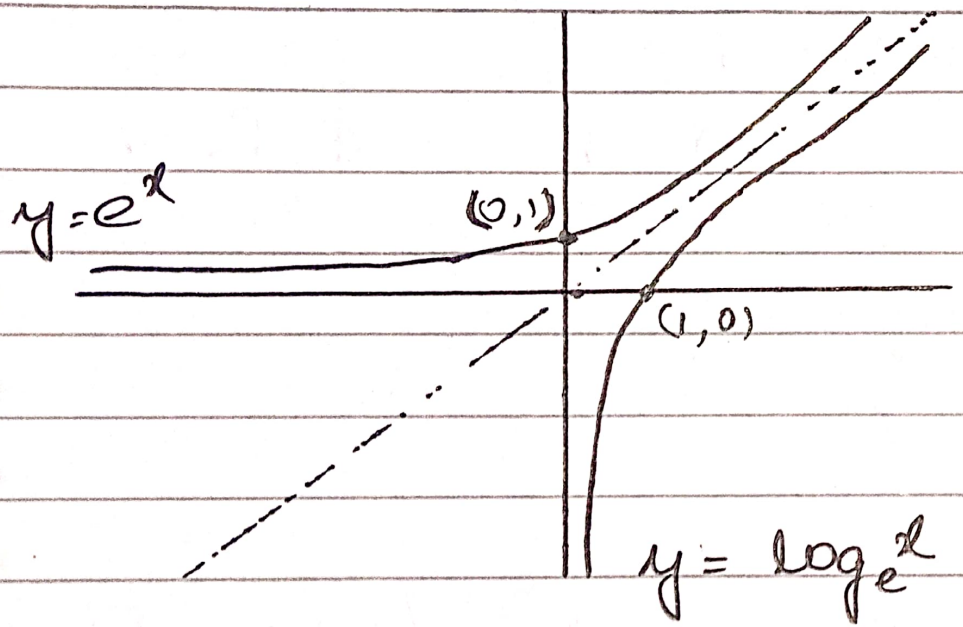
⑤ $y = 1/x$; $1/x^3$; $1/x^5$; $1/x^7$; ....



⑥ $y = 1/x^2$; $\frac{1}{x^4}$; $\frac{1}{x^6}$; ....



(7) Inverse functions & functions graphs :-



$f(x)$ & $f^{-1}(x)$ are mirror images of each other about $y = x$