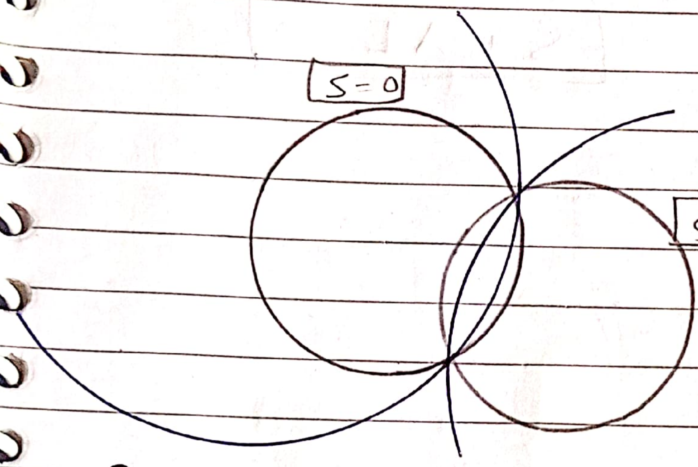
(12) Family of Circles :-

Case - I) Family of circles passing ~~to them~~ through point of Intersection of 2 circle



$$\boxed{S + \lambda S' = 0}$$

Q. $C_1 = x^2+y^2-6x+2y+4=0$; $C_2 = x^2+y^2+2x-4y-6=$
Find a circle passing through point of Intersection of $\boxed{S=0}$ & $\boxed{S'=0}$ and origin

Sol^n $S+\lambda S'=0$

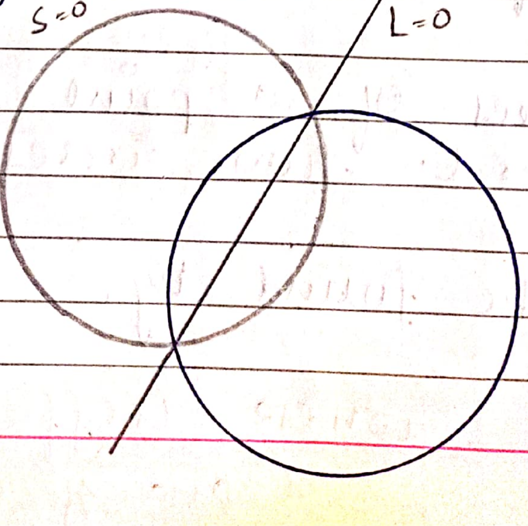
$\Rightarrow (x^2+y^2-6x+2y+4) + \lambda(x^2+y^2-6+2x-4y)=0$

Since, the circle formed is passing through

$\Rightarrow$ origin :-

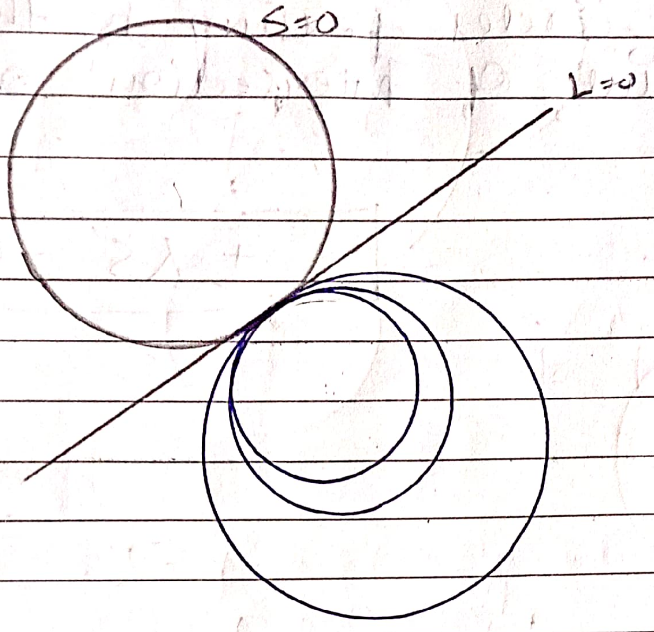
$\lambda(-6)+4=0 \Rightarrow \boxed{\lambda = +2/3}$

Case - II) Family of circles passing through P.O.I. of circle and one line.



$$\boxed{S + \lambda L = 0}$$

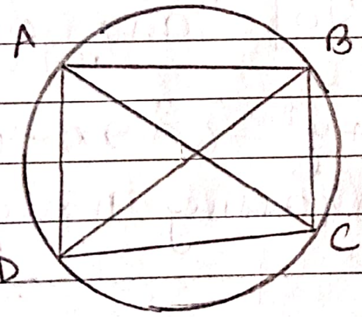
Case – III) Family of circles of touching the P.O.I. of circle and straight line.



$$\boxed{S + \lambda L = 0}$$

(13) Ptolemy's Theorem :–

$$(AB)(CD) + (BC)(AD) = AC \times BD$$



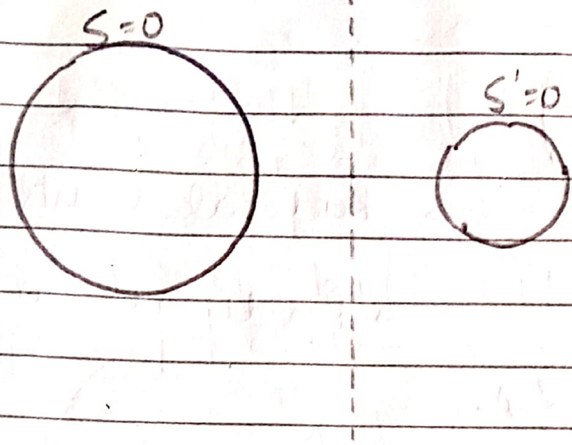
(14) Power of a Point :–
for a circle, $S = 0$, $P(x_1, y_1)$,

$S_1 \longrightarrow$ Power of a point.

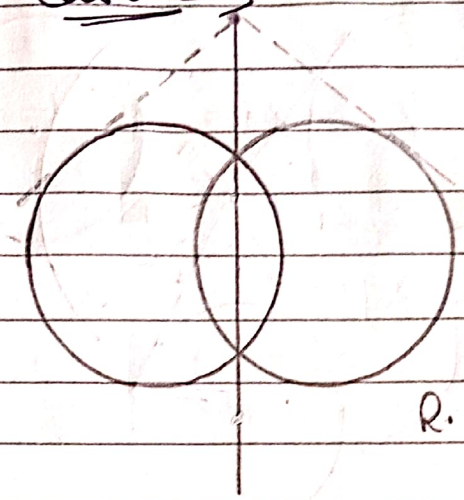
(15) Radical Axis :– Locus of a point such that powers w.r.t. two circles are equal.
Its equation can be found by,

$$\boxed{S - S' = 0}$$, where coeff. of x^2 and y^2 are same

Case - I)

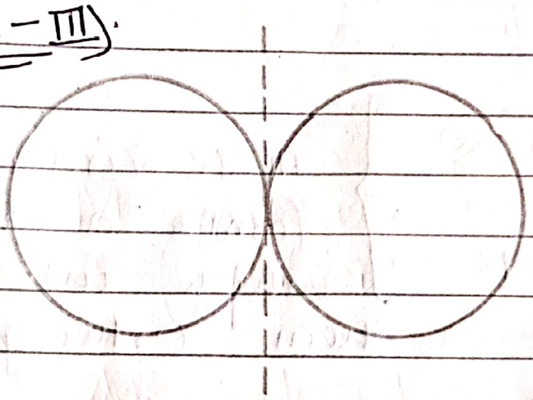


Case - II)

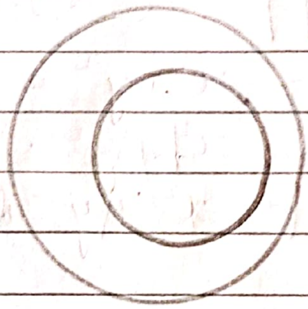


R.A. = Common chord.

Case - III).



Case - IV)



No Radical Axis

When, $\boxed{S_1 = S_1'}$, then the locus of these points form Radical Axis.

Also, Sometimes, like in Case - II), Tangents too get formed, thus,

$$\boxed{\sqrt{S_1} = \sqrt{S_1'}}.$$

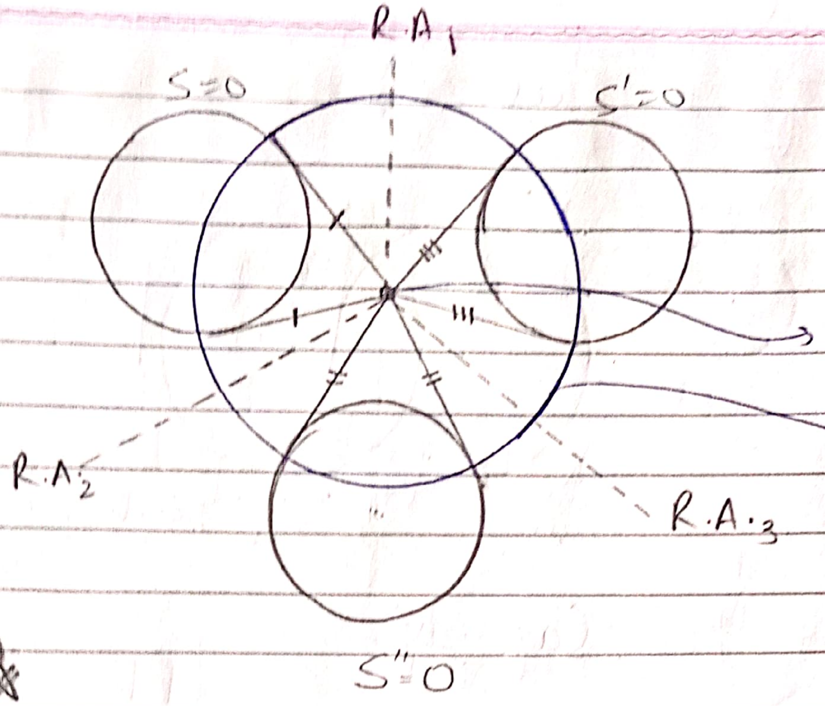
- Properties of Radical Axis :-

(i) Radical Axis is ⊥ar to C_1 & C_2.

V. Imp.

(ii) Radical ~~Axis~~ Circle formed by $S=0$, $S'=0$ and $S''=0$, intersects the circles Orthogonally.

(iii)



V. Imp

Q. $C_1 = x^2+y^2+3x+2y+1=0$
$C_2 = x^2+y^2-x+6y+5=0$
$C_3 = x^2+y^2+8x-8y+15=0$

These circles form a new circle which intersects them orthogonally.

Sol^n.

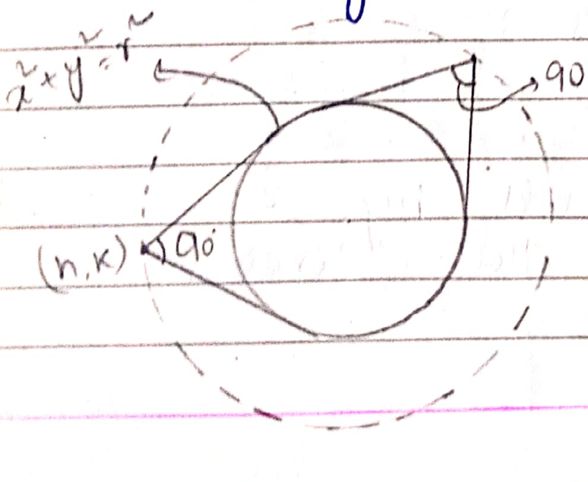
$R.A_1 = x-y-1=0$; $R.A_2 = 3x-7y+5=0$

$R.A_3 = 2x-10y+14=0$

Thus, the Centre of Circle formed = $(3,2)$

So, $\sqrt{S_1}$ = Radius of Circle formed = $\boxed{3\sqrt{3}}$ Ans.

(16) Director Circle :- (May or may not be circle)
Locus of a point such that tangents drawn from it to a circle are always ⊥ar.



Since, the equat^n of a pair of Tangents from a point can be written by:-

$(S \cdot S_1) = T^2$

$\therefore (x^2+y^2-r^2)(h^2+k^2-r^2) = (xh+yk-r^2)^2$

For a, pair of straight lines (Tangents) to be perpendicular, $\boxed{a + b = 0}$

x^2 & y^2 → coefficients.

$\therefore (\cancel{h^2} + k^2 - r^2 - \cancel{h^2}) + (h^2 + \cancel{k^2} - r^2 - \cancel{k^2})$

$\Rightarrow h^2 + k^2 = 2r^2$

$\Rightarrow x^2 + y^2 = 2r^2 \Rightarrow \boxed{(x)^2 + (y)^2 = (\sqrt{2}r)^2}$

Thus, the Director circle formed is Concentric to the Original Circle with $\boxed{\text{radius} = \sqrt{2}r}$

Q. Consider a family of circles passing through two fixed points A(3, 7) & B(6, 5). The chord ~~of~~ in which the circle $x^2 + y^2 - 4x - 6y - 3$ cuts each member of f.o.c. passes through a fixed point (a, b). Then, the value of a + 3b is _______.

Solⁿ

Q. The centre of the circle passing through the points of intersectⁿ of the curves $(2x + 3y + 4)(3x + 2y - 1) = 0$ and $xy = 0$ is?

Solⁿ By $C_1 + \lambda C_2 = 0$, we have:-

$(2x + 3y + 4)(3x + 2y - 1) + \lambda(xy) = 0$ — $\lambda = -13$

$\therefore$ Eqn. of circle :- $6x^2 + 6y^2 + 10x + 5y - 4 = 0$

$\therefore$ Centre → $\left(-\frac{5}{6}, -\frac{5}{12}\right)$.

Q. The locus of the centre of circle which cuts the circle $x^2 - 20x + y^2$ orthogonally and also touches the line $x = 2$ is ______

Sol^n Let the circle be $x^2 + y^2 + 2gx + 2fy + C = 0$

~~for orthogonal intersection $\Rightarrow 2(-10)g + 2f(0) = C + 4$~~

$\Rightarrow \boxed{20g + C + 4 = 0}$

& since it touch line $x = 2$, so,

$\left|\dfrac{-g-2}{\sqrt{1}}\right| = \sqrt{g^2+f^2-C} \Rightarrow 4 + 4g = f^2 - C$

$\Rightarrow C = f^2 - 4g - 4$

So, $20g + f^2 - 4g - \cancel{4} + \cancel{4} = 0 \Rightarrow f^2 + 16g = 0$

Since, Centre of the circle $\Rightarrow (-g, -f) \Rightarrow (h, k)$

$\therefore (-f)^2 - 16(-g) = 0 \Rightarrow \boxed{k^2 - 16h = 0}$ → locus of centre of circle.

Q. Let C_1 and C_2 be two circles of radius r_1 & r_2 resp. $(r_1 > r_2)$ touching both the axes. If ~~the~~ circles are orthogonal, then $\dfrac{r_1}{r_2} =$ ______.

(A) $\sqrt{3} + 2$ (C) 2

(B) $\sqrt{2} + 3$ (D) None

Sol^n

Let eqn. of circle be :- $(x - r_1)^2 + (y - r_1)^2 = r_1^2$

and $(x - r_2)^2 + (y - r_2)^2 = r_2^2$

Since, orthogonal, so,

$2(-r_1)(r_2) + 2(-r_1)(-r_2) = r_1^2 + r_2^2$

$\Rightarrow r_1^2 + r_2^2 = 4r_1r_2 \Rightarrow \left(\dfrac{r_1}{r_2}\right) + \left(\dfrac{r_2}{r_1}\right) = 4$

$\Rightarrow t + 1/t = 4 \Rightarrow \boxed{\dfrac{r_1}{r_2} = 2 + \sqrt{3}}$

Q. The radius of the circle touching the line $x + y = 4$ at $(1, 3)$ and intersecting $x^2 + y^2 = 4$ orthogonally is ______.