Q. $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$, then, find $x \in (-\pi, \pi)$, where, $x \neq 0, \pi/2, -\pi/2$ sum of all ther solutions ?

Sol$^n$ $\frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x} + 2\left(\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\right) = 0$

$$= \frac{\sqrt{3}\sin x + \cos x}{\sin x \cos x} + 2\frac{(\sin^2 x - \cos^2 x)}{\sin x \cdot \cos x} = 0$$

$$= \sqrt{3}\sin x + \cos x - 2(\cos^2 x - \sin^2 x) = 0$$

$$= 2\left(\frac{\sqrt{3}}{2} x + \frac{1}{2}\cos x\right) - 2\cos 2x = 0$$

$$= 2\left(\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right) - 2\cos 2x = 0$$

$$= \cancel{2}\left(\sin x \cdot \sin\frac{\pi}{3} + \cos x \cdot \cos\frac{\pi}{3}\right) = \cancel{2}\cos 2x$$

$$= \boxed{\cos\left(x - \frac{\pi}{3}\right) = \cos 2x}$$

$$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm 2x$$

$$\Rightarrow x - \frac{\pi}{3} = 2n\pi + 2x$$

$$\Rightarrow -x = 2n\pi + \frac{\pi}{3}$$

$$\Rightarrow x = -(2n\pi + \pi/3)$$

$$\therefore \boxed{x \in -\pi/3}$$

$$x - \frac{\pi}{3} = 2n\pi - 2x$$

$$\Rightarrow 3x = \frac{\pi}{3} + 2n\pi$$

$$\Rightarrow x = \frac{2n\pi}{3} + \frac{\pi}{9}$$

$$\Rightarrow x \in \left\{\frac{\pi}{9}, \frac{7\pi}{9}, \frac{-5\pi}{9}\right\}$$

$$\therefore \underline{\Sigma x = 0}$$

Q. Find no. of solutions for $\sin x + \sin 3x = 0$, in $x \in (0, 2\pi)$ ?

Sol$^n$. $2 \sin 2x \cdot \cos x = 0$

$\sin 2x = 0 = \sin 0$

$x = \frac{n\pi}{2} \rightarrow x \in \{\frac{\pi}{2}, \pi\}$

$\cos x = 0 = \cos \pi/2$

$\boxed{x \in \pi/2, 3\pi/2}$

$\therefore x \in \{\frac{\pi}{2}, \pi, \frac{3\pi}{2}\} \rightarrow 3$ solutions

Q. Find general solutions for $\tan\theta + \tan 2\theta + \tan 3\theta = 0$

Sol$^n$. $\tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \cdot \tan\theta}$ $\rightarrow \tan 3\theta - \tan 3\theta \cdot \tan 2\theta \cdot \tan\theta = \tan 2\theta + \tan\theta$

$\therefore \tan\theta + \tan 2\theta + \tan 3\theta \Rightarrow \tan 3\theta - \tan 3\theta \tan 2\theta \tan\theta + \tan 3\theta = 0$

$\Rightarrow 2\tan 3\theta - \tan 3\theta \cdot \tan 2\theta \cdot \tan\theta = 0$

$\Rightarrow \tan 3\theta \cdot (2 - \tan 2\theta \cdot \tan\theta) = 0$

$\tan 3\theta = 0 = \tan 0$

$\theta = \frac{n\pi}{3}$

$\tan\theta \cdot \tan 2\theta = 2$

$\Rightarrow \tan\theta \left(\frac{2\tan\theta}{1 - \tan^2\theta}\right) = 2$

$\Rightarrow 2\tan^2\theta = 1 \Rightarrow \boxed{\tan^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2}$

$\boxed{\theta = n\pi \pm \alpha}$

where, $\boxed{\tan\alpha = 1/\sqrt{2}}$

NOTE :-

If $\left.\begin{aligned}\sin^2\theta &= \sin^2\alpha \\ \cos^2\theta &= \cos^2\alpha \\ \tan^2\theta &= \tan^2\alpha\end{aligned}\right\}$ $\boxed{\theta = n\pi \pm \alpha}$ ; $\boxed{n \in Z}$

Q. find general solutⁿ for
$3 - 2\cos x - 4\sin x - \cos 2x + \sin 2x = 0$

Solⁿ $3 - 2\cos x - 4\sin x - (2\cos^2 x - 1) + 2\sin x \cos x = 0$

$\Rightarrow 4(1 - \sin x) - 2\cos x(1 - \sin x) - 2(1 - \sin^2 x) = 0$

$\Rightarrow [4 - 2\cos x - 2(1 + \sin x)](1 - \sin x) = 0$

$1 - \sin x = 0$

$\sin x = 1 = \sin \pi/2$

$\sin x = \sin \pi/2$

$\boxed{x = n\pi + (-1)^n \pi/2}$

$4 - 2\cos x - 2 - 2\sin x = 0$

$\cancel{2} = \cancel{2}\sin x + \cancel{2}\cos x$

$\sin x + \cos x = 1$

$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

$\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\boxed{x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}}$

Q. find no. of solutions for $\cos x + \cos 2x + \cos 4x =$
for $x \in [0, \pi]$ ?

Solⁿ $\cos x + 2\cos(3x) \cdot \cos x = 0$

$\Rightarrow \cos x(1 + 2\cos 3x) = 0$

$\Rightarrow$

$\cos x = 0 = \cos \pi/2$

$\boxed{x = \pi/2}$

$1 + 2\cos 3x = 0$

$\cos 3x = -1/2 = \cos 2\pi/3$

$3x = 2n\pi \pm 2\pi/3$

$\boxed{x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}}$

$\therefore$ for $x = \frac{2n\pi}{3} \pm \frac{2\pi}{9} \longrightarrow x = \left\{\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}\right\}$

$\therefore \boxed{x \rightarrow 4 \text{ solutions}}$

Q. $\sec 4x - \sec 2x = 2$, find solutions for eqn. in $-\pi \leq x \leq \pi$.

Soln $\Rightarrow \cos 2x - \cos 4x = 2\cos 2x \cdot \cos 4x$

$\Rightarrow \cancel{\cos 2x} - \cos 4x = \cos 6x + \cancel{\cos 2x}$

$\Rightarrow \cos 6x = -\cos 4x$

$\Rightarrow \cos 6x = \cos(\pi - 4x)$

$\Rightarrow 6x = 2n\pi \pm (\pi - 4x)$

$\boxed{x = (2n+1)\frac{\pi}{10}}$

$x \in \left\{\frac{\pi}{10}, \frac{3\pi}{10}, \frac{-\pi}{10}, \frac{5\pi}{10}, \frac{-3\pi}{10}, \frac{7\pi}{10}, \frac{-5\pi}{10}, \frac{9\pi}{10}, \frac{-9\pi}{10}, -\frac{7\pi}{10}\right\}$

$\boxed{x = (2n-1)\frac{\pi}{2}}$

$x \in \left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

$\therefore \boxed{x \rightarrow 10 \text{ solutions}}$

Q. If $\sin\theta = 1/2$ and $\cos\theta = \sqrt{3}/2$, then $\theta = ??$

Soln $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad \theta = \frac{\pi}{6}, \frac{\pi}{6}\pi$

$\therefore \boxed{\theta = \pi/6} \Longrightarrow \boxed{\theta = 2n\pi + \frac{\pi}{6}}$

Q. If $\sin\theta = -1/2$ and $\cos\theta = \sqrt{3}/2$, then $\theta = ??$

Sol$^n$ $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\qquad$ $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

$\therefore \theta = \frac{11\pi}{6} \implies \boxed{\theta = 2n\pi + \frac{11\pi}{6}}$

Q. Find general solutions for $\sin x + \cos x = \sin 2x - 1$

Sol$^n$. $\sin x + \cos x = 2\sin x \cos x - 1$

Now, let $\sin x + \cos x = t$

$1 + 2\sin x \cos x = t^2$

$\therefore t = 2(t^2 - 1) - 1 \implies 2t^2 - 2 - 1 - t = 0$

$\implies \boxed{2t^2 - t - 3 = 0}$

$\therefore \boxed{t = 2}$

$\cancel{\sin x + \cos x = 2}$

$\boxed{t = -1}$

$\sin x + \cos x = -1$

$\frac{1}{\sqrt{2}}\sin x + \frac{\cos x}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\implies \boxed{x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}}$

Q. General solution of the equation, $\tan^2\theta + \sec 2\theta = 1$

Sol$^n$. $\tan^2\theta + \frac{1}{\cos 2\theta} = 1 \implies \tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1$

$\implies \tan^4\theta - 3\tan^2\theta = 0$

$\tan^2\theta = 0$

$\tan^2\theta = \tan^2 0$

$\boxed{\theta = m\pi}$

$\tan^2\theta = 3$

$\tan^2\theta = \tan^2\frac{\pi}{3}$

$\boxed{\theta = n\pi \pm \frac{\pi}{3}}$

Q. The general solution of the system of equations
$$\sin^3 x + \sin^3\left(\frac{2\pi}{3} + x\right) + \sin^3\left(\frac{4\pi}{3} + x\right) + \frac{3}{4}\cos 2x = 0$$
and $\cos x \neq 0$ is ____________

Q. The set $\{x \in R :- \cos 2x + 2\cos^2 x = 2\}$ is equal to :-

Sol$^n$. $2\cos^2 x - 1 + 2\cos^2 x = 2 \Rightarrow \cos^2 x = \cos^2 \frac{\pi}{6}$

$\therefore \boxed{x = n\pi \pm \frac{\pi}{6}}$

Q. The no. of ordered pairs $(x, y)$ satisfying the equations $y = 2\sin x$ and $y = 5x^2 + 2x + 3$

Sol$^n$. $\underbrace{2\sin x}_{\text{max value} = 2} = 5x^2 + 2x + 3$

$= \underbrace{5\left(x + \frac{1}{5}\right)^2 + \frac{14}{5}}_{\text{min. value} = \frac{14}{5}}$

$\therefore n(x, y) = 0$

Q. If $x$ and $y$ are the solution of the eqn. $5 + 8\sin^2 x = 2y^2 - 8y + 21$ then the least possible value of $x^2 y^3$ is :-

Sol$^n$. $\underbrace{5 + 8\sin^2 x}_{\text{max value} = 13} = 2y^2 - 8y + 21$

$= \underbrace{2(y - 2)^2 + 13}_{\text{min. value} = 13 \; (y = 2)}$

$\therefore 5 + 8\sin^2 x = 13 \Rightarrow \sin^2 x = \pm 1 \Rightarrow \boxed{x = \frac{\pi}{2}}$

$\therefore x^2 y^3 = 2\pi^2$

Q. If $0 \le x \le 2\pi$, then the no. of real value of $x$ satisfying the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ is ________

Sol$^n$. $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

$\Rightarrow 81^{\sin^2 x} + 81^{(1 - \sin^2 x)} = 30$

$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30 \Longrightarrow$ Let $81^{\sin^2 x} = t$

$\Rightarrow t + \frac{81}{t} = 30 \Rightarrow \boxed{t = 27}$ & $\boxed{t = 3}$

| | |
|---|---|
| $\Rightarrow 81^{\sin^2 x} = 3$ | $\Rightarrow 81^{\sin^2 x} = 27$ |
| $\Rightarrow 81^{\sin^2 x} = (81)^{1/4}$ | $\Rightarrow 81^{\sin^2 x} = (81)^{3/4}$ |
| $\Rightarrow \sin^2 x = 1/4$ | $\Rightarrow \sin^2 x = 3/4$ |
| $\sin x = \pm 1/2$ | $\sin x = \pm \sqrt{3}/2$ |
| $x \in \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ | $x \in \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ |

Q. The sum of the roots of the equat$^n$ $|\sqrt{3}\cos x - \sin x| = 2$ in $[0, 4\pi]$ is $k\pi$, then the value of $6k$ is ________

Sol$^n$. $\sqrt{3}\cos x - \sin x = \pm 2$

$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \pm 1 \Rightarrow \cos\left(x + \frac{\pi}{6}\right) = \pm 1$$

$$\Rightarrow \cos\theta = \pm 1 \Rightarrow \boxed{\theta = n\pi}$$

$$\Rightarrow x + \frac{\pi}{6} = n\pi \Rightarrow \boxed{x = n\pi - \frac{\pi}{6}}$$

$$\therefore x = \pi - \frac{\pi}{6},\ 2\pi - \frac{\pi}{6},\ 3\pi - \frac{\pi}{6},\ 4\pi - \frac{\pi}{6}$$

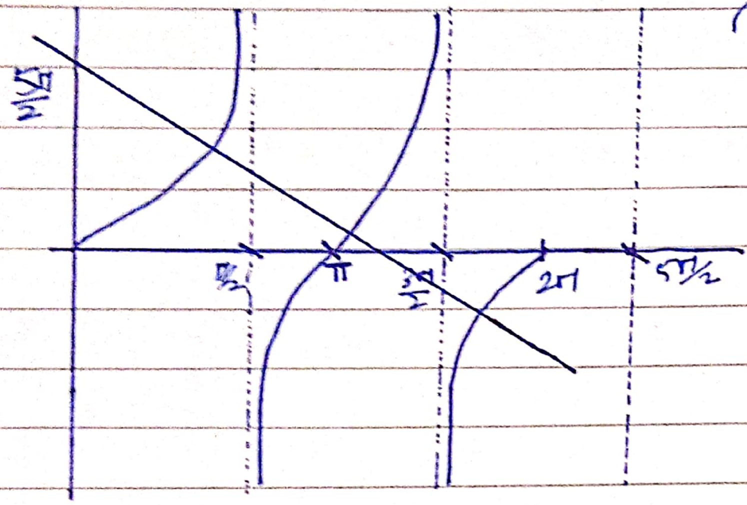
$$\Rightarrow \sum x = \frac{28\pi}{3} = k\pi$$

$$\therefore \boxed{6k = 56}$$

Q. Total no. of solutions of the equation $2x + 3\tan x = \frac{5\pi}{2}$ in $x \in [0, 2\pi]$ is ______

Sol$^n$. $3\tan x = \frac{5\pi}{2} - 2x$

→ $\boxed{\text{3 solutions}}$




Q. The no. of roots of the equation $\tan x + \sec x = 2\cos x$ in $[0, 4\pi]$ is ______

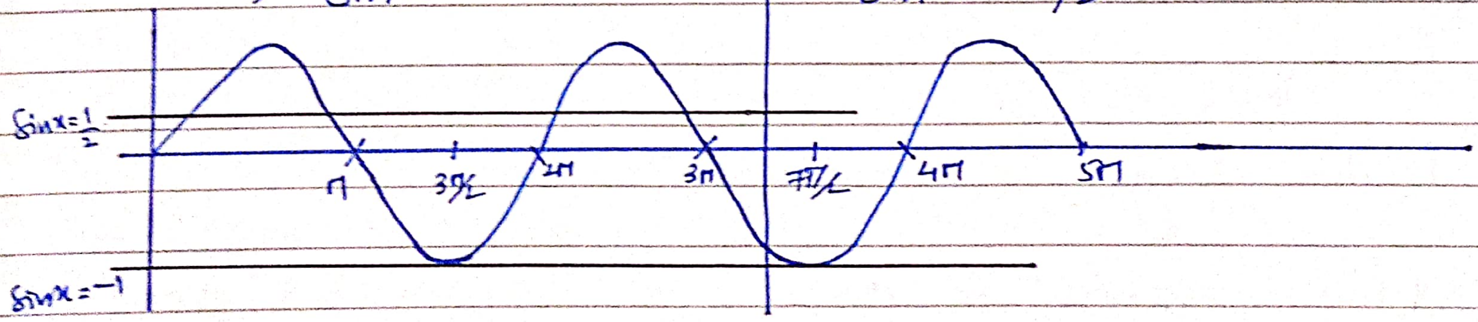
Sol$^n$ $\tan x + \sec x = 2\cos x$ ⇒ $\sin x + 1 = 2\cos^2 x$

$\sin x = \cos 2x$ or $2\sin^2 x + \sin x - 1 = 0$

$2\sin^2 x + 2\sin x - \sin x - 1 = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

⇒ $\sin x = -1$ $\quad$ $\sin x = 1/2$




∴ Total solutions = 6, but since, $\tan x$ & $\sec x$ are not defined at $x = \frac{3\pi}{2}$ & $\frac{7\pi}{2}$

∴ Exact total solutions = 4

Q. For $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, the no. of points of intersec-tion of curves $y = \cos x$ and $y = \sin 3x$ is ____

Sol^n. At the point of intersection,

$\sin 3x = \cos x = \cos\left(\frac{\pi}{2} - 3x\right)$

$\Rightarrow x = 2n\pi \pm \left(\frac{\pi}{2} - 3x\right)$

$x = 2n\pi - \frac{\pi}{2} + 3x$

$\Rightarrow -2x = 2n\pi - \pi/2$

$\Rightarrow \boxed{x = -n\pi + \pi/4}$

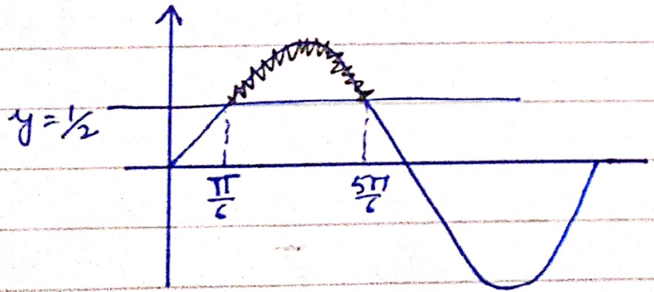
$x = 2n\pi + \frac{\pi}{2} - 3x$

$4x = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \boxed{x = \frac{n\pi}{2} + \frac{\pi}{8}}$

$\therefore x \in \left(\frac{\pi}{4}, \frac{\pi}{8}, -\frac{3\pi}{8}\right) \rightarrow$ 3 solutions

Q. Find general solutions for $\sin\theta \geq 1/2$ for $\theta \in [0, 2\pi]$ ?

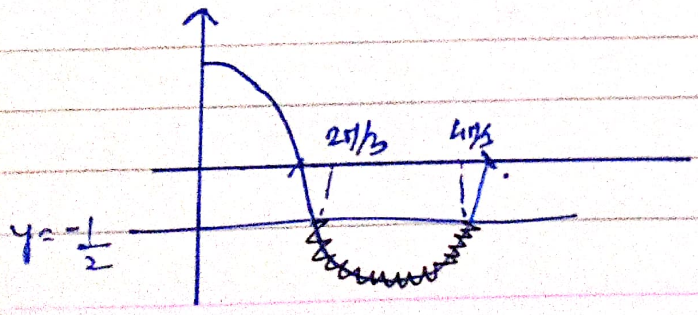



$\theta \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

general solution

$\theta \in \left[2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right]$

Q. Find general solutions for $\cos\theta < -1/2$ for $\theta \in [0, 2\pi]$ ?




$\theta \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

general solution,

$\theta \in \left[2n\pi + \frac{2\pi}{3}, 2n\pi + \frac{4\pi}{3}\right]$

Q. $\cos\theta \geq \frac{\sqrt{3}}{2}$ $\Leftarrow$ find general solution for $\theta \in [0, 2\pi]$ ?

$[0, \frac{\pi}{6}] \cup [\frac{11\pi}{6}, 2\pi]$