(4) Geometric Progression :- Series of terms where ratio of consecutive terms will be constant.

i.e. $a, ar, ar^2, ar^3, ar^4, \ldots$

$\boxed{\text{General Term :- } a_n = a \cdot r^{n-1}}$

where,

a = first term

r = Common Ratio

n = no. of ~~factor~~ terms

If $r > 1 \rightarrow$ Increasing G.P.

If $r < 1 \rightarrow$ Alternate +ve & −ve

If $0 < r < 1 \rightarrow$ Decreasing G.P.

Q. If $T_3 = 2$ and $T_6 = -1/4$, then find $T_{10} = ??$

Sol^n

$T_3 = ar^2 = 2$ — (1)

$T_6 = ar^5 = -1/4$ — (2)

(2) ÷ (1)

$r^3 = -1/8 \longrightarrow \boxed{r = -1/2}$

$\therefore a\left(\frac{-1}{2}\right)^2 = 2$

$\Rightarrow a\left(\frac{1}{4}\right) = 2 \longrightarrow \boxed{a = 8}$

Now, $T_{10} = ar^9 = 8\left(\frac{-1}{2}\right)^9 \Rightarrow \boxed{\frac{-1}{64}}$

Q. If $a_p = x$; $a_q = y$; $a_r = z$ and are in G.P. Then, find the value of

$$x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = ??$$

Solⁿ.

$$AR^{p-1} = x$$
$$AR^{q-1} = y$$
$$AR^{r-1} = z$$

$$\Rightarrow x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$$

$$= (AR^{p-1})^{q-r} \cdot (AR^{q-1})^{r-p} \cdot (AR^{r-1})^{p-q}$$

$$= A^{q-r+r-p+p-q} \cdot R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$

V. Imp

$$= A^0 \cdot R^0 = \boxed{1}$$

(4) Sum of G.P. :-

$$S = a + ar + ar^2 + ar^3 \ldots ar^{n-1}$$
$$rS = ar + ar^2 + \ldots ar^n$$

$$\rightarrow S - rS = a - ar^n$$
$$S(1-r) = a(1-r^n)$$

$$\therefore \boxed{S = \frac{a(1-r^n)}{(1-r)}}$$

where, $\boxed{r \neq 1}$

(6) Select^n of terms in G.P.

(A) 3 terms → a/r, a, ar

(B) 4 terms → a/r^3, a/r, ar, ar^3

(C) 5 terms → a/r^2, a/r, a, ar, ar^2

Q. Product of 3 nos. in G.P. → 125. and Sum of products in pairs is 175/2. find nos.

Sol^n $\frac{a}{r}$, a, ar → $\frac{a}{r} \times a \times ar = 125$

→ $\boxed{a^3 = 125}$

$\boxed{a = 5}$

Also,

$$\Rightarrow \frac{a}{r} \cdot a + ar \cdot a + \frac{a}{r} \cdot ar = \frac{175}{2}$$

$$\Rightarrow \frac{a^2}{r} + a^2 r + a^2 = \frac{175}{2}$$

$$\Rightarrow a^2\left(\frac{1}{r} + r + 1\right) = \frac{175}{2}$$

$$\Rightarrow \cancel{25}\left(\frac{r^2 + r + 1}{r}\right) = \frac{\cancel{175}\ 7}{2}$$

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$\boxed{r = 2}$ $\boxed{r = 1/2}$

$5/2, 5, 10$ ← ↳ $10, 5, 5/2$

Q. Four +ve nos. are in G.P. and if their Sum = 85 and P = 4096
find the nos.

Sol^n. Let the four nos. be a, ar, ar^2, ar^3

Then,

$$a \times ar \times ar^2 \times ar^3 = 4096$$

$$a^4 \cdot r^6 = 4096$$

$$a^2 \cdot r^3 = 64$$

$\therefore$ $\boxed{a = 1}$ and $\boxed{r = 4}$

$\therefore$ $\boxed{\text{Numbers} \rightarrow 1, 4, 16, 64}$

Q. $S_n = 255$; $a_n = 128$ and $\boxed{r = 2}$

Then, find $n = ??$

Sol^n.

$$a \cdot r^{n-1} = 128$$

$$a \cdot (2)^{n-1} = 128$$

$$\boxed{a \cdot 2^n = 256} \quad \text{--- } (1)$$

Also, $S_n = 255$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 255$$

$$\Rightarrow a(2^n - 1) = 255$$

$$\Rightarrow a2^n - a = 255$$

$$\Rightarrow \boxed{a = 1} \quad \text{--- } (2)$$

$\therefore$ $\boxed{n = 16}$

Q. Find the sum of :-

$$\left(x+\frac{1}{x}\right)^2, \left(x^2+\frac{1}{x^2}\right)^2, \left(x^3+\frac{1}{x^3}\right)^2 \text{ ---- n terms}$$

Sol^n $\left(x^2+\frac{1}{x^2}+2\right)+\left(x^4+\frac{1}{x^4}+2\right)$ ------

$$\left(x^2+x^4+x^6 \text{ ----}\right)+\left(\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^6} \text{ ----}\right) + 2n$$

$$\Rightarrow \frac{x^2\left((x^2)^n-1\right)}{x^2-1} + \frac{1}{x^2}\left(\left(\frac{1}{x^2}\right)^n-1\right) + 2n$$

Q. If $x, 2x+2, 3x+3$ ---- are in G.P., then find $a_4 = ??$

Sol^n

Common Ratio :- $\frac{2x+2}{x} = \frac{3x+3}{2x+2}$

$\Rightarrow (2x+2)^2 = 3x^2+3x$

$\Rightarrow 4x^2+4+8x = 3x^2+3x$

$\Rightarrow x^2+4x+x+4=0$

$\Rightarrow (x+4)(x+1)=0$

$\Rightarrow \boxed{x \neq -1}\ \boxed{x=-4}$

$\therefore \boxed{r = -3/2}$

Now, $ar^{4-1} = -9 \times \frac{3}{2} = \frac{-27}{2}$

Q. $x(x+y) + x^2(x^2+y^2) + x^3(x^3+y^3)$ ...... n term

Solⁿ $(x^2+xy) + (x^4+x^2y^2) + (x^6+x^3y^3)$ + ... n term

$\Rightarrow (x^2+x^4+x^6+ \ldots\ldots) + (xy + x^2y^2 + x^3y^3$

$\therefore$ Sum $= \dfrac{x^2(x^{2n}-1)}{(x^2-1)} + xy\left(\dfrac{(xy)^n-1}{xy-1}\right)$

(7) Sum of infinite G.P. :-

$$\boxed{S_\infty = \frac{a}{1-r}}, \quad \text{where } \boxed{-1<r<1}$$

Q. Sum of first three consecutive terms of G.P. is 19. And Product = 216. find S_∞.

Solⁿ $\Rightarrow \dfrac{a}{r} \times a \times ar = 216 \rightarrow \boxed{a=6}$

$\Rightarrow \dfrac{a}{r} + a + ar = 19 \Rightarrow a\left(\dfrac{1+r+r^2}{r}\right) = 19$

$\Rightarrow a(1+r+r^2) = 19r$

$\Rightarrow 6(1+r+r^2) = 19r$

$\Rightarrow 6 + 6r + 6r^2 = 19r$

$\Rightarrow \boxed{\cancel{r = 3/2}} \quad \boxed{r = \frac{2}{3}}$

$\Rightarrow \dfrac{a}{r} = \dfrac{6}{2/3}$, 9 $\rightarrow S_\infty = \dfrac{A}{1-r}$

$\boxed{= 27}$

Imp ①

Q. If $S_\infty = 15$ and Sum of their sq. is 45 (S_∞). Find common ratio
*

Sol^n. Given,

$$S_\infty = \frac{a}{1-r} = 15$$

$$\Rightarrow \boxed{\frac{a^2}{(1-r)^2} = 225} \quad — ①$$

Also,

$$S'_\infty = \frac{a^2}{1-r^2} = 45 \quad — ②$$

Now, ② ÷ ①

$$\frac{(1-r)^2}{(1-r)(1+r)} = \frac{45}{225}$$

$$\frac{1-r}{1+r} = \frac{\cancel{45}\ 3}{\cancel{15}\times\cancel{15}\ 5} \Rightarrow 5-5r = \cancel{3+3r}\ 1+r$$

$$\therefore \boxed{r = 2/3}$$

Q. Find the value of :-

$$S = \frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} \text{------}\infty$$

Sol^n.
$$S = \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \text{—}\right) + \left(\frac{1}{3^2} + \frac{1}{3^4} + \text{—}\right.$$

$$= \left(\frac{1/2}{-1/4+1}\right) + \left(\frac{1/3^2}{1-1/9}\right)$$

$$= \frac{2}{3} + \frac{1}{8} = \frac{16+3}{24} = \boxed{\frac{19}{24}}$$

Imp

Q. Find the value of :-

$S = 7 + 77 + 777 + 7777 \ldots n$ terms

Sol^n.

$\Rightarrow 7 [1 + 11 + 111 + 1111 + \ldots]$

$\Rightarrow \frac{7}{9} [9 + 99 + 999 + \ldots]$

$\Rightarrow \frac{7}{9} [(10-1) + (10^2-1) + (10^3-1) \ldots]$

$\Rightarrow \frac{7}{9} \left[\frac{10(10^n-1)}{(10-1)} - n\right]$

Imp

Q. $S = 0.7 + 0.77 + 0.777 + \ldots n$ terms.

Sol^n $\Rightarrow \frac{7}{9} (0.9 + 0.99 + 0.999 + \ldots)$

$\Rightarrow$

$\frac{7}{9} [(1-0.1) + (1-(0.1)^2) + \ldots]$

$\Rightarrow \frac{7}{9} \left[n - \frac{(0.1)(1-(0.1)^n)}{1-0.1}\right]$

Q. $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P.

Then, find the relat^n b/w a, b, c.

Sol^n. Since these are in A.P.

$\therefore 2\left(\frac{1}{2b}\right) = \frac{1}{a+b} + \frac{1}{b+c}$

$\Rightarrow \dfrac{1}{b} = \dfrac{b+c+a+b}{(a+b)(b+c)}$

$\Rightarrow \boxed{b^2 = ac}$

Hence, they are in G.P.

(8) Some Important Properties of G.P. :-

(A) $a_1 k, a_2 k, a_3 k, a_4 k$ ...... $\rightarrow$ G.P. $(r' = r)$

(B) $\dfrac{a_1}{k}, \dfrac{a_2}{k}, \dfrac{a_3}{k}, \dfrac{a_4}{k}$, ... $\rightarrow$ G.P $(r' = r)$

(C) $a_1^k, a_2^k, a_3^k \longrightarrow$ G.P.

where $r' = (r)^k$

(D) Product of terms equidistant from beginning and end, is eq. to product of I^{st} and last terms :-

$a, ar, ar^2 \ldots \ldots ar^{n-3}, ar^{n-2}, ar^{n-1}$

$a \cdot ar^{n-1} = a^2 r^{n-1} = a^2 r^{n-1}$

(E) If a_1, a_2, a_3, a_4 ...... $\rightarrow$ G.P.

Then, $\log a_1, \log a_2, \log a_3$ ...... $\rightarrow$ A.P.

Imp

Q. If $x^a = x^{b/2} \cdot z^{b/2} = z^c$. Then, find the value / relat^n b/w $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$.

Sol^n. $x^a = x^{b/2} \cdot z^{b/2}$

$\Rightarrow \left(\frac{x^a}{z^{b/2}}\right) = z^{b/2} \Rightarrow x^{a-b/2} = z^{b/2}$

$\Rightarrow z = \left(x^{a-b/2}\right)^{2/b} = x^{2a/b - 1}$

Also, $x^{b/2} \cdot z^{b/2} = z^c$

$x^{b/2} = \frac{z^c}{z^{b/2}} = z^{c-b/2}$

$\Rightarrow x^{b/2} = \left(x^{2a/b - 1}\right)^{c-b/2}$

$\Rightarrow x^{b/2} = x^{(2a/b - 1)(c - b/2)}$

$\Rightarrow \frac{b}{2} = \left(\frac{2a}{b} - 1\right)\left(c - \frac{b}{2}\right)$

$\Rightarrow \cancel{\frac{b}{2}} = \frac{2ac}{b} - a - c + \cancel{\frac{b}{2}}$

$\Rightarrow ab + bc = 2ac$

$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.