PERMUTATIONS AND COMBINAT"

#-1

- **Permutations :-** Ways of arranging any data according to its specifications

Q. Arrange 0, 1, 2, 4, 5 as to make a four digit no. when repitition is allowed ?

$\_\_ \quad \_\_ \quad \_\_ \quad \_\_$

$4 \times 5 \times 5 \times 5 \rightarrow 500$ ways

Q. Arrange 0, 1, 2, 4, 5 and find the ways to form a four digit no. when repetition is not allowed ?

Sol^n (case-1) $\_\_ \quad \_\_ \quad \_\_ \quad \_\_$

$4 \times 3 \times 2 \quad 0 \rightarrow 24$

(case-2) $\_\_ \quad \_\_ \quad \_\_ \quad \_\_$

$3 \times 3 \times 2 \quad 5 \rightarrow 18$

$+$

42 ways

(1) **Factorial :-** To arrange 'n' distinct Objects at 'n' different places the no. of ways to arrange is called $n!$

Q. Place 4 Mothers and 4 daughters on 4 chairs such that 1 women holds 1 child in her lap.

Sol^n $\rightarrow$ $(4!) \times (4!) \rightarrow 24 \times 24 \rightarrow 576$

Q. Arrange the alphabets of the word 'KNIFE' such that the vowels are at terminal places.

Sol^n $\underline{I} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{E}$

$\overleftarrow{3 \times 2 \times 1} \times 2 \rightarrow 12$ ways

Q. Arrange 'ORANGE' for vowels at terminal places.

$\underline{\overset{\uparrow}{3}} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\quad} \quad \underline{\overset{\uparrow}{2}}$

$3 \times 4 \times 3 \times 2 \times 1 \times 2 \Rightarrow 144$

Q. Form a 3-digit no. less than 600 from 0, 1, 2, 3, 4, 5, 9

(i) When repetition is not allowed

$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$

$5 \times 6 \times 5 \Rightarrow \boxed{150}$

ii) When repitition is allowed :-

$\underline{\quad} \quad \underline{\quad} \quad \underline{\quad}$

$5 \times 7 \times 7 \rightarrow 245$

iii) When rep. not allowed and divisible by 5.

Case - I) $\underline{\quad} \quad \underline{\quad} \quad \underline{0}$

$5 \times 4 \times 1 \rightarrow 20$

Case - II) $\underline{\quad} \quad \underline{\quad} \quad \underline{5}$

$4 \times 5 \rightarrow 20$

45

iv). Divisible by 5 but rep allowed.

$5 \times 7 \times 2 \Rightarrow (70)$

- Number of ways of Arrangement of Letter of Words with Repitition

For Example :-

(A) IITJEE → $\frac{6!}{2!\,2!}$ ways

(B) VINEET → $\frac{6!}{2!}$

(C) ASSASSINATION → $\frac{13!}{4!\,3!\,2!\,2!}$

- Dictionary Problems :-

(A) RANDOM :-

↳ ~~A D M N O R~~

↳

A ——— → 5! → 120
D ——— → 5! → 120
M ——— → 5! → 120
N ——— → 5! → 120
O ——— → 5! → 120
R A D ——— → 3! → 6
R A M ——— → 3! → 6
R A N D M ——— → 11 → 1
R A N D O M → 0! → 1

$(n-1)^{th}$ rank

(B) MOTHER :-

↳ EHMORT

↳ E _____ → 5!
H _____ → 5!
ME _____ → 4!
MH _____ → 4!
MOE _____ → 3!
MOH _____ → 3!
MOR _____ → 3!
MOTE _____ → 2!
MOTHER → 0!

309th Rank

(C) GAURAV :-

↳ AAGRUV →

↳ A(GARUV) → 5!

GAA _____ → 3!
GAR _____ → 3!
GAUA _____ → 2!
GAURAV → 0!

135th Rank.

(D) VINEET →

↳ EEINTV →

↳ E(_____) → 5!
I(ENEETV) → 5!/2!
N(_____) → 5!/2!
T(_____) → 5!/2!

V E (_ _ _ _) → 4!

V I E (E N T) → 3!

V I N E E T → 0! → 1

331st Rank

(2) Selection / Combination → order does not matter.

n distinct items → Choose 'r' item

$= {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

For Example :-

↳ Place 3 persons out of 10 on 3 Chairs :-

_ _ _ → ${}^{10}C_{3} \times 3! \rightarrow$ 720

- ${}^{n}P_{r}$ formula :-

n Persons → r places

↓

${}^{n}C_{r} \times r! = \frac{n!}{(n-r)! \times r!} \times r!$

$= \frac{n!}{(n-r)!} \rightarrow {}^{n}P_{r}$

Imp

Q. How many words can be formed by the word DELHI such that vowels occupy Odd places?

Sol^n

O E O E O

$[{}^{3}C_{2} \times 2! \times 3!]$

Imp

Q. How many 4-letter words can be formed from word "HISTORY" such that all vowels should be present?

× 2!

I O

HSTRY → $[{}^{5}C_{2} \times 2!]$

∴ Ans → $[{}^{5}C_{2} \times \times 4!]$

Imp

18 persons

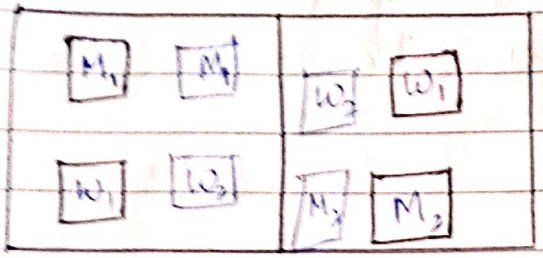
4 persons → V.I.P.

18 Chair

$[{}^{9}C_{4} \times 4! \times 14!]$

V. Imp

10. There are six married couple 6H - 6W. Find the no. of Matches played b/w them such that no husband and wife play in same game.



$${}^{6}C_2 \times {}^{4}C_2 \times 2! \rightarrow \frac{6\times5}{2} \times \frac{4\times3}{2} \times 2$$

$$= 180$$

11. If there are 'n' point on a circle

i) How many straight line can be form-ed by these points :-

$${}^{n}C_2 \times 1 \rightarrow {}^{n}C_2$$

ii). No. of △'s :-

$$\boxed{{}^{n}C_3}$$

V. Imp

12. If there exists 10 points out of which 5 are Co-Linear.

a). How many △'s can be formed

$${}^{10}C_3 - {}^{5}C_3$$

(b) No. of Straight lines :-

$$\left[{}^{10}C_{2+1} - {}^{5}C_{2+1}\right]$$

Imp.

Q. Find the no. of diagonals that can be formed out in an n-sided polygon!

Soln

$$\left[{}^{n}C_2 - n\right]$$

- Bag-Method ; String Method ; Gathri Concept ; Grouping Method :-

For Example :-

→ Arrange three different types of books such as 3P, 2C, 4M such that Books of same subject are together :-

Soln

$$(3!) \times 3! \times 2! \times 4! \rightarrow \text{Ans.}$$

(3!) → for Bags.

→ Arrange 5G-5B on 10 chairs such that 1 Brother - 1 Sister are together :-

(B S) _ _ _ _ _ _ _ _

$9! \times 2!$ → Ans.

• Gap Method :-

↳ for Example :-

→ If there exists 4B - 4G.

(a) Arrange Boys on odd places & Girls on even places.

_ _ _ _ _ _ _ _

$4! \times 4!$ → 576

(b) Place Boys and Girls Alternatively :-

Case - I) B_1 G_1 B_2 G_2 _ _ _ _

Case - II) G_1 B_1 G_2 B_2 _ _ _ _

$4! \times 4! \times 2$

(c) Neither two boys and nor two girls are placed together :-

$4! \times 4! \times 2$

- Cross Method :-

↳ For Example

Q. If there exists 4 Boys and 4 girls.

(a) No two girls are together :-

$$\underline{X}\ B_1\ \underline{X}\ B_2\ \underline{X}\ B_3\ \underline{X}\ B_4\ \underline{X}$$

$${}^5C_4 \times 4! \times 4! \rightarrow \boxed{5 \times 4!}$$

Q. Arrange the letters of the words 'ASSASSINATION' such that :-

(a) All S are together :-

$$(SSSS)\ AAA\ I\ N\ T\ I\ ON$$

$$\boxed{\frac{10!}{3! \times 2! \times 2!} \times \frac{4!}{4!}} \rightarrow \text{Ans}$$

(b) All S are not together :-

$$\left(\begin{matrix}\text{Total no. of} \\ \text{words formed}\end{matrix}\right) - \left(\begin{matrix}\text{words having all} \\ \text{S together}\end{matrix}\right)$$

$$\frac{13!}{4! \times 3! \times 2! \times 2!} - \frac{10!}{3! \times 2! \times 2!} \times \frac{4!}{4!}$$

No two S are together :-

X A X A X A X I X N X T X I X O X N X

$$\left[{}^{10}C_4 \times \frac{4!}{4!} \times \frac{9!}{3! \times 2! \times 2!} \right] \rightarrow \text{Ans}$$

- Formation of groups :-
  - ↳ only on Distinct Items
  - ↳ Distribution ways of certain no. of items into groups.

For Example :-

↳ Arrange 10 items in groups of 2, 3, 5 items :-

$$10 \leftarrow \begin{matrix} 2 \\ 3 \\ 5 \end{matrix} \rightarrow \left[\frac{10!}{2! 3! \times 5!} \right]$$

ways of Distributing 10 items

↳ Arrange 10 items in groups of 2, 2, 2, 4 items :-

$$10 \leftarrow \begin{matrix} 2 \\ 2 \\ 2 \\ 4 \end{matrix} \rightarrow \left[\frac{10!}{2! \cdot 2! \cdot 2! \cdot 4! \, (3!)} \right]$$

Due to repitition

Way

Q. Arrange 6 Books in groups of [4,1,1] and distribute the group with 4 book Ram and the other group of 1-1 Books b/w Sham and Aam?

Sol^{n}

4 → Ram (Pre-decided)

6 < 1, 1] → Anyone can get among Aam & Sham

$$\frac{6!}{4! \times 1! \times 1! \times (2!)} \times 2! \rightarrow 6 \times 5 \rightarrow \boxed{30} \text{ ways}$$

Q. Arrange 8 different people in 3 diff teams such that each team contains atleast 2 members.

8 < 3, 3, 2 **or** 8 < 3, 2, 3

$$\frac{8!}{2! \times 2! \times 4! (2!)} + \frac{8!}{(3!)(2!)(3!)(2!)}$$

Q. Arrange 8 different mangoes among A, B, C such that each get atleast one Mango and atmost 4 mangoes.

Sol^{n}

8 < 4, 3, 1 ; 8 < 4, 2, 2 ; 8 < 3, 3, 2

$$\frac{8!}{4!\,3!\,1!} \times 3! + \frac{8!}{4! \times 2!\,2!\,(2!)} \times 3! + \frac{8!\,(3!)}{3! \times 3! \times 2! \times (2!)}$$