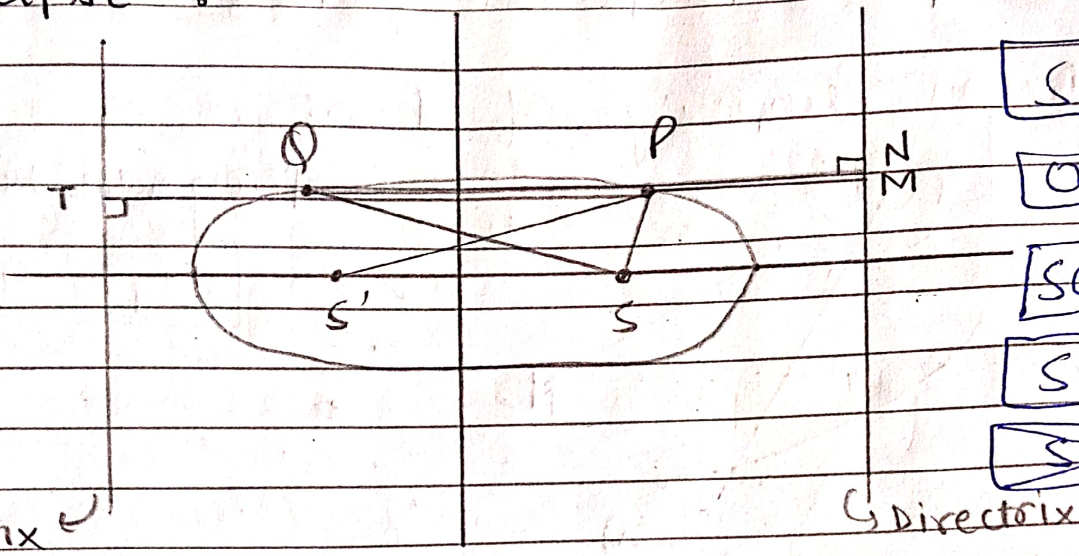
(19) Ellipse :-



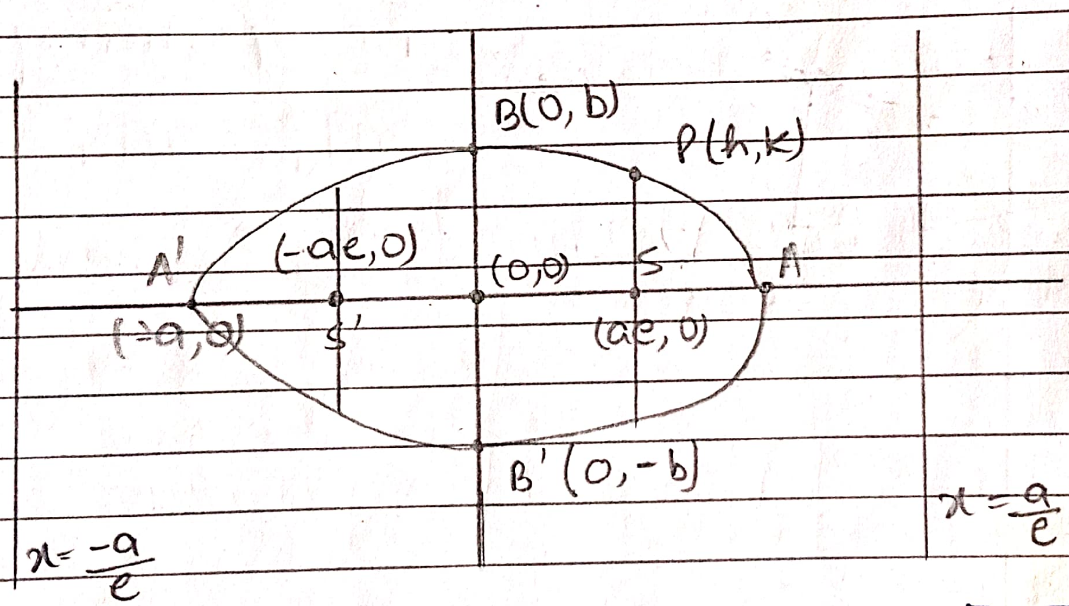
$\boxed{SP = ePM}$

$\boxed{0 < e < 1}$

$\boxed{SQ = eQN}$

$\boxed{S'P = ePT}$

$\boxed{\cancel{SP = ePT}}$



→ General Eqn. Of ~~Parabola~~ Ellipse :- $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$
where, $\boxed{a > b}$

→ Coordinates Of Focii = $S(\pm ae, 0)$

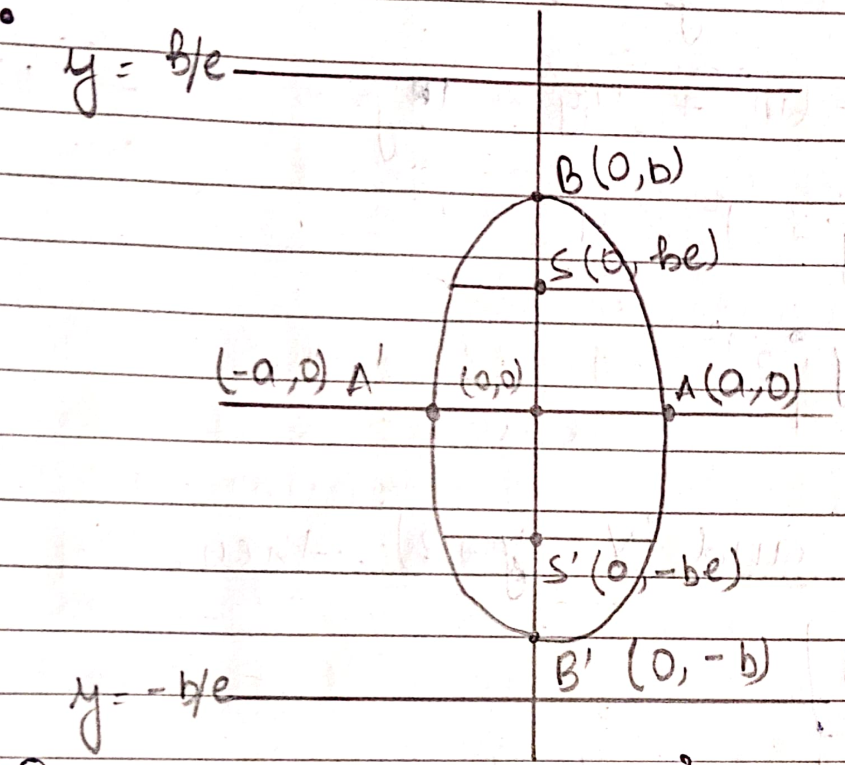
→ Major Axis = $AA' = 2a$ | → Minor Axis = $BB' = 2b$

→ Latus Rectum = $\boxed{\frac{2b^2}{a}}$

→ Eccentricity, $e = \sqrt{1 - \frac{b^2}{a^2}}$

Or, Eccentricity $= \sqrt{1 - \frac{(\text{Semi Minor Axis})^2}{(\text{Semi Major Axis})^2}}$

and Latus Rectum $= \frac{2b^2}{a} = \frac{2(\text{Semi Minor Axis})^2}{\text{Semi Major Axis}}$



$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$ $\boxed{a < b}$

- $e = \sqrt{1 - \frac{a^2}{b^2}}$
- $L.R. = \frac{2a^2}{b}$
- Major Axis $= BB' = 2b$
- Minor Axis $= AA' = 2a$

Q. For the Ellipse, $\frac{x^2}{4} + \frac{y^2}{12} = 1$, find its L.R., Eccentricity, focus, Directrix.

Sol^n Here, on Comparing $\boxed{\frac{x^2}{4} + \frac{y^2}{12} = 1}$ & $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

we get, $\boxed{a = 2}$ and $\boxed{b = 2\sqrt{3}}$. Thus,

focus $= S = \left(0 \pm 2\sqrt{3} \times \frac{\sqrt{2}}{\sqrt{3}}\right) = (0, \pm 2\sqrt{2})$

Directrix $\equiv y = \pm \frac{2\sqrt{3}}{\sqrt{2}} \times \sqrt{3} = \pm \frac{6}{\sqrt{2}} = \boxed{\pm 3\sqrt{2}}$

Eccentricity (e) $= \sqrt{1 - \frac{4}{12}} = \sqrt{\frac{2}{3}}$

$L.R. = \frac{2(4)}{2\sqrt{3}} = \boxed{\frac{4}{\sqrt{3}}}$

Shifted Ellipse :-

Q. Find Major Axis, Minor Axis, eccentricity, Latus Rectum of the Ellipse, $x^2 + 4y^2 + 2x + 16y + 13 = 0$.

Solⁿ. $\Rightarrow x^2 + 4y^2 + 2x + 16y + 13 = 0$

$\Rightarrow x^2 + 2x + (1)^2 - (1)^2 + 4(y^2 + 4y + (2)^2 - (2)^2) + 13$

$\Rightarrow (x+1)^2 + 4(y+2)^2 = 4$

$\Rightarrow \frac{(x+1)^2}{4} + \frac{(y+2)^2}{1} = 1$

Let, $\boxed{X = x+1}$ and $(Y = y+2)$, then,

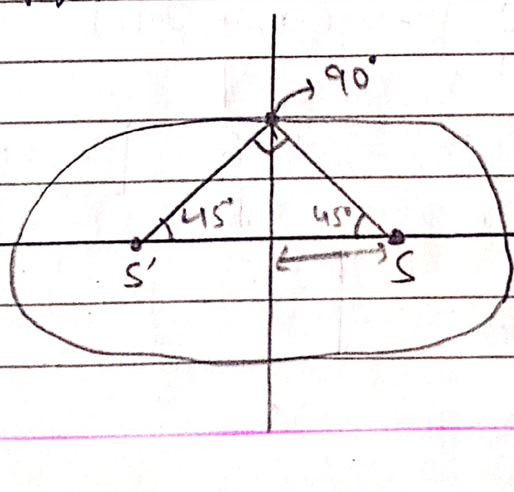
$$\boxed{\frac{X^2}{4} + \frac{Y^2}{1} = 1}$$

$\therefore$ Eccentricity $= \sqrt{1 - \frac{1}{4}} = \boxed{\frac{\sqrt{3}}{2}}$

$\therefore$ Major Axis $= 2 \times 2 = 4$ | $\therefore$ Minor Axis $= 2$

$\therefore$ Latus Rectum $= 1$

Q. Find Eccentricity = ??



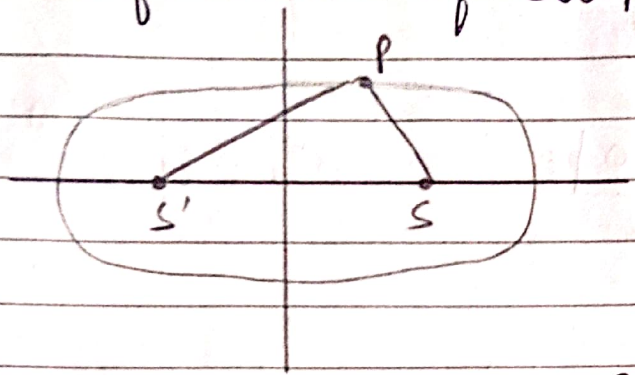
Since, $\boxed{ae = b}$

$\therefore \boxed{e = \frac{b}{a}}$

Also, $e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$

$\Rightarrow e^2 = 1 - e^2 \Rightarrow \boxed{e = \frac{1}{\sqrt{2}}}$

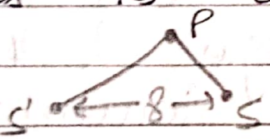
(20) 2nd Defination Of Ellipse :-



If $\boxed{S'P + SP = \text{constant}}$ then, P will move in Elliptical Path, ie. $\boxed{S'P + SP = 2a}$

Q. If a man running in Race Course, having two flagposts, and the sum of distance of flagpost from him is always 10 m and Distance b/w flag-posts is 8 m. Find the equation of Ellipse?

Sol^n.



Since, $S'P + SP = 10 = 2a$

$\therefore \boxed{a = 5}$

Also, $2ae = 8$, then, $\boxed{e = 4/5}$

Now, for 'b', we have :-

$$e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow e^2 = \frac{a^2 - b^2}{a^2} \Rightarrow 16 = 25 - b^2$$

$$\Rightarrow \boxed{b^2 = 9}.$$

$\therefore$ Eqn. of Ellipse $= \boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$

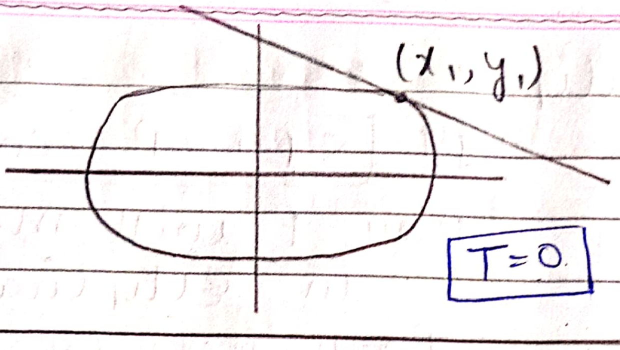
(21) Position Of a Point :-

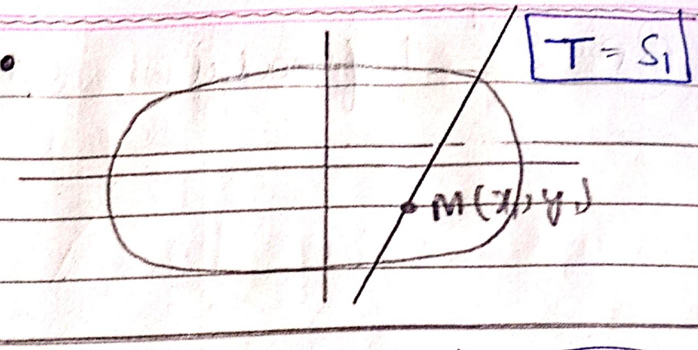
↳ find $\boxed{S_1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1}$,

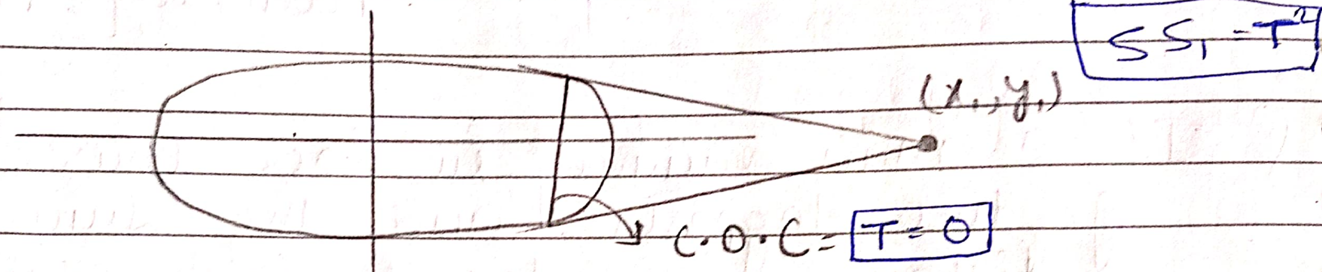
If $S_1 < 0 \longrightarrow$ Inside

$S_1 = 0 \longrightarrow$ On

$S_1 > 0 \longrightarrow$ Outside

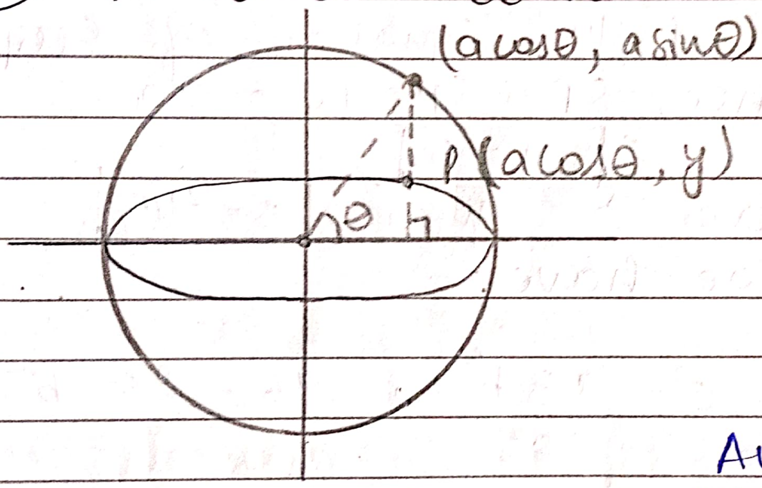
- 

- 

- 

★-4

(22) Parametric Coordinates :-



Circle with Centre at Origin & Radius as Semimajor or Diameter = Major Axis is Called Auxillary Circle, i.e, Auxillary Circle of Ellipse

θ = eccentric Angle

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \Rightarrow \boxed{x^2 + y^2 = a^2}$$

Now, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{(a\cos\theta)^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow y^2 = b^2\sin^2\theta \Rightarrow \boxed{y = b\sin\theta}$

$\therefore$ $\boxed{\text{Parametric Coordinates} = (a\cos\theta, b\sin\theta)}$

- Tangent at Parametric Point 'P' :-

By $\boxed{T=0}$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

$$\therefore \boxed{\frac{x(\cancel{a}\cos\theta)}{a^{\cancel{2}}} + \frac{y\cancel{b}\sin\theta}{b^{\cancel{2}}} = 1}$$

where, $\boxed{\text{Slope} = -\frac{b}{a}\cot\theta}$

~~23~~ Position of line w.r.t. Ellipse :-

For a line, $\boxed{y = mx + C}$ and Ellipse $= \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

$$\Rightarrow \frac{x^2}{a^2} + \frac{(mx+C)^2}{b^2} = 1$$

$$\Rightarrow (b^2 + a^2m^2)x^2 + 2a^2Cmx + (a^2c^2 - a^2b^2) = 0$$

Now, if $\boxed{D = 0}$, then, the line will be Tangent to the Ellipse.

$$\therefore 4a^4m^2c^2 - 4(b^2 - a^2m^2)(a^2c^2 - a^2b^2) = 0$$

$$\Rightarrow \boxed{C = \pm\sqrt{a^2m^2 + b^2}}$$

$$\therefore \boxed{y = mx \pm \sqrt{a^2m^2 + b^2}} \longrightarrow \text{Always Tangent to Ellipse.}$$

Q. If line $\boxed{lx + my + n = 0}$ is tangent to Ellipse $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$, then, prove $\boxed{a^2l^2 + b^2m^2 = n^2}$

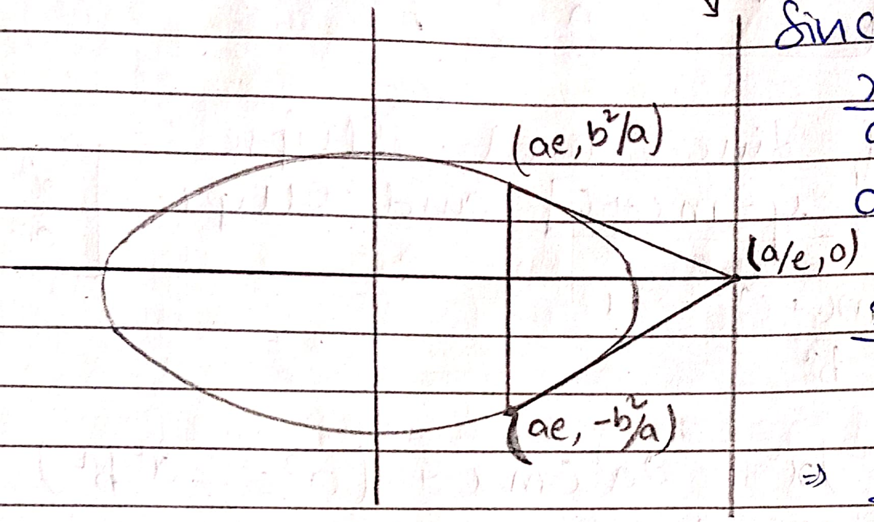
Sol^n: Here, $lx + my + n = 0 \Rightarrow y = -\frac{l}{m}x - \frac{n}{m}$

and also, $C^2 = a^2m^2 + b^2$

$$\Rightarrow \frac{n^2}{m^2} = a^2\left(\frac{l^2}{m^2}\right) + b^2$$

$$\Rightarrow \boxed{n^2 = a^2l^2 + b^2m^2}$$

NOTE :- Tangents at the end points of Latus Rectum intersects at x-axis and at Directrix.



Since, for a Ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, by applying $\boxed{T=0}$

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

$$\Rightarrow \frac{x\,ae}{a^2} + \frac{y\,b^2}{a\,b^2} = 1$$

$$\Rightarrow \boxed{xe + y = a}$$

Also, similarily other eqn., $\boxed{xe - y = a}$, thus

Tangents intersects at $\boxed{x = a/e}$, $\boxed{y=0}$ on Directrix.

(24) Normal to an Ellipse :-

For a Ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, Eqn of Tangent, can be written by, $\boxed{T=0}$, thus,

$$\boxed{\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1} \Rightarrow \boxed{\text{Slope (Tangent)} = \frac{-b^2}{a^2}\frac{x_1}{y_1}}$$

$$\therefore \boxed{\text{Slope(Normal)} \Rightarrow \frac{a^2 y_1}{b^2 x_1}}$$

Now, By Slope - Point form, eqn. of Normal,

$$y - y_1 = \left(\frac{a^2 y_1}{b^2 x_1}\right)(x - x_1)$$

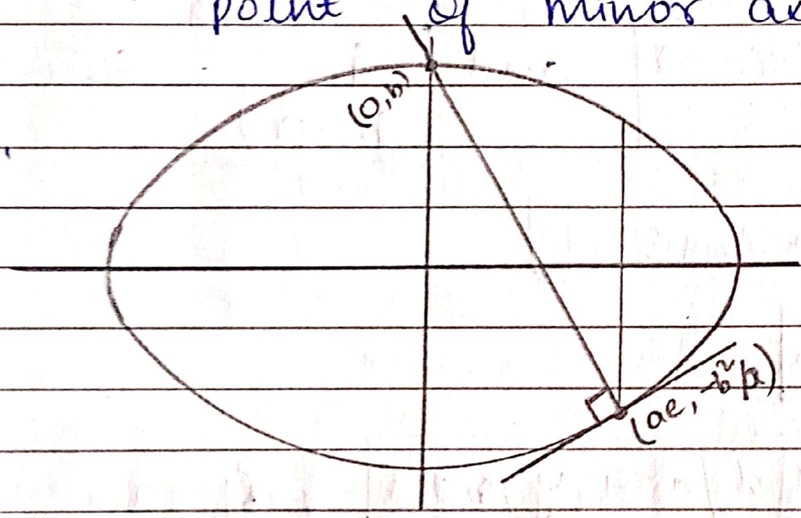
$\Rightarrow b^2x_1y - y_1b^2x_1 = a^2y_1x - a^2x_1y_1$

$$\Rightarrow \boxed{\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2}.$$

Now, $(x_1, y_1) = (a\cos\theta, b\sin\theta)$, assumed, then

$$\frac{a^2x}{a\cos\theta} - \frac{b^2y}{b\sin\theta} = a^2 - b^2 \Rightarrow \underbrace{\boxed{ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2}}_{\text{Eqn. Of Normal.}}$$

NOTE:- If a Normal is drawn at the end point Of Latus Rectum, then, it passes through end point Of minor axis.



By general eqn. Of Normal

$$\Rightarrow \frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

$$\Rightarrow \frac{a^2x}{ae} + \frac{b^2y}{b^2/a} = a^2 - b^2$$

$$\Rightarrow 0 + ab = a^2 - b^2$$

$$\Rightarrow \frac{b}{a} = 1 - \frac{b^2}{a^2} \Rightarrow \boxed{\frac{b}{a} = e^2}$$

$$\therefore e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - e^4 \Rightarrow \boxed{e^4 + e^2 - 1 = 0}$$

$$\therefore \boxed{e = \sqrt{\frac{\sqrt{5}-1}{2}}}$$

Q. For a Ellipse, $\boxed{\frac{x^2}{16} + \frac{y^2}{9} = 1}$ find the equation of Normal parallel to $\boxed{y = 2x}$?

Sol^n. Let the line be $\boxed{y = 2x + C}$ and let the point of contact of Normal and Ellipse be a $(a\cos\theta, b\sin\theta)$. So,

$$\boxed{ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2}$$

and $\boxed{2x - y + C = 0}$ be represent same line.

Thus, $\frac{a\sec\theta}{2} = \frac{-b\,\mathrm{cosec}\,\theta}{-1} = \frac{b^2 - a^2}{C}$

$\Rightarrow \boxed{\frac{aC}{2(b^2 - a^2)} = \cos\theta}$ and $\boxed{\frac{bC}{(b^2 - a^2)} = \sin\theta}$

Since, $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$\therefore \boxed{C = \pm 7/\sqrt{3}}$

Q. For an Ellipse, $\boxed{x^2/a^2 + y^2/b^2 = 1}$, if an COC_1 from $P(x_1, y_1)$ and COC_2 from $Q(x_2, y_2)$ intersect at Right angles i.e. $COC_1 \perp C.O.C_2$ Then, find the value of $\frac{x_1 x_2}{y_1 y_2} = ??$

Sol^n. From $\boxed{T = 0}$, $\boxed{\frac{x x_1}{a^2} + \frac{y y_1}{b^2} = 1}$

and $\boxed{\frac{x x_2}{a^2} + \frac{y y_2}{b^2} = 1}$

$\therefore \left(\frac{-x_1}{a^2} \cdot \frac{b^2}{y_1}\right) \cdot \left(\frac{-x_2 b^2}{a^2 y_2}\right) = -1 \Rightarrow \left(\frac{x_1 x_2}{y_1 y_2}\right) = \frac{-a^4}{b^3}$

25 Director Circle (Ellipse) :-

For an Ellipse, $\boxed{x^2/a^2 + y^2/b^2 = 1}$ by $\boxed{SS_1 = T^2}$

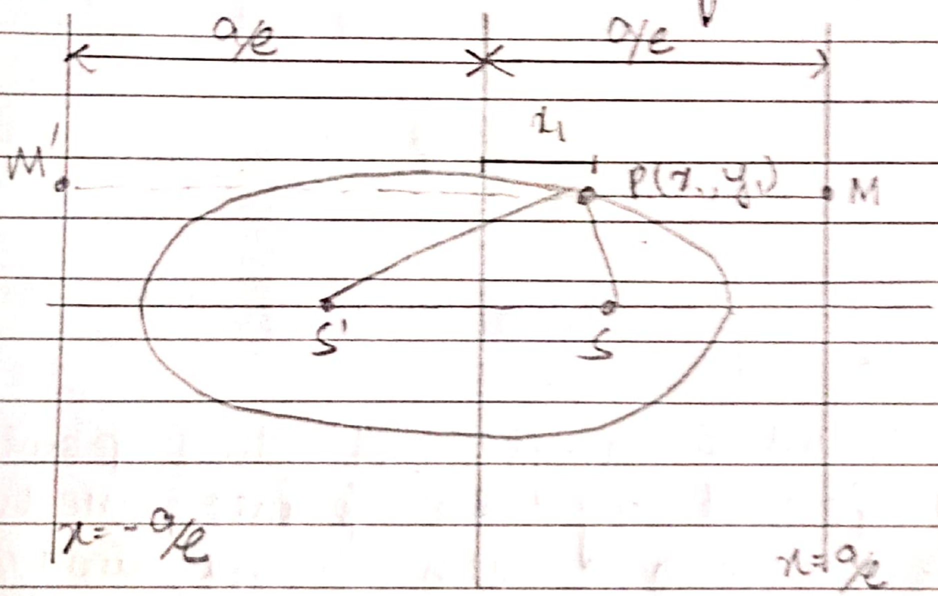
$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) = \left(\frac{hx}{a^2} + \frac{ky}{b^2} - 1\right)^2$$

$\therefore$ Coefficient of x^2 + Coeff. of $y^2 = 0$

$$\Rightarrow \left(\frac{-h^2}{a^2}\right)\left(\frac{1}{a^2}\right)\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) + \left(\frac{1}{b^2}\right)\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) - \frac{k^2}{b^4} = 0$$

$$\Rightarrow \boxed{h^2 + k^2 = a^2 + b^2} \Longrightarrow \boxed{x^2 + y^2 = a^2 + b^2}$$

26 Focal Distance of a point on Ellipse :-



Here,

$(SP) = e(PM)$

$= e\left(\frac{a}{e} - x_1\right)$

$\boxed{SP = a - ex_1}$

Also,

$(S'P) = e(PM')$

$(S'P) = e\left(\frac{a}{e} + x_1\right)$

$\boxed{(S'P) = a + ex_1}$

$\therefore \boxed{S'P + SP = 2a}$

where, SP ; $S'P$ $\longrightarrow$ Focal Distance.

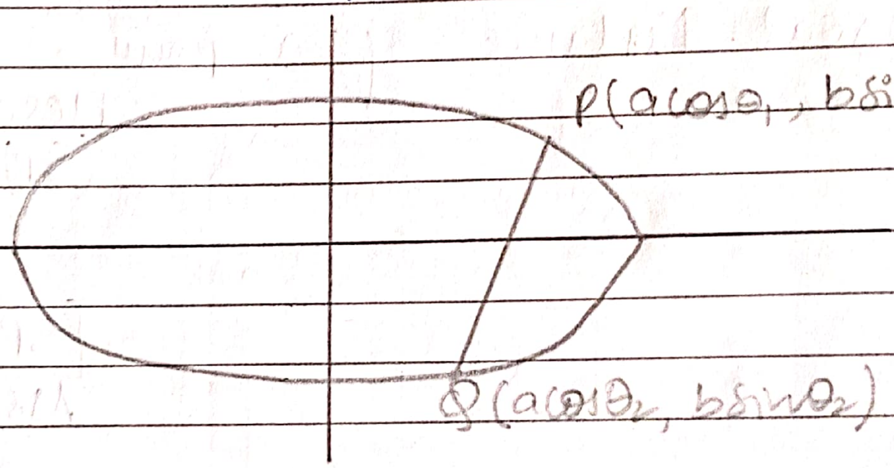
NOTE :- Equation of Chord drawn by joining two Parametric points $P(a\cos\theta_1, b\sin\theta_1)$ and $Q(a\cos\theta_2, b\sin\theta_2)$

$\therefore$ Eqn. of Chord $= \frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right) = \cos\left(\frac{\theta_1-\theta_2}{2}\right)$

NOTE :- The condition which is to be satify when the chord formed by joining two Parametric Points $P(a\cos\theta_1, b\sin\theta_1)$ and $Q(a\cos\theta_2, b\sin\theta_2)$ which passes from focus $(ae, 0)$ or $(-ae, 0)$ is :-

For $\boxed{(ae, 0)} \rightarrow \tan\left(\frac{\theta_1}{2}\right) \cdot \tan\left(\frac{\theta_2}{2}\right) = \frac{e-1}{e+1}$

For $\boxed{(-ae, 0)} \rightarrow \tan\left(\frac{\theta_1}{2}\right) \cdot \tan\left(\frac{\theta_2}{2}\right) = \frac{-e-1}{-e+1} = \frac{e+1}{e-1}$



Q. If α ; β be eccentric angle of two points and the chord formed by the points meets the major axis at a Distance 'd' units. from Centre. Now, find the value of $\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} = ??$

M-1 Since, for two eccentric angles passing through focus, $\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} = \frac{e-1}{e+1}$

Now, multiply & divide by a, thus,

$$\frac{a(e-1)}{a(e+1)} = \frac{ae-a}{ae+a} = \boxed{\frac{d-a}{d+a}}$$ Ans.

M-2 $\frac{x}{a}\cos\left(\frac{\theta_1+\theta_2}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1-\theta_2}{2}\right)$

Since, it passes from $(d, 0)$, thus,

$$\Rightarrow \frac{d}{a}\cos\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

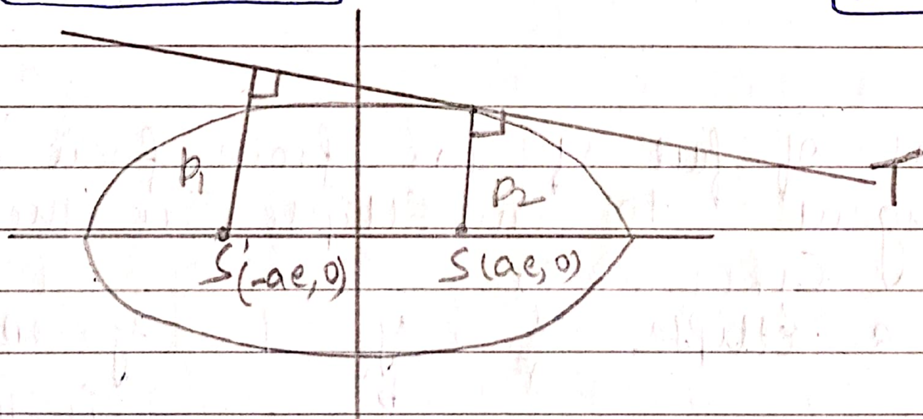
$$\Rightarrow \frac{d}{a}=\frac{\cos\left(\frac{\theta_1-\theta_2}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)}$$

Now, by componendo & dividendo, method

Ans. $$\frac{d+a}{d-a}=\frac{\cos\left(\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)-\cos\left(\frac{\alpha+\beta}{2}\right)}=\boxed{\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$$

NOTE:- If perpendiculars are drawn from the two focus of an Ellipse to a Tangent, then

$$\boxed{P_1 \cdot P_2 = b^2} \rightarrow \text{applicable for } \boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$



V. Imp.

Q. Find locus of Centre of Ellipse sliding b/w 2 ⊥ar axis.

Sol^n. $\frac{x_1+x_2}{2}=h \Rightarrow \boxed{x_1+x_2=2h}$

& $\frac{y_1+y_2}{2}=k \Rightarrow \boxed{y_1+y_2=2k}$

Also, $(x_1 - x_2)^2 + (y_1 - y_2)^2 = (2ae)^2$

$\Rightarrow (x_1+x_2)^2 - 4x_1x_2 + (y_1+y_2)^2 - 4y_1y_2 = 4a^2e^2$

$\Rightarrow (2h)^2 + (2k)^2 = 4a^2e^2 + 4(x_1x_2 + y_1y_2)$

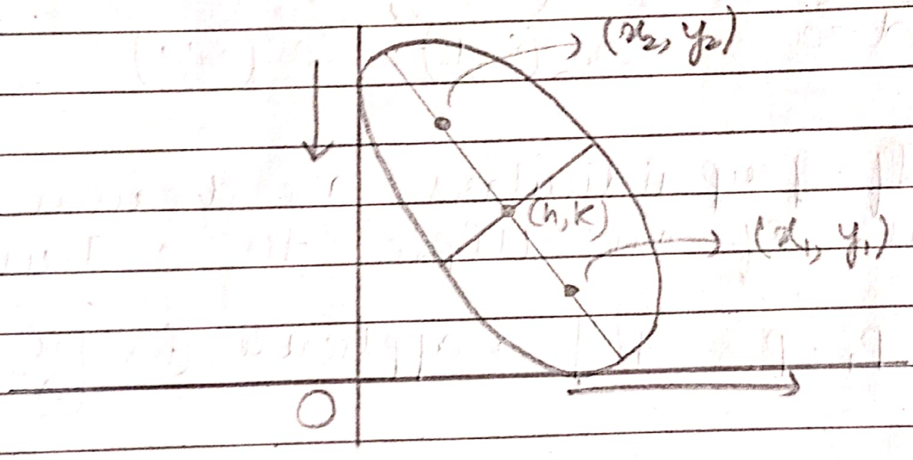
$\Rightarrow h^2 + k^2 = a^2e^2 + (x_1x_2 + y_1y_2)$

$\Rightarrow h^2 + k^2 = a^2e^2 + (b^2 + b^2)$

$\Rightarrow x^2 + y^2 = a^2e^2 + b^2 + b^2$

Since, $e^2 = a^2 - b^2/a^2$, thus, $\boxed{a^2e^2 = a^2 - b^2}$

$\therefore \boxed{x^2 + y^2 = a^2 + b^2}$ Ans.



(27) Locus of foot of ⊥ar from focii to any Tangent to an Ellipse is the AUXILLARY CIRCLE :-

For a Ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $y = mx \pm \sqrt{a^2m^2 + b^2}$ is the eqn. of Tangent.

$\therefore \boxed{y = -\frac{1}{m}x + K}$ → Eqn. of ⊥ Tangent

Since, it passes from $(ae, 0)$, thus,

$0 = -\frac{ae}{m} + k \Rightarrow \boxed{k = \frac{ae}{m}} \Rightarrow \boxed{y = \frac{-1}{m}x + \frac{ae}{m}}$

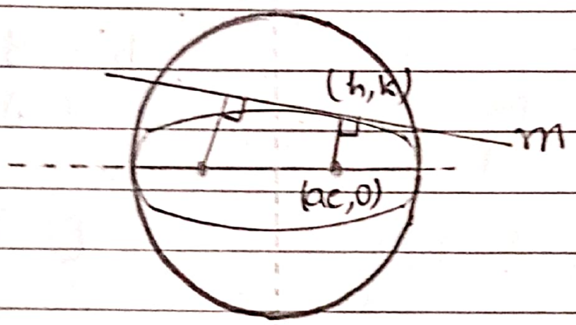
Since, $y = mx \pm \sqrt{a^2m^2+b^2}$ & $\boxed{my = -x + ae}$ intersect at (h,k), thus,

→ $k = mh \pm \sqrt{a^2m^2+b^2} \Rightarrow (k-mh)^2 = a^2m^2+b^2$

→ $mk = -h + ae \Rightarrow (mk+h)^2 = (ae)^2$

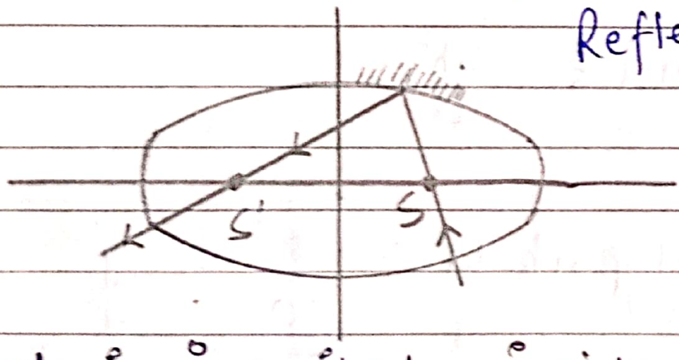
By adding both eqns. ; we get :-

$\therefore \boxed{x^2 + y^2 = a^2}$



NOTE :- A ray passing from one focus of the ellipse after striking to the inner surface of the Ellipse must have to pass from the other focus.

Reflection Property.



Q. If a Circle is inscribed inside a Ellipse with its focus as Centre and touching the Ellipse internally. Then, find the radius if $\boxed{a = 5}$ and $\boxed{b = 4}$ for an standard Ellipse.

Sol^n. By the figure shown below, $r = a - ae$

$= 5\left(1 - \frac{3}{5}\right)$

$= 2$ units.