②

Q. If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $\alpha - \beta = 2\theta$, then $\frac{\cos 3\theta}{\cos\theta}$ is equal to,

Solⁿ $\frac{\cos 3\theta}{\cos\theta} = \frac{4\cos^3\theta - 3\cos\theta}{\cos\theta} = 4\cos^2\theta - 3$

$= 2(2\cos^2\theta) - 3$

$= 2(1+\cos 2\theta) - 3$

$= 2 + 2\cos 2\theta - 3$

$= 2\cos 2\theta - 1$

$\Rightarrow 2\cos(\alpha-\beta) - 1 = a^2 + b^2 - 2 - 1 = \boxed{a^2+b^2-3}$ Ans

$a^2+b^2 = \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cdot\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$

$= 1 + 1 + 2(\cos(\alpha-\beta))$.

$= 2 + 2\cos(\alpha-\beta) \Rightarrow 2\cos(\alpha-\beta) = \boxed{a^2+b^2-2}$

Q. $\frac{2((\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin 89^\circ))}{2(\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots \cos 44^\circ) + 1} = ??$

Solⁿ Here, $2[(\sin 1^\circ + \sin 89^\circ) + (\sin 2^\circ + \sin 88^\circ) + \cdots (\cdot n)]$

$\Rightarrow 2[2\sin 45^\circ \cos 44^\circ + 2\sin 45^\circ\cdot\cos 43^\circ + 2\sin 45^\circ \cos 42^\circ)$

$\Rightarrow \frac{2[2\sin 45^\circ(\cos 44^\circ + \cos 43^\circ + \cos 42^\circ + \cdots + \cos 1^\circ) + \sin 45^\circ]}{2[\cos 1^\circ + \cos 2^\circ + \cdots + \cos 44^\circ] + 1}$

$\Rightarrow 2\sin 45^\circ \frac{[\cancel{2(x)+1}]}{\cancel{2(x)+1}} = 2\left(\frac{1}{\sqrt{2}}\right) = \sqrt{2}$

Q. The value of $2\sin^2\theta + 4\cos(\theta+\alpha)\cdot\sin\alpha\sin\theta + \cos 2(\alpha+\theta)$ is :-

(A) $\cos\theta + \cos\alpha$
(B) Independent of θ
(C) Independent of α
(D) Indep. of both θ & α.

Solⁿ $2\sin^2\theta + 2\cos(\theta+\alpha)[\cos(\alpha-\theta) - \cos(\alpha+\theta)] + (2\cos^2(\alpha+\theta) - 1)$

$$\Rightarrow 2\sin^2\theta + 2\cos(\theta+\alpha)\cdot \overset{\cos(\theta-\alpha)}{\cancel{\cos(\alpha-\theta)}} - \cancel{2\cos^2(\alpha+\theta)} + \cancel{2\cos^2(\alpha+\theta)} - 1$$

$$\Rightarrow 2\sin^2\theta + 2(\cos^2\theta - \sin^2\alpha) - 1$$

$$= 2(1) - 2\sin^2\alpha - 1 = 1 - 2\sin^2\alpha = \boxed{\cos 2\alpha}$$

Q* If $p = \sin^2 x + \cos^4 x$, then,

(A) $\frac{3}{4} \le p \le 1$ (B) $\frac{3}{10} \le p \le \frac{1}{4}$

(C) $\frac{1}{4} \le p \le \frac{1}{2}$ (D) None of these.

Solⁿ. $p = \sin^2 x + \cos^4 x$

$= 1 - \cos^2 x + (\cos^2 x)^2$

$= (\cos^2 x)^2 - \cos^2 x + 1 \quad = \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\because \cos^2 x \in [0, 1] \Rightarrow \cos^2 x - \frac{1}{2} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\Rightarrow \left(\cos^2 x - \frac{1}{2}\right)^2 \in \left[0, \frac{1}{4}\right] \Rightarrow \boxed{\left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \in \left[\frac{3}{4}, 1\right]}$

(13) Conditional Identities :-

→ If $A + B + C = \pi$ or A, B, C are angles of $\triangle$

(A) $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$:-

→ $\sin 2A + \sin 2B + \sin 2C$

$= 2\sin(A+B)\cdot\cos(A-B) + 2\sin C\cdot\cos C$

$= 2\sin(\pi - C)\cdot\cos(A-B) + 2\sin C\cdot\cos C$ $[\because A+B = \pi - C]$

$= 2\sin C\cdot\cos(A-B) + 2\sin C\cdot\cos C$

$= 2\sin C[\cos(A-B) + \cos C]$

$= 2\sin C[\cos(A-B) + \cos(\pi-(A+B))]$ $[\because C = \pi - (A+B)]$

$= 2\sin C[\cos(A-B) - \cos(A+B)]$

$= 2\sin C\cdot 2\sin A\cdot\sin B \quad = \underline{4\sin A\sin B\sin C}$

(B) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A \cos B \cos C$

$\Rightarrow 2\cos(A+B)\cdot\cos(A-B) + \cos 2C$

$\Rightarrow 2\cos(\pi - C)\cdot\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C \cdot \cos(A-B) + 2\cos^2 C - 1$

$= -1 - 2\cos C\,[\cos(A-B) - \cos C]$

$= -1 - 2\cos C\,[\cos(A-B) - \cos(\pi-(A+B))]$

$= -1 - 2\cos C\,[\cos(A-B) + \cos(A+B)]$

$= -1 - 4\cos A\cdot\cos B\cos C.$

(C) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}$

(D) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

NOTE :-

For $\tan(A_1 + A_2 + A_3 + A_4 + \cdots A_n) = \dfrac{S_1 - S_3 + S_5 - S_7 \cdots}{1 - S_2 + S_4 - S_6 + \cdots}$

where,

$S_1 = \tan A_1 + \tan A_2 + \tan A_3 + \cdots$

$S_2 = \tan A_1 \cdot \tan A_2 + \tan A_1 \cdot \tan A_3 + \cdots$

$S_3 = \tan A_1 \cdot \tan A_2 \cdot \tan A_3 + \cdots$

Eg. :- $\tan(A+B) = \dfrac{S_1}{1-S_2} = \dfrac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$

Eg. :- $\tan(A+B+C) = \dfrac{S_1 - S_3}{1 - S_2} = \dfrac{\tan A + \tan B + \tan C - \tan A\cdot\tan B\cdot\tan C}{1 - \tan A \tan B - \tan B\cdot\tan C - \tan C\cdot\tan A}$

(E) $\tan(A+B+C) = \tan\pi = 0$

$\dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\cdot\tan C - \tan C\cdot\tan A} = 0$

$$= \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$ for a $\triangle$

(F) $\cot A \cdot \cot B + \cot B \cdot \cot C + \cot C \cdot \cot A = 1$

(G) $\tan\frac{A}{2} \cdot \tan\frac{B}{2} + \tan\frac{B}{2} \cdot \tan\frac{C}{2} + \tan\frac{C}{2} \cdot \tan\frac{A}{2} = 1$

(H) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2} \cdot \cot\frac{B}{2} \cdot \cot\frac{C}{2}$

Q. If $A + B + C = \pi$, then, $\sin^2 A + \sin^2 B + \sin^2 C - 2\cos A \cos B \cos C$??=

Sol. $\frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} + \frac{1 - \cos 2C}{2} - 2\cos A \cdot \cos B \cdot \cos C$

$$= \frac{3}{2} - \frac{\cos 2A + \cos 2B + \cos 2C + 4\cos A \cos B \cos C}{2}$$

$$= \frac{3}{2} - \frac{(-1 - 4\cancel{\cos A \cos B} \cdot \cos C + 4\cancel{\cos A \cos B}\cos C)}{2} = \underline{\underline{2}}$$

Q. $\tan^6 20^\circ - 33\tan^4 20^\circ + 27\tan^2 20^\circ + 4 = ??$

Sol. $\because \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$$\Rightarrow \sqrt{3} = \tan 3(20^\circ) = \frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ}$$

$$\Rightarrow \sqrt{3} - 3\sqrt{3}\tan^2 20^\circ = 3\tan 20^\circ - \tan^3 20^\circ$$

Let $\boxed{\tan 20^\circ = t}$

$$\Rightarrow [\sqrt{3}(1 - 3t^2)] = [3(t) - t^3]$$

Now, squaring both sides, we have,

$$3(1 - 3t^2)^2 = (3t - t^3)^2$$

$$= \boxed{t^6 - 33t^4 + 27t^2 = 3}$$

$\therefore \quad 3 + 4 = \underline{7}$

Q. Let the Max. and Minimum Value of the exp. $2\cos^2\theta + \cos\theta + 1 = M$ and m resp., then, the value of $\left[\frac{M}{m}\right]$ is

Soln. $2\cos^2\theta + \cos\theta + 1 = 2\left[\left(\cos\theta + \frac{1}{4}\right)^2 - \frac{1}{16}\right] + 1$

$\Rightarrow 2\left(\cos\theta + \frac{1}{4}\right)^2 + \frac{7}{8}$

Also, $-1 \leq \cos\theta \leq 1$

$\Rightarrow -\frac{3}{4} \leq \cos\theta + \frac{1}{4} \leq \frac{5}{4} \Rightarrow 0 \leq \left(\cos\theta + \frac{1}{4}\right)^2 \leq \frac{25}{16}$

$\Rightarrow 0 \leq 2\left(\cos\theta + \frac{1}{4}\right)^2 \leq \frac{25}{8}$

$\Rightarrow \boxed{\frac{7}{8} \leq 2\left(\cos\theta + \frac{1}{4}\right)^2 + \frac{7}{8} \leq 4}$

$\therefore \boxed{M = 4}$ & $\boxed{m = \frac{7}{8}} \Rightarrow \left[\frac{M}{m}\right] = 4$ Ans

Q. If $\pi < \theta < \frac{3\pi}{2}$ and $\cos\theta = \frac{-3}{5}$, then, $\tan\frac{\theta}{4} = ?$

Soln. $\because \cos\theta = -\frac{3}{5} \Rightarrow 2\cos^2\frac{\theta}{2} - 1 = -\frac{3}{5} \Rightarrow \boxed{\cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}}$

$\Rightarrow \cos\frac{\theta}{2} = \frac{-1}{\sqrt{5}} = \frac{1 - \tan^2\theta/4}{1 + \tan^2\theta/4} \Rightarrow \tan^2\frac{\theta}{4} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

$\Rightarrow \tan^2\theta/4 = \frac{(\sqrt{5}+1)^2}{4} \Rightarrow \boxed{\tan\frac{\theta}{4} = \frac{\sqrt{5}+1}{2}}$ Ans.

Q. Let $f(n) = (\sin 1^\circ) \times \sin(2) \times \cdots\cdots \sin n \quad \forall\, n \in N$, then the no. of elements in the set $A = \{f(1), f(2), \cdots\cdots f(6)\}$ that are positive are :-

Sol^n. $\because 1^c = 57^\circ$, so,

$\sin 1 = +ve$ | $\sin 4 = -ve$
$\sin 2 = +ve$ | $\sin 5 = -ve$
$\sin 3 = +ve$ | $\sin 6 = -ve$

$\therefore f(1) = \sin 1 \quad (+)$
$f(2) = \sin 1 \cdot \sin 2 \cdot \quad (+)$
$f(3) = \sin 1 \; \sin 2 \; \sin 3 \quad (+)$
$f(4) = \sin 1 \; \sin 2 \; \sin 3 \; \sin 4 \quad (-)$
$f(5) = \sin 1 \; \sin 2 \; \sin 3 \; \sin 4 \; \sin 5 \quad (+)$
$f(6) = \sin 1 \; \sin 2 \; \sin 3 \; \sin 4 \; \sin 5 \; \sin 6 \quad (-)$

$\therefore$ +ve elements are — 4

Q. The diff. b/w the greatest and the least possible value of the expression $3 - \cos x + \sin^2 x$ is

Sol^n. $= 3 - \cos x + \sin^2 x$
$= 3 - \cos x + 1 - \cos^2 x = 4 - \cos x - \cos^2 x$
$= -\left[\left(\cos x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 4$

So, $-1 \leq \cos x \leq 1$

$\frac{17}{4} \geq \frac{17}{4} - \left(\cos x + \frac{1}{2}\right)^2 \geq \frac{17}{4} - \frac{9}{4} = 2$

$\therefore \frac{17}{4} - 2 = \underline{\frac{9}{4}}$

Q. If $E = \cos^2 71^\circ + \cos^2 49^\circ + \cos 71^\circ \cos 49^\circ$, then, $10E = ?$

Sol^n. $E = \frac{1}{2}\left[2\cos^2 71^\circ + 2\cos^2 49 + 2\cos 71 \cdot \cos 49^\circ\right]$

$\Rightarrow E = \frac{1}{2}\left[1 + \cos 142^\circ + 1 + \cos 98^\circ + \cos 120^\circ + \cos 22^\circ\right]$

$\Rightarrow E = \frac{1}{2}\left[\frac{3}{2} + \cos 142^\circ + \cos 98^\circ + \cos 22^\circ\right]$

$\Rightarrow E = \frac{1}{2}\left[3/2 + 2\cos 120^\circ - \cos 22^\circ + \cos 22^\circ\right]$

$\Rightarrow \boxed{E = \frac{3}{4}} \Rightarrow \boxed{10E = 7.5}$

Q. The value of the expression $1 + \operatorname{cosec}\frac{\pi}{4} + \operatorname{cosec}\frac{\pi}{8} + \operatorname{cosec}\frac{\pi}{16} = ??$

(A) $\cot \frac{\pi}{8}$ (B) $\cot \pi/16$

(C) $\cot \pi/32$ (D) $\operatorname{cosec}^2 \frac{\pi}{16}$

Sol^n. $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1 + \cos\theta}{\sin\theta}$

$= \frac{2\cos^2 \theta/2}{2\sin\theta/2 \cdot \cos\theta/2} = \cot\theta/2$

$\therefore \operatorname{cosec}\theta = \cot\theta/2 - \cot\theta$

So, $\operatorname{cosec}\frac{\pi}{4} = \cot\frac{\pi}{8} - \cot\frac{\pi}{4}$

$+ \operatorname{cosec}\frac{\pi}{8} = \cot\frac{\pi}{16} - \cot\frac{\pi}{8}$ $= \cot\frac{\pi}{32} - \cancel{\cot\frac{\pi}{4}} + \cancel{1}$

$+ \operatorname{cosec}\frac{\pi}{16} = \cot\frac{\pi}{32} - \cot\frac{\pi}{16}$

$\Rightarrow 1 + \operatorname{cosec}\frac{\pi}{4} + \operatorname{cosec}\frac{\pi}{8} + \operatorname{cosec}\frac{\pi}{16} = \cot\frac{\pi}{32}$

Q. If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = \dfrac{3}{4}$, then the value of

$\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = ??$

Soln Let $x = \dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$; $y = \dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$

$\therefore \dfrac{x}{y} = \dfrac{\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}}{\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}} = \dfrac{2\sin\alpha \times (1+\sin\alpha)}{(1+\cos\alpha+\sin\alpha)(1-\cos\alpha+\sin\alpha)}$

$\Rightarrow$

$\dfrac{x}{y} = \dfrac{2\sin\alpha(1+\sin\alpha)}{[(1+\sin\alpha)+\cos\alpha][(1+\sin\alpha)-\cos\alpha]}$

$\dfrac{x}{y} = \dfrac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)^2-\cos^2\alpha} = \dfrac{2\sin\alpha(1+\sin\alpha)}{1+\sin^2\alpha+2\sin\alpha-\cos^2\alpha}$

$\dfrac{x}{y} = \dfrac{2\sin\alpha(1+\cancel{\sin\alpha})}{2\cancel{\sin^2\alpha}+2\sin\alpha} = 1 \Rightarrow \boxed{x = y = \frac{3}{4}}$

Q. If $\cos 5\theta = 5\cos\theta - 20\cos^3\theta + a\cos^5\theta + b$, then, the value of $a+b$ is equal to :-

Soln $\cos 5\theta = \cos(3\theta + 2\theta)$

$= \cos 3\theta \cdot \cos 2\theta - \sin 3\theta \cdot \sin 2\theta$

$= (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - (3\sin\theta - 4\sin^3\theta)(2\sin\theta\cos\theta)$

$= (8\cos^5\theta - 4\cos^3\theta - 6\cos^3\theta + 3\cos\theta)$
$-(3-(4\sin^2\theta))(2)\sin^2\theta\cos\theta$

$$= \left(8\cos^5\theta - 10\cos^3\theta + 3\cos\theta\right) - 2\left(3 - 4\left(1-\cos^2\theta\right)\right)\frac{\left(1-\cos^2\theta\right)}{\cos\theta}$$

$$= \left(8\cos^5\theta - 10\cos^3\theta + 3\cos\theta\right) + \left(-6 + 8\left(1-\cos^2\theta\right)\right)\frac{\left(1-\cos^2\theta\right)}{(\cos\theta)}$$

$$= \left(8\cos^5\theta - 10\cos^3\theta + 3\cos\theta\right) + \left[2 - 8\cos^2\theta\right]\left[\cos\theta - \cos^3\theta\right]$$

$$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta + 2\cos\theta - 8\cos^3\theta - 2\cos^3\theta + 8\cos^5\theta$$

$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

$\therefore$ $\boxed{a = 16}$ and $\boxed{b = 0}$ $\Rightarrow$ $\boxed{a + b = 16}$

Q. Let $f(x) = \max(\tan x, \cot x)$. Then, the no. of roots of the equation $f(x) = \frac{1}{2}$ in $(0, 2\pi)$ is $\underline{0.00}$