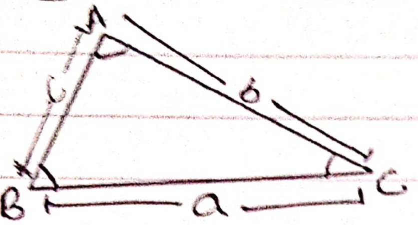
SOLUTIONS OF TRIANGLES

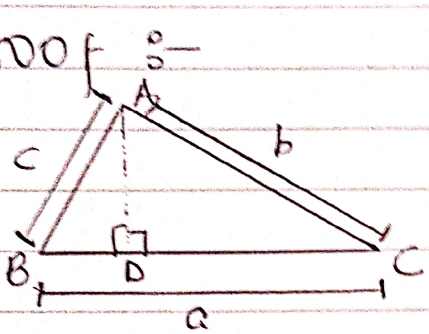
8-1

① Sine Rule :-



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

• Proof :-



In $\triangle ABD$, $\frac{AD}{AB} = \sin B$

$\Rightarrow \boxed{AD = c \cdot \sin B}$ —①

In $\triangle ADC$, $\frac{AD}{AC} = \sin C$

$\Rightarrow \boxed{AD = b \sin C}$ —②

$$\therefore \boxed{\frac{c}{\sin C} = \frac{b}{\sin B}}$$

Similarly, $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

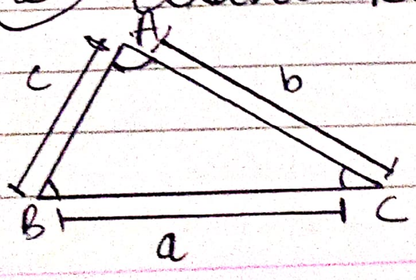
• For conversions of sides ⟶ Angles :-

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k_1 \Rightarrow \begin{aligned} a &= k_1 \sin A \\ b &= k_1 \sin B \\ c &= k_1 \sin C \end{aligned}$$

• For conversions of Angles ⟶ Sides :-

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k_2 \Rightarrow \begin{aligned} \sin A &= a\, k_2 \\ \sin B &= b\, k_2 \\ \sin C &= c\, k_2 \end{aligned}$$

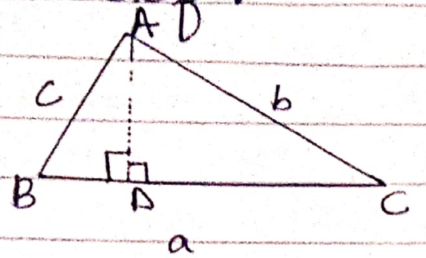
② Cosine Rule :-



$$\cos A = \frac{b^2+c^2-a^2}{2bc} \quad ; \quad \cos C = \frac{b^2+a^2-c^2}{2ab}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

• Proof :–



In ΔABD,

$AB^2 = AD^2 + BD^2$

$c^2 = AC^2 - CD^2 + BD^2$

$c^2 = b^2 - CD^2 + (BC - CD)^2$

$c^2 = b^2 - \cancel{CD^2} + BC^2 + \cancel{CD^2} - 2BC \cdot CD$

$c^2 = b^2 + a^2 - 2(a)(CD)$

$c^2 = b^2 + a^2 - 2ab\cos C$

In ΔADC,

$AD^2 + CD^2 = AC^2$

$AD^2 = AC^2 - CD^2$

$\cos C = \frac{CD}{AC} = \frac{CD}{b}$

$\therefore \boxed{CD = b\cos C}$

$\therefore \boxed{\cos C = \frac{a^2 + b^2 - c^2}{2ab}}$

Q. Angles of a Δ are in A.P., also, $a = 2, c = 4$, then, find 'b'.

Sol^n $A; B; C$ in A.P.

$\therefore 2B = A + C$

Also, $A + B + C = 180°$ $\Rightarrow \boxed{B = 60°}$

So, $\cos B = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \frac{1}{\cancel{2}} = \frac{4 + 16 - b^2}{\cancel{2}(2)(4)}$

$\therefore \boxed{b = 2\sqrt{3}}$

Q. If two angles of a Δ are $45°$ and $60°$, then find the ratio of smallest & greatest sides.

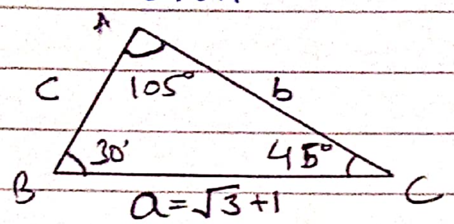
Sol^n Also, 3rd Angle $= 180° - 45° - 60° = 75°$

Since, we know that larger side is opp. to a larger angle, thus,

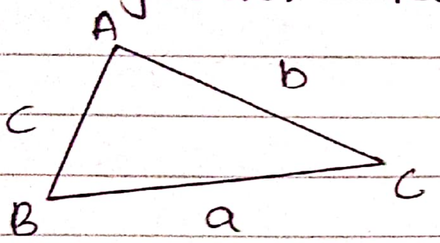
$\frac{a}{\sin 45°} = \frac{b}{\sin 75°} \Rightarrow \frac{a}{b} = \frac{\sin 45°}{\sin 75°} = \frac{2}{\sqrt{3} + 1}$

Q. Find the remaining 2 sides, if $30°, 45°$ and included side is $\sqrt{3}+1$.

Soln. $\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sqrt{3}+1}{\sin 105°} = \frac{b}{\sin 30°} \Rightarrow \boxed{b = \sqrt{2} \text{ \& } c = 2}$



③ Projection Rule :-



$a = c\cos B + b\cos C$

$b = c\cos A + a\cos C$

$c = a\cos B + b\cos A$

Q. Prove that :- $\frac{c - a\cos B}{b - a\cos C} = \frac{\sin B}{\sin C}$

Soln. $\frac{c - a\cos B}{b - a\cos C} = \frac{\cancel{a\cos B} + b\cos A - \cancel{a\cos B}}{c\cos A + \cancel{a\cos C} - \cancel{a\cos C}} = \frac{b}{c} = \frac{\sin B}{\sin C}$

Q. Prove that :- $2[a\sin^2 C/2 + c\sin^2 A/2] = a+c-b$

Soln. $2\sin^2 C/2 \cdot a + c \cdot 2\sin^2 A/2 \Rightarrow a(1-\cos C) + c(1-\cos A)$

$\Rightarrow a + c - [a\cos C + c\cos A] \Rightarrow \boxed{a+c-b}$

④ Tangent Rule (Napier's Analogy) :-

→ $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$

→ $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$

→ $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$

- Proof :-

$\because \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{c} = \frac{\sin B}{\sin C}$ —— (1)

Now, applying componendo & dividendo, we

$$\Rightarrow \frac{b+c}{b-c} = \frac{\sin B + \sin C}{\sin B - \sin C} = \frac{\cancel{2}\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)}{\cancel{2}\sin\left(\frac{B-C}{2}\right)\cdot\cos\left(\frac{B+C}{2}\right)}$$

$$\Rightarrow \frac{b+c}{b-c} = \frac{\tan\left(\frac{B+C}{2}\right)}{\tan\left(\frac{B-C}{2}\right)}$$

$\because A+B+C = 180°$

$\boxed{\frac{B+C}{2} = 90 - \frac{A}{2}}$

$$\Rightarrow \boxed{\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}}$$

Q. Prove that :- $a\cos\left(\frac{B-C}{2}\right) = (b+c)\sin\frac{A}{2}$

Sol^n $\frac{b+c}{a} = \frac{\cancel{k}\sin B + \cancel{k}\sin C}{\cancel{k}\sin A}$

$a = k\sin A$
$b = k\sin B$
$c = k\sin C$

$$\frac{b+c}{a} = \frac{\cancel{2}\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)}{\cancel{2}\sin\frac{A}{2}\cdot\cos\frac{A}{2}}$$

$\boxed{\frac{B+C}{2} = 90 - \frac{A}{2}}$

$$\frac{b+c}{a} = \frac{\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}} \Rightarrow \boxed{a\cos\frac{B-C}{2} = (b+c)\sin\frac{A}{2}}$$

Hence, Proved.

Q. $(b^2-c^2)\cot A + (c^2-a^2)\cot B + (a^2-b^2)\cot C = ??$

Sol^n $(b^2-c^2)\frac{\cos A}{\sin A} = \frac{(b^2-c^2)}{(ak)}\left(\frac{b^2+c^2-a^2}{2bc}\right) = \frac{(b^2-c^2)(b^2+c^2-a^2)}{2abck}$

Similarly, $\frac{(c^2-a^2)(c^2+a^2-b^2)}{2abck}$; $\frac{(a^2-b^2)(a^2+b^2-c^2)}{2abck}$

$= 0$

$\therefore \dfrac{(b^2-c^2)(b^2+c^2-a^2)+(c^2-a^2)(c^2+a^2-b^2)+(a^2-b^2)(a^2+b^2-c^2)}{2abck}$

Q. If a^2, b^2, c^2 are in A.P., then check whether $\cot A, \cot B, \cot C \rightarrow$ A.P.

Sol.

$b^2 - a^2 = c^2 - b^2$

$\cancel{k^2}\sin^2 B - \cancel{k^2}\sin^2 A = \cancel{k^2}\sin^2 C - \cancel{k^2}\sin^2 B$

$\Rightarrow \sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$

$\Rightarrow \sin(B+A)\cdot\sin(B-A) = \sin(C+B)\cdot\sin(C-B)$

$\Rightarrow \sin(180-C)\cdot\sin(B-A) = \sin(180-A)\cdot\sin(C-B)$

$A+B = 180-C$

$C+B = 180-A$

$\Rightarrow \sin C\cdot\sin(B-A) = \sin A\cdot\sin(C-B)$

$\Rightarrow \sin C(\sin B\cdot\cos A - \cos B\cdot\sin A) = \sin A(\cos B\cdot\sin C - \cos C\cdot\sin B)$

$\Rightarrow \sin C\cdot\sin B\cdot\cos A + \sin A\cos C\cdot\sin B = 2\sin A\cdot\cos B\cdot\sin C$

Now, Dividing both sides by $\sin A\sin B\sin C$,

$\cot A + \cot C = 2\cot B \Rightarrow$ A.P.

Q. Find angle A, if $\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ac}$

Sol. $\dfrac{\cos A}{a} + \dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c} + \dfrac{\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ac}$

$\Rightarrow \dfrac{\cos A}{a} + \dfrac{c\cos A + a\cos C}{ac} + \dfrac{c\cos B + b\cos C}{bc} = \dfrac{a}{bc} + \dfrac{b}{ac}$

$\Rightarrow \dfrac{\cos A}{a} + \cancel{\dfrac{b}{ac}} + \cancel{\dfrac{a}{bc}} = \cancel{\dfrac{a}{bc}} + \cancel{\dfrac{b}{ac}}$

$\Rightarrow \dfrac{\cos A}{a} = 0 \Rightarrow \boxed{A = \pi/2}$

Q. find C, if $c^4 - 2(a^2+b^2)c^2 + a^4 + b^4 + a^2b^2 = 0$

Sol^n $a^4 + b^4 + c^4 - 2a^2c^2 - 2b^2c^2 + 2a^2b^2 = a^2b^2$

$\Rightarrow (a^2)^2 + (b^2)^2 + (c^2)^2 + 2(a^2)(-c^2) + 2b^2(-c^2) + 2(a^2)(b^2) = a^2b^2$

$\Rightarrow (a^2+b^2-c^2)^2 = (ab)^2 \Rightarrow \left(\frac{a^2+b^2-c^2}{ab}\right)^2 = 1$

$\Rightarrow \left(\frac{a^2+b^2-c^2}{2ab}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow (\cos C)^2 = (1/2)^2$

$\therefore \cos C = 1/2$ and $\cos C = -1/2$

$\boxed{C = \pi/3}$ and $\boxed{C = 2\pi/3}$

P-2

(5) Half angle formulas :-

* $\cos A = \dfrac{b^2+c^2-a^2}{2bc} \Rightarrow 1 - 2\sin^2\frac{A}{2} = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow 2\sin^2\frac{A}{2} = 1 - \dfrac{(b^2+c^2-a^2)}{2bc} \Rightarrow 2\sin^2\frac{A}{2} = \dfrac{a^2 - (b^2+c^2-2bc)}{2bc}$

$\Rightarrow 2\sin^2 A/2 = \dfrac{(a+c-b)(a+b-c)}{2bc}$

$\Rightarrow 2\sin^2\frac{A}{2} = \dfrac{(2s-b-b)(2s-c-c)}{2bc}$

$\frac{a+b+c}{2} = s$ (semi-perimeter)

$\Rightarrow a+b+c = 2s$

$\boxed{a+c = 2s-b}$ $\boxed{a+b = 2s-c}$

$\Rightarrow \sin\frac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$

$\Rightarrow \sin\frac{B}{2} = \sqrt{\dfrac{(s-a)(s-c)}{ac}}$

$\Rightarrow \sin\frac{C}{2} = \sqrt{\dfrac{(s-b)(s-a)}{ab}}$

* $\cos A = \dfrac{c^2+b^2-a^2}{2bc} \Rightarrow 2\cos^2\frac{A}{2} - 1 = \dfrac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos^2\frac{A}{2} = \dfrac{(2s-2a)(2s)}{4bc} \Rightarrow \boxed{\cos\frac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}}$

* $\tan\frac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

* $\sin A = 2\sin\frac{A}{2}\cdot\cos\frac{A}{2} = 2\sqrt{\dfrac{(s-b)(s-c)}{bc}}\cdot\sqrt{\dfrac{s\cdot(s-a)}{bc}}$

$= \dfrac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)} = \dfrac{2\Delta}{bc}$ Area of Δ

* $\Delta = \frac{1}{2}bc\sin A$; $\Delta = \frac{1}{2}ca\sin B$; $\Delta = \frac{1}{2}ab\sin C$

Q. $(a+b+c)\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 2c\cot\frac{C}{2}$ ← Prove

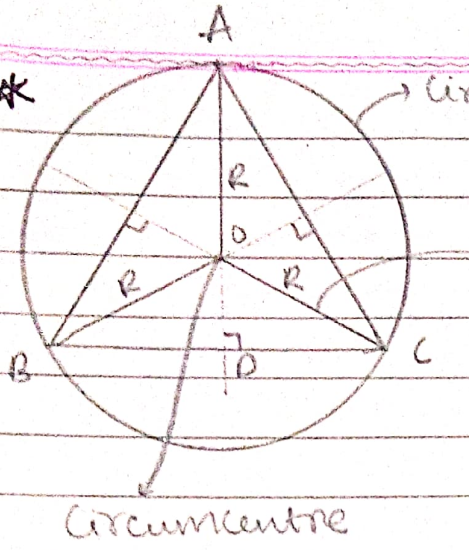
Solⁿ $(2s)\left(\sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}} + \sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}\right)$

$\Rightarrow 2s\sqrt{\dfrac{s-c}{s}}\left(\sqrt{\dfrac{s-b}{s-a}} + \sqrt{\dfrac{s-a}{s-b}}\right)$

$\Rightarrow 2\sqrt{s(s-c)}\left(\dfrac{s-b+s-a}{\sqrt{(s-a)(s-b)}}\right) = \dfrac{2\sqrt{s(s-c)}\,(c)}{\sqrt{(s-a)}\sqrt{s-b}} = \dfrac{2c}{\tan\frac{C}{2}}$

* $\Delta = \frac{1}{2}bc\cdot\sin A \Rightarrow \dfrac{\sin A}{a} = \dfrac{2\Delta}{abc}$

$\therefore \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = \dfrac{abc}{2\Delta}$

* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{abc}{2\Delta} = 2R$$

$$\boxed{R = \frac{abc}{4\Delta}}$$

Proof :-

$\angle BOD = \angle COD = \frac{1}{2}\angle BOC = \frac{1}{\cancel{2}}(\cancel{2}\angle A)$

$\sin A = \frac{BD}{OB} = \frac{a/2}{R} \Rightarrow R = \frac{a}{2\sin A}$

$\therefore \boxed{\frac{a}{\sin A} = 2R}$

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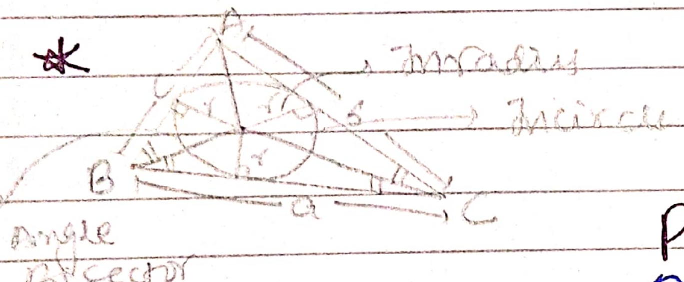
* 

$$\boxed{r = \frac{\Delta}{s}}$$

Proof :-

$\Delta OBC = \frac{1}{2}(a)r$ ——— (1)

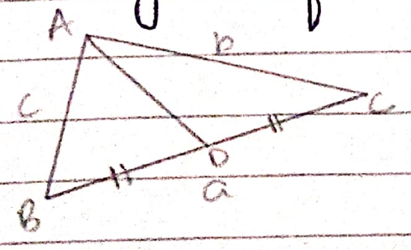
$\Delta OAC = \frac{1}{2}(b)r$ ——— (2)

$\Delta OAB = \frac{1}{2}(c)r$ ——— (3)

$\therefore \Delta = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = \frac{1}{2}(a+b+c)r = \frac{1}{\cancel{2}}(\cancel{2}s)r$

$\therefore \boxed{\Delta = sr}$

(6) Length of Median :-



$$AD(\text{median}) = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

- Proof :- In ΔADC, we have :-

$\cos C = \frac{AC^2 + CD^2 - AD^2}{2(AC)(CD)} \Rightarrow 2\cos C\,(AC)(CD) = b^2 + \frac{a^2}{4} - AD^2$

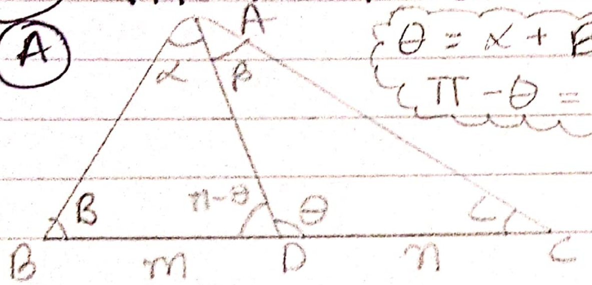
$$\Rightarrow AD^2 = b^2 + \frac{a^2}{4} - 2\cos C \cdot (b) \cdot \frac{a}{2}$$

$$= b^2 + \frac{a^2}{4} - \cancel{ab}\left(\frac{b^2 + a^2 - c^2}{2\cancel{ab}}\right)$$

$$\sqrt{AD^2} = \sqrt{\frac{2b^2 + 2c^2 - a^2}{4}}$$

(7) m-n Theorem :-

(A)



$\theta = \alpha + B$

$\pi - \theta = \beta + C$

In $\triangle ABD$,

$$\frac{BD}{\sin\alpha} = \frac{AD}{\sin B}$$

In $\triangle ACD$,

$$\frac{CD}{\sin\beta} = \frac{AD}{\sin C}$$

$$\therefore \frac{BD}{AD} = \frac{\sin\alpha}{\sin B} \quad \text{—(1)} \quad \& \quad \frac{CD}{AD} = \frac{\sin\beta}{\sin C} \quad \text{—(2)}$$

$$\text{So,} \quad \boxed{\frac{BD}{CD} = \frac{\sin\alpha}{\sin B} \cdot \frac{\sin C}{\sin\beta} = \frac{m}{n}} \quad \text{—(3)}$$

Now, since, $\theta = \alpha + B \Rightarrow \boxed{B = \theta - \alpha}$

and $\pi - \theta = \beta + C \Rightarrow \boxed{C = \pi - (\theta + \beta)}$

$$\therefore \frac{m}{n} = \frac{\sin\alpha \cdot \sin(\pi - (\theta + \beta))}{\sin(\theta - \alpha) \cdot \sin\beta} = \frac{\sin\alpha \cdot \sin(\theta + \beta)}{\sin(\theta - \alpha) \cdot \sin\beta}$$

$$\Rightarrow \frac{m \sin(\theta - \alpha)}{\sin\theta \cdot \sin\alpha} = \frac{n \sin(\theta + \beta)}{\sin\theta \cdot \sin\beta}$$

$$\Rightarrow m(\cot\alpha - \cot\theta) = n(\cot\beta + \cot\theta)$$

$$\Rightarrow (m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

(B) Now, since, $\theta = \alpha + \beta \Rightarrow \boxed{\alpha = \theta - B}$

$\pi - \theta = B + C \Rightarrow \boxed{\beta = \pi - (\theta + C)}$

$\therefore \dfrac{\sin\alpha \cdot \sin C}{\sin\beta \cdot \sin B} = \dfrac{m}{n} \Rightarrow \dfrac{\sin(\theta - B)\cdot \sin C}{\sin B \cdot \sin(\pi - (\theta + C))} = \dfrac{m}{n}$

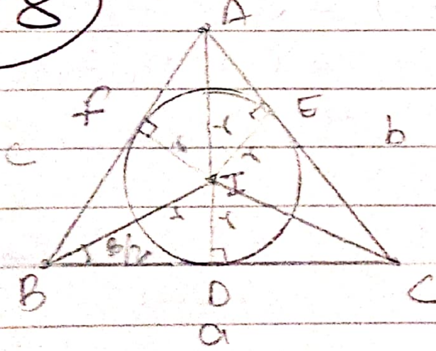
$\Rightarrow \dfrac{m \cdot \sin(\theta + C)}{\sin\theta \cdot \sin C} = \dfrac{n \cdot \sin(\theta - B)}{\sin\theta \cdot \sin B}$

$\Rightarrow m\left(\dfrac{\sin\theta \cdot \cos C + \cos\theta \cdot \sin C}{\sin\theta \cdot \sin C}\right) = n\left(\dfrac{\sin\theta \cdot \cos B - \cos\theta \cdot \sin B}{\sin\theta \cdot \sin B}\right)$

$\Rightarrow m(\cot C + \cot\theta) = n(\cot B - \cot\theta)$

$\Rightarrow (m+n)\cot\theta = n\cot B - m\cot C$

(8)



Here, $BD = BF$

$\Rightarrow 2BD = BD + BF$

$2AE = AE + AF$

$2CE = CE + CD$

So,

$\Rightarrow 2(BD + AE + CE) = (BD + CD) + (AE + CE) + (BF + AF)$

$\Rightarrow 2BD + 2AC = BC + AC + AB$

$\Rightarrow 2BD + 2b = a + b + c$

$\Rightarrow 2BD = a + c - b$

$2BD = 2s - 2b \Rightarrow \boxed{BD = s - b}$

Similarity, $CE = s - c$; $AF = s - a$

Now, in $\triangle IBD$, $\tan(IBD) = \dfrac{r}{BD} \Rightarrow \boxed{r = BD \cdot \tan\frac{B}{2}}$

So, $\boxed{r = (s-b)\tan\frac{B}{2}}$; $\boxed{r = (s-a)\tan\frac{A}{2}}$; $\boxed{r = (s-c)\tan}$

$\therefore \quad a = BD + CD$

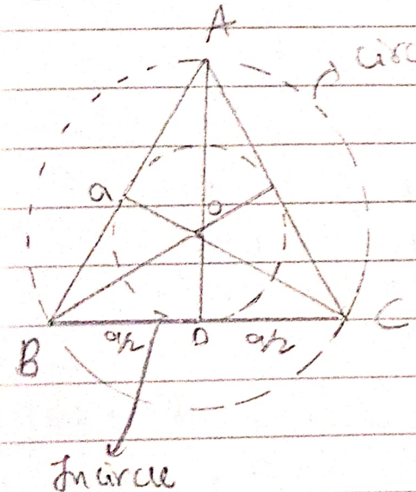
$\Rightarrow a = \frac{r}{\tan B/2} + \frac{r}{\tan \frac{C}{2}} = r\left[\frac{\cos\frac{B}{2}\cdot\sin\frac{C}{2} + \sin\frac{B}{2}\cdot\cos\frac{C}{2}}{\sin\frac{B}{2}\cdot\sin C/2}\right]$

$\Rightarrow a = r\cdot\frac{\sin\left(\frac{C+B}{2}\right)}{\sin B/2\cdot\sin\frac{C}{2}} \Rightarrow r = \frac{a\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}}{\cos A/2}$

$\Rightarrow r = \frac{2R\cdot\sin A\cdot\sin\frac{B}{2}\cdot\sin C/2}{\cos A/2} = \frac{2R\left(2\sin\frac{A}{2}\cdot\cancel{\cos\frac{A}{2}}\right)\sin\frac{B}{2}\ldots}{\cancel{\cos A/2}}$

$\therefore \quad r = 4R\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

(9) Some Imp. results for Equilateral $\triangle$:-



→ Centroid = Incentre = Circum Centre

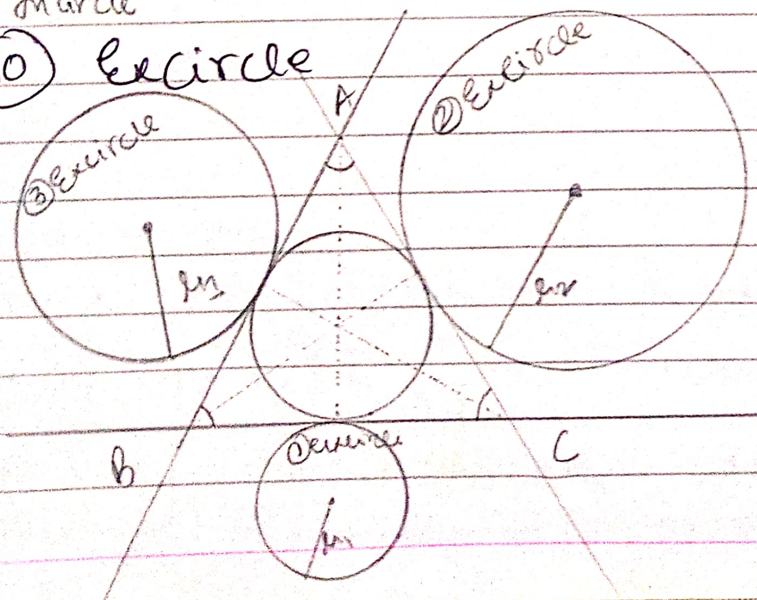
→ OA (circumradius) $= R = 2/3\ AD$

→ OD (Inradius) $= r = 1/3\ AD$

$R = a/\sqrt{3}$; $r = a/2\sqrt{3}$

→ If $R = 2r \Longleftrightarrow$ Equilateral $\triangle$

(10) Excircle



4 circles which touch all the three st. lines.

$r_1 = \frac{\triangle}{s-a}$

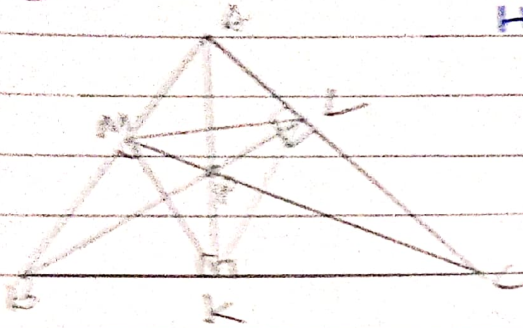
$r_2 = \frac{\triangle}{s-b}$

$r_3 = \frac{\triangle}{s-c}$

Also, $r_1 = s \tan A/2$
$r_2 = s \tan B/2$
$r_3 = s \tan C/2$

$r_1 = 4R \sin\frac{A}{2} \cos B/2 \cos C/2$
$r_2 = 4R \cos\frac{A}{2} \sin B/2 \cos C/2$
$r_3 = 4R \cos\frac{A}{2} \cos B/2 \sin C/2$

(11) Pedal Triangle :-



Here, P → Orthocentre ⇒ P.O.I. of Altitude
$\triangle KLM$ = Pedal Triangle.

Here,

☆ $PK = 2R \cos B \cdot \cos C$
↳ Distance of orthocentre from side.

☆ $LM = a \cdot \cos A$
↳ Length of side of Pedal's $\triangle$.

☆ $AP = 2R \cos A$
↳ Distance of orthocentre from vertex A.

Q. Find the type of $\triangle$ formed, by :-
$$\frac{a^2 - b^2}{a^2 + b^2} = \frac{\sin(A-B)}{\sin(A+B)}$$

Sol^n. $\frac{k^2\sin^2 A - k^2\sin^2 B}{k^2\sin^2 A + k^2\sin^2 B} = \frac{\sin(A-B)}{\sin(A+B)}$

$\Rightarrow \frac{\sin(A+B)\sin(A-B)}{\sin^2 A + \sin^2 B} = \frac{\sin(A-B)}{\sin(A+B)}$

$\Rightarrow \frac{\sin(A+B) \cdot \sin(A-B)}{\sin^2 A + \sin^2 B} - \frac{\sin(A-B)}{\sin(A+B)} = 0$

$\Rightarrow \sin A - B = 0$
$\Rightarrow A - B = 0$
$\Rightarrow \boxed{A = B}$ ↳ Isosceles.

$\frac{\sin(A+B)}{\sin^2 A + \sin^2 B} = \frac{1}{\sin(A+B)}$
$\Rightarrow \sin^2(A+B) = \sin^2 A + \sin^2 B$
$\Rightarrow \sin^2 C = \sin^2 A + \sin^2 B$
$\Rightarrow k^2c^2 = k^2a^2 + k^2b^2$
$\Rightarrow a^2 + b^2 = c^2$ → Right

$\therefore$ The $\triangle$ could be right angled or isosceles.

Q. $(a-b)\cos^2\frac{C}{2} + (a+b)^2 \sin^2\frac{C}{2} = ??$

Sol^n $(a^2+b^2-2ab)\cos^2\frac{C}{2} + (a^2+b^2+2ab)\sin^2\frac{C}{2}$

$\Rightarrow a^2+b^2 - 2ab\left(\cos^2\frac{C}{2} - \sin^2\frac{C}{2}\right)$

$\Rightarrow a^2+b^2 - 2ab(\cos C)$

$\Rightarrow \cancel{a^2}+\cancel{b^2} - \cancel{2ab}\left(\frac{\cancel{b^2}+\cancel{a^2}-c^2}{\cancel{2ab}}\right) = c^2$