1

Q. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan A}{\tan B}$; then $A - B = ?$

Solⁿ. $\dfrac{\frac{1}{\cos 8\theta} - 1}{\frac{1}{\cos 4\theta} - 1} = \dfrac{\frac{1-\cos 8\theta}{\cos 8\theta}}{\frac{1-\cos 4\theta}{\cos 4\theta}} = \dfrac{1-\cos 8\theta}{\cos 8\theta} \cdot \dfrac{\cos 4\theta}{1-\cos 4\theta}$

$\Rightarrow \dfrac{1-\cos 8\theta}{1-\cos 4\theta} \cdot \dfrac{\cos 4\theta}{\cos 8\theta} \Rightarrow \dfrac{2\sin^2 4\theta}{2\sin^2 2\theta} \cdot \dfrac{\cos 4\theta}{\cos 8\theta}$

$\Rightarrow \dfrac{2\sin 4\theta \cdot \sin 4\theta \cdot \cos 4\theta}{2\sin^2 2\theta \cdot \cos 8\theta} = \dfrac{\sin 4\theta}{2\sin^2 2\theta} \cdot \dfrac{\sin 8\theta}{\cos 8\theta}$

$\Rightarrow \dfrac{2\cancel{\sin 2\theta} \cdot \cos 2\theta}{2\sin^{\cancel{2}} 2\theta} \cdot \tan 8\theta = \dfrac{\tan 8\theta}{\tan 2\theta} = \dfrac{\tan A}{\tan B}$

$\therefore \boxed{A - B = 6\theta}$

9. Triple angle formulas :-

(A) $\sin 3A = 3\sin A - 4\sin^3 A$

(B) $\cos 3A = 4\cos^3 A - 3\cos A$

(C) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

10. Max. & Min. Value of Trig. functions :-

Q. Find the Range of the following functions :-

(A) $y = f(x) = 3\sin x \longrightarrow y \in [-3, 3]$

Solⁿ $\because -1 \le \sin x \le 1 \Rightarrow -3 \le 3\sin x \le 3$

(B) $y = f(x) = \sin x \cdot \cos x$

Sol^n: $y = \frac{\sin 2x}{2} \Rightarrow \because -1 \le \sin 2x \le 1$

$\Rightarrow -\frac{1}{2} \le \frac{\sin 2x}{2} \le \frac{1}{2}$

$\therefore y \in [-\frac{1}{2}, \frac{1}{2}]$

(C) $y = f(x) = 3\sin^2 x$

Sol^n: $y = 3\sin^2 x \longrightarrow \because -1 \le \sin x \le 1$

$\Rightarrow 0 \le \sin^2 x \le 1$

$\Rightarrow 0 \le 3\sin^2 x \le 3$

$\therefore \boxed{y \in [0, 3]}$

(D) $y = 16\sin^2 x \cdot \cos^2 x$

Sol^n: $y = 4(2\sin x \cdot \cos x)^2$

$y = \boxed{4\sin^2 2x}$

$\because -1 \le \sin 2x \le 1 \Rightarrow 0 \le \sin^2 2x \le 1$

$\Rightarrow 0 \le 4\sin^2 2x \le 4$

$\therefore \boxed{y \in [0, 4]}$

(E)* $y = f(x) = \sin^2 x + 2\sin x + 5$

Sol^n*: $y = (\sin^2 x + 2\sin x + 1) + 4$

$y = (\sin x + 1)^2 + 4$

$\because -1 \le \sin x \le 1$

$\Rightarrow 0 \le \sin x + 1 \le 2$

$\Rightarrow 0 \le (\sin x + 1)^2 \le 4$

$\Rightarrow 4 \le (\sin x + 1)^2 + 4 \le 8$

$\therefore \boxed{y \in [4, 8]}$

(F) $y = f(x) = \cos^2 x - 4\cos x + 16$

Sol^n $y = (\cos^2 x - 4\cos x + 4) + 12$

$y = (\cos x - 2)^2 + 12$

$\because -1 \le \cos x \le 1 \Rightarrow -3 \le \cos x - 2 \le -1$

$\Rightarrow 1 \le (\cos x - 2)^2 \le 9$

$\Rightarrow 1 + 12 \le (\cos x - 2)^2 + 12 \le 21$

$\therefore \boxed{y \in [13, 21]}$

→ For $f(x) = a\sin x \pm b\cos x$, Max. & min. Value can be determined by :-

$\left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$

eg :- $y = 3\sin\theta - 5\cos\theta \longrightarrow [-\sqrt{34}, \sqrt{34}]$

Q. Find out Range of the following functions :-

(A) $y = (6\sin x - 8\cos x) - 15$

Sol^n $[-10, 10] - 15 \Rightarrow [-25, -5]$

(B) $y = 2\cos^2 x - 4\sqrt{2}\sin x\cos x + 7$

Sol^n $y = 1 + \cos 2x - 2\sqrt{2}\sin 2x + 7$

$y = 8 + (\cos 2x - 2\sqrt{2}\sin 2x)$

$8 + [-3, 3] \Rightarrow [5, 11]$

(C) $y = 2\sin(x + 30^\circ) - 4\cos x$

Sol^n $y = 2[\sin x \cdot \cos 30^\circ + \sin 30^\circ \cos x] - 4\cos x$

$y = 2\cos 30^\circ \sin x + 2\sin 30^\circ \cos x - 4\cos x$

$y = \sqrt{3}\sin x - 4\cos x + \cos x$

$y = \sqrt{3}\sin x - 3\cos x \Rightarrow [-\sqrt{12}, \sqrt{12}]$

Q. Find out the largest -ve value of x for which $y = \sqrt{3}\cos x - \sin x$ is max.

Sol.
$$y = 2\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right] = 2[\cos x \cdot \cos 30^\circ - \sin x \sin 30^\circ]$$

$$y = 2[\cos(x+30^\circ)] \longrightarrow \boxed{x = -30^\circ} \Rightarrow \boxed{2y}$$

$$\therefore \boxed{x = -30^\circ}$$

NOTE :- Some Important Values of Angle :-

- $\sin 18^\circ$ ($\cos 72^\circ$) $\Rightarrow$ $\theta = 18^\circ \longrightarrow 5\theta = 2\theta + 3\theta = 18^\circ \times 5 = 90^\circ$

$$\Rightarrow \boxed{2\theta = 90^\circ - 3\theta}$$

$$\therefore \sin 2\theta = \sin(90 - 3\theta) = \cos 3\theta$$
$$\Rightarrow 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$$
$$\Rightarrow 2\sin\theta = 4\cos^2\theta - 3$$
$$\Rightarrow 2\sin\theta = 4(1 - \sin^2\theta) - 3$$
$$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0 \quad \Rightarrow \sin(\theta / 18) = \frac{-1 \pm \sqrt{5}}{4}$$

- $\cos 18^\circ$ ($\sin 72^\circ$) $= \frac{\sqrt{10 + 2\sqrt{5}}}{4}$; $\cos 36^\circ$ ($\sin 54^\circ$) $= \frac{\sqrt{5} + 1}{4}$

- $\sin 36^\circ$ ($\cos 54^\circ$) $= \frac{\sqrt{10 - 2\sqrt{5}}}{4}$; $\tan 22.5^\circ = \sqrt{2} - 1$

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(11) Double Angle Product Series in Cosines :-

for $\cos\theta \cdot \cos 2\theta \cdot \cos 4\theta \cdot \cos 8\theta \cdots\cdots \cos 2^n\theta$

$$= \frac{\sin(2^n\theta)}{2^n \sin\theta}$$

Q. $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ \cdot \cos 160^\circ \cdot \cos 320^\circ \cdot \cos 640^\circ = ??$

Solⁿ. $\dfrac{\sin 2^6 (20^\circ)}{2^6 \cdot \sin 20^\circ} = \dfrac{\sin (1280^\circ)}{64 \sin 20^\circ} = \dfrac{\sin (1280^\circ - 1080^\circ)}{64 \cdot \sin 20^\circ}$

$= \dfrac{\sin 200^\circ}{64 \cdot \sin 20^\circ} = \dfrac{\sin (180^\circ + 20^\circ)}{64 \cdot \sin 20^\circ} = \dfrac{-\sin 20^\circ}{64 \cdot \sin 20^\circ} = \dfrac{-1}{64}$

Q. $\cos \frac{\pi}{33} \cdot \cos \frac{2\pi}{33} \cdot \cos \frac{4\pi}{33} \cdot \cos \frac{8\pi}{33} \cdot \cos \frac{16\pi}{33} \cdot \cos \frac{32\pi}{33} = ?$

Solⁿ. $\dfrac{\sin 2^6 (\pi/33)}{2^6 \sin (\pi/33)} = \dfrac{\sin \frac{64\pi}{33}}{64 \cdot \sin \frac{\pi}{33}} = \dfrac{\sin \left(\frac{64\pi}{33} - 2\pi\right)}{64 \cdot \sin \pi/33}$

$\Rightarrow \dfrac{\sin \left(\frac{64\pi - 66\pi}{33}\right)}{64 \cdot \sin \pi/33} = \dfrac{\sin \left(-\frac{2\pi}{33}\right)}{64 \sin \pi/33} = \dfrac{-\sin 2\pi/33}{64 \sin \pi/33}$

$= \dfrac{-2 \cancel{\sin \frac{\pi}{33}} \cdot \cos \frac{\pi}{33}}{64 \cancel{\sin \pi/33}} = \boxed{-\dfrac{\cos \frac{\pi}{33}}{32}}$

Q. $\cos \frac{\pi}{9} \cdot \cos \frac{2\pi}{9} \cdot \cos \frac{\pi}{3} \cdot \cos \frac{4\pi}{9} = ??$

Solⁿ. $\dfrac{1}{2} \left[\cos \frac{\pi}{9} \cdot \cos \frac{2\pi}{9} \cdot \cos 4\pi/9\right] = \dfrac{1}{2} \left[\dfrac{\sin 2^3 (\pi/9)}{2^3 \cdot \sin \pi/9}\right]$

$= \dfrac{1}{2} \left[\dfrac{\cancel{\sin 8\pi/9}}{2^3 \cancel{\sin \pi/9}}\right] = \dfrac{1}{16}$

Q. $\cos 12^\circ \cdot \cos 24^\circ \cdot \cos 36^\circ \cdot \cos 48^\circ \cdot \cos 72^\circ \cdot \cos 84^\circ = ??$

Solⁿ. $(\cos 12^\circ \cos 24^\circ \cos 48) \cdot (\cos 36^\circ \cos 72^\circ) \cos 84$

$= \dfrac{\sin 2^3 (12)}{2^3 \sin 12^\circ} \cdot \dfrac{\sin 2^2 (36)}{2^2 \sin 36^\circ} \cdot \cos 84$

$$= \frac{\sin 96^\circ}{32 \sin 12^\circ} \cdot \frac{\sin 144^\circ}{\sin 36^\circ} \cdot \cos 84^\circ$$

$$= \frac{\sin(90^\circ + 6^\circ)}{32} \cdot \frac{\sin(180^\circ - 36^\circ)}{\sin 12^\circ \ \sin 36^\circ} \sin 84^\circ$$

$$= \frac{\cos 6^\circ \ \cancel{\sin 36} \ \cancel{\sin 6^\circ}}{32 (2 \cancel{\sin 6^\circ} \cos 6^\circ) \cancel{\sin 36}} = \frac{1}{64}$$

(12) (A) Sine Series :- (Addition) – (Angles in A.P)

for $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \sin(\alpha+3\beta) \dots$

$$= \frac{\sin(n\beta/2) \cdot \sin(\alpha + (n-1)\beta/2)}{\sin \beta/2}$$

(B) Cosine Series :-

for $\cos\alpha + \cos(\alpha+2\beta) + \cos(\alpha+\beta) + \dots$ n terms

$$= \frac{\sin(n\beta/2) \cdot \cos(\alpha + (n-1)\beta/2)}{\sin(\beta/2)}$$

Q. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = ??$

Solⁿ $\frac{\sin(3)(\frac{\pi}{7}) \cdot \cos\left(\frac{2\pi}{7} + (3-1)\frac{\pi}{7}\right)}{\sin \pi/7}$

$$= \frac{\sin 3\pi/7 \cdot \cos\frac{4\pi}{7}}{\sin \pi/7} = \frac{\sin\frac{3\pi}{7} \cdot \cos\left(\pi - \frac{3\pi}{7}\right)}{\sin \pi/7}$$

$$= \frac{-2\sin\frac{3\pi}{7} \cdot \cos\frac{3\pi}{7}}{2\sin \pi/7} = \frac{-\sin\left(\frac{6\pi}{7}\right)}{2\sin(\pi/7)} = \frac{-1}{2}$$

Q. If A, B, C are in arithmetic progression & $B = \frac{\pi}{4}$, then, $\tan A \cdot \tan B \cdot \tan C = ??$

Solⁿ. Since, $B = \pi/4$ $\Rightarrow$ Let $A = \frac{\pi}{4} - \theta$ & $C = \frac{\pi}{4} + \theta$

$\therefore \tan\left(\frac{\pi}{4} - \theta\right) \cdot \tan\frac{\pi}{4} \cdot \tan\left(\frac{\pi}{4} + \theta\right) =$

$= \cancel{\frac{1 - \tan\theta}{1 + \tan\theta}} \cdot 1 \cdot \cancel{\frac{1 + \tan\theta}{1 - \tan\theta}} = \underline{\underline{1}}$

Q. If the value of $(1 + \tan 1°)(1 + \tan 2°)(1 + \tan 3°) \ldots$

* .... $(1 + \tan 44°) \cdot (1 + \tan 45°) = 2^{\lambda}$, then the sum of digits of no. λ is $\underline{5.00}$.

Solⁿ. If $A + B = 45°$

$\Rightarrow \tan(A + B) = \tan 45° = 1$

$\Rightarrow \tan A + \tan B + \tan A \cdot \tan B = 1$

$\Rightarrow 1 + \tan A + \tan B + \tan A \tan B = 2$

$\Rightarrow (1 + \tan A) + \tan B(1 + \tan A) = 2$

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$

So, $[(1 + \tan 1°)(1 + \tan 44°)] \cdot [(1 + \tan 2°)(1 + \tan 43°)]$

$\ldots [(1 + \tan 22°)(1 + \tan 23°)][(1 + \tan 45)] = 2^{22} \cdot 2^{1}$

$= 2^{23}$

$\therefore \boxed{\lambda = 23} \rightarrow \boxed{\text{sum of digits} = 5}$

Q. $(1 + \cos\pi/8)(1 + \cos 3\pi/8)(1 + \cos 5\pi/8)(1 + \cos 7\pi/8) = ??$

Solⁿ. $\left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right) = \sin^2\frac{\pi}{8} \cdot \sin^2\frac{3\pi}{8}$

$= \frac{1}{4}\left(2 \sin\frac{\pi}{8} \cdot \cos\frac{\pi}{8}\right)^2 = \frac{1}{4}\left[\sin^2\left(2 \cdot \frac{\pi}{8}\right)\right]$ $\left[\sin\frac{3\pi}{8} = \cos\frac{\pi}{8}\right]$

$= \underline{\underline{\frac{1}{8}}}$