TRIGONOMETRY

① Basics Of Trigonometry :- (I)

• System Of measurements Of angles :-

(A) Sexagesimal System | Degree System

$1^\circ = 60'$ minute

$1' = 60''$ seconds

For Example :-

$8.5^\circ = 8^\circ + 0.5^\circ$

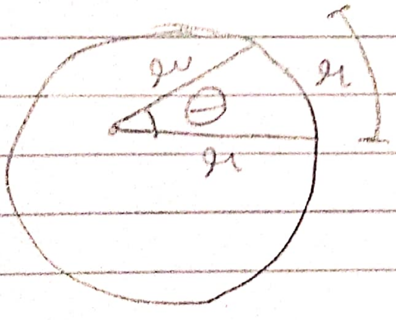
$= 8^\circ + 0.5 \times 60'$

$= \boxed{8^\circ 30'}$

(B) Circular | Radian Measure :-

↳ 1^c or $1 = 1$ Radian

↳ 1 Radian is a measure of angle which is subtended by an arc of circle whose length is equal to the radius of circular.



$\theta = 1^c$

Length Of Arc = Radius Of Circle

↳ $\theta = \frac{l}{r}$; where θ is in Radian

If $l = r \Rightarrow \theta = 1^c$

$\theta = \frac{l}{R} \Rightarrow \theta = \frac{\pi r}{r}$

$\boxed{\theta = \pi = 180^\circ}$

↳ $180^\circ = \pi^c$ Radian

↳ For ~~Degree~~ Radian → ~~Radian~~ Degree conversion

$\frac{180^\circ}{\pi} \times x^c$ Radian $\Rightarrow y^\circ$

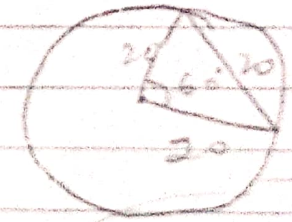
↳ For Degree → Radian convert

$\frac{\pi}{180^\circ} \times x^\circ \Rightarrow y^c$

↳ $0^\circ = 0$ Radian

Q. A circle with diameter = 40 cm and has a chord = 20 cm in length. Find the length of minor Arc.

Soln Here,



$\boxed{\theta = l/r}$

$60^\circ = \frac{\pi^c}{3}$

$\frac{\pi}{3} = \frac{l}{20}$

$l = \frac{20\pi}{3}$

- Basics of Trigonometry :- (I)
  - $\sin^2 x + \cos^2 x = 1$

    valid for $\boxed{x \in R.}$
  - $\sec^2 x - \tan^2 x = 1$
  - $\operatorname{cosec}^2 x - \cot^2 x = 1$

→ Substitutions in Trigonometric Equations :-

→ $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = (1 - \cos x)(1 + \cos x)$

$$\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$$

→ $\cos^2 x = (1 - \sin x)(1 + \sin x)$

$$\frac{\cos x}{1 - \sin x} = \frac{1 - \sin x}{\cos x}$$

→ $\sec^2 x - \tan^2 x = 1$

$(\sec x - \tan x)(\sec x + \tan x) = 1$

$$(\sec x - \tan x) = \frac{(1)}{\sec x + \tan x}$$

→ $$\operatorname{cosec} x - \cot x = \frac{1}{\operatorname{cosec} x + \cot x}$$

$\rightarrow$ $\tan^2 x \sin^2 x = \tan^2 x (1 - \cos^2\theta)$

$= \tan^2 x - \tan^2 x \cos^2 x$

$= \tan^2 x - \dfrac{\sin^2 x}{\cancel{\cos^2} x} \times \cancel{\cos^2} x$

$$\boxed{\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x}$$

$\rightarrow$ $\cot^2 x \cos^2 x = \cot^2 x (1 - \sin^2 x)$

$= \cot^2 x - \cot^2 x \sin^2 x$

$\cot^2 x \cos^2 x = \cot^2 x - \cos^2 x$

$\rightarrow$ $\sec^2\theta + \operatorname{cosec}^2\theta = \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta \cos^2\theta}$

$\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$

- $\sqrt{x^2} = |x|$

$\sqrt{\sin^2\theta} = |\sin\theta|$

$\sqrt{1 - \sin^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta|$

- Elimination Process :-

  - For example :-

$$a \sec\theta = 1 - b\tan\theta \quad \text{— (1)}$$
$$a^2 \sec^2\theta = 5 + b^2\tan^2\theta \quad \text{— (2)}$$

Eliminate θ.

Multiply eq. (1) by $a \sec = 1 - b\tan\theta$,

$\therefore$ $a^2 \sec^2\theta = 1 + b^2\tan^2\theta - 2b\tan\theta$ — (3)

Subtract (1) − (3) ; then,

$$0 = 4 + 2b\tan\theta$$

$$\therefore \boxed{\tan\theta = \frac{-2}{b}}$$

Now,

$\Rightarrow a^2 \sec^2\theta = 5 + b^2\tan^2\theta$

$\Rightarrow a^2 (1 + \tan^2\theta) = 5 + b^2\tan^2\theta$

$\Rightarrow a^2 \left(\frac{b^2+4}{b^2}\right) = 5 + \cancel{b^2}\left(\frac{4}{\cancel{b^2}}\right)$

$\Rightarrow \frac{a^2(b^2+4)}{b^2} = 9$

$\Rightarrow \boxed{a^2(b^2+4) = 9b^2}$

② Trigonometry in convex Polygon :-

- Convex Polygon :-

↳ Angles (Internal) $< 180^\circ$

↳ Sum of all internal angles of convex Polygon $= (n-2)\,180^\circ$

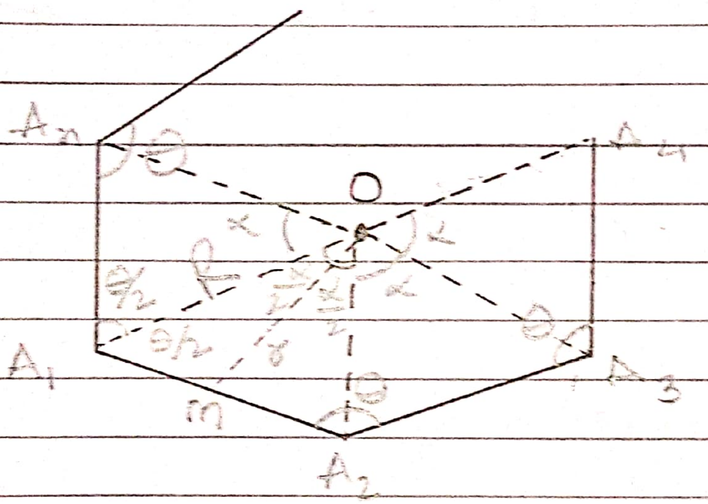
- Regular Polygon :-

↳ All sides equal.

↳ All internal angle equal

↳ Each internal angle $= \frac{(n-2)180}{n}$

↳



Here,

$\Rightarrow n\alpha = 2\pi = 360^\circ$

$\Rightarrow \alpha = \frac{2\pi}{n}$

$\Rightarrow \boxed{\frac{\alpha}{2} = \frac{\cancel{2}\pi}{n}}$

Now, in $\triangle OMA_1$; we get :-

$\Rightarrow \tan\frac{\alpha}{2} = \frac{A_1M}{OM} = \frac{a/2}{r}$

$\Rightarrow r = \frac{a/2}{\tan\alpha/2}$ $\Rightarrow \boxed{r = \frac{a}{2}\cot\frac{\alpha}{2}}$

↑ Distance of side of Polygon from center

$\Rightarrow \boxed{r = \frac{a}{2}\cot\frac{\pi}{n}}$ ←

↳ Now,

$$\Rightarrow \sin\frac{\alpha}{2} = \frac{A_1M}{OA_1} = \frac{a/2}{R}$$

$$\Rightarrow R = \frac{a/2}{\sin\alpha/2} = \frac{a}{2}\ \operatorname{cosec}\frac{\alpha}{2}$$

$$\therefore \boxed{R = \frac{a}{2}\operatorname{cosec}\frac{\pi}{n}}$$

↑ Distance Of center from Vertex

↳ Also,

$$\underset{(\Delta)}{\text{Area}} = \frac{1}{2}\times a\times r$$

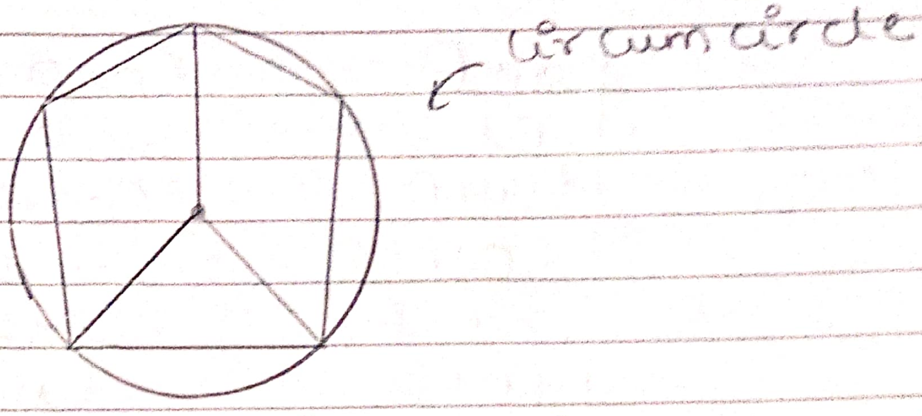
$$= \frac{1}{2}\times a\times\frac{a}{2}\cot\frac{\pi}{n}$$

$$= \frac{a^2}{4}\cot\frac{\pi}{n}$$

Then, Area Of Polygon $= n(\Delta)$

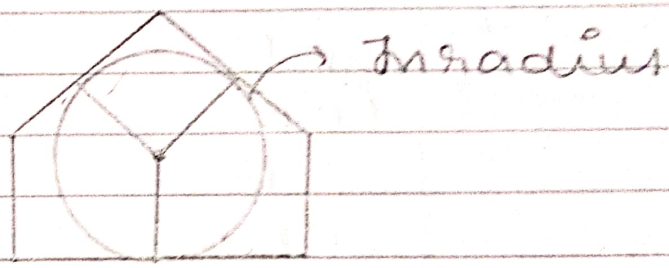
$$= n\left(\frac{a^2}{4}\cot\frac{\pi}{4}\right)$$

• CIRCUM RADIUS :-



Circum circle's Radius = $\frac{a}{2} \operatorname{cosec} \frac{\pi}{n}$

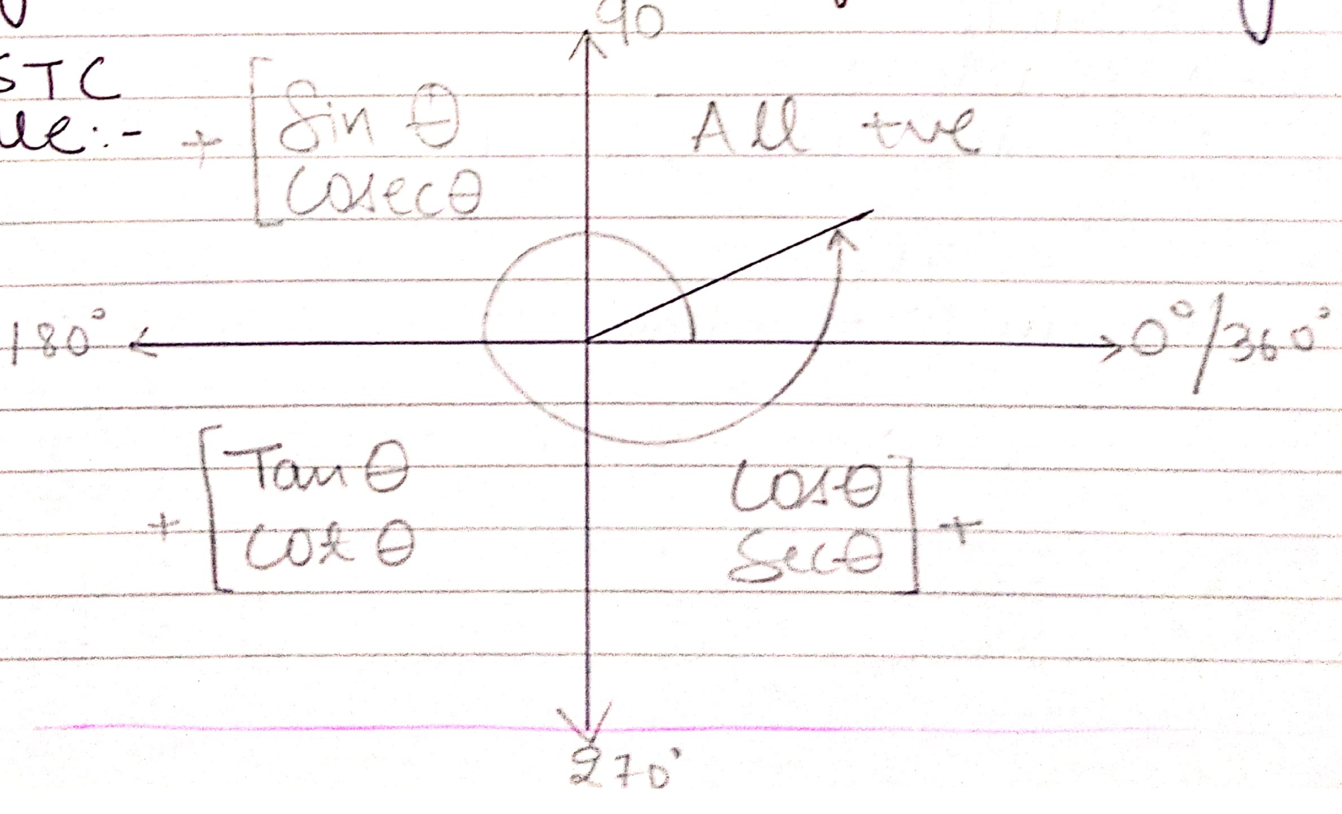
• Inradius :-



Radius Of Circle :- $\frac{a}{2} \cot \frac{\pi}{n}$

③ Trigonometric Ratios Of Allied Angles:

Ⓐ ASTC Rule :-



(B) Vertical Rule :-

For $(90^\circ \pm \theta)$ & $(270^\circ \pm \theta)$

$\sin\theta \longleftrightarrow \cos\theta$
$\cos\theta \longleftrightarrow \sin\theta$
$\tan\theta \longleftrightarrow \cot\theta$
$\text{cosec}\,\theta \longleftrightarrow \cancel{\sin\theta}\ \sec\theta$
$\sec\theta \longleftrightarrow \cancel{\cos\theta}\ \text{cosec}\,\theta$
$\cot\theta \longleftrightarrow \tan\theta$

(C) Horizontal Rule !-

For $(180^\circ \pm \theta)$ & $(360^\circ \pm \theta)$

$\sin\theta \longleftrightarrow \sin\theta$
$\cos\theta \longleftrightarrow \cos\theta$
$\tan\theta \longleftrightarrow \tan\theta$
$\text{cosec}\,\theta \longleftrightarrow \text{cosec}\,\theta$
$\sec\theta \longleftrightarrow \sec\theta$
$\cot\theta \longleftrightarrow \cot\theta$

Q. Find the value of the following T-Ratios.

i) $\sin 10^\circ$
$\hookrightarrow \sin 370^\circ = \sin(10^\circ + 360^\circ)$
$\Rightarrow \sin(370^\circ - 360^\circ) = \sin 10^\circ$

ii) $\sin 750^\circ$
$\hookrightarrow \sin(750 - 360^\circ) = \sin 390^\circ$
$\Rightarrow \sin(390^\circ - 360^\circ) \Rightarrow \sin 30^\circ = \frac{1}{2}$

iii) $\sin(4350^\circ)$
$\hookrightarrow \sin(4350^\circ - 360^\circ \times 12) = \sin 30^\circ = \frac{1}{2}$

iv). $\cos 570^\circ \sin 510^\circ - \sin 330^\circ \times \cos 390^\circ$

$\hookrightarrow \cos(570^\circ - 360^\circ)\sin(510 - 360^\circ)$
$- \sin(360^\circ - 30^\circ)\cos(360^\circ + 30^\circ)$

$\Rightarrow \cos 210^\circ \cdot \sin 150 - (-\sin 30^\circ)\cos 30^\circ$

$\Rightarrow \cos(180^\circ + 30^\circ)\sin(180^\circ - 30^\circ)$
$+ \sin 30^\circ \cos 30^\circ$

$\Rightarrow - \cancel{\cos 30^\circ \sin 30^\circ} + \cancel{\sin 30^\circ \cos 30^\circ}$

$\Rightarrow 0$

v). $\sin 420^\circ \cos 390^\circ + \cos(-300^\circ)\sin(-330^\circ)$

$\hookrightarrow \sin 60^\circ \cos 30^\circ - \cos(360^\circ - 60^\circ)$
$\sin(360^\circ - 30^\circ)$

$\Rightarrow \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{4} \Rightarrow \boxed{1}$

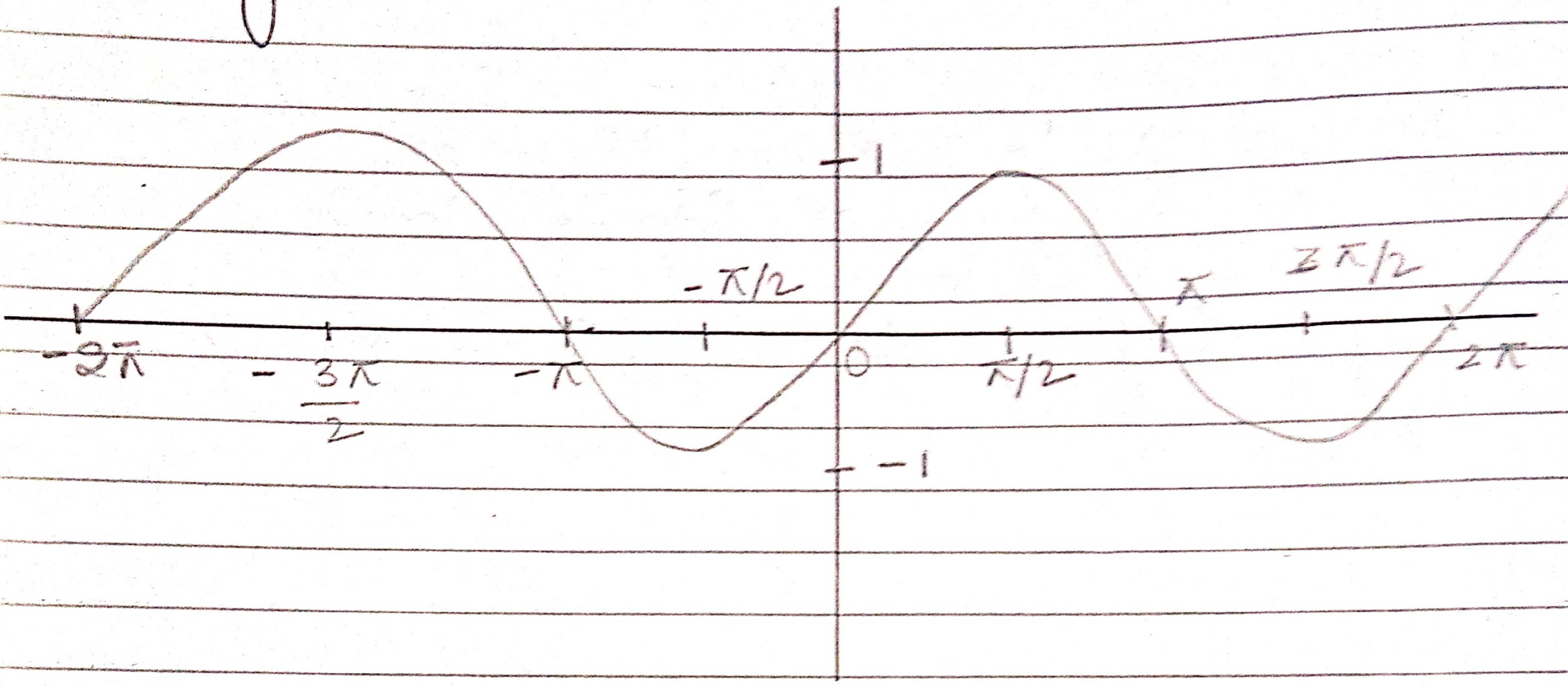
vi). $\sec(270 - A)\sec(90^\circ - A) - \tan(270 - A)$
$\tan(90^\circ + A)$

$\hookrightarrow -\operatorname{cosec} A \operatorname{cosec} A - \cot A(-\cot A)$

$\Rightarrow \cot^2 A - \operatorname{cosec}^2 A = -1$

④ Graph of Trigonometry Ratio :-

Ⓐ $y = \sin x.$

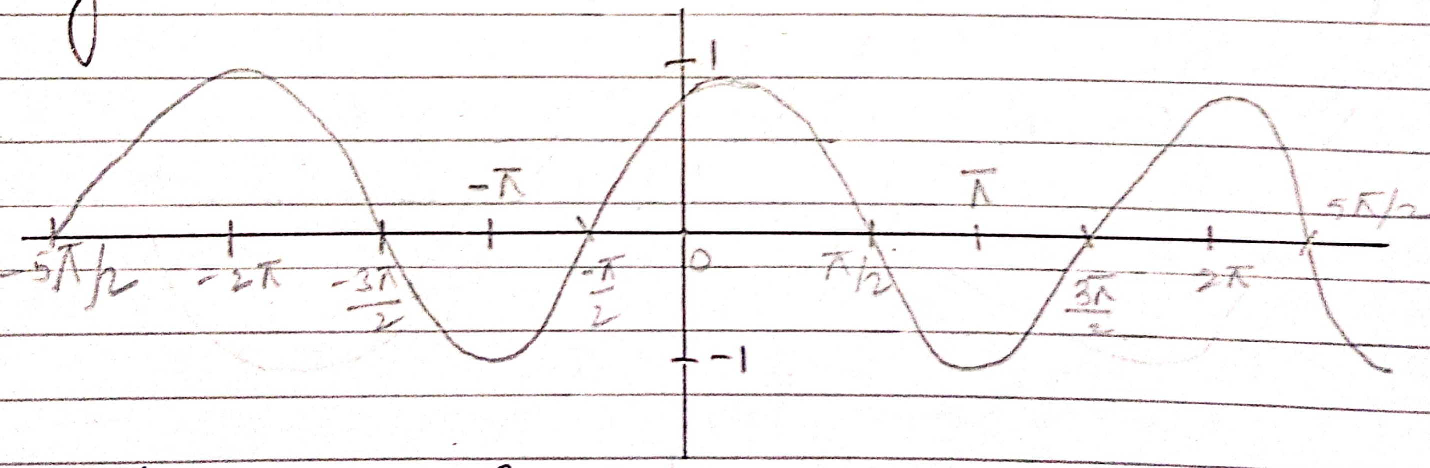


↳ $x = D_f \in R$

↳ $R_f = y \in [-1, 1]$

↳ $-1 \leq \sin x \leq 1$

Ⓑ $y = \cos x.$

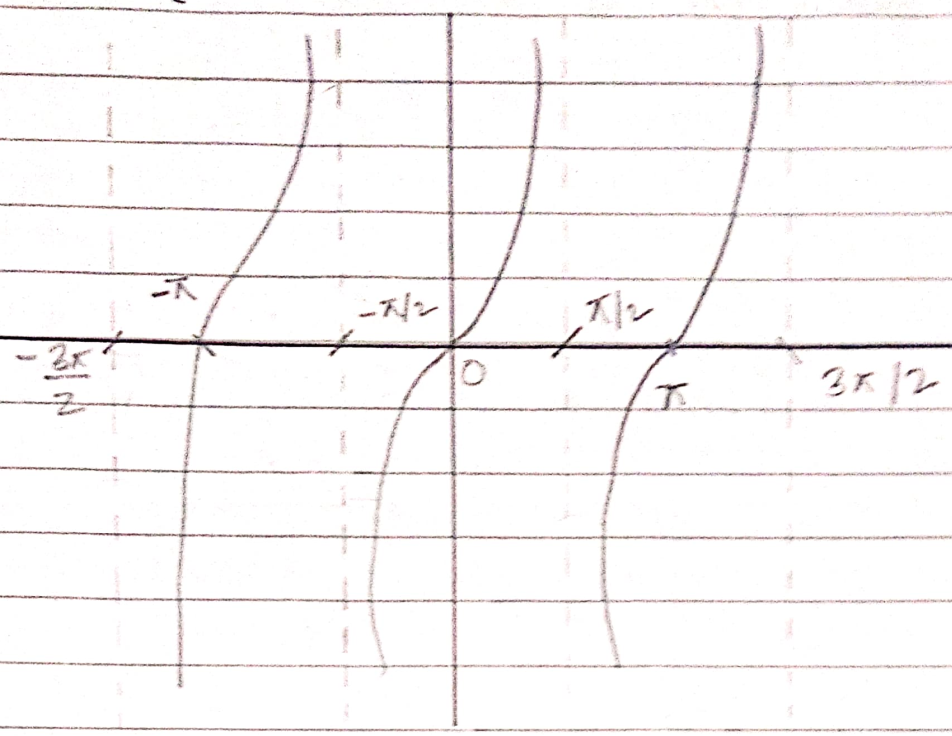


↳ $x = D_f \in R$

↳ $y = R_f \in [-1, 1]$

↳ $-1 \leq \cos x \leq 1$

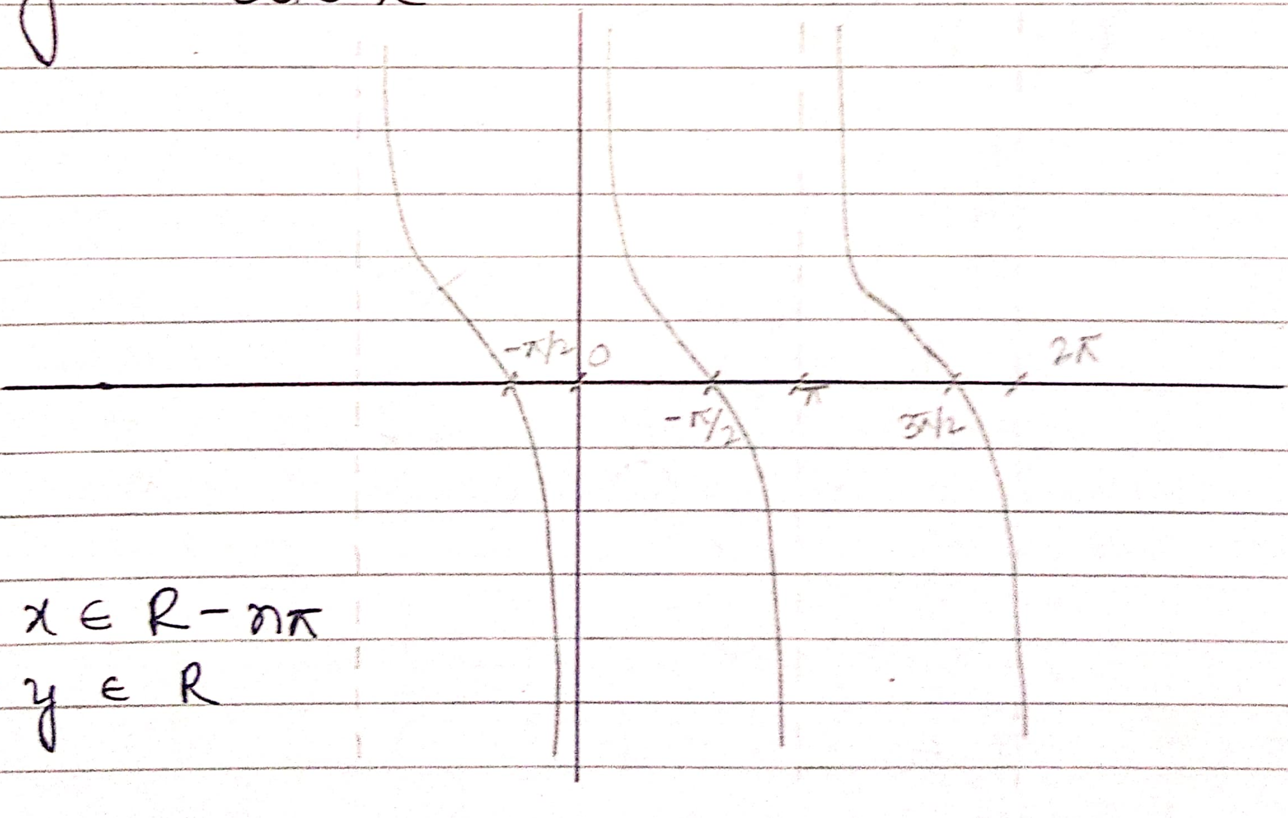
Ⓒ $y = \tan x$



↳ $D_f = x \in R - (2n+1)\frac{\pi}{2}$

↳ $R_f = y \in R$

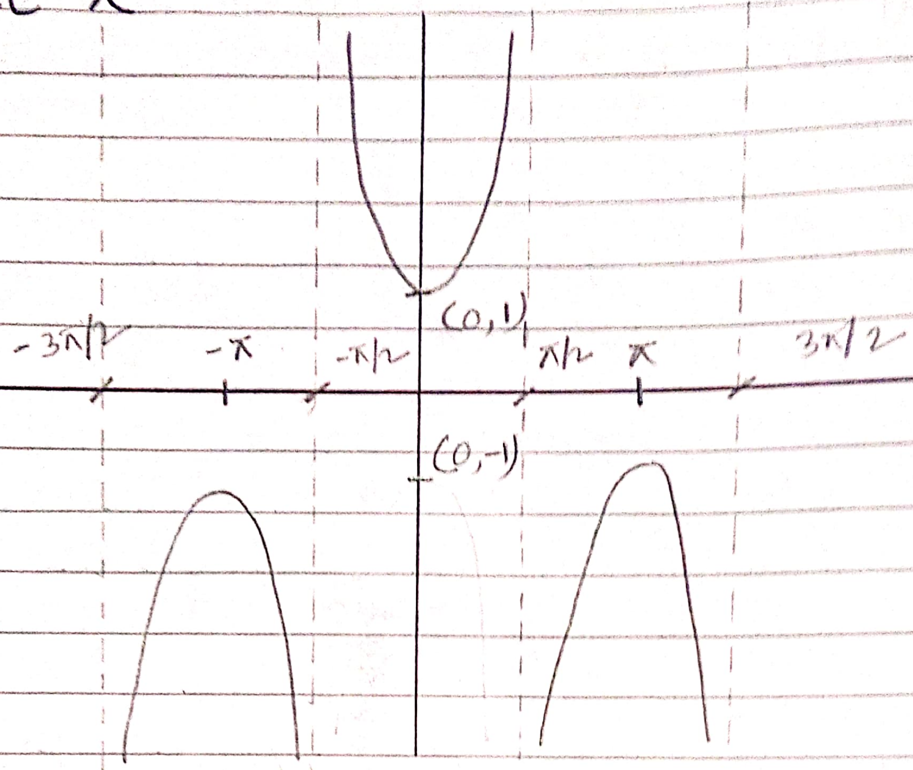
Ⓓ $y = \cot x$



$D_f = x \in R - n\pi$

$R_f = y \in R$

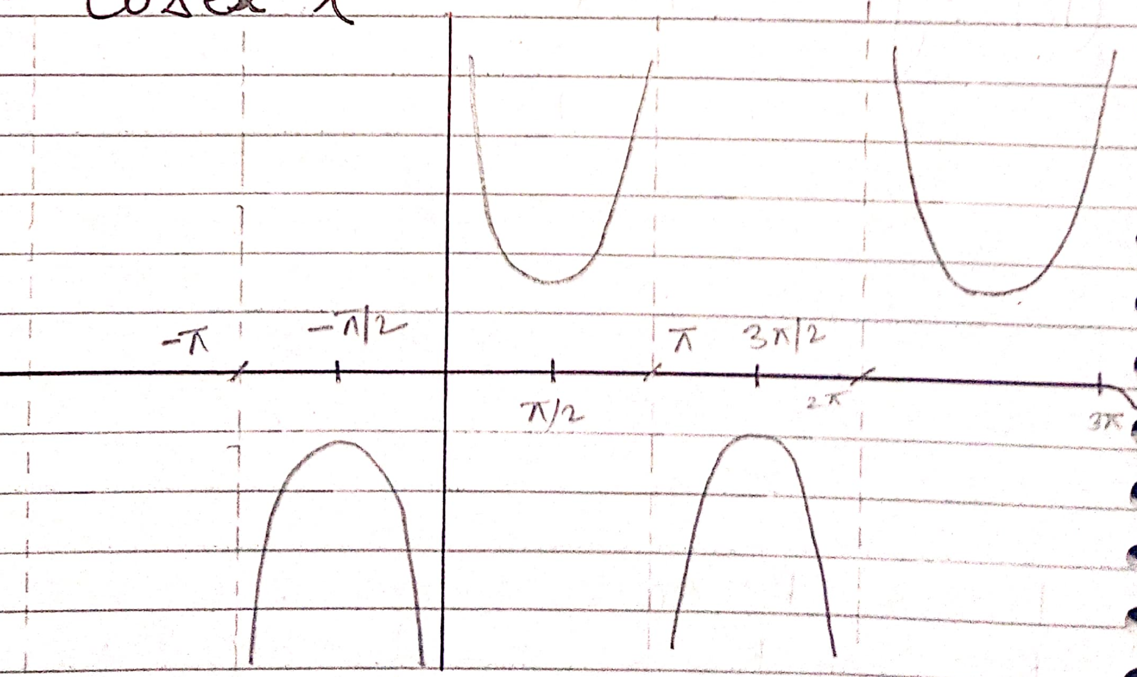
(E) $y = \sec x$



↳ $D_f = x \in R - (2n+1)\pi/2$

↳ $R_f = y \in (-\infty, -1] \cup [1, \infty)$

(F) $y = \operatorname{cosec} x$



↳ $D_f = x \in R - n\pi$

$R_f = y \in (-\infty, -1] \cup [1, \infty)$