CH-TRIGONOMETRIC EQUATIONS

① Trig. Identity

$\sin^2 x + \cos^2 x = 1$

$\boxed{x \in R}$

(Infinite solution)

Trig. Equation.

$\sin\theta = 1/2$

where,

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6} + 2\pi + \frac{5\pi}{6}$

Infinite solution, but not all

② Particular solutions & Principal solutions:-

eg:- Let for $\sin\theta = 1/2$,

Particular Solutions $\Rightarrow \theta \in \{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{-17\pi}{6}, --\}$

&

Principal solutions $\Rightarrow \theta \in \{\frac{\pi}{6}, \frac{5\pi}{6}\}$

$(0 \leq \theta < 2\pi)$

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eg:- Let for $\sin\theta = 0$

Particular Solutions $\Rightarrow \theta \in \{0, \pi, 2\pi, 3\pi, -\pi, -2\pi --\}$

& Principal Solutions :- $\theta \in \{0, \pi\}$

Also,

General Solutions :- $\theta = n\pi \quad ; n \in Z$

(Compact form of particular solutions)

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eg:- Let for $\sin\theta = 1$

Particular Solutions :- $\theta \in \{\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, ---\}$

; Principal Solution :- $\theta \in \{\frac{\pi}{2}\}$

; General Solution :- $\theta \in (4n+1)\frac{\pi}{2} \quad ; n \in Z$

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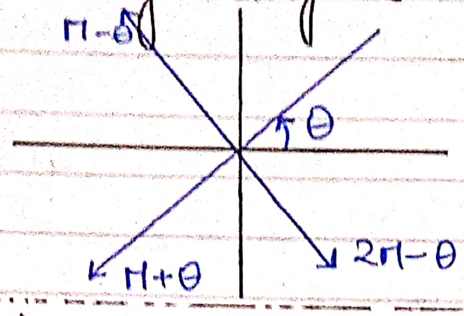
eg:- Let for $\cos\theta = 1/2$

Particular Solutions :- $\theta \in \{\frac{\pi}{3}, \frac{5\pi}{3}, ---\}$

Principal Solutions :- $\theta \in \{\frac{\pi}{3}, \frac{5\pi}{3}\}$

③ For finding angles in II, III & IV Quadrant :-



Eg :- for principal sol^n of $\tan\theta = +1$

$\theta \in \left\{\frac{\pi}{4}, \pi+\frac{\pi}{4}\right\}$

Eg :- Principal solutions of $\tan\theta = -1/\sqrt{3}$

$\theta \in \left\{\pi-\frac{\pi}{6}, 2\pi-\frac{\pi}{6}\right\} \Leftrightarrow \left\{\frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

④ General Solutions for $\sin\theta$; $\cos\theta$; $\tan\theta$:-

Ⓐ $\sin\theta$:-

General Solution = $\theta = n\pi + (-1)^n \underbrace{\alpha}_{\text{Least principal Value}}$; $n \in Z$

Eg :- Find general solutions of $\sin\theta = 1/2$

$\Rightarrow \sin\theta = \sin\alpha = \sin\frac{\pi}{6}$

$\therefore \boxed{\theta = n\pi + (-1)^n \frac{\pi}{6}}$ → General solution for $\sin\theta = \frac{1}{2}$

Eg :- Find general solution of $\sin\theta = \frac{-\sqrt{3}}{2}$

$\Rightarrow \sin\theta = \sin\alpha = \sin\left(\frac{4\pi}{3}\right) = \cancel{\sin\left(\frac{5\pi}{3}\right)}$

$\therefore \boxed{\theta = n\pi + (-1)^n \frac{4\pi}{3}}$ → General sol^n for $\sin\theta = \frac{-\sqrt{3}}{2}$

Eg :- Find general solution of $s\ \theta = 0$

$\Rightarrow \sin\theta = \sin\alpha = \sin 0°$

$\therefore \theta = n\pi + (-1)^n (0) = \underline{\underline{n\pi}}$; $\boxed{n \in Z}$

(B) $\cos\theta$:-

General Solution :- $\theta = 2n\pi \pm \alpha$; $\boxed{n \in Z}$

α : least principal value

Eg :- Find general solution for $\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta = \cos\alpha = \cos\left(\frac{\pi}{6}\right)$

$\therefore \boxed{\theta = 2n\pi \pm \frac{\pi}{6}}$ → general sol^n for $\cos\theta = \frac{\sqrt{3}}{2}$

Eg :- Find general solution for $\cos\theta = -\frac{1}{2}$

$\Rightarrow \cos\theta = \cos\alpha = \cos\left(\frac{2\pi}{3}\right)$

$\therefore \boxed{\theta = 2n\pi \pm \frac{2\pi}{3}}$

Eg :- Find general solution for $\cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\theta = \cos\alpha = \cos 3\pi/4$

$\therefore \boxed{\theta = 2n\pi \pm \frac{3\pi}{4}}$

(C) $\tan\theta$:-

General Solution :- $\theta = n\pi + \alpha$; $\boxed{n \in Z}$

Eg :- Find general solutions for $\tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan\alpha = \tan\frac{\pi}{6}$

$\therefore \boxed{\theta = n\pi + \frac{\pi}{6}}$

(1)

eg. :- $\tan\theta = -\sqrt{3}$:-

$\tan\theta = \tan\alpha = \tan 2\pi/3 \Rightarrow \boxed{\theta = n\pi + \frac{2\pi}{3}}$

Q. Find the general solutions of $\cos^2 x - \sin x - \frac{1}{4} = 0$

Solⁿ. $(1-\sin^2 x) - \sin x - \frac{1}{4} = 0$

$\Rightarrow -\sin^2 x - \sin x + \frac{3}{4} = 0 \Rightarrow \sin^2 x + \sin x - \frac{3}{4} = 0$

$\Rightarrow 4\sin^2 x + 4\sin x - 3 = 0 \Rightarrow 4\sin^2 x + 6\sin x - 2\sin x - 3 = 0$

$\Rightarrow 2\sin x(2\sin x + 3) - 1(2\sin x + 3) = 0$

$\Rightarrow (2\sin x - 1)\cancel{(2\sin x + 3)} = 0$

$\Rightarrow \sin\theta = \sin\alpha = \sin x = \frac{1}{2} = \sin\frac{\pi}{6}$

$\therefore \boxed{\theta = n\pi + (-1)^n \frac{\pi}{6}}$

Q. Find general solutions of $\tan 6\theta - \tan 4\theta = 0$

Solⁿ. $\tan(\underbrace{6\theta}_{\omega}) = \tan(\underbrace{4\theta}_{\phi})$

$\Rightarrow \omega = n\pi + \phi$

$\Rightarrow 6\theta = n\pi + 4\theta$

$\Rightarrow 2\theta = n\pi \Rightarrow \boxed{\theta = \frac{n\pi}{2}}$

Q. * Find general solutions of $-2\cos 6x + \cos 9x = 2$

Solⁿ. $\cos 3(3x) - 2\cdot\cos 2\cdot(3x) = 2$

$\Rightarrow 4\cos^3 3x - 3\cos 3x - 2(2\cos^2 3x - 1) = 2$

$\Rightarrow 4\cos^3 3x - 3\cos 3x - 4\cos^2 3x + \cancel{2} = \cancel{2}$

$\Rightarrow \cos 3x(4\cos^2 3x - 4\cos 3x - 3) = 0$

$\Rightarrow \boxed{\cos 3x = 0}$ or $4\cos^2 3x - 4\cos 3x - 3 = 0$

$\cos 3x = \cos\frac{\pi}{2}$

$\boxed{x = \frac{2n\pi}{3} \pm \frac{\pi}{6}}$

$\Rightarrow (2\cos 3x + 1)\cancel{(2\cos 3x - 3)} = 0$

$\Rightarrow \boxed{\cos 3x = -1/2}$

$\therefore \cos 3x = -\frac{1}{2} = \cos \frac{2\pi}{3} \Rightarrow \boxed{x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}}$

Q. Find general solutions for $\sin 2\theta = \frac{-\sqrt{3}}{2}$

Sol^n. $\sin\left(\frac{4\pi}{3}\right) = \frac{-\sqrt{3}}{2} = \sin 2\theta$

$\Rightarrow 2\theta = n\pi + (-1)^n\left(\frac{4\pi}{3}\right) \Rightarrow \boxed{\theta = \frac{n\pi}{2} + (-1)^n\left(\frac{2\pi}{3}\right)}$

Q. Find principal solution for $\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}}$.

Sol^n. $\cos\theta = -\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = -\cos 75^\circ = \cos(180^\circ - 75^\circ)$

$= \cos(105^\circ)$

$\therefore$ Other principal solution $= \cos(180^\circ + 75^\circ)$

$\boxed{= \cos 255^\circ}$

Q. Find General Solutions of $4\sin x \cdot \cos x = \sqrt{3}$

Sol^n. $4\sin x \cos x = \sqrt{3} \Rightarrow 2\sin 2x = \sqrt{3} \Rightarrow \boxed{\sin 2x = \sqrt{3}/2}$

$\Rightarrow \sin 2x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\therefore 2x = n\pi + (-1)^n\frac{\pi}{3} \Rightarrow \boxed{x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{6}}$

Q. Find General Solutions of $\cos 2\theta = \frac{\sqrt{5}+1}{4}$.

Sol^n. $\cos 2\theta = \frac{\sqrt{5}+1}{4} = \cos 36^\circ = \cos\left(\frac{\pi}{5}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{5} \Longrightarrow \boxed{\theta = n\pi \pm \frac{\pi}{10}}$

*1

Q. Find principal solutions of $2 + \operatorname{cosec} x = 0$.

Sol^n. $\operatorname{cosec}\theta = -2 \Rightarrow \sin x = -\frac{1}{2} = \sin\left(\frac{7\pi}{6}\right)$

$\therefore$ Principal solution $= \frac{7\pi}{6}, \frac{11\pi}{6}$ $\Big|$ $= \sin\left(\frac{11\pi}{6}\right)$

Q. The set of values of 'x' for which

* $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ is ______ ??

Sol^n. $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \cdot \tan 2x} = \tan(3x - 2x) = \tan\frac{\pi}{4} = 1$

$\therefore$ $\boxed{x = n\pi + \frac{\pi}{4}}$

But, at $\tan 2\left(\frac{\pi}{4}\right) \Rightarrow$ not defined

$\therefore$ $\boxed{x \in \phi}$

Q. If $\tan 5x = \cot 3x$, then $x \in$ ______ ?

Sol^n. $\tan 5x = \tan\left(\frac{\pi}{2} - 3x\right)$

$\Rightarrow 5x = n\pi + \frac{\pi}{2} - 3x \Rightarrow \boxed{x \in \frac{n\pi}{8} + \frac{\pi}{16}}$

Q. $\sin 2\theta = \cos 3\theta$ & find general solutions

Sol^n. $\boxed{M-1}$ $\sin 2\theta = \sin\left(\frac{\pi}{2} - 3\theta\right)$

$2\theta = n\pi + (-1)^n\left(\frac{\pi}{2} - 3\theta\right)$

Case-I) If $n = $ odd $\Rightarrow n = 2m+1$ (or) $2m-1$

$\theta = -(4m+1)\frac{\pi}{2}$ $\Big|$ $\theta = -(4m-3)\frac{\pi}{2}$

Case-II) If $n \to$ even, i.e. $n = 2m$

$$2\theta = 2m\pi + \left(\frac{\pi}{2} - 3\theta\right) \Rightarrow \boxed{\theta = \frac{2m\pi}{5} + \frac{\pi}{10}}$$

Q. Find general solution for $\sin 5x \cdot \cos 3x = \sin 6x \cdot \cos$

Sol^n. $2\sin 5x \cdot \cos 3x = 2\sin 6x \cdot \cos 2x$

$\Rightarrow \cancel{\sin 8x} + \sin 2x = \cancel{\sin 8x} + \sin 4x$

$\Rightarrow \sin 2x = \sin 4x$

$\Rightarrow \boxed{4x = n\pi + (-1)^n 2x}$

Case-I) $n =$ odd, $n = 2m+1$

$\Rightarrow \boxed{x = (2m+1)\frac{\pi}{6}}$

Case-II) $n =$ even, $n = 2m$

$\Rightarrow \boxed{x = m\pi}$

Also, $\sin 4x = \sin 2x \Rightarrow \sin 4x - \sin 2x = 0$

$\Rightarrow 2\cos 3x \cdot \sin x = 0$

$\Rightarrow \cos 3x = 0$

$\Rightarrow 3x = 2n\pi \pm \frac{\pi}{2}$

$\Rightarrow \boxed{x = \frac{2n\pi}{3} \pm \frac{\pi}{6}}$

$\sin x = 0$

$\boxed{x = n\pi}$

Q. Find general solutions for $\sqrt{2}\sec\theta + \tan\theta = 1$?

Sol^n $\frac{\sqrt{2}}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \boxed{\sqrt{2} = \cos\theta - \sin\theta}$

$\Rightarrow \sqrt{2} = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right) \Rightarrow 1 = \left(\cos\left(\theta + \frac{\pi}{4}\right)\right) = \cos$

$\Rightarrow \theta + \frac{\pi}{4} = 2n\pi \pm 0 \Rightarrow \theta = 2n\pi - \frac{\pi}{4}$

Q. $\sin 2\theta + 5\sin\theta + 5\cos\theta + 1 = 0 \in$ find general solⁿ.

Solⁿ $2\sin\theta\cos\theta + 5(\sin\theta + \cos\theta) + 1 = 0$ —— (1)

Now, Let $\sin\theta + \cos\theta = t$

$\Rightarrow (\sin\theta + \cos\theta)^2 = t^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta = t^2$

$\Rightarrow 2\sin\theta\cos\theta = t^2 - 1$

So, eqn. (1) can be written as,

$\Rightarrow t^2 - \cancel{1} + 5t + \cancel{1} = 0$

$\Rightarrow t(t+5) = 0$

$\boxed{t = 0}$ $\boxed{\cancel{t = -5}}$

$\Rightarrow \sin\theta + \cos\theta = 0$

$\Rightarrow \sin\theta = -\cos\theta \Rightarrow \boxed{\tan\theta = -1} = \tan\left(\frac{3\pi}{4}\right)$

$$\boxed{\theta = n\pi + \frac{3\pi}{4} = n\pi - \frac{\pi}{4}}$$

Q. If $\sin\theta\cdot\cos\phi = 1$, find the ordered pairs of (θ, ϕ) in $[0, 2\pi]$?

Solⁿ. Case-I) $\sin\theta = 1 \rightarrow \theta = \pi/2$

$\cos\phi = 1$ $\quad \phi = 0, 2\pi$

Case-II) $\sin\theta = -1 \rightarrow \theta = 3\pi/2$

$\cos\phi = -1$ $\quad \phi = \pi$

$\therefore$ Ordered pairs $= \left\{\left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 2\pi\right), \left(\frac{3\pi}{2}, \pi\right)\right\}$

Q. Find $\theta = ?$ for which $\cos\theta \cdot \cos 4\theta = -1$?
in $\theta \in [0, 2\pi]$.

Sol^n case-I) $\cos\theta = 1 \rightarrow \theta = 0, 2\pi$

$\cos 4\theta = -1 \rightarrow 4\theta = \pi \Rightarrow \theta = \pi/4$

$4\theta = 3\pi \Rightarrow \theta = \frac{3\pi}{4}$

$4\theta = 5\pi \Rightarrow \theta = 5\pi/4$

$4\theta = 7\pi \Rightarrow \theta = 7\pi/4$

$\therefore \theta \in \phi$ (No solution)

Case-II) $\cos\theta = -1 \longrightarrow \theta = \pi$

$\cos 4\theta = 1 \rightarrow 4\theta = 0 \Rightarrow \theta = 0$

$4\theta = 2\pi \Rightarrow \theta = \pi/2$

$4\theta = 4\pi \Rightarrow \theta = \pi$

$4\theta = 8\pi \Rightarrow \theta = 2\pi$

$4\theta = 6\pi \Rightarrow \theta = 3\pi/2$

$\therefore \boxed{\theta = \pi}$

Q. Find $\theta = ?$ for which $\cos\theta \cdot \cos 6\theta = -1$
in $\theta \in [-\pi, \pi]$?

Sol^n Case-I) $\cos\theta = -1 \Rightarrow \theta = \pi, -\pi$

$\cos 0 = \cos 6\theta = 1 \Rightarrow 6\theta = 2n\pi \pm 0 \Rightarrow \boxed{\theta = \frac{n\pi}{3}}$

So, $n = 0 \longrightarrow \theta = 0$

$n = 1 \longrightarrow \theta = \pi/3$

$n = -1 \longrightarrow \theta = -\pi/3$

$n = 2 \longrightarrow \theta = 2\pi/3$

$n = -2 \longrightarrow \theta = -2\pi/3$

$n = 3 \longrightarrow \theta = \pi$

$n = -3 \longrightarrow \theta = -\pi$

$\therefore \boxed{\theta \in \pi \ \& \ -\pi}$

Case-II) $\cos\theta = +1$ $\qquad$ $\cos 6\theta = -1 = \cos\pi$

$\theta = 0$ $\qquad$ $\theta = (2n \pm 1)\frac{\pi}{6}$

$\therefore \theta \in \phi$ (no solution)

Q. Find 'x' & 't' for which $12\sin x + 5\cos x = 2t^2 - 4t + 15$?

Sol^n

$12\sin x + 5\cos x$	$2t^2 - 4t + 15$
$\downarrow$	$\downarrow$
$[-13, 13]$	$2[t^2 - 2t + 1 - 1] + 15$
	$= 2(t-1)^2 + 13$
[Max. = 13]	[Min. = 13]

$\Rightarrow 12\sin x + 5\cos x = 13$	$2t^2 - 4t + 15 = 13$
$\Rightarrow \frac{12}{13}\cdot\sin x + \frac{5}{13}\cos x = 1$	$2t^2 - 4t + 2 = 0$
	$t^2 - 2t + 1 = 0$
	$(t-1)^2 = 0$
$\Rightarrow \cos x \cdot \cos\alpha + \sin\alpha \cdot \sin x = 1$	$\boxed{t = 1}$
$\Rightarrow \cos(x - \alpha) = \cos\theta = \cos 0$	
$\Rightarrow x - \alpha = 2n\pi$	
$\Rightarrow \boxed{x = 2n\pi + \alpha}$	
where, $\boxed{\cos\alpha = \frac{5}{13}}$	

Q. Find general solutions of $\tan\theta + \cot 2\theta = 0$

Sol^n $\tan\theta = -\cot 2\theta$

$\tan\theta = \tan\left(\frac{\pi}{2} + 2\theta\right)$

$\Rightarrow \theta = n\pi + \frac{\pi}{2} + 2\theta \rightarrow \boxed{\cancel{\theta = -n\pi \pm \frac{\pi}{2}}}$

$\therefore \theta \in \phi$ (no sol^n)