SKA

SPIRAL NOTEBOOK

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Class : XI

Subject : Mathematics

Institute :

CH- BINOMIAL THEOREM

K-1

① Expansion of Binomial Expression:-

$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} \cdot y^2 + \ldots$
$\ldots {}^nC_n x^0 y^n$

$T_{r+1} = {}^nC_r x^{n-r} \cdot y^r$ → No. of Terms :- $n+1$

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* $(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 \ldots + {}^nC_n x^0 y^n$
* $(x-y)^n = {}^nC_0 x^n y^0 - {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 - {}^nC_3 x^{n-3} y^3 + \ldots$

So, $(x+y)^n + (x-y)^n = 2[{}^nC_0 x^n y^0 + {}^nC_2 x^{n-2} y^2 + \ldots]$

$(x+y)^n - (x-y)^n = 2[{}^nC_1 x^{n-1} y^1 + {}^nC_3 x^{n-3} y^3 + \ldots]$

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* $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \ldots {}^nC_n x^n$
* $(1-x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 - {}^nC_3 x^3 + \ldots$

So, $(1+x)^n + (1-x)^n = 2[{}^nC_0 + {}^nC_2 x^2 + {}^nC_4 x^4 + \ldots]$

$(1+x)^n - (1-x)^n = 2[{}^nC_1 + {}^nC_3 x^3 + {}^nC_5 x^5 + \ldots]$

Q. Find T_{11} for $\left(3x - \frac{1}{x\sqrt{3}}\right)^{20}$

Sol^n ${}^{20}C_{10} (3x)^{20-10} \left(\frac{-1}{\sqrt{3}x}\right)^{10} = T_{11}$

$= {}^{20}C_{10} 3^{10} \cancel{x^{10}} \frac{1}{3^5 \cancel{x^{10}}} = \boxed{{}^{20}C_{10} 3^5}$

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Q. Find Term independent of x for $\left(2x - \frac{1}{x}\right)^{10}$?

Solⁿ. $T_{r+1} = {}^{10}C_r (2x)^{10-r} \left(-\frac{1}{x}\right)^r = {}^{10}C_r \, 2^{10-r} \cdot x^{10-r} \frac{(-1)^r}{x^r}$

$= {}^{10}C_r \, 2^{10-r} (-1)^r \, x^{10-2r}$ $\left\{ \begin{array}{l} 10 - 2r = 0 \\ \boxed{r = 5} \end{array} \right.$

$\therefore \boxed{T_6 = {}^{10}C_5 \, 2^5 (-1)^5 x^0}$

Q. Find out the max power of x in
$\left[x + (x^3-1)^{1/2}\right]^6 + \left[x - (x^3-1)^{1/2}\right]^6$

Solⁿ Let $X = x$; $Y = (x^3-1)^{1/2}$

$\therefore (X+Y)^6 + (X-Y)^6 \longrightarrow 7 \quad \to 8$

$\Rightarrow 2\Big[{}^6C_6 \, x^6 + {}^6C_2 \, x^4 \left((x^3-1)^{1/2}\right)^2 + {}^6C_4 \, x^2 \left[(x^3-1)^{1/2}\right]^{4} + {}^6C_6 \, x^{0} \left[(x^3-1)^{1/2}\right]^{6}\Big]$

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NOTE :- ${}^nC_r = {}^nC_{n-r}$

$\rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$ $\rightarrow$ eg.:- $\frac{{}^{12}C_7}{{}^{12}C_6} = \frac{6}{7}$

$\rightarrow$ eg.:- $\frac{{}^{15}C_5}{{}^{15}C_6} = \frac{6}{10}$

eg.:- $\frac{{}^{62}C_{41}}{{}^{62}C_{41}} = \frac{{}^{62}C_{41}}{{}^{62}C_{42}} \cdot \frac{{}^{62}C_{42}}{{}^{62}C_{41}} = \frac{20}{41} \cdot \frac{21}{42}$

Q. $1 \cdot \frac{{}^{30}C_1}{{}^{30}C_0} + 2 \cdot \frac{{}^{30}C_2}{{}^{30}C_1} + 3 \cdot \frac{{}^{30}C_3}{{}^{30}C_2} + 4 \frac{{}^{30}C_4}{{}^{30}C_3} + \text{---}$

$\text{---} + 30 \cdot \frac{{}^{30}C_{30}}{{}^{30}C_{29}}$

Solⁿ. $1 \cdot \left(\frac{30}{1}\right) + \cancel{2}\left(\frac{30-1}{\cancel{2}}\right) + \cancel{3}\left(\frac{30-2}{\cancel{3}}\right) + \cdots \cancel{30}\left(\frac{30-29}{\cancel{30}}\right)$

$= 30 + 29 + 28 + 27 + \ldots\ldots + 1 = \underline{465}$

NOTE :- $\boxed{{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r}$

↳ sum of consecutive Binomial Coeff.

Eg. :— ${}^{41}C_{29} + {}^{41}C_{28} + {}^{42}C_{30} = {}^{42}C_{29} + {}^{42}C_{30} = {}^{43}C_{30}$

Q. Find the term in which x^{-3} is present $\Rightarrow \left(x^2 + \frac{3a}{x}\right)^{15}$

Solⁿ. $T_{r+1} = {}^{15}C_r \, (x^2)^{15-r} \left(\frac{3a}{x}\right)^r$

$= {}^{15}C_r \cdot x^{30-2r} \cdot \frac{(3a)^r}{x^r}$

$30 - 3r = -3$

$\boxed{r = 11}$

$\boxed{T_{12} = {}^{15}C_{11} \, (3a)^{11} \, x^{-3}}$

☆ Q. Find out coeff. of x^3 in $(1 + x + x^2)^n$

Solⁿ. ${}^{n}C_0 \underbrace{(1+x)^n}_{x^3} + {}^{n}C_1 \underbrace{(1+x)^{n-1}}_{x^1} \underbrace{(x^2)^1}_{x^2} + \cancel{{}^{n}C_2 (1+x)^{n-2} (x^2)^2} \cancel{+ \cdots}$

$= {}^{n}C_0 \cdot \left[{}^{n}C_3\right] + {}^{n}C_1 \left[{}^{n-1}C_1\right] = \boxed{{}^{n}C_3 + n(n-1)}$

Q. Find 'a' such that term independent of 'x' in $\left(\sqrt{x} + \frac{a}{x^2}\right)^{10}$ is 405.

Solⁿ. $T_{r+1} = {}^{10}C_r \, (\sqrt{x})^{10-r} \left(\frac{a}{x^2}\right)^r = {}^{10}C_r \, x^{\frac{10-r}{2}} \cdot \frac{a^2}{x^{2r}}$

②

So, term independent of x; $\frac{10-r}{2} - 2r = 0 \Rightarrow \boxed{r=2}$

$\therefore T_3 = {}^{10}C_2\, a^2 = 405 \longrightarrow \boxed{a = \pm 3}$

Q. Find out term independent of x in,

$(1+x+2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.

Solⁿ. $1\cdot\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 + x\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 + 2x^3\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

So, $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9 \Rightarrow T_{r+1} = {}^9C_r\left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r} x^{18-2r}\left(-\frac{1}{3}\right)^r\left(\frac{1}{x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$

(i) $18-3r = 0 \Rightarrow \boxed{r=6} \Rightarrow {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6$ — ①

(ii) $18-3r = -1 \Rightarrow \cancel{r = \frac{19}{3}}$

(iii) $18-3r = -3 \Rightarrow \boxed{r=7} \Rightarrow 2\left({}^9C_7\left(\frac{3}{2}\right)^2\left(-\frac{1}{3}\right)^7\right)$ — ②

Q. Find out coeff. of x^{12} in $(1+x^6)^2(1+x)^{24}$

Solⁿ. $(1+2x^6+x^{12})(1+x)^{24}$

$= 1\cdot(1+x)^{24} + x^{12}(1+x)^{24} + 2x^6(1+x)^{24}$

$= {}^{24}C_{12} + {}^{24}C_0 + 2\cdot{}^{24}C_6$

Q. Coeff. of x^3 in $(1+2x)^6(1-x)^7 = ?$

Solⁿ.
${}^6C_3(2x)^3\cdot{}^7C_0(-x)^0 \longleftarrow x^3 \quad x^0$
${}^6C_2(2x)^2\cdot{}^7C_1(-x)^1 \longleftarrow x^2 \quad x^1$
${}^6C_1(2x)^1\cdot{}^7C_2(-x)^2 \longleftarrow x^1 \quad x^2$
${}^6C_0(2x)^0\cdot{}^7C_3(-x)^3 \longleftarrow x^0 \quad x^3$

$= -143$

Q. Coeff. of x^5 in $(1+x)^3(1-x)^6 = ??$

Solⁿ
$$\left.\begin{array}{lll} x^3 & x^2 & \rightarrow {}^3C_3 \times {}^6C_2 (-1)^2 \\ x^2 & x^3 & {}^3C_2 \times {}^6C_3 (-1)^3 \\ x^1 & x^4 & {}^3C_1 \times {}^6C_4 (-1)^4 \\ x^0 & x^5 & {}^3C_0 \times {}^6C_5 (-1)^5 \end{array}\right\} \underline{\underline{=6}}$$

Q. Coeff. of x^8 in expression $(1+x+x^2+x^3)^4 = ??$

Solⁿ $(1+x)^4(1+x^2)^4$
$$\left.\begin{array}{lll} x^4 & x^4 & = {}^4C_4 \times {}^4C_2 \\ \cancel{x^3} & \cancel{x^5} & \\ x^2 & x^6 & = {}^4C_2 \times {}^4C_3 \\ \cancel{x^1} & \cancel{x^7} & \\ x^0 & x^8 & = {}^4C_0 \; {}^4C_4 \end{array}\right\} \underline{\underline{31}}$$

Q. If 2^{nd}, 3^{rd}, 4^{th} term in expansion of $(x+a)^n$ are 240, 720, and 1080, resp., then find a, x, and n.

Solⁿ
$T_2 = {}^nC_1 \cdot x^{n-1} \cdot a^1 = 240$ —— (1)
$T_3 = {}^nC_2 \; x^{n-2} \; a^2 = 720$ —— (2)
$T_4 = {}^nC_3 \; x^{n-3} \; a^3 = 1080$ —— (3)

$\therefore$ $\boxed{x=2}$, $\boxed{a=3}$, $\boxed{n=5}$

(2) Divisibility :-

Q. Prove that $x^{2n+1} + y^{2n+1}$ is divisible by $x+y$.

Solⁿ. $x^{2n+1} = (x+y-y)^{2n+1}$
$= {}^{2n+1}C_0 (x+y)^{2n+1} + {}^{2n+1}C_1 (x+y)^{2n} (-y)^1 + \dots {}^{2n+1}C_{2n+1} (-y)^{2n}$

$\rightarrow (x^{2n+1} + y^{2n+1}) = (x+y)$ [Rem. Qty]

$\therefore$ Divisible by $x+y$.

①

Q. $2^{4n} + 15n - 1$ is not divisible by :-

(a) 5 (b) 15 (c) 225 (d) 625

Solⁿ.

$2^{4n} = 16^n = (1+15)^n$

$= {}^nC_0 15^0 + {}^nC_1 (15)^1 + {}^nC_2 (15)^2 + \ldots {}^nC_n (15)^n$

$\Rightarrow 2^{4n} = 1 + 15n + 15^2 [{}^nC_2 + 15\, {}^nC_3 + \ldots]$

$\Rightarrow 2^{4n} - 15n - 1 = 15^2 [X]$

③ Greatest Binomial Coefficient :-

→ Binomial Coeff. of Middle term → G.B.C.

④ Numerically Greatest Term :-
(Absolute Value)

M-1) for $(x+y)^n$:-

Assume T_{r+1} to be the greatest term, then,

$$\left|\frac{T_{r+1}}{T_r}\right| = \left|\frac{{}^nC_r\, x^{n-r}\, y^r}{{}^nC_{r-1}\, x^{n-r+1} \cdot y^{r-1}}\right| = \left(\frac{n-r+1}{r}\right)\left|\frac{y}{x}\right|$$

$\Rightarrow$ Also, $\left|\frac{T_{r+1}}{T_r}\right| \geq 1 \Rightarrow \boxed{r \leq k}$

If $r \leq k$, $k \notin I \Rightarrow T_{[k]+1}$

, $k \in I \Rightarrow T_k$ and T_{k+1}

M-2) For $(x+y)^n$:

$$k = \frac{(n+1)\,|y|}{|x| + |y|}$$

→ $k \notin I \rightarrow T_{[k]+1}$

→ $k \in I \rightarrow T_k$ & T_{k+1}

Q. $(4+3x)^7$ at $x=\frac{2}{3}$ → Which term is N.G.T.?

Sol^n $\left|\frac{T_{r+1}}{T_r}\right| = \left|\frac{{}^7C_r\,(4)^{7-r}\,(3x)^r}{{}^7C_{r-1}\,(4)^{7-r+1}\,(3x)^{r-1}}\right| = \left(\frac{7-r+1}{r}\right)\left|\frac{3x}{4}\right| \geq 1$

$\Rightarrow \left(\frac{7-r+1}{r}\right)\left(\frac{1}{2}\right) \geq 1 \Rightarrow \boxed{r \leq \frac{8}{3}}$

So, $\boxed{k = 8/3 \neq I}$ $\Rightarrow T_{[8/3]+1} = T_{2+1} = \underline{\underline{T_3}}$

Q. $(3-5x)^{15}$ at $x = 1/5$ → which term is N.G.T.?

Sol^n $\left|\frac{T_{r+1}}{T_r}\right| = \left(\frac{15-r+1}{r}\right)\left|\frac{-5x}{3}\right| = \left(\frac{16-r}{r}\right)\left(\frac{1}{3}\right) \geq 1$

$\Rightarrow \boxed{r \leq 4} \Rightarrow \boxed{K = I}$ → $T_K = T_4$, $T_{K+1} = T_5$

(5) Sum of Binomial Coefficients :-

* $(1+x)^n = {}^nC_0 + {}^nC_1\,x + {}^nC_2\,x^2 + {}^nC_3\,x^3 + \dots + {}^nC_n\,x^n$

→ On $\boxed{x=1}$,

$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$

(or)

$$\boxed{2^n = \sum_{r=0}^{n} {}^nC_r}$$

→ On $\boxed{x=-1}$,

$0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 + \dots$

(or)

$$\boxed{\sum_{r=0}^{n} (-1)^r\,{}^nC_r = 0}$$

Q. Sum of all coeff. of $(1-2x+3x^2)^{10}$

Sol^n $(1-2x+3x^2)^{10} = k_1 + k_2 x + k_3 x^2 + \dots$

On $\boxed{x=1}$, $2^{10} = k_1 + k_2 + k_3 + k_4 \dots$

T.I.P. :- $\sum_{r=0}^{n} r\,{}^nC_r = n(2^{n-1})$

Q. Find the sum of all coeff. $(1+x-3x^2)^{2163}$?

Solⁿ. $(1+1-3)^{2163} = (-1)^{2163} = \underline{\underline{-1}}$

Q. If sum of coeff. of $(\alpha x^3 - 2x + 1)^{35}$ is S_1 and $(x-\alpha y)^{35}$ is S_2, and $S_1 = S_2$, then find out, α.

Solⁿ. On $\boxed{x=1}$, $(\alpha x^3 - 2x + 1)^{35} \Rightarrow (\alpha - 1)^{35}$

On $(x=y=1)$, $(x-\alpha y)^{35} \to (1-\alpha)^{35}$

$\therefore (\alpha-1)^{35} = (1-\alpha)^{35} \Rightarrow (\alpha-1)^{35} = -(\alpha-1)^{35}$

$\Rightarrow 2(\alpha-1) = 0 \Rightarrow \boxed{\alpha = 1}$

NOTE :- $\boxed{r\,{}^nC_r = n\,{}^{n-1}C_{r-1}}$

Eg :- $35\,{}^{61}C_{35} = 61\,{}^{60}C_{34}$

Eg :- $17\,{}^{23}C_{6} = 17\,{}^{23}C_{17} = 23\,{}^{22}C_{16}$

Q. $2\,{}^nC_0 + 5\,{}^nC_1 + 8\,{}^nC_2 + \ldots\ldots$ complete.

Solⁿ. $T_{r+1} = \sum_{r=0}^{n} {}^nC_r(3r+2) = \sum_{r=0}^{n} 3r\,{}^nC_r + 2\sum_{r=0}^{n} {}^nC_r$

$= 3\sum_{r=0}^{n} r\,{}^nC_r + 2\cdot(2^n)$

$= \boxed{3(n2^{n-1}) + 2^{n+1}}$

Q. $1\cdot{}^{20}C_0 + 5\,{}^{20}C_1 + 9\,{}^{20}C_2 + \ldots\ldots$

Solⁿ. $\sum_{r=0}^{20}(4r+1)\,{}^{20}C_r = 4\sum_{r=0}^{20} r\,{}^{20}C_r + \sum_{r=0}^{20} {}^{20}C_r$

$= 4(20)2^{19} + (2^{20})$

T.I.P :- $\sum_{r=1}^{n} (r^2)\, {}^nC_r = n(n+1)\cdot 2^{n-2}$

$\sum_{r=0}^{n} r\, {}^nC_r = n(2^{n-1})$

Q. $(1^2+3)\,{}^nC_1 + (2^2+7)\,{}^nC_2 + (3^2+11)\,{}^nC_3 + \cdots$

Solⁿ. $\sum_{r=1}^{n} (r^2+4r-1)\,{}^nC_r$

$= \sum_{r=1}^{n} r^2\,{}^nC_r + 4\sum_{r=1}^{n} r\,{}^nC_r - \sum_{r=1}^{n} {}^nC_r$

$= n(n+1)\,2^{n-2} + 4\left(n\,(2^{n-1})\right) - (2^n-1)$

Q. $(1^2+2)\,{}^nC_1 + (2^2+5)\,{}^nC_2 + (3^2+8)\,{}^nC_3 + \cdots$

Solⁿ. $\sum_{r=1}^{n} (r^2+3r-1)\,{}^nC_r$

$= \sum_{r=1}^{n} r^2\,{}^nC_r + 3\sum_{r=1}^{n} r\cdot{}^nC_r - \sum_{r=1}^{n} {}^nC_r$

$= n(n+1)\,2^{n-2} + 3\cdot n(2)^{n-1} - (2^n-1)$

NOTE :-

$$\boxed{\sum_{r=0}^{n} r(r-1)\,{}^nC_r = n(n-1)\,2^{n-2}}$$

Q. $1\cdot3\cdot{}^{30}C_1 + 3\cdot7\cdot{}^{30}C_2 + 5\cdot11\,{}^{30}C_3 + \cdots$

Solⁿ. $\sum_{r=1}^{30} (2r-1)(4r-1)\,{}^{30}C_r = \sum_{r=1}^{30} (8r^2-6r+1)\,{}^{30}C_r$

$= 8\sum_{r=1}^{30} r^2\,{}^{30}C_r - 6\sum_{r=1}^{30} r\cdot{}^{30}C_r + \sum_{r=1}^{30} {}^{30}C_r$

$= 8\left[(30)(30+1)\,2^{28}\right] - 6\left[(30)(2^{29})\right] + (2^{30}-1)$

NOTE :-

${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \cdots (-1)^n\,{}^nC_r = 0$

T.I.P. :- $\sum_{r=0 \text{ or } r=1}^{n} (-1)^r r \, {}^nC_r = 0$

Q. $^{31}C_2 - {}^{31}C_3 + {}^{31}C_4 - {}^{31}C_5 + \ldots\ldots + {}^{31}C_{30} = ??$

Soln. $0 - ({}^{31}C_0 - {}^{31}C_1 - {}^{31}C_{31}) = 0 - (1 - 31 - 1) = \underline{\underline{31}}$

Q. $3\,{}^{30}C_0 - 10\,{}^{30}C_1 + 17\,{}^{30}C_2 - \ldots\ldots$ complete series.

Soln. $\sum_{r=0}^{30} (-1)^r (7r+3)\,{}^{30}C_r$

$= \sum_{r=0}^{30} (-1)^r 7r\,{}^{30}C_r + \cancel{3 \cdot \sum_{r=0}^{30} (-1)^r\,{}^{30}C_r} \to 0$

$= 7\sum_{r=1}^{30} (-1)^r r\,{}^{30}C_r = 7 \times 0 = \underline{\underline{0}}$

NOTE :- $\sum_{r=0}^{n} (-1)^r\,{}^nC_r = 0$

$\sum_{r=0}^{n} (-1)^r r\,{}^nC_r = 0$

$\sum_{r=0}^{n} (-1)^r r(r-1)\,{}^nC_r = 0$

$\sum_{r=0}^{n} (-1)^r r^2\,{}^nC_r = 0$

NOTE :- $\therefore r\,{}^nC_r = n\,{}^{n-1}C_{r-1}$

$\Rightarrow \frac{{}^nC_r}{n} = \frac{{}^{n-1}C_{r-1}}{r}$ $\xrightarrow{\text{Similarly}}$ $\frac{{}^nC_r}{r+1} = \frac{{}^{n+1}C_{r+1}}{n+1}$

Q. $\frac{{}^{35}C_1}{2} + \frac{{}^{35}C_2}{3} + \frac{{}^{35}C_3}{4} + \ldots\ldots \frac{{}^{35}C_{35}}{36} = ??$

Soln. $\sum_{r=1}^{35} \frac{{}^{35}C_r}{r+1} = \sum_{r=1}^{35} \frac{{}^{36}C_{r+1}}{36} = \frac{1}{36}\sum_{r=1}^{35} {}^{36}C_{r+1}$

$= \frac{1}{36}\left(2^{36} - {}^{36}C_0 - {}^{36}C_1\right)$

V. Imp.

Q. $\frac{{}^{50}C_0}{2} + \frac{{}^{50}C_1}{3} + \frac{{}^{50}C_2}{4} + \cdots \frac{{}^{50}C_{49}}{51} = ??$

Sol^n. $\sum_{r=0}^{49} \frac{{}^{50}C_r}{r+2} = \sum_{r=0}^{49} \frac{{}^{50}C_r}{(r+1)} \frac{(r+1)}{(r+2)} = \sum_{r=0}^{49} \frac{{}^{51}C_{r+1}}{51} \frac{(r+1)}{(r+2)}$

$= \sum_{r=0}^{49} \frac{{}^{52}C_{r+2}}{(51)(52)} (r+1) = \frac{1}{(51)(52)} \sum_{r=0}^{49} (r+1)\, {}^{52}C_{r+2}$

$= \frac{1}{(51)(52)} \sum_{r=0}^{49} (r+2-1)\, {}^{52}C_{r+2} =$

$= \frac{1}{(51)(52)} \left[\sum_{r=0}^{49} (r+2)\, {}^{52}C_{r+2} - \sum_{r=0}^{49} {}^{52}C_{r+2} \right]$

$= \frac{1}{(51)(52)} \left[(52) \cdot 2^{51} - (1 \cdot {}^{52}C_1 + 52 \cdot {}^{52}C_{52}) \right]$

$- \left[2^{52} - ({}^{52}C_0 + {}^{52}C_1 + {}^{52}C_{52}) \right]$

Q. ${}^nC_0 \cdot {}^{n+2}C_2 + {}^nC_1 \cdot {}^{n+2}C_3 + {}^nC_2 \cdot {}^{n+2}C_4 + \cdots$

Sol^n. $\sum {}^nC_r \cdot {}^{n+2}C_{r+2} = \sum {}^nC_{n-r} \cdot {}^{n+2}C_{r+2} \Rightarrow \underline{{}^{2n+2}C_{n+2}}$

Q. ${}^nC_0\, {}^nC_3 + {}^nC_1 \cdot {}^nC_4 + {}^nC_2 \cdot {}^nC_5 + \cdots$

Sol^n. ${}^nC_r\, {}^nC_{r+3} = {}^nC_{n-r} \cdot {}^nC_{r+3} \Rightarrow {}^{2n}C_{n+3}$ (or) ${}^{2n}C_{n-3}$

Q. ${}^nC_0^2 + {}^nC_1^2 + {}^nC_2^2 + {}^nC_3^2 + \cdots + {}^nC_n^2$

Sol^n ${}^nC_0 \cdot {}^nC_0 + {}^nC_1 \cdot {}^nC_1 \Rightarrow {}^nC_r \cdot {}^nC_r = \underline{{}^{2n}C_n}$

Q. Find the no. of Rational terms in $(9^{1/4} + 8^{1/6})^{1000}$

Sol^n. ${}^{1000}C_r\, (3^{1/2})^{1000-r} (2^{1/2})^r = {}^{1000}C_r\, (3)^{\frac{1000-r}{2}} \cdot (2)^{r/2}$

$\therefore r = 0, 2, 4, 6, \ldots\ldots 1000 = \boxed{501}$

1

Q. Find the no. of rational terms in $(5^{1/8} + 3^{1/6})^{100}$

Sol^n. ${}^{100}C_r \cdot (5)^{\frac{100-r}{8}} \cdot 3^{r/6} \longrightarrow r = 12, 36, 60, 84 \Rightarrow (4)$

Q. If 5^{97} is divided by 52, then the remainder obtained is :-

Sol^n. $5^{97} = (5^4)^{24} \cdot 5 = (625)^{24} \cdot 5$

$= 5[52\lambda + 1]^{24}$ $[\because \lambda = 12]$

$= 5[1 + 52\lambda]^{24}$

$5^{97} = 5[{}^{24}C_0 + {}^{24}C_1 (52\lambda) + {}^{24}C_2 (52\lambda)^2 + \cdots + {}^{24}C_{24} (52\lambda)^{24}]$

$\Rightarrow 5^{97} = 5 + {}^{24}C_1 (52\lambda) + {}^{24}C_2 (52\lambda)^2 + \cdots {}^{24}C_{24} (52\lambda)^{24}$

$\Rightarrow \frac{5^{97}}{52} = \frac{5 + 52k}{52} \Rightarrow \boxed{\frac{5^{97}}{52} = \frac{5}{52} + k}$

$\therefore$ $\boxed{\text{Remainder} = 5}$

NOTE :-

$${}^kC_k + {}^{k+1}C_k + {}^{k+2}C_k + \cdots {}^nC_k = {}^{n+1}C_{k+1}$$

Q. ${}^{20}C_{20} + {}^{21}C_{20} + {}^{22}C_{20} + {}^{23}C_{20} + \cdots {}^{29}C_{20} = {}^{30}C_{21}$

${}^{21}C_{21}$ ${}^{22}C_{21}$ ${}^{23}C_{21}$ ${}^{24}C_{21}$ ${}^{30}C_{21}$

6) Binomial Theorem for any Index :-

If $|x| < 1 \Rightarrow -1 < x < 1$

then, $(1+x)^n = 1 + nx + \dfrac{n(n-1)x^2}{1\cdot 2} + \dfrac{n(n-1)(n-2)x^3}{1\cdot 2\cdot 3} + \ldots$

n may/may not be a +ve integer.

eg:- $(1+x)^{-1/5} = 1 + \left(\dfrac{-x}{5}\right) + \dfrac{\left(-\frac{1}{5}\right)\left(-\frac{1}{5}-1\right)x^2}{1\cdot 2} + \ldots$

7) Integral and fractional part :-

Q. If $N = (2+\sqrt{3})^6$, find its Integral part & $N(1-f) =$

Sol^n.

$(2+\sqrt{3})^6 = {}^6C_0 (2)^6(\sqrt{3})^0 + {}^6C_1 (2)^5(\sqrt{3})^1 + \ldots \; {}^6C_6 (2)^0(\sqrt{3})^6$

$(2-\sqrt{3})^6 = {}^6C_0 (2)^6(-\sqrt{3})^0 - {}^6C_1 (2)^5(\sqrt{3})^1 + \ldots \; {}^6C_6 (2)^0(-\sqrt{3})^6$

$(2+\sqrt{3})^6 + (2-\sqrt{3})^6 = 2\underbrace{\left[{}^6C_0 (2)^6 + {}^6C_2 (2)^4(\sqrt{3})^2 + \ldots \; {}^6C_6 (2)^0(\sqrt{3})^6\right]}_{I}$

$\therefore (2+\sqrt{3})^6 + (2-\sqrt{3})^6 = 2I \;\; \& \;\; (2+\sqrt{3})^6 = N = 2I - \underbrace{(2-\sqrt{3})^6}_{<1}$

$\Rightarrow (2+\sqrt{3})^6 = N = \underbrace{2I-1}_{I} + \underbrace{1-(2-\sqrt{3})^6}_{f}$

$\therefore \boxed{I = 2701}$ & $(1-f)(N) \Rightarrow 1$

Q. If $N = (3\sqrt{3}+5)^{11}$, find if Integral part is even/odd, and $Nf =$ ______?

Sol^n. $N = (3\sqrt{3}+5)^{11} = {}^{11}C_0 (3\sqrt{3})^{11}(5)^0 + {}^{11}C_1 (3\sqrt{3})^{10}(5)^1 + \ldots \; {}^{11}C_{11}(3\sqrt{3})^0(5)^{11}$

$(3\sqrt{3}-5)^{11} = {}^{11}C_0 (3\sqrt{3})^{11}(-5)^0 - {}^{11}C_1 (3\sqrt{3})^{10}(5)^1 + \ldots - {}^{11}C_{11}(3\sqrt{3})^0(+5)^{11}$

$(3\sqrt{3}+5)^{11} - (3\sqrt{3}-5)^{11} = 2\underbrace{\left[{}^{11}C_1 (3\sqrt{3})^{10}(5)^1 + \ldots \; {}^{11}C_{11}(5)^{11}\right]}_{I}$

$\therefore (3\sqrt{3}+5) = \underbrace{2I}_{\text{Integral part = even}} + \underbrace{(3\sqrt{3}-5)^{11}}_{<1}$

Integral part = even

Also, $N - f = (3\sqrt{3}+5)^{11}(3\sqrt{3}-5)^{11} = \underline{\underline{(2)^{11}}}$ Ans.

⑧ Multinomial Theorem :-

→ For $(k_1 x + k_2 y + k_3 z)^n$,

Coeff. of $x^\alpha y^\beta z^\gamma$ in above expression,

if $\alpha + \beta + \gamma \neq n \longrightarrow$ Coeff. $= 0$

if $\alpha + \beta + \gamma = n \rightarrow$ coeff. $= \dfrac{n!\,(k_1)^\alpha (k_2)^\beta (k_3)^\gamma}{\alpha!\,\beta!\,\gamma!}$

where, x, y, z are diff. variables with each having degree $= 1$.

Eg :- Find coeff. of $x^5 y^7 z^8$ in $(2x - 3y + 4z)^{20}$.

⇒ Coeff. $= \dfrac{20!}{5!\,7!\,8!}(2)^5(-3)^7(4)^8$

Eg :- Find coeff. of x^5 in $(4 - 3x + 7x^2)^{20}$.

Soln Let $X = 1$; $Y = x$; $Z = x^2$

$\therefore$ Eqn. $= (4X - 3Y + 7Z)^{20}$

X	Y	Z
15	5	0
16	3	1
17	1	2

$\therefore$ Coeff. $= \dfrac{20!\,(4)^{15}(-3)^5(7)^0}{15!\,5!\,0!} + \dfrac{20!\,(4)^{16}(-3)^3(7)^1}{16!\,3!\,1!} + \dfrac{20!\,(4)^{17}(-3)^1(7)^2}{17!\,1!\,2!}$

→ for $(d_1 x_1 + d_2 x_2 + d_3 x_3 + d_4 x_4 + \ldots d_k x_k)^n$

* General Term ⇒ $\dfrac{n!}{\alpha_1!\,\alpha_2!\,\alpha_3!\ldots\alpha_k!}(d_1 x_1)^{\alpha_1}(d_2 x_2)^{\alpha_2}\ldots(d_k x_k)^{\alpha_k}$

* No. of Terms :- ${}^{n+k-1}C_{k-1}$

Q. The coeff. of x^5 in the expansion of $\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}\right)^2$ is ____

Soln. $\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}\right)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}\right)$

⇒ $\frac{1}{0!}\times\frac{1}{5!}+\frac{1}{1!}\cdot\frac{1}{4!}+\frac{1}{2!}\cdot\frac{1}{3!}+\frac{1}{3!}\cdot\frac{1}{2!}+\frac{1}{4!}\cdot\frac{1}{1!}+\frac{1}{5!}\cdot\frac{1}{0!}$

$= \frac{4}{15}$

Q. The coeff. of x^{50} in $(1+x)^{41}(1-x+x^2)^{40}$ is

Soln. $(1+x)(1+x)^{40}(1-x+x^2)^{40} = (1+x)(1+x^3)^{40}$

⇒ $(1+x^3)^{40} + x(1+x^3)^{40}$

$\downarrow \qquad \downarrow$

$\times \qquad \times \qquad = 0$

Q. The coeff. of x^2 in the exp. of $(1-x+2x^2)\left(x+\frac{1}{x}\right)^{10}$

Soln. $\left(x+\frac{1}{x}\right)^{10} - x\left(x+\frac{1}{x}\right)^{10} + 2x^2\left(x+\frac{1}{x}\right)^{10}$

$(r=4)$ ${}^{10}C_4 + 0 + 2\,{}^{10}C_5 = 714$

Q. The coeff. of x^{10} in the expansion of $(1+x)^{15} + (1+x)^{16} + (1+x)^{17} + \cdots (1+x)^{30}$ is ____

Sol^n. $^{15}C_{10} + {}^{16}C_{10} + {}^{17}C_{10} + {}^{18}C_{10} + \cdots {}^{30}C_{10}$

$= {}^{31}C_{11} - \left({}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + {}^{13}C_{10} + {}^{14}C_{10}\right)$

$= {}^{31}C_{11} - {}^{15}C_{11}$