- Distribution Of Like Objects (Beggar's Method)

Like → $\boxed{\text{Similiar}}$

10 coins (Similiar) → 3 Beggars (Diff.)

Now, Impose 2 walls → 3 partitions → 3 Beggars.

0 0 0 | 0 0 0 0 | 0 0 0 0 → $\boxed{{}^{10+2}C_2 \text{ ways}}$

Q. Distribute 15 coins b/w 4 Beggars.

3 walls

$^{15+3}C_3 \rightarrow \boxed{^{18}C_3}$ ways

Q. If there are 15 coins and they are to be distributed among 3 beggar such that each should get atleast One coin.

Sol^n Let us first give 1 coin to each

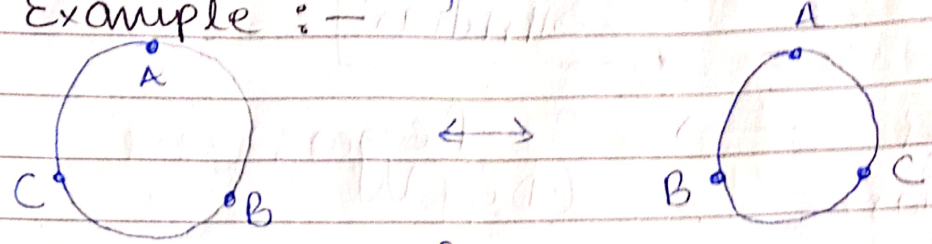
Then, $15-1-1-1 = \boxed{12 \text{ coins}}$

$^{12+2}C_2 \rightarrow \boxed{^{14}C_2}$ ways.

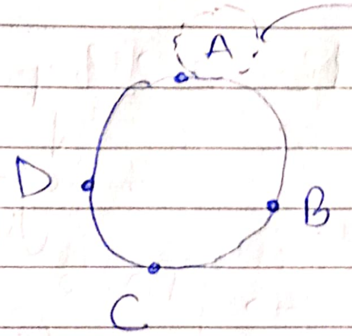
- Circular Permutations :-

↳ Requires 1 frame of Referance.

Example :-



→ $\boxed{2!}$ ways



frame of reference

→ $\boxed{3!\ \text{ways}}$

∴ To place 'n' distinct objects on Circular Table → $\boxed{(n-1)!}$ ways

- Ways of Arranging Beads of Garland

$\boxed{\dfrac{(n-1)!}{2}}$ ways

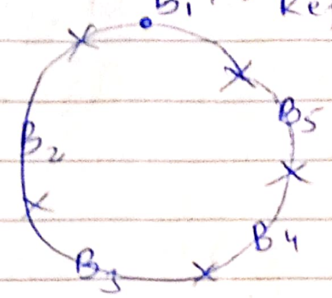
Imp

Q. Find the no. of ways of forming a garland having 6 flowers from a pack of 10 flowers of diff. colours?

Ans → $\boxed{{}^{10}C_6 \times \dfrac{5!}{2}}$ ways.

Q. If there are 5 B & 5 G, then place them on Alternate positions around a circular Table.

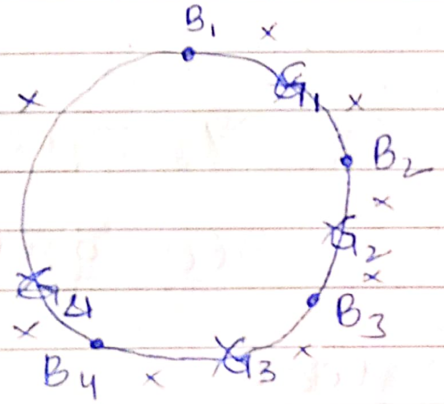
Ans



$\rightarrow \boxed{4! \times 5! \text{ ways}}$

Q. If there are 5 B & 5 G and we have to place 1B-1G - Together as well as other B - G in Alternate positions.

Sol^n



$\boxed{G_5 - B_5}$

$\boxed{3! \times 4! \times {}^8C_1 \times 1}$

Q. Form a four letter words from the alphabets of word 'ASSASSINATION'

(Can come instead in numerals form)

Sol^n Here, $\boxed{S \rightarrow 4}$ $\boxed{A \rightarrow 3}$ $\boxed{I \rightarrow 2}$ $\boxed{N \rightarrow 2}$ $\boxed{T \rightarrow 1}$ $\boxed{O \rightarrow 1}$

Case-I) All letter same → 4 same

x x x x → 1 word (S)

Case-II) 3 S 1 Diff.

x x x y

$^2C_1 \times {}^5C_1 \times 4!/3! \rightarrow 40$

$\boxed{3S}$ $\boxed{1D}$

Case - III) 2S 2D

X X Y W

$^4C_1 \times {}^5C_2 \times \frac{4!}{2!} \rightarrow 480$ words

2S

Case - IV) 2S 2S

X X Y Y

$^4C_2 \times \frac{4!}{2! \times 2!} \rightarrow 36$ words

Case - V) 4 Diff.

W X Y Z

$^6C_4 \times 4! = 360$ words

$\therefore$ Total $\rightarrow$ 917 words

(3) Selectⁿ of Items (One/more/none) :-

If 'n' distinct Objects,

No. of ways of selecting one or more items :-

$= {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 \ldots\ldots + {}^nC_n$

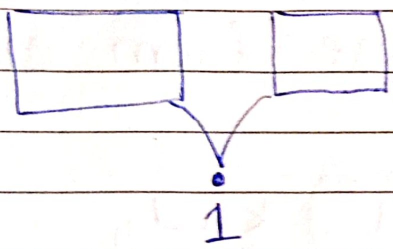
$= 2 \times 2 \times 2 \times 2 \ldots\ldots \rightarrow \boxed{2^n - 1}$

Imp. 1

Q. Place 'n' distinct objects in two identical boxes such that no-box should remain empty.

*

Soln



$\longrightarrow 2\times2\times2\times \cdots \longrightarrow \boxed{\frac{2^n-2}{2}}$

(4) Identical Objects :-

If, n objects $\rightarrow$ r Objects $\rightarrow$ No. of ways = 1

then, No. of Ways of selecting any no. of objects from 'n' identical objects (including selectn of none)

0 objects $\rightarrow$ 1 Way
1 Object $\rightarrow$ 1 Way
2 Object $\rightarrow$ 1 Way
3 Object $\rightarrow$ 1 Way
⋮
n Object $\rightarrow$ 1 way $\rightarrow$ Total no. of ways $\Downarrow$ $n+1$

Q. *
a_1 objects $\rightarrow$ 1st type.
a_2 Objects $\rightarrow$ 2nd type.
a_3 Objects $\rightarrow$ 3rd type.

Select atleast one of $(a_1+a_2+a_3)$ object.

Soln $(a_1+1)(a_2+1)(a_3+1)-1$ ways.

Q. a_1 Object → 1^{st} Kind
* a_2 Objects → 2^{nd} Kind
a_3 Objects → 3^{rd} Kind.

Select atleast one from each category

Sol^n $(a_1)(a_2)(a_3)$

Q. (A) a_1 Objects → 1^{st} Kind
** a_2 " → 2^{nd} "
* a_3 " → 3^{rd} "

K Objects distinct.
Now, select atleast one of $(a_1+a_2+a_3+k)$

Sol^n
$\Rightarrow (a_1+1)(a_2+1)(a_3+1)(2\cdot2\cdot2\cdots2)-1$

$\Rightarrow \boxed{(a_1+1)(a_2+1)(a_3+1)(2^k)-1}$

**
Q. (B) Also, select atleast one of a_1, a_2, a_3
* (each) and atleast one of the distinct.

Sol^n $(a_1)(a_2)(a_3)(2^k-1)$

Q. Find Sum of Numbers formed by
* the digits 2, 3, 5, 7, 8?

$5! \rightarrow 120$ words → Sum

Sol^n
$\_\ \_\ \_\ \_\ \underline{2} = 4! \times 2 \times 1$
$\_\ \_\ \_\ \_\ \underline{5} = 4! \times 5 \times 1$
$\_\ \_\ \_\ \_\ \underline{3} = 4! \times 3 \times 1$
$\_\ \_\ \_\ \_\ \underline{7} = 4! \times 7 \times 1$
$\_\ \_\ \_\ \_\ \underline{8} = 4! \times 8 \times 1$

$\hookrightarrow 4!(2+5+7+3+8)\times 1$

_ _ _ _ 2 _ → $4! \times 2 \times 10$
_ _ _ _ 3 _ → $4! \times 3 \times 10$
_ _ _ _ 5 _ → $4! \times 5 \times 10$
_ _ _ _ 7 _ → $4! \times 7 \times 10$
_ _ _ _ 8 _ → $4! \times 8 \times 10$

Same for other places also,

$$\therefore \text{Sum} = (4!)(2+3+5+7+8)(1+10+100+1000+10000+100000)$$

$$\Rightarrow \boxed{(4!)(25)(111111)}$$

$$\boxed{\text{Trick :- } (n-1)!\,(\text{Sum of Digits})\,(\underbrace{11111111}_{n \text{ times}})}$$

Q. Find the sum of Numbers formed by the digits 0, 1, 2, 3, 5.

Sol^n

_ _ _ _ 0 → $4! \times 0 \times 1 \to 0$
_ _ _ _ 1 → $18 \times 1 \times 1 \to$
3 × 3 × 2 × 1 2 → $18 \times 2 \times 1 \to$
_ _ _ _ 3 → $18 \times 3 \times 1 \to$
_ _ _ _ 5 → $18 \times 5 \times 1 \to$

For Ten's Place,

_ _ _ 0 _ → $4! \times 0 \times 10 \to$
_ _ _ 1 _ → $18 \times 01 \times 10 \to$
_ _ _ 2 _ → $18 \times 2 \times 10 \to$
_ _ _ 3 _ → $18 \times 3 \times 10 \to$
_ _ _ 5 _ → $18 \times 5 \times 10 \to$

$$\text{Sum} \to 4!\,(1+2+3+5) \times 10000 + 18\,(1+2+3+5)\,(1111)$$

(2)

(5) Grid problems :-

V. Imp

Q. If 8 parallel Horizontally and 8 parallel Vertical lines have formed a grid. Then find :-

(1) How many rectangles ??

→ two vertical lines → $^8C_2$

$\times$

two Horizontal lines :- $^8C_2$

$\times 1$

↓

Ans.

(2) How many Squares are possible ??

→ 1×1 side → $7\times7 = 7^2$

2×2 side → $6\times6 = 6^2$

3×3 side → $5\times5 = 5^2$

4×4 side → $4\times4 = 4^2$

5×5 " → $3\times3 = 3^2$

6×6 " → $2\times2 = 2^2$

7×7 " → $1\times1 = 1^2$

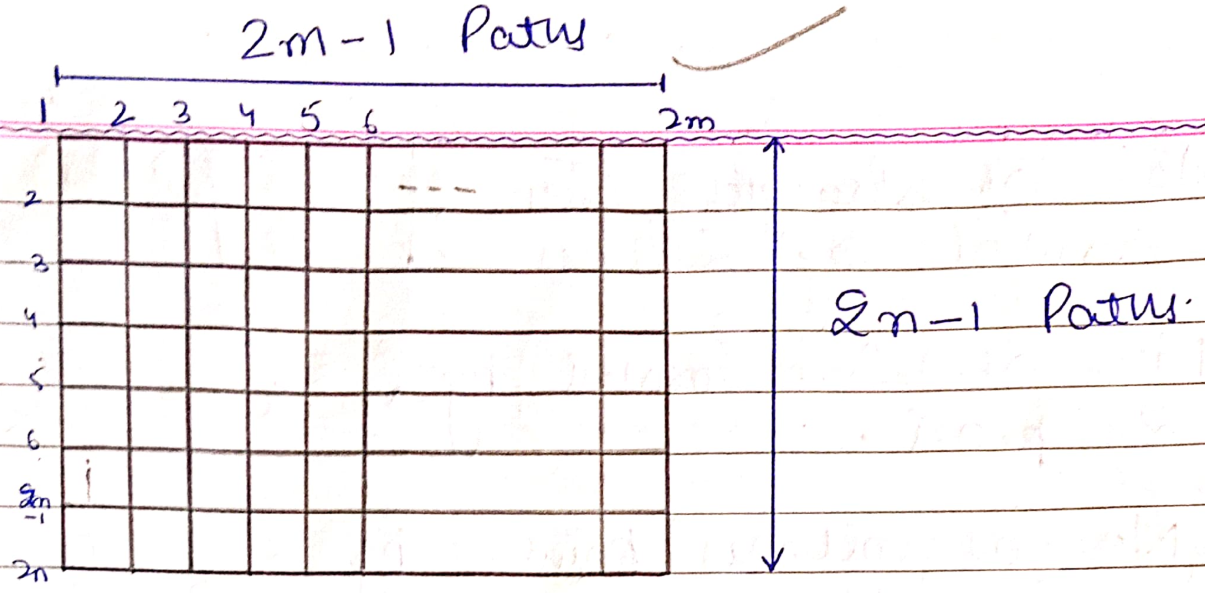
$\boxed{\Sigma 7^2}$ squares

V. Imp

Q. How many rectangles with odd side lengths can be made from

$2m$ → Vertical lines.

$2n$ → Horizontal lines. ?



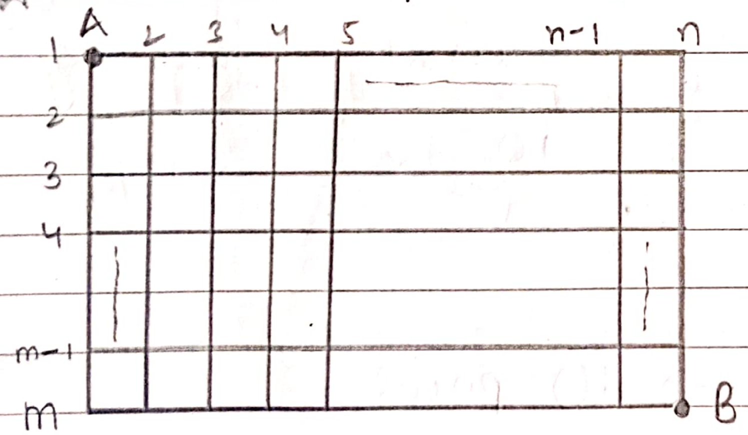
∴ Total no. of Rectangles with Odd side lengths ⇒ ${}^mC_1 \cdot {}^mC_1 \cdot {}^nC_1 \cdot {}^nC_1$

$\downarrow$

$\boxed{m^2 n^2}$

V. Imp.

Q. Find the number of Shortest paths possible b/w A ⟶ B ?



Vertical Paths ⟶ (m − 1) Paths

⟶ Horizontal Paths ⟶ (n − 1) Paths

∴ Total Shortest Paths ⇒ $\dfrac{(m+n-2)!}{(m-1)!\,(n-1)!}$

Imp.

Q. If there are 8 lines in a plane, no two of which are parallel and no three of which are concurrent. How many Δs can be formed with point of intersectⁿ as vertices ?

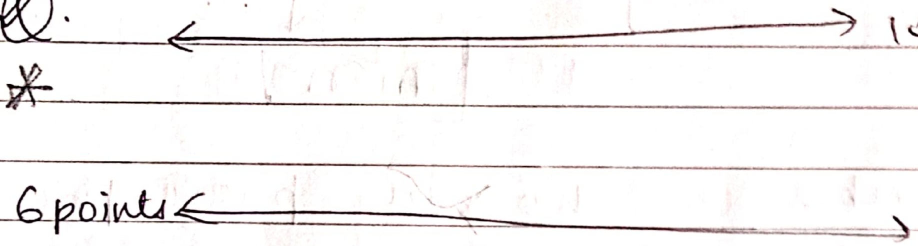
Solⁿ. No. of ~~Straight~~ p.o.i. lines formed by 8 lines → $\boxed{{}^8C_2}$ → 28

No. of Δs formed by 28 p.o.i. → ${}^{28}C_3$

No. of collinear points by 8 lines → $8 \times {}^7C_3$

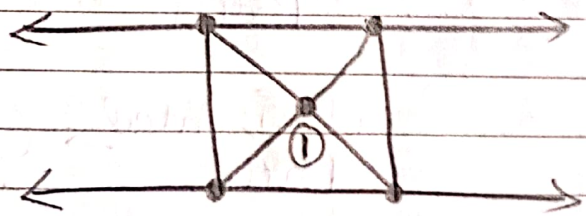
$\therefore \boxed{\text{Total } \Delta s = {}^{28}C_3 - 8\,{}^7C_3}$

Q.



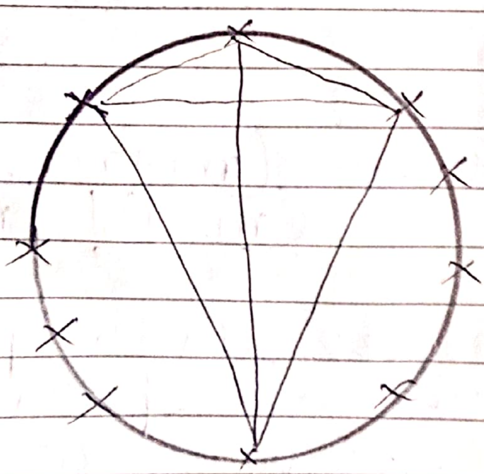
→ How many point of intersectⁿ will you obtain inside this region

Solⁿ.



$\boxed{{}^{10}C_2 \times {}^6C_2 \times 1}$

Q.



→ 10 points

Max. points which can be formed inside the circle?

Solⁿ. $\boxed{{}^{10}C_4 \times 1}$

(6) Divisor Problems :-

(A) Find the no of divisors for $N = 75600$.

Sol^n. $N = 75600$
$= 2^4 \times 3^3 \times 5^2 \times 7^1$

$\Rightarrow (4+1) \times (3+1) \times (2+1) \times (1+1) \Rightarrow 120$ Divisors

(B) No of Divisors divisible by 2 :-

Sol^n. $75600 = 2^4 \times 3^3 \times 5^2 \times 7^1$

$(4)(3+1)(3)(2) \rightarrow 96$ Even Divisor

& Odd Divisors $\rightarrow 120 - 96 \rightarrow 24$ Odd Divisor

(C) No. of Divisors divisible by 6 :-

Sol^n. $4 \times 3 \times 3 \times 2 \rightarrow 72$

- PROPER DIVISORS $\rightarrow$ Divisors excluding 1 & the no. itself

i.e. Total Divisors $-$ 2 $=$ Proper Divisors.

(7) Dearrangement :- No one occupies its orignal position.

- Ways of Dearranging any items:-

$$n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \cdots\cdots (-1)^n \frac{1}{n!}\right]$$

1

Q. Find the no. of ways of Dearranging 3 Letters given to three people :-

Sol^n.

$$3!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right) = \boxed{2}$$

(8) Exponents of prime 'p' in $n!$:-

For $E_p(n!) = \left[\frac{n!}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \left[\frac{n}{p^4}\right] + \ldots$

Q (A) Find the exponents of 2, 3, 6 in $100!$?

Sol^n.

$$E_2(100!) = \left[\frac{100}{2}\right] + \left[\frac{100}{4}\right] + \left[\frac{100}{8}\right] + \left[\frac{100}{16}\right] + \left[\frac{100}{32}\right] + \left[\frac{100}{64}\right] + \ldots$$

$$\boxed{E_2(100!) = 97}$$

$$E_3(100!) = \left[\frac{100}{3}\right] + \left[\frac{100}{9}\right] + \left[\frac{100}{27}\right] + \left[\frac{100}{81}\right] + \ldots$$

$$\boxed{E_3(100!) = 48}$$

For '6', $\boxed{6^{48}}$

(B) Maximum value of K for which $\frac{100!}{2^K}$ is divisible is 97.00.

(C) On expansion of $100!$, how many zeros will be present

Sol^n For Exponents of 10, we need 2 & 5,

So, $2^{97} \times 5^{24} \Rightarrow 2^{73} \times 10^{24}$

→ 24 zeros