SEQUENCES AND SERIES

(1) A.P. → Series of Terms where Difference of consecutive term is constant.

$$\boxed{\text{General Term :- } a_n = \underbrace{a}_{\text{first Term}} + (\underbrace{n}_{\text{no of Terms}} - 1)\underbrace{d}_{\text{c.d.}}}$$

↳ If C. Diff. → +ve → Increasing A.P.
↳ If C. Diff. → −ve → Decreasing A.P.
↳ If C. Diff. → 0 → Constant A.P.

$$\boxed{D = t_n - t_{n-1}}$$

Q. Show that progression log a, log(ab), log (ab^2), log (ab^3) ...... is an A.P., find its general Term :-

Sol^n Here, $a = \log a$

$d = ?? = a_2 - a_1$

$= \log(ab) - \log a$

$= \boxed{\log b}$

$\therefore$ $a_{n^{th}} = \log a + (n-1)\log b$

Q. Which term of the A.P. 30, 29 1/4, 28 1/2, 27 3/4 --- is the first negative term?

Sol^n Here $a = 30$

$d = -3/4$

Let its n^{th} term be first −ve term.

Then, $T_{n^{th}} < 0$

$\Rightarrow a + (n-1)d < 0$

$\Rightarrow 30 + (n-1)(-0.75) < 0$

$\Rightarrow 30.75 < 0.75n$

$\Rightarrow \boxed{n > 41}$ Ans

Q. Find the 10^{th} common term b/w the arithmetic series 3, 7, 11, 15, and 1, 6, 11, 16, 21, ----

Sol^n On Observat^n, the series formed by the two different A.P. is :-

11, 31, 51, -----

$\therefore$ $a = 11$
$d = 20$

$\therefore T_{10} = 11 + (9)20 = \boxed{191}$

Q. If a_1, a_2, a_3, a_4 ---- are in A.P. with common difference 'd', then prove that :-

$$\sin d \,(\operatorname{cosec} a_1 \cdot \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \cdot \operatorname{cosec} a_3 + \text{----} + \operatorname{cosec} a_{n-1} \cdot \operatorname{cosec} a_n)$$ is equal to $\rightarrow$ $(\cot a_1 - \cot a_n)$.

Sol^n

$$\frac{\sin d}{\sin a_1 \cdot \sin a_2} + \frac{\sin d}{\sin a_2 \cdot \sin a_3} + \frac{\sin d}{\sin a_3 \cdot \sin a_4} \text{----} + \frac{\sin d}{\sin a_{n-1} \cdot \sin a_n}$$

$$\Rightarrow \frac{\sin(a_2 - a_1)}{\sin a_1 \cdot \sin a_2} + \frac{\sin(a_3 - a_2)}{\sin a_2 \cdot \sin a_3} - - - - -$$

$$+ \frac{\sin(a_n - a_{n-1})}{\sin a_{n-1} \cdot \sin a_n}$$

$$\Rightarrow \left(\frac{\cos a_1}{\sin a_1} - \cancel{\frac{\cos a_2}{\sin a_2}}\right) + \left(\cancel{\frac{\cos a_2}{\sin a_2}} - \cancel{\frac{\cos a_3}{\sin a_3}}\right)$$

$$+ \left(\cancel{\frac{\cos a_3}{\sin a_3}} - \cancel{\frac{\cos a_4}{\sin a_4}}\right) - - - - + \left(\cancel{\frac{\cos a_{n-1}}{\sin a_{n-1}}} - \frac{\cos}{\sin a}\right.$$

$$\Rightarrow \boxed{\cot a_1 - \cot a_n}$$

Q. If $a_1, a_2, a_3 - - - -$ are in A.P.

* then, show that :-

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + - - - - \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$$

$$= \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$$

Sol^n Rationalision of Denominator :-

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} \times \frac{\sqrt{a_1} \quad \sqrt{a_2}}{\sqrt{a_1} - \sqrt{2}} + \frac{(\sqrt{a_2} - \sqrt{a_3})}{(\sqrt{a_2} + \sqrt{a_3})(\sqrt{a_2} - \sqrt{a_3})}$$

$$- - - - - \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{(\sqrt{a_n} + \sqrt{a_{n-1}})(\sqrt{a_n} - \sqrt{a_{n-1}})}$$

1

$$\Rightarrow \frac{\sqrt{a_2} - \sqrt{a_1}}{(a_2 - a_1)} + \frac{\sqrt{a_3} - \sqrt{a_2}}{(a_3 - a_2)} \cdots$$

$$\frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{(a_n - a_{n-1})}$$

$$\Rightarrow \frac{\sqrt{a_2} - \sqrt{a_1}}{d} + \frac{\sqrt{a_3} - \sqrt{a_2}}{d} \cdots \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{d}$$

$$\Rightarrow \frac{1}{d}\left[\cancel{\sqrt{a_2}} - \sqrt{a_1} + \cancel{\sqrt{a_3}} - \cancel{\sqrt{a_2}} \cdots + \sqrt{a_n} - \cancel{\sqrt{a_{n-1}}}\right]$$

$$\Rightarrow \frac{1}{d}\left[\sqrt{a_n} - a_1\right]$$

$$\Rightarrow \frac{1}{d}\left[\sqrt{a_n} - \sqrt{a_1}\right] \frac{\left[\sqrt{a_n} + \sqrt{a_1}\right]}{\left[\sqrt{a_n} + \sqrt{a_1}\right]}$$

$$\Rightarrow \frac{1}{d}\left[\frac{a_n - a_1}{\sqrt{a_n} + \sqrt{a_1}}\right] \Rightarrow \frac{1}{\cancel{d}}\left[\frac{(n-1)\cancel{d}}{\sqrt{a_n} + \sqrt{a_1}}\right]$$

$$\Rightarrow \frac{(n-1)}{\sqrt{a_n} + \sqrt{a_1}}$$

V. Imp

Q. If $a_1, a_2, a_3 \text{---} a_n$ are in A.P. then, prove that :-

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$$\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + \cdots + \frac{1}{a_{n-1} \cdot a_n} = \frac{n-1}{a_1 a_n}$$

Sol^n $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} ---- + \frac{1}{a_{n-1} \cdot a_n}$

$\Rightarrow \frac{1}{d}\left[\frac{d}{a_1 a_2} + \frac{d}{a_2 a_3} + \frac{d}{a_3 a_4} + \frac{d}{a_{n-1} \cdot a_n}\right.$

$\Rightarrow \frac{1}{d}\left[\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 \cdot a_3} + --- + \frac{a_n - a_{n-1}}{a_{n-1} \cdot a_n}\right.$

$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1} - \cancel{\frac{1}{a_2}} + \cancel{\frac{1}{a_2}} - \frac{1}{a_3} ---- \cancel{\frac{1}{a_{n-1}}} - \frac{1}{a_n}\right.$

$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_n}\right] \Rightarrow \frac{1}{d}\left[\frac{a_n - a_1}{a_1 a_n}\right]$

$\Rightarrow \boxed{\frac{(n-1)}{a_1 a_n}}$

Hence, Proved.

Q. If $m T_m = n T_n$. Then, find $T_{m+n} = ?$

Sol^n Given that :-

$\Rightarrow m(a + (m-1)d) = n(a + (n-1)d)$

$\Rightarrow ma + (m^2 - m)d = na + (n^2 - n)d$

$\Rightarrow (ma - na) + (m^2 - m + n - n^2)d = 0$

$\Rightarrow a(m-n) + [(m-n)(m+n) - (m-n)]d = 0$

$\Rightarrow [a + (m+n-1)d]\,(\cancel{m-n}) = 0$

$\therefore \boxed{T_{m+n} = 0}$

- Important Properties of an A.P.

↳ If a constant is added to each term of an A.P. then the resulting progression is an A.P.

↳ If a constant is subtracted from each term of an A.P. then the resulting progression is A.P.

↳ If each term of an A.P. is multiplied or divided by the same non-zero no. then, the resulting progression is A.P.

Imp. ↳ In a finite A.P., sum of first terms equidistant from beginning and end always remain constant and is equal to sum of I^{st} and last term.

→ For Example :-

V. Imp. For an A.P. such as :-

$a_1, a_2, a_3, a_4 \text{ ------ } a_{n-2}, a_{n-1}, a_n$

$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2}$

$= 2 \times \text{Middle Term}$

$= \text{Sum of two M.T.}$

(2) Sum of n^{th} Terms of an A.P. :-

$$S_{n^{th}} = \frac{n}{2}\left[2a + (n-1)d\right]$$

$\boxed{\text{OR}}$

$$S_{n^{th}} = \frac{n}{2}\left[a + l\right]$$

Q. Sum of natural nos. which are multiples of 5 and lie b/w 100 and 1000 ?

Sol^n. The above A.P. formed is :- 105, 110, 115 995

Let a_{nth} be → 995. Then,

$$995 = 105 + (n-1)5$$

$$\boxed{890 = (n-1)5} \Rightarrow \boxed{n = 179}$$

now,

$$S_n = \frac{179}{2}\left[105 + 995\right] \rightarrow 98450$$

Q. Sum of all natural nos which are divisible by 3 but not by 5 from 1 to 100.

Sol^n. 3, 6, 9 ..... 99 → $\boxed{n = 33}$

$$S_{n^{th}} = \frac{33}{2}\left[2(3) + (33-1)3\right] = \boxed{1683}$$

Also, 15, 30, 45, ... 90 → $n = 6$

$$S_6 = \frac{6}{2}\left[30 + (5)15\right] \rightarrow 315$$

∴ $\boxed{\text{Ans} \rightarrow}$ 1368

Q. If $a_m = \frac{1}{n}$ | $a_n = \frac{1}{m}$. Then, prove that, $S_{mn} = \frac{1}{2}(mn+1)$

Soln

$a_m = a + (m-1)d = 1/n$

$a_n = a + (n-1)d = 1/m$

$D\,\cancel{(m-n)} = \frac{\cancel{m-n}}{mn} \Rightarrow \boxed{D = \frac{1}{mn}}$

Then, $A + (m-1)\frac{1}{mn} = \frac{1}{n}$

$\boxed{A = 1/mn}$

$\therefore S_{mn} = \frac{mn}{2}\left[2\left(1/mn\right) + (mn-1)\left(1/mn\right)\right]$

$= \frac{\cancel{mn}}{2}\left[\frac{2+mn-1}{\cancel{mn}}\right]$

$= \boxed{\frac{1+mn}{2}}$

Hence, Proved.

Q. If $40 + 35 + 30 + 25 \text{ ------ } = 165$. Then, find the total no. of n?

Soln $a = 40$; $d = -5$

$S_{mn} = \frac{n}{2}\left[2(40) + (n-1)(-5)\right] = 165$

$\Rightarrow n\,[80 + (-5n) + 5] = 330$

$\Rightarrow n\,[85 - 5n] = 330$

$\Rightarrow n^2 - 17n + 66 = 0$

$\Rightarrow \boxed{n = 6}$ and $\boxed{n = 11}$

Q. Find the value of :- $100^2 - 99^2 + 98^2 - 97^2 + 96^2 ------ + 2^2 - 1^2 = ??$

Sol^n The above A.P. can be written as

$\Rightarrow (100 - 99)(100 + 99) + (98 - 97)(98 + 97) + (96 - 95)(96 + 95) ----$

$\Rightarrow 199 + 195 + 191 + ------ + 3$

$\therefore 199 = 3 + (n-1)4$

$\boxed{n = 50}$

Also, $\frac{50}{2}[199 + 3] \Rightarrow \boxed{5050}$

Q. If $S_{2n} = 3S_n$, then find $\frac{S_{3n}}{S_n} = ??$

Sol^n

$\Rightarrow \frac{\cancel{2}n}{\cancel{2}}[2a + (2n-1)d] = 3\left(\frac{n}{\cancel{2}}\right)[2a + (n-1)d]$

$\Rightarrow 4a + (4n-2)d = 6a + (3n-3)d$

$\Rightarrow 2a + d(3n - 3 - 4n + 2) = 0$

$\Rightarrow 2a = (n+1)d$

$\therefore \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{n}{2}[2a + (n-1)d]}$

$$\frac{S_{3n}}{S_n} = 3\frac{\left[(n+1)\cancel{d} + (3n-1)\cancel{d}\right]}{\left[(n+1)\cancel{d} + (n-1)\cancel{d}\right]}$$

$$= 3\left[\frac{n+1+3n-1}{n+1+n-1}\right] = 3\left(\frac{4n}{2n}\right)$$

$$\therefore \boxed{\frac{S_{3n}}{S_n} = \frac{6}{1}}$$

Q. Prove that :- If $S_n = an + bn^2 \rightarrow$ is an A.P.

Soln

$S_1 = a + b = A_1$ — (1)

$S_2 = 2a + 4b = A_1 + A_2$ — (2)

$S_3 = 3a + 9b = A_1 + A_2 + A_3$ — (3)

Now, $S_2 - S_1 = A_2 = \boxed{a + 3b}$

Also, $S_3 - S_2 = a + 5b = A_3$

And, $A_2 - A_1 = 2b$

$A_3 - A_2 = 2b$

$\therefore$ The S_n is A.P.

V. Imp.

$\boxed{\text{Trick :- If } S_n = an + bn^2 \text{ Then, C.d. } = 2b}$

(3) Selection of terms in A.P. :-

(A) 3 terms $\longrightarrow$ $a-d,\ a,\ a+d$ [C.D. = d]

(B) 4 terms $\rightarrow$ $a-3d,\ a-d,\ a+d,\ a+3d$ [C.D. = 2d]

(C) 5 terms $\rightarrow$ $a-2d,\ a-d,\ a,\ a+d,\ a+2d$ [C.D. → d]

Q. If 3 numbers are in A.P. and their sum = 18, sum of their sq. is 158. Find the largest no.

Sol^n Let the three terms be $a-d$, a, $a+d$

$\therefore (a-d)+(a)+(a+d) = 18$

$\cancel{3}a = \cancel{18}\,6$

$\boxed{a = 6}$

Also, $(6-d)^2 + 6^2 + (6+d)^2 = 158$

$\Rightarrow 36 + d^2 - 12d + 36 + 36 + d^2 + 12d$

$108 + 2d^2 = 158$

$2d^2 = 50$

$\boxed{d = \pm 5}$

$\therefore$ A.P.$_1$ $\longrightarrow$ 1, 6, 11

A.P.$_2$ $\longrightarrow$ 11, 6, 1

Q. Divide 20 into 4 parts such that they are in A.P. Product of Ist and 4th is to " of 2nd and 3rd are in ratio 2:3. Find the A.P.

Sol^n Given,

$(a-3d)+(a-d)+(a+d)+(a+3d) = 20$

$\cancel{4}a = \cancel{20}\,5 \Rightarrow \boxed{a=5}$

Also, $\frac{(5-3d)(5+3d)}{(5-d)(5+d)} = \frac{2}{3}$

$\rightarrow \frac{25-9d^2}{25-d^2} = \frac{2}{3} \longrightarrow \boxed{d = \pm 1}$

Q. If $a_1 + a_2 + a_3 ---- + a_{24} = ??$
and If $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24}$
$= 225$

Soln.
$$\cancel{3}(a_1 + a_{24}) = \cancel{225}\ 75$$
$$\boxed{(a_1 + a_{24}) = 75}$$
$$\therefore S_{24} = \frac{24}{2}[a_1 + a_{24}] = 12 \times 75 = \boxed{900}$$

Q. Consider an A.P. $a_1, a_2, a_3, a_4 ---$ such that $a_3 + a_5 + a_8 = 11$ and $a_4 + a_2 = -2$ then find the value of $a_1 + a_6 + a_7 = ??$

A). -8
B). 5
C) 7
D) 9

Soln. $(a + 2d) + (a + 4d) + (a + 7d) = 11$
$$\Rightarrow \boxed{3a + 13d = 11} \quad \text{--- (1)}$$

Also, $(a + 3d) + (a + d) = -2$
$$(a + 2d) = -1 \quad \text{--- (2)}$$

Now, Multiplying eq. (2) by (3), we have
$$3a + 6d = -3 \quad \text{--- (3)}$$

Now, (1) − (3); we get :-
$$7d = 14$$
$$\boxed{d = 2} \quad \text{and} \quad \boxed{a = -5}$$
$$\therefore \boxed{a_1 + a_6 + a_7 = 7}$$