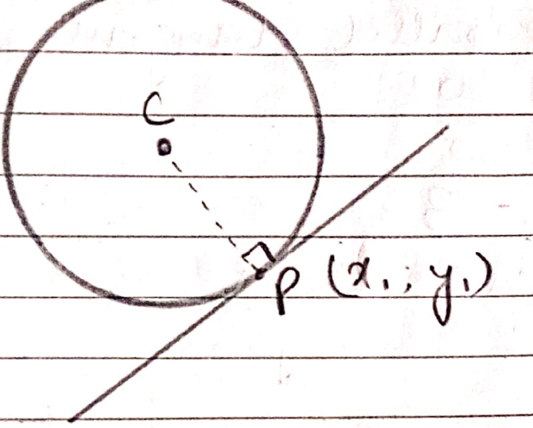
v. Imp

(4) Eqn. Of Tangent at a point on the circle :-

<u>Method</u> → Find $\boxed{T = 0}$ w.r.t. (x_1, y_1)



Where, $\boxed{T = 0}$ can be formed by,

$x^2 \rightarrow x \cdot x_1$	$c \rightarrow c$
$y^2 \rightarrow y \cdot y_1$	$y = \frac{y + y_1}{2}$
$x \rightarrow \frac{x + x_1}{2}$	

$$xy = \frac{xy_1 + yx_1}{2}$$

Q. A tangent on a circle passes from (3, 4). Find Eqn. Of tangent if the eqn. of circle is $x^2 + y^2 = 25$

<u>Soln</u>

$\Rightarrow (3)(x) + (y)(4) = 25$

$\Rightarrow \boxed{3x + 4y = 25}$ → Eqn. of Tangent.

Q. A tangent on a circle $x^2 + y^2 - 2x + 4y + 1 = 0$ passes from (01, 0). Find Eqn. of Tangent :-

<u>Soln</u>

$$x(1) + y(0) - 2\left(\frac{x+1}{2}\right) + 4\left(\frac{y+0}{2}\right) + 1$$

$\Rightarrow \boxed{y = 0}$ → $\boxed{x\text{-axis}}$ is the tangent to circles.

Q. A tangent $3x - 4y - 1 = 0$ is passes through a circle $x^2 + y^2 - 2x + 4y + 1 = 0$. Find their point of contact.

Soln. Let the point of contact be (h, k)

$\therefore x(h) + y(k) - 2\left(\frac{x+h}{2}\right) + 2\left(\frac{y+k}{2}\right) + 1$

$\Rightarrow \boxed{(h-1)x + (k+2)y - h + 2k + 1 = 0}$

Since, this this line is similiar to $3x - 4y - 1 = 0$.

$\therefore \frac{h-1}{3} = \frac{k+2}{-4} = \frac{-h+2k+1}{-1}$

$\Rightarrow \frac{h-1}{3} = \frac{-h+2k+1}{-1}$

$\Rightarrow h - \cancel{3} = 3h - 6k - \cancel{3}$

$\Rightarrow 2h - 6k = 0$

$\Rightarrow \boxed{h = 3k}$

$\boxed{h = -\frac{1}{5}}$

$\Rightarrow \frac{k+2}{+4} = \frac{-h+2k+1}{+1}$

$\Rightarrow k + 2 = 8k - 4h + 4$

$\Rightarrow 7k - 4h + 2 = 0$

$\Rightarrow 7k - 4(3k) + 2 = 0$

$\Rightarrow 7k - 12k + 2 = 0$

$\Rightarrow 5k = 2$

$\Rightarrow \boxed{k = \frac{2}{5}}$

Q. Find the eqn. of tangents drawn from (1, 4) to a circle $x^2 + y^2 = 16$

Sol^n Here, Slope ⇒

$$\frac{(y-4)}{(x-1)} = m \Rightarrow \boxed{mx - y + 4 - m = 0}$$

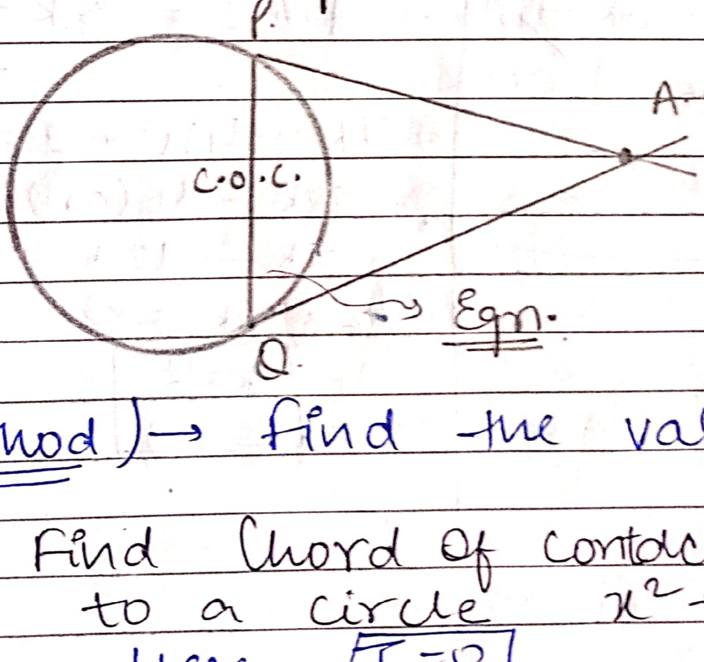
∴ distance from Centre :-

$$\Rightarrow \left|\frac{0+0+4-m}{\sqrt{m^2+1}}\right| = 4$$

$$\Rightarrow \boxed{m=0} \qquad \boxed{m = -\frac{8}{15}}$$

$$\boxed{y=4} \qquad (y-4) = \frac{-8}{15}(x-1)$$

(5) Chord Of Contact :-



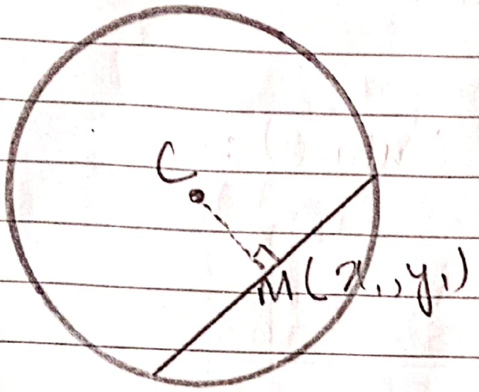
Method) → find the value of $\boxed{T=0}$

Q. Find Chord of contact formed by (4,3) to a circle $x^2 + y^2 = 9$.

Sol^n Here, $\boxed{T=0}$

So, $x(4) + y(3) = 9 \Rightarrow \boxed{4x + 3y = 9}$

(6) Eqn. Of Chord bisected at a Point :-
(Mid-point form of chord)



<u>Method</u>) → Find $\boxed{T = S_1}$

Q. Find the eqn. Of chord bisected at (5, -3) in a circle $x^2 + y^2 - 6x + 8y = 0$

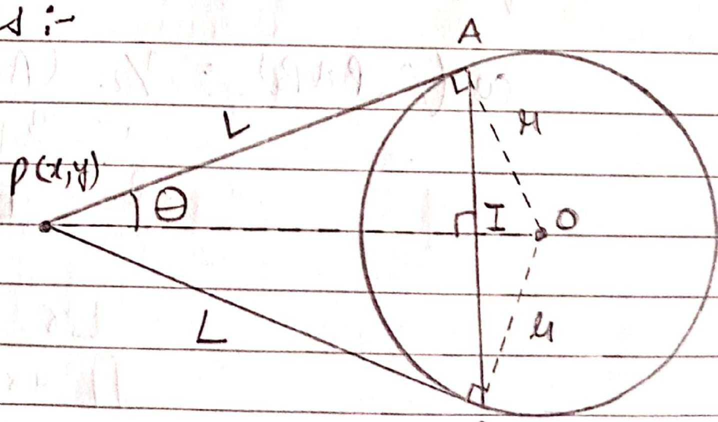
<u>Sol^n</u> Here, $\boxed{T = S_1}$ is to be applied.

$$\Rightarrow x(5) + y.(-3) + 8\left(\frac{y-3}{2}\right) - 6\left(\frac{x+5}{2}\right) = (5)^2 + (-3)^2 - 6(5) + 8(-3)$$

$$\Rightarrow 5x - 3y + 4y - 12 - 3x - 15 = -20$$

$$\Rightarrow \boxed{2x + y - 7 = 0}$$

(7) Geometry of circles :-



(A) Lengths Of Tangents drawn from a point :-
Since, Length of Tangents drawn from a point are equal. Thus, its length can be found by $\boxed{L = \sqrt{S_1}}$

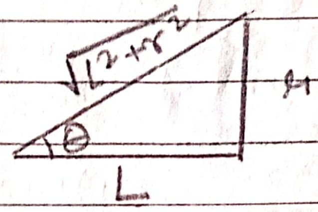
(B) Area Of Quad. PAOB :-

$$\text{Area} = \left(\frac{1}{2} \times L \times r\right) \times 2 = \boxed{Lr}$$

(C) Length of AB (Chord Of Contact) :-

Since, $\tan\theta = \dfrac{r}{L}$

and $\sin\theta = \dfrac{r}{\sqrt{L^2+r^2}}$,



$$\cos\theta = \frac{L}{\sqrt{L^2+r^2}}$$

Now, in ΔPAI, $\boxed{\dfrac{AI}{L} = \sin\theta}$ $\Rightarrow$ $\boxed{AI = L\sin\theta}$

Since, $\boxed{AB = 2AI = 2L\sin\theta}$

$$\therefore \boxed{AB = \frac{2Lr}{\sqrt{L^2+r^2}}}$$

(D) Area Of ΔPAB :-

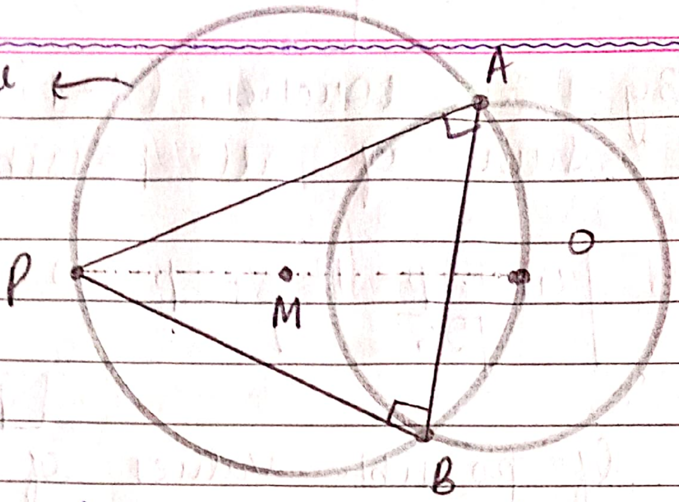
$$ar(\Delta PAB) = \tfrac{1}{2}(AB)(PI)$$

$$= \frac{1}{2} \times \frac{2Lr}{\sqrt{L^2+r^2}} \times L \times \cos\theta$$

$$= \frac{Lr}{\sqrt{L^2+r^2}} \times L \times \frac{L}{\sqrt{L^2+r^2}}$$

$$\boxed{ar(\Delta PAB) = \frac{L^3 r}{L^2+r^2}}$$

(E) Circle



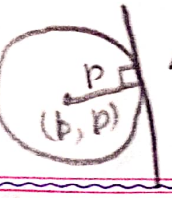
The figure drawn above, has a circumcircle of PAOB (quad.). Also, it has a circumcircle of $\triangle PAB$.
Where, the Centre = Midpoint of P & O.
and Radius = OM = MP = OP/2.

Q. Abscissa of A and B are the roots of the eqn $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $y^2 + 2py - q^2 = 0$. The equatn of the circle with AB as its diameter :-

Soln. Using Diametric form, $x^2 + 2ax - b^2 + y^2 + 2py - q^2 = 0$ is the equatn of circle.

Q. The foot of the normal from the point (4,3) to a circle is (2,1) and the diameter of the circle has eqn. $2x - y - 2 = 0$, then the equatn of circle.

Soln. Here, $y - 1 = \frac{3-1}{4-2} =$



Q. If $4x+3y-12=0$ touches $(x-p)^2+(y-p)^2=p^2$, then the sum of all possible values of 'p' is :-

Sol^n. Here, $\left|\frac{4p+3p-12}{\sqrt{25}}\right| = p \Rightarrow |7p-12| = 5p$

$\boxed{p=6}$ $\boxed{p=1}$

$\therefore$ Sum of possible values of $p = 7$

Q. Which of the following equations of a line intercepts on a circle $x^2+y^2-6x-8y=0$ a chord of longest length?

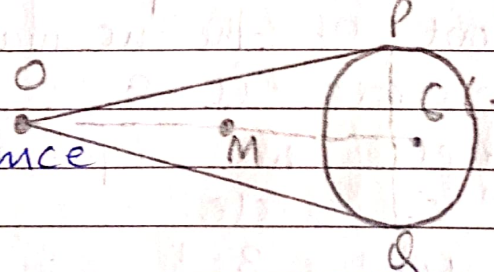
(A) $y-x=0$ (B) $x+y=0$

(C) $x+y=2$ (D) $3x+4y-2=0$

Sol^n From the circle's eqn., Centre $=(3,4)$

Thus, the eqn which is more nearby to $(3,4)$ is the longest chord.

Q. If O is the origin and OP, OQ are distinct tangents to the circle $x^2+y^2+2gx+2fy+c=0$, then the circumcentre of the Δ OPQ is:-



Here, Mid point of OC, is the circumcentre of ΔOPQ.

$\therefore \left(\frac{-g+0}{2}, \frac{-f+0}{2}\right) \Rightarrow \boxed{\left(\frac{-g}{2}, \frac{-f}{2}\right)}$ Ans.

Q. Consider circles C_1 & C_2 touching both the axes and passing through $(4,4)$, then the product of radii of these circles is :-

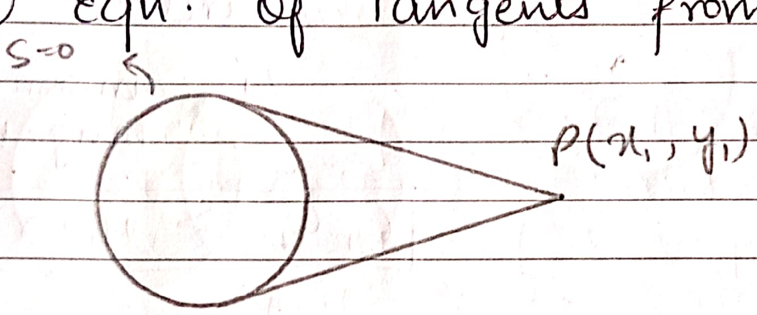
Sol^n Since, the circle touching both sides (axes) has a eqn. of form, $r^2 = (x-r)^2 + (y-r)^2$

$$\therefore \quad (4-r)^2 + (4-r)^2 = r^2$$

$$\Rightarrow \quad r^2 - 16r + 32 = 0 \; \begin{matrix} r_1 \\ r_2 \end{matrix}$$

$$\therefore \boxed{\text{Product of radii} = 32}$$

(8) Eqn. of Tangents from a point as POSL :- Method) →

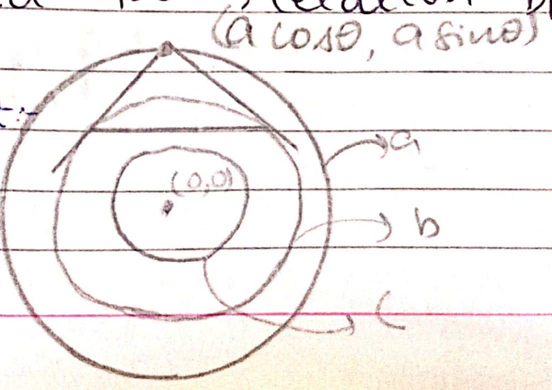


$$\boxed{S \cdot S_1 = T^2}$$

for Example :-
Eqn of Circle → $x^2+y^2=4$ & $P(5,3)$:-
$\therefore$ Pair of Tangents =

$$\underbrace{(x^2+y^2-4)}_{S}\,\underbrace{(5^2+3^2-4)}_{S_1} = \underbrace{(5x+3y-4)^2}_{T^2}$$

Q. Chord of contact of tangents drawn from a point on $x^2+y^2=a^2$, to $x^2+y^2=b^2$ touches $x^2+y^2=c^2$. Find the relation b/w a, b, c.



Sol^n from, $x^2+y^2=b^2$, we get:-

$$ax\cos\theta + ay\sin\theta = b^2$$

$$ax\cos\theta - ay\sin\theta - b^2 = 0$$

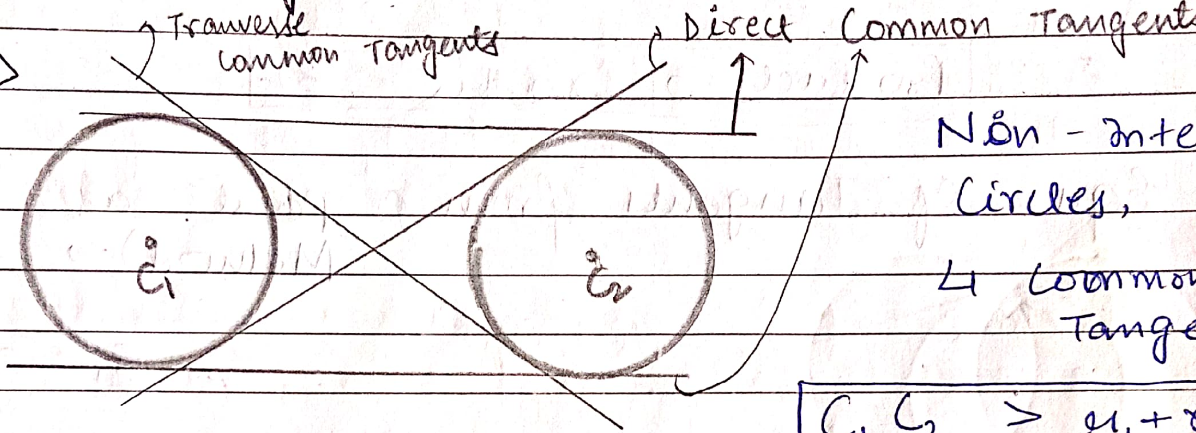
$\therefore \quad \left|\dfrac{0+0-b^2}{\sqrt{a^2\cos^2\theta+a^2\sin^2\theta}}\right| = c$

$\Rightarrow \quad \boxed{b^2 = ac}$

$\therefore$ a, b, c, are in G.P.

(9) Common Tangents & Circles :-

Case-I)

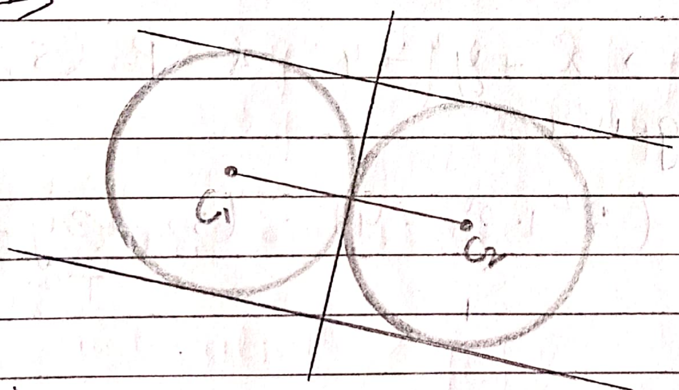


Non - Intersecting Circles,

4 Common - Tangents

$\boxed{C_1C_2 > r_1 + r_2}$

Case-II)

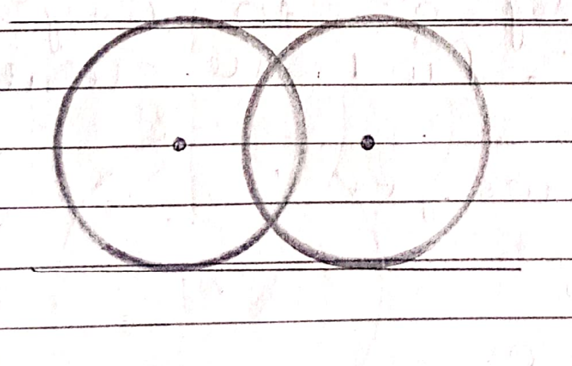


External Touching,

3 Common Tangents

$\boxed{C_1C_2 = r_1 + r_2}$

Case-III)

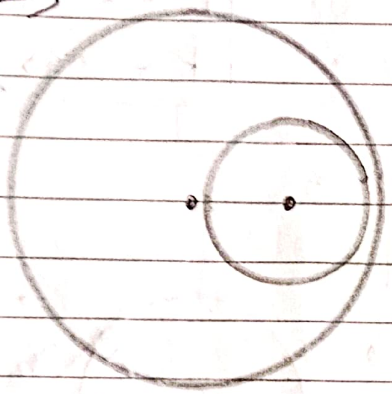


$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

Case-IV)

$$\boxed{C_1C_2 = |r_1 - r_2|}$$

Case-V)



$$\boxed{C_1C_2 < |r_1 - r_2|}$$

Q. Find the Total no. of Common Tangent b/w the two given Circles as,

$x^2+y^2-6x-2y+1=0$ and ——— C_1

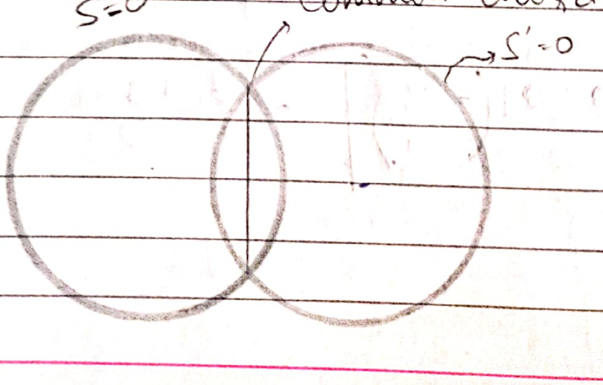
$x^2+y^2+2x-8y+13=0$. ——— C_2

Sol^n The centre of $C_1 = (3,1)$ and $C_2 = (-1,4)$. So Distance b/w their centres $= \sqrt{16+9} = \boxed{5}$

Also, $r_1 = \sqrt{9+1-1} = 3$ unit & $r_2 = 2$ unit

Thus, $r_1 + r_2 = C_1C_2$, then $\boxed{N.O.C.T. = 3}$

(10) Equation Of Common Chord :-

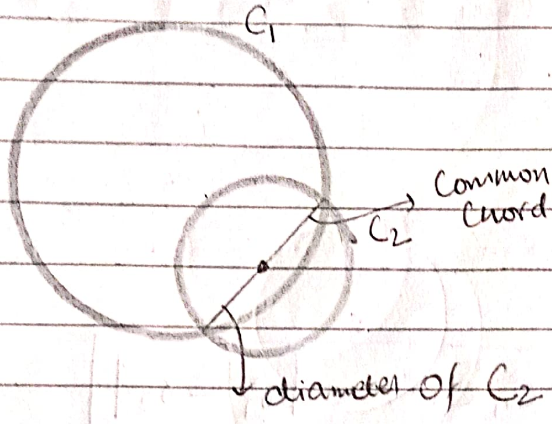


Make Coefficient of x^2 & y^2 same in both eqns, then,

$$\boxed{S - S' = 0}$$

Q. C_1 bisects the circumference of C_2, then $k =$
if $C_1 = x^2 + y^2 + 6x - 2y + k = 0$
$C_2 = x^2 + y^2 - 2x - 6y - 15 = 0$

Sol^n.



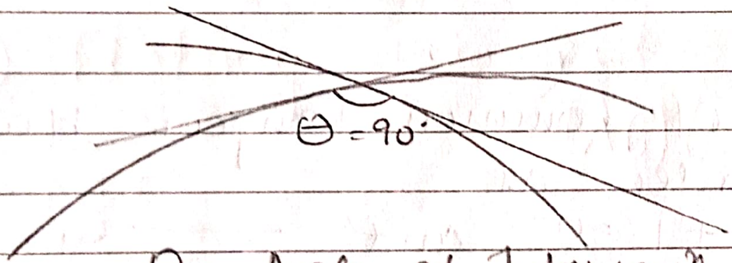
$$\boxed{S_1 - S_2 = 0}$$

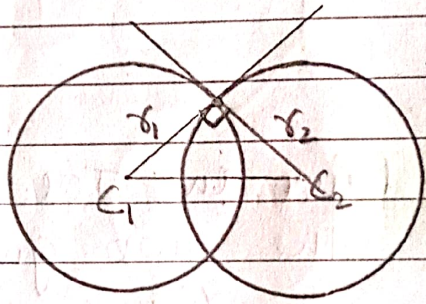
$$8x + 4y + (k + 15) = 0$$

$$8 + 12 + k + 15 = 0$$

$$\boxed{k = -35}$$

(11). Orthogonal Circles :-





θ = Angle of Intersect^n of Tangents.

$S = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S' = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

$\therefore r_1^2 + r_2^2 = (C_1C_2)^2$

and also, $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

Q. Check whether the two given circles are Orthogonal to each other for which valu[e]
k ?

Sol^n. $2\left[\left(\frac{k}{2}\right)(-1) + 2(2)\left(-\frac{3}{4}\right)\right] = 2 + \frac{k}{2}$

$$\Rightarrow \boxed{k = \left(\frac{-10}{3}\right)}$$