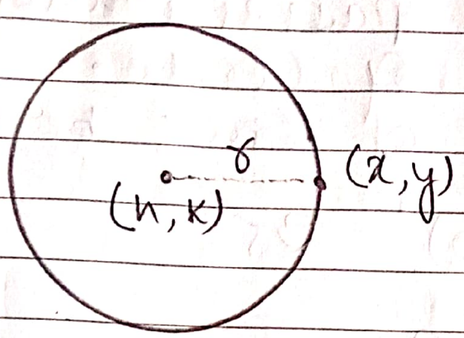
CIRCLES [3D Geometry — II]

$\cancel{8}$-1

① Standard form of eqn. of circle :-



$$\sqrt{(x-h)^2+(y-k)^2} = r$$

$$\Rightarrow (x-h)^2 + (y-k)^2 = r^2$$

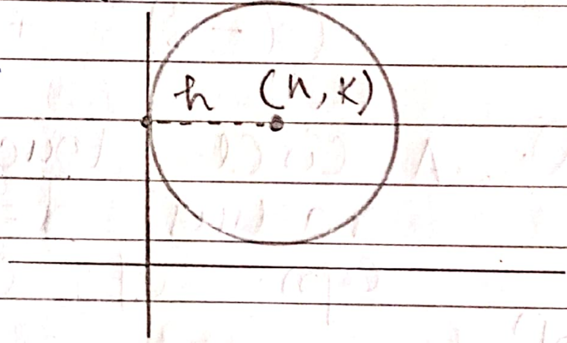
$$\Rightarrow \boxed{x^2 + y^2 = r^2} \rightarrow \text{Standard form.}$$

where, $(h, k) = (0, 0)$ (Origin)

• Some Special Cases :-

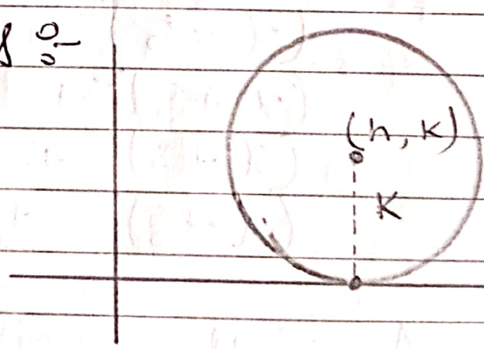
Ⓐ Circles touching y-axis :-

$$\Rightarrow (x-h)^2 + (y-k)^2 = (h)^2$$



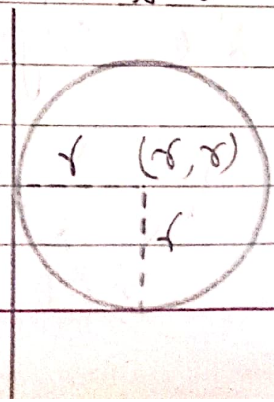
Ⓑ Circle touching x-axis :-

$$\Rightarrow (x-h)^2 + (y-k)^2 = (k)^2$$



Ⓒ Circles touching both the axis :-

$$(x-r)^2 + (y-r)^2 = r^2$$



Q. A circle of radius $r = 5$ whose cent[re] lies on x-axis ; passes through $(2, 3)$. Find the eqn. of circle.

Sol^n. Let the coordinates of circle be $(h, [0])$

Then,

$$(x - h)^2 + (y - 0)^2 = 25$$

$$\Rightarrow (2 - h)^2 + (3)^2 = 25$$

$$\Rightarrow (2 - h)^2 = 16$$

$$\Rightarrow (2 - h) = \pm 4 \Rightarrow \boxed{h = -2},\ \boxed{h = 6}$$

Q. Find the eqn. of circle which touch the x-axis at $P(3, 4)$?

Sol^n.

$$(x - 3)^2 + (y - 4)^2 = (4)^2$$

Q. A circle touches both the axis having radius $r = 5$ units. Then, find the eqn. of circles formed.

Sol^n.

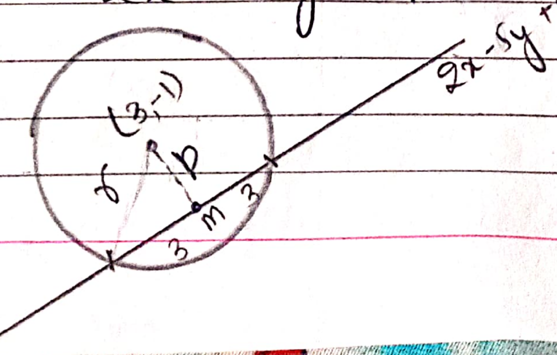
$$(x - 5)^2 + (y - 5)^2 = 25 \rightarrow I^{st} \text{ Quad.}$$

$$(x + 5)^2 + (y + 5)^2 = 25 \rightarrow III^{rd} \text{ Quad.}$$

$$(x + 5)^2 + (y - 5)^2 = 25 \rightarrow II^{nd} \text{ Quad.}$$

$$(x - 5)^2 + (y + 5)^2 = 25 \rightarrow IV^{th} \text{ Quad.}$$

Q. A circle with centre $C(3, -1)$ has a intercept of length 6 cm from line $2x - 5y + 18 = 0$. Then, find its radius.



$$p = \frac{2(3) - 5(-1) + 18}{\sqrt{2^2 + 5^2}}$$

$$= \sqrt{29}$$

By pythagoras' theorem; we have :-

$$\gamma^2 = p^2 + 3^2 \Rightarrow \boxed{\gamma = \sqrt{38}} \text{ units.}$$

Q. Convert this equat^n in its standard form :- $x^2 + y^2 + 4x - 6y + 2 = 0$

Sol^n.

$\rightarrow (x^2 + 4x + 4) + (y^2 - 6x + 9) = -2 + 4 + 9$

$\Rightarrow (x+2)^2 + (y-3)^2 = (\sqrt{11})^2 \rightarrow$ Ans.

- Standard form $\longrightarrow$ Extended eqn.

$\Rightarrow (x-h)^2 + (y-k)^2 = \gamma^2$

$\Rightarrow x^2 + h^2 - 2hx + y^2 + k^2 - 2ky = \gamma^2$

$\Rightarrow x^2 + y^2 - 2hx - 2ky + \{h^2 + k^2 - \gamma^2\} = 0$

Let $\boxed{g = -h}$ & $\boxed{f = -k}$; we get :-

$\Rightarrow x^2 + y^2 + 2gx + 2fy + c = 0$

$\boxed{h = -g}$ $\boxed{k = -f}$

Also,

$\Rightarrow h^2 + k^2 - \gamma^2 = c$

$\Rightarrow g^2 + f^2 - \gamma^2 = c$

$\Rightarrow \gamma^2 = g^2 + f^2 + (-c)$

$\rightarrow \gamma = \sqrt{g^2 + f^2 - c}$

Q. Find the coordinates of centre of circle of the following equa^ns :-

(a) $x^2 + y^2 + 20x - 32y + C = 0$

$\hookrightarrow (-10, 16)$

(b) $x^2 + y^2 - 10x + 14y + C = 0$

$\hookrightarrow (5, -7)$

(c) $2x^2 + 2y^2 + 20x + 14y + C = 0$

$\hookrightarrow (-5, -7/2)$

Q. Find the radius of circle having eqn

$x^2 + y^2 - 8x + 6y + 21 = 0$

$\Rightarrow$ Here, $(h, k) = (4, -3)$

$\therefore \; r = \sqrt{g^2 + f^2 - c} = \sqrt{16 + 9 - 21} = \boxed{2}$

- For $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ be a equa^n of circle :-

$\hookrightarrow \boxed{a = b}$ and $\boxed{h = 0}$

Q.

Q. Find the eqn. of circle concentric to $x^2 + y^2 - 4x - 6y - 3 = 0$ and touching y-axis.

Sol^n Centre of circle :- $C\ (2, 3)$

$\therefore$ Eqn. of circles $= (x-2)^2 + (y-3)^2 = (2$

- Values of r :-

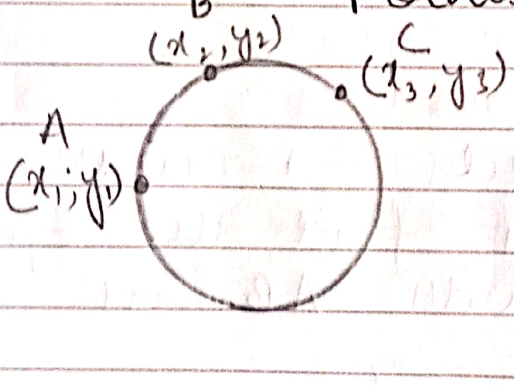
↳ If $r = \sqrt{g^2 + f^2 - c} \Rightarrow g^2 + f^2 - c > 0$

$\Rightarrow r \rightarrow$ real circle (where, $r = k$)

↳ If $g^2 + f^2 - c = 0 \Rightarrow \boxed{r = 0} \rightarrow$ Point circle.

↳ If $g^2 + f^2 - c < 0 \Rightarrow r =$ imaginary circ

- Circle passing through 3-non-collinear points :-

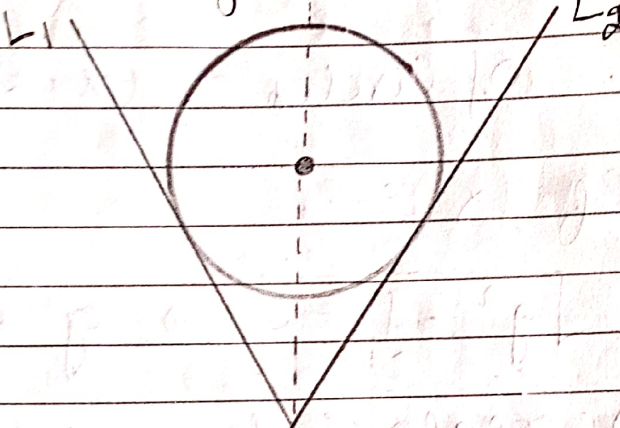


Let the eqn. be
$x^2 + y^2 + 2gx + 2fy + c = 0$

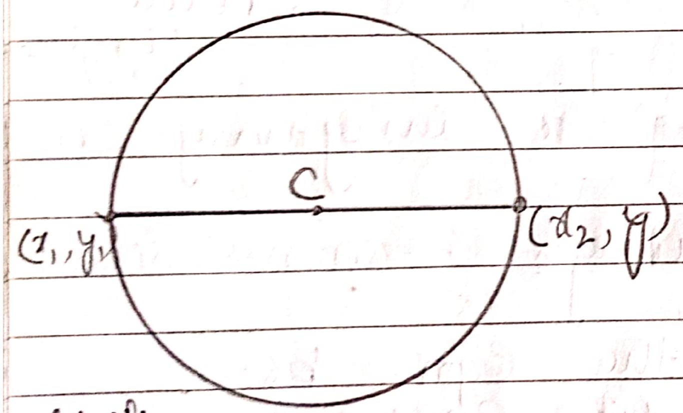
Then,
Substitute A; B; C; in the eqn. and find the value of g; f; c

Thus, form a eqn of circle.

- If a circle is present b/w two inter-secting lines L_1 & L_2, then, its Centre lies at the Angle Bisector of L_1 & L_2.



- Diametric form :-

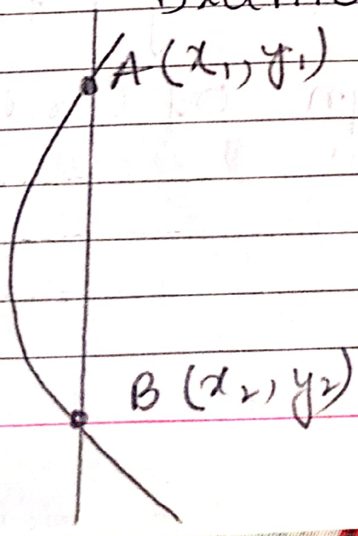


∴ Eqn. Of Circle. ⇒

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

Imp.

Q. A line $x + y = 8$ intersects a curve $y^2 = x$ at A & B. Then, find the Eqn. of circle formed with AB as Diameter.



Here, $\boxed{y = 8 - x}$

and $\boxed{y^2 = x}$

∴ $(8 - x)^2 = x$

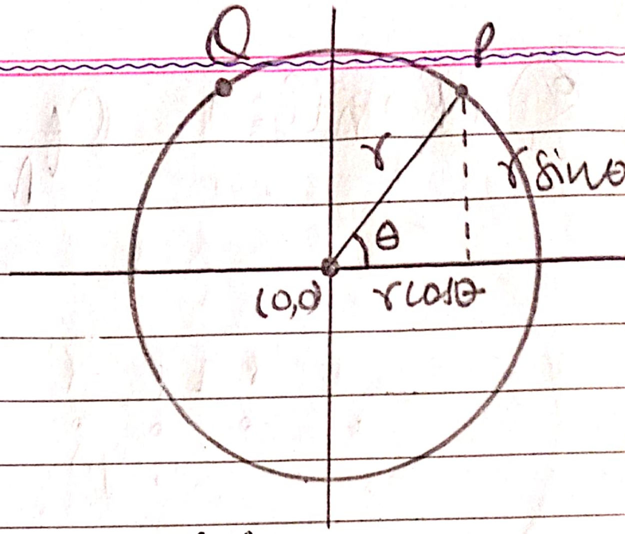
⇒ $x^2 - 17x + 64 = 0$

Also, $\boxed{y^2 = 8 - y}$

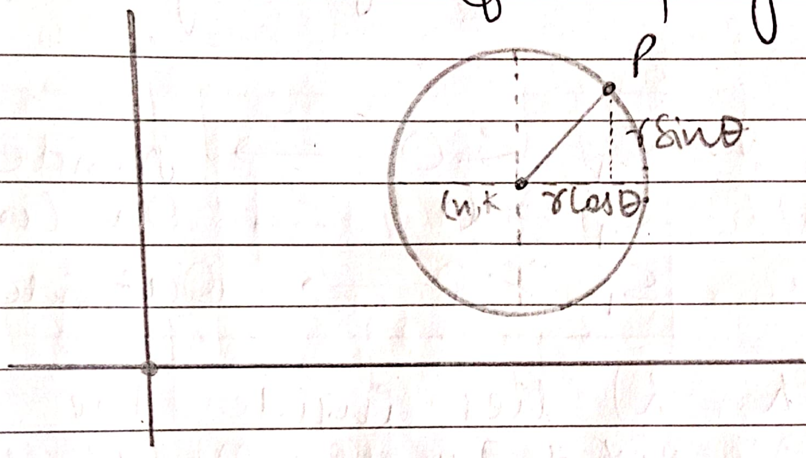
∴ Eqn. Of Circle:- $x^2 + y^2 - 17x + y + 56 = 0$

• Parametric Point :-

$P(r\cos\theta, r\sin\theta)$



→ In case of shifting of origin;



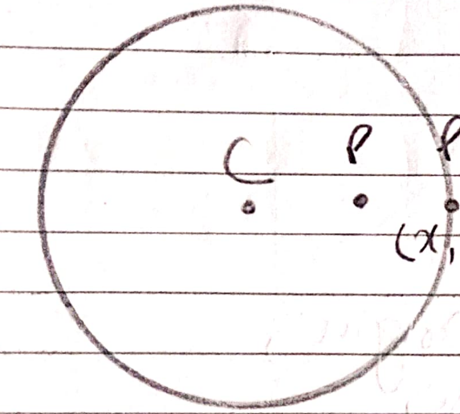
Here,

$x = h + r\cos\theta$

$y = k + r\sin\theta$

(2) Position Of a point w.r.t. Centre :-

<u>Method</u> $\rightarrow$ find the value Of S_1.



where, $S = x^2 + y^2 + 2gx + 2fy + c$

$\therefore S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

Thus, if

$$\boxed{\begin{array}{l} S_1 < 0 \longrightarrow \text{Inside} \\ S_1 = 0 \longrightarrow \text{On Circle} \\ S_1 > 0 \longrightarrow \text{Outside} \end{array}}$$

Q. A point $P(\lambda, -\lambda)$ lies inside the circle $x^2 + y^2 - 4x + 2y - 8 = 0$. find the range of λ

<u>Sol^n</u> Since, P lies inside the Circle.

so, $S_1 < 0$

$\therefore \lambda^2 + (-\lambda)^2 - 4(\lambda) + 2(-\lambda) - 8 < 0$

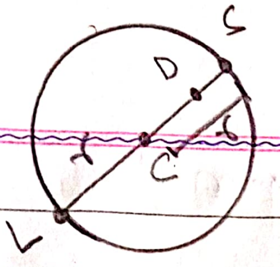
$\Rightarrow \lambda^2 + \lambda^2 - 6\lambda - 8 < 0$

$\Rightarrow \lambda^2 - 3\lambda - 4 < 0$

$\therefore \boxed{\lambda \in (-1, 4)}$

- Shortest and Longest Distance Of a Point from Circle :-

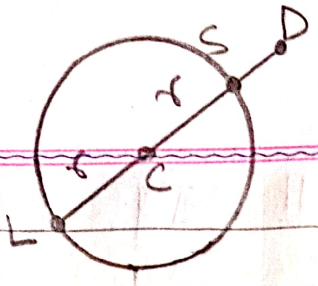
(Case-I)



$SD = r - C.D.$

$LD = CD + r$

(Case-II)



$SD = CD - r$

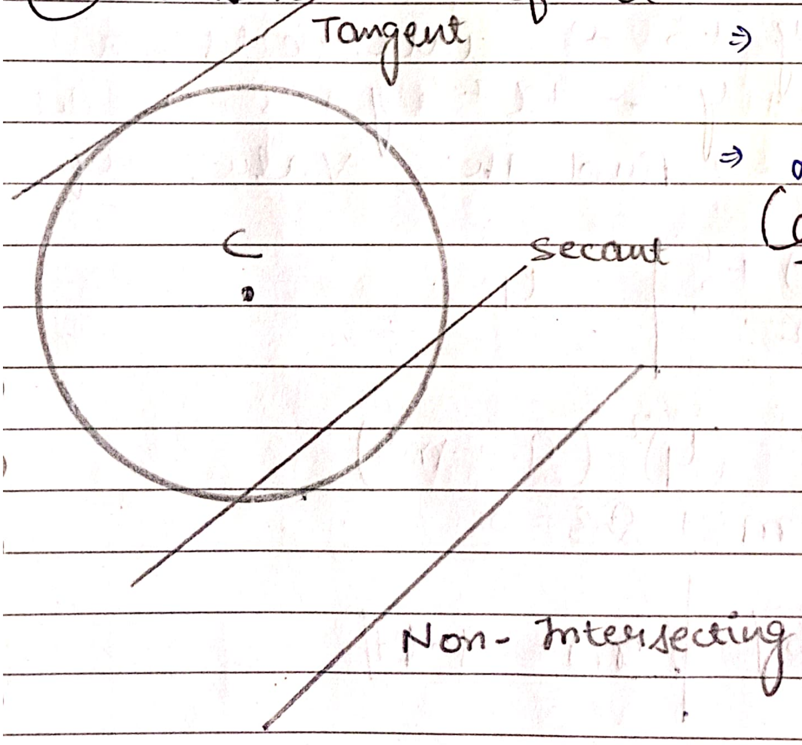
$LD = CD + r$

Q. Find the shortest and longest Distance of point $(2,-7)$ from the circle.
$x^2 + y^2 - 14x - 10y - 151 = 0$

Solⁿ. $S.D = 2$ units.
$L.D. = 28$ units.

Imp.

③ Position of a Line w.r.t. Circle :-



⇒ Let the centre of circle be at $C(0,0)$

⇒ Let the line be $y = mx + c$

(Case - I)

Here, $y = mx + c$
$\Downarrow$
$mx + c - y = 0$

$\therefore d = \left| \dfrac{0 + 0 + c}{\sqrt{(m^2+1)}} \right| > r$

$\Rightarrow \boxed{c^2 > r^2(1+m^2)}$
↓
Non-Intersecting Line.

Case-II) If $c^2 < r^2(1+m^2)$

$\Rightarrow$ line intersecting is a secant.

Case-III) If $c^2 = (m^2+1)r^2$

$\Rightarrow \boxed{c = \pm r\sqrt{1+m^2}}$

↳ Condition of Tangency

$\therefore$ from $y = mx + c$; we get :-

$\Rightarrow \boxed{y = mx \pm a\sqrt{1+m^2}}$

↳ Always a tangent of slope m to circle $x^2+y^2=a^2$

Q. A line $3x - my + 5 = 0$ intersects a circle $x^2+y^2-4x+6y-3=0$ as a tangent. Find the value of 'm'.

Sol^n. $d = \left|\dfrac{6 - m(-3)+5}{\sqrt{9+m^2}}\right| = 4$

$\Rightarrow (11+3m)^2 = (4)^2(9+m^2)$

$\Rightarrow 7m^2 - 66m + 23 = 0$

$\Rightarrow \boxed{m = 0.36} \quad \boxed{m = 9.07}$

Q. A ~~tangent~~ line $3x - 4y - 7 = 0$ is drawn to a circle $x^2+y^2-2x-4y-4=0$. Find the eqn. of tangent parallel to $3x - 4y - 7 = 0$

Sol^n. Here, Centre of Circle $\Rightarrow$ $(1, 2)$
& Radius $\Rightarrow$ 3 units.

Let the Eqn. of Parallel tangent be
$3x - 4y + c = 0$

Also, $\left| \frac{3 - 8 + c}{\sqrt{25}} \right| = 3$

$c - 5 = 15$
$\boxed{c = 20}$

$c - 5 = -15$
$\boxed{c = -10}$

$\therefore$ Tangents formed $= \boxed{\begin{matrix} 3x - 4y + 20 = 0 \\ 3x - 4y - 10 = 0 \end{matrix}}$

Q. A tangent $3x - 4y - 7 = 0$ to circle $x^2 + y^2 - 2x - 4y - 4 = 0$. Find Lar$^{\text{tangent}}$ to this tangent. for the circle.

Sol^n Let the Lars be of form,
$4x + 3y + k = 0$

Also, $\left| \frac{4(1) + 3(2) + k}{5} \right| = 3$

$\Rightarrow$ $|k + 10| = 15$ $\boxed{k = 5}$ $\boxed{k = -25}$

$\therefore$ Lars formed are :- $\boxed{\begin{matrix} 4x + 3y + 5 = 0 \\ 4x + 3y - 25 = 0 \end{matrix}}$