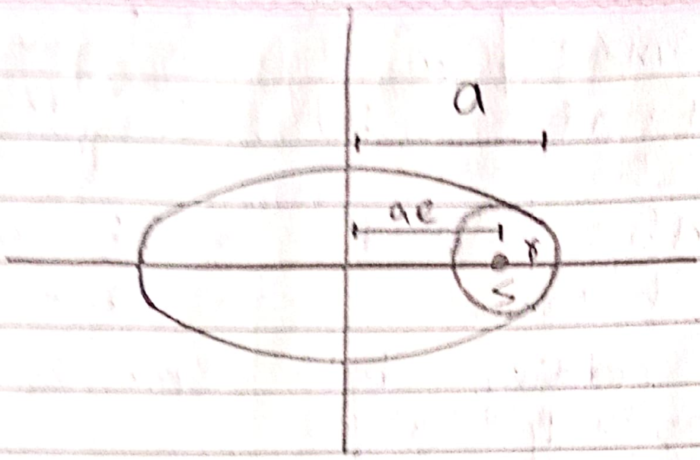
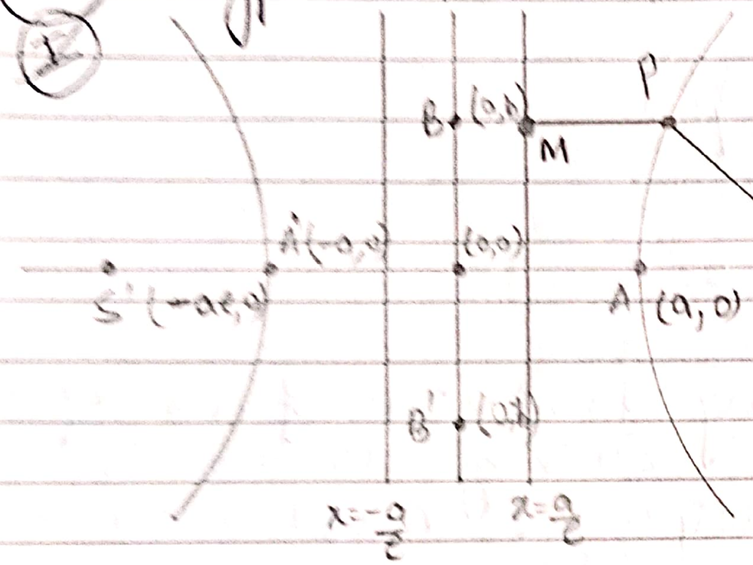


28 Hyperbola :-

I



(2b) = BB' = Conjugate Axis

(2a) = AA' = Transverse Axis

$\boxed{SP = e(PM)}$

$\boxed{S'P = e PM'}$

where $\boxed{e > 1}$

$$\boxed{e = \sqrt{1 + \frac{b^2}{a^2}}}$$

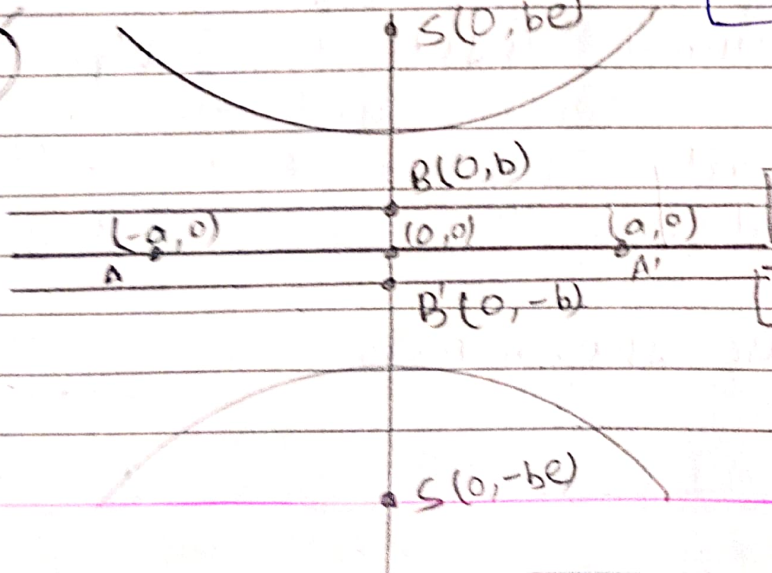
- Latus Rectum :- $\frac{2b^2}{a}$

- Eqn. Of Hyperbola :- $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

Doesnt matter

$\boxed{\begin{matrix} a > b \\ b > a \\ a = b \end{matrix}}$

II



$\boxed{y = \frac{b}{e}}$

$\boxed{y = -b/e}$

$$\boxed{e = \sqrt{1 + \frac{a^2}{b^2}}}$$

$$\boxed{L.R. = \frac{2a^2}{b}}$$

T.I.P. :- $\boxed{\frac{1}{e^2} + \frac{1}{(e')^2} = 1}$ → Valid for conjugate Hyperbola.

• Eqn. of (II) Hyperbola :- $\boxed{-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

or $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1}$

Q. If in a Hyperbola, Conjugate Axis = 5 units & Distance b/w focii = 13 units, then find the eqn. of Hyperbola?

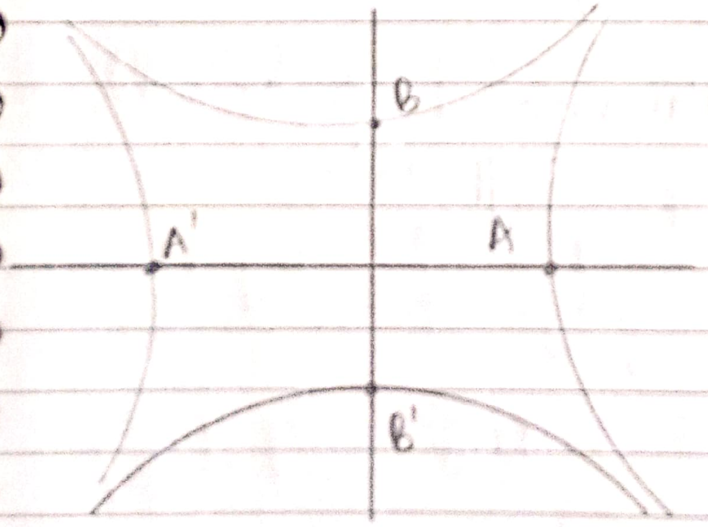
Sol^n. Given that, $2b$ (Conjugate Axis) = 5 → $\boxed{b = \frac{5}{2}}$

and $2ae = 13$ ⇒ $\boxed{a^2e^2 = 169/4}$

Also, $e^2 = \frac{a^2 + b^2}{a^2}$ ⇒ $\boxed{a^2 + b^2 = a^2e^2}$ ⇒ $\boxed{a^2 = 36}$

∴ Eqn. of Hyperbola - $\boxed{\frac{x^2}{36} - \frac{4y^2}{25} = 1}$

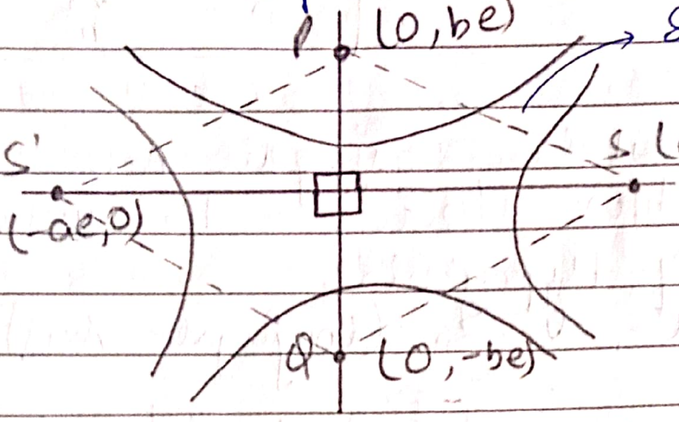
(29) Conjugate Hyperbola :-
A Hyperbola where $\boxed{AA' = C.A.}$ and $\boxed{BB' = T.A}$
i.e. opposite of Original Hyperbola.



$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$ → e

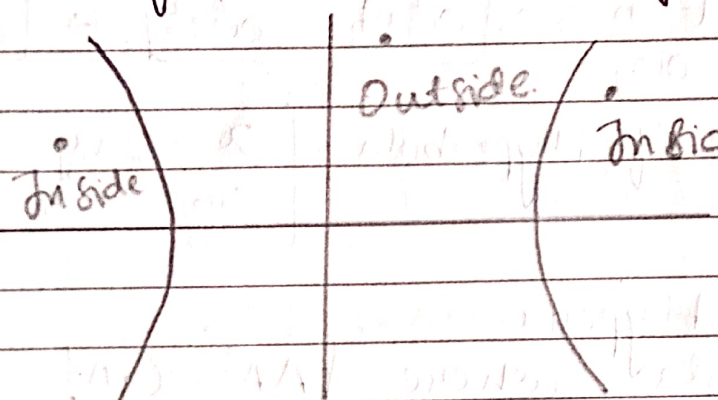
$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1}$ → e'

NOTE :- If Foci Of Hyperbola and Conjugated Hyperbola are joined, it resu in a Square.



$$\boxed{SQ = SP}$$
$$\boxed{SQ \perp SP}$$

(30) Position Of a Point in Hyperbola :-

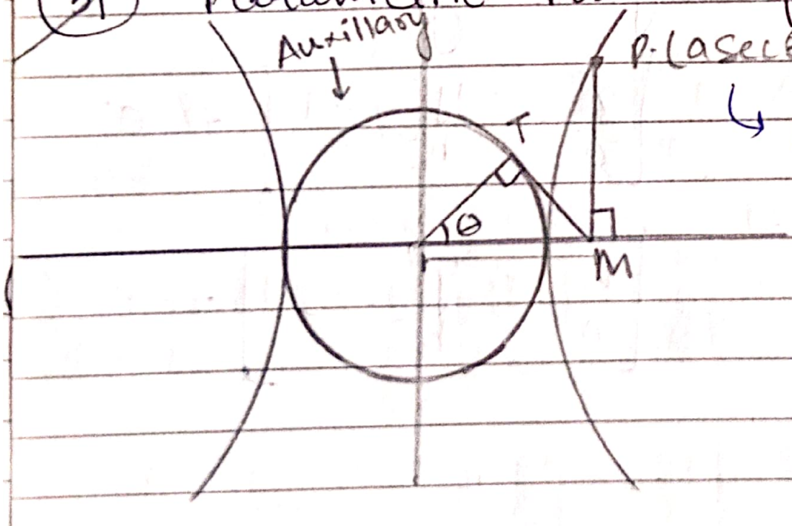


↳ $S_1 < 0$ (Outside)

↳ $S_1 = 0$ (On)

↳ $S_1 > 0$ (Inside)

(31) Parametric Point Of Hyperbola :-



P $(a\sec\theta, b\tan\theta)$

↳ Parametric Point on Hyperbola.

(32) Position of a line w.r.t hyperbola :-

For a line, $\boxed{y = mx + c}$ and hyperbola, $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

$$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1 \Rightarrow (b^2 - a^2m^2)x^2 - 2a^2mcx - (a^2)(c^2+b^2) =$$

For a Tangent at Hyperbola, $\boxed{D = 0}$, thus,

$$\Rightarrow 4a^4m^2c^2 + 4(b^2 - a^2m^2)(a^2c^2 + a^2b^2) = 0$$

$$\Rightarrow \boxed{c^2 = a^2m^2 - b^2}$$

→ condition for Tangency

Only for $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

- For Chord Of Contact, apply $\boxed{T = 0}$

(33) Eqn. Of a Chord Bisected at a point :-

Apply $\boxed{T = S_1}$

(34) Eqn. Of External Tangents (Combined) :-

Apply $\boxed{SS_1 = T^2}$

(35) Director Circle :-

For Hyperbola, $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$, Director Circle is,

$$\boxed{x^2 + y^2 = a^2 - b^2}$$

where, if $a < b$ ⟶ Director Circle not possible

$a = b$ ⟶ Point Circle

$a > b$ ⟶ Real Circle.

T.I.P.:- Director Circle for $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, is

$$\boxed{x^2 + y^2 = b^2 - a^2}$$

(36) Eqn. Of Chord joining θ_1 & θ_2 :-

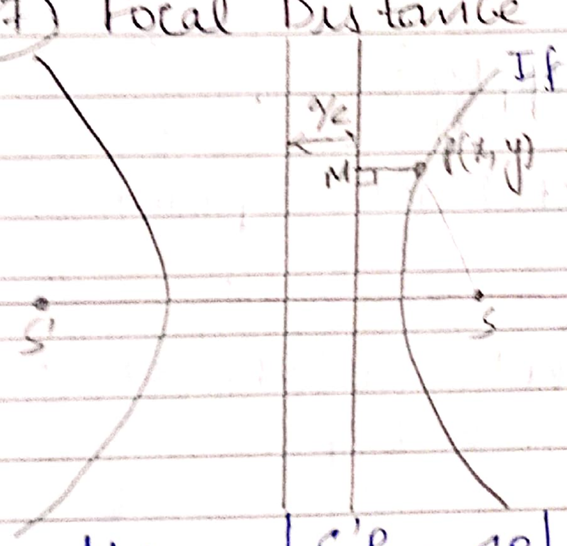
↳ $\frac{x}{a}\cos\left(\frac{\theta_1 - \theta_2}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 + \theta_2}{2}\right)$

If it passes from focal chord, it must satisfy the given Conditions.

If $(ae, 0) \longrightarrow \left(\tan\frac{\theta_1}{2}\right)\left(\tan\frac{\theta_2}{2}\right) = \frac{1-e}{1+e}$

If $(-ae, 0) \rightarrow \left(\tan\frac{\theta_1}{2}\right)\left(\tan\frac{\theta_2}{2}\right) = \frac{1-(-e)}{1+(-e)}$

(37) Focal Distance of a Point :-



If $(ae, 0)$ SP = Focal Distance

$= e(PM)$

$= e\left(\left|x - \frac{a}{e}\right|\right)$

$= \boxed{|ex - a|}$

If $(-ae, 0)$ S'P $= (|ex + a|)$

Also, $|S'P - SP| = 2a$ = Length of Transverse Axis.

(38) Normal to the Hyperbola :-

for Hyperbola, $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$, by $\boxed{T = 0}$,

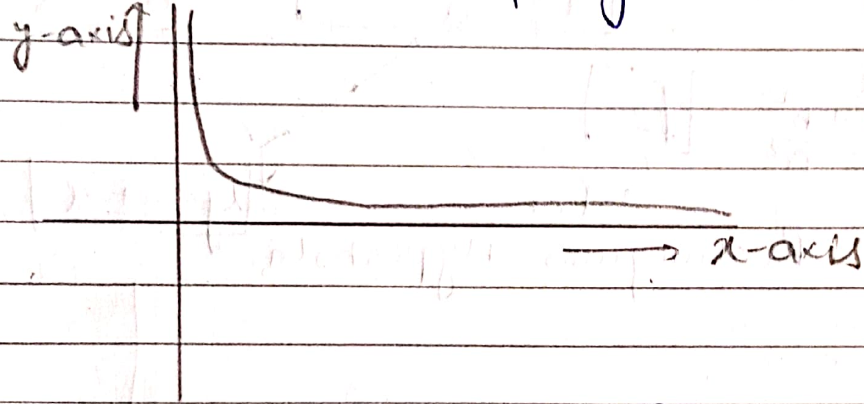
$$\boxed{\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1} \longrightarrow m_N = \frac{-y_1}{x_1}\left(\frac{a^2}{b^2}\right)$$

Now, by Slope - Point form, we get :-

$$\Rightarrow y - y_1 = \frac{-y_1 a^2}{x_1 b^2}(x - x_1) \Rightarrow \frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$$
$$= a^2 e^2$$

If Normal is drawn from Parametric Point then, $\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2 = a^2 e^2 \Rightarrow \boxed{\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2}$

(39) Asymtotes :- Asymptotes are tangents formed at infinity.



- Eqn. Of Asym totes :- For Hyperbola, $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

and line $\boxed{y = mx + c}$,

$$\frac{x^2}{a^2} - \frac{(mx + c)^2}{b^2} = 1 \Rightarrow b^2 x^2 - a^2 (m^2 x^2 + c^2 + 2mcx) = a^2 b^2$$

$$\Rightarrow (b^2 - a^2 m^2) x^2 - 2mca^2 x - (a^2)(c^2 + b^2) = 0$$

$\Rightarrow$ Since, the line meets at ∞, thus, $\boxed{S = \infty}$

$\therefore$ ~~Bco~~ $-2mca^2 = 0 \Rightarrow \boxed{c = 0}$

Also, $b^2 - a^2 m^2 = 0 \Rightarrow \boxed{m = \pm b/a}$

$\therefore$ Eqn. of Tangent $= y = \pm \frac{b}{a} x \Rightarrow \frac{x}{a} \pm \frac{y}{b} = 0$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \Rightarrow \boxed{\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{a} + \frac{y}{b}\right) = 0}$

Combined Eqn.

(10) Rectangular Hyperbola :-

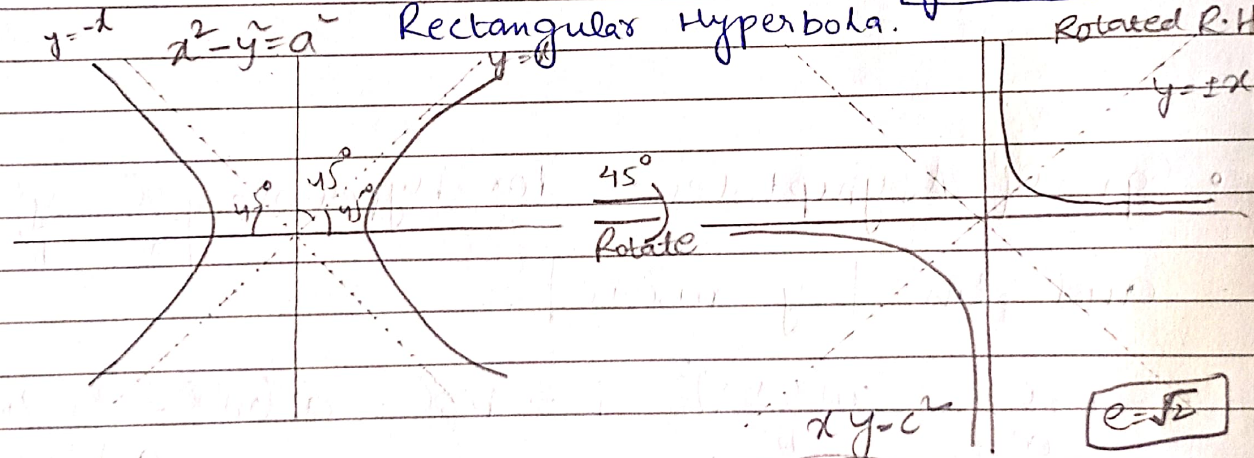
↳ for Hyperbola, $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$, if $\boxed{a = b}$

then, Rectangular Hyperbola is formed, i.e.

↳ $\boxed{x^2 - y^2 = a^2} \Rightarrow \boxed{\frac{x}{a} + \frac{y}{a} = 0}$ & $\boxed{\frac{x}{a} - \frac{y}{a} = 0}$

$\boxed{y = \pm x}$

↳ $e = \sqrt{1 + \frac{b^2}{a^2}} = \boxed{\sqrt{2}}$ ↳ For Rectangular Hyperbola.



$c^2 = a/2$

• For $\boxed{xy = c^2}$, parametric point $= \left(ct, \frac{c}{t}\right)$

• Some
→ Hype
→ Asym
→ Angle
is
of
de
the
→ Sinc

Q. If
Asymp
Som

- Some Important Points on Asmptotes :-

→ Hyperbola & Conjugate Hyperbola have Same asymptotes.

→ Asymptotes pass through centre of Hyp.

→ Angle Bisectors of Asymptotes are the axis of the hyperbola (Vice-Versa is not true) (it is only true in case of Rect. Hyperbola), i.e., Asymptotes do not bisect the axis, but axes, bisect the asymptotes.

→ Since, $H \Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0$

Asym. $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Conjugate $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$

$\boxed{A = \frac{H+C}{2}}$

Q. If Eqn. $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$ represent a hyperbola, then, find the eqn. of Asymptote, conjugate hyperbola, for the given Hyperbola ?

Soln Eqn. of Asymptote will be similiar to

$3x^2 - 5xy - 2y^2 + 5x + 11y + K = 0$ → Represents POSL

$\therefore \Delta = 0 \Rightarrow K = -12$

Thus, Eqn. of Conjugate Hyperbola, $\Rightarrow C = 2A - H$

$\Rightarrow C \Rightarrow 3x^2 - 5xy - 2y^2 + 5x + 11y - 16$

Q. The eqn. $xy - 2x - 3y = 0$ represents a hyperbola; find the eqn. of Asymptote, & Conj. Hyperbola

Sol^n Asymptote's eqn. will be similar to:

$$xy - 2x - 3y + k = 0 \longrightarrow \text{POSL}$$

$\therefore \Delta = 0 \Longrightarrow \boxed{k = 6}$

$\therefore$ Asymptote's eqn. $\Rightarrow \boxed{xy - 2x - 3y + 6 = 0} \to \boxed{y = 2}, \boxed{x = 3}$

(41) Centre of Central Conics :- Circle / Ellipse / Hyperbola

$\boxed{\frac{\partial}{\partial x} = 0}$ & $\boxed{\frac{\partial}{\partial y} = 0} \longrightarrow \boxed{\text{Solve}}$