(9) Harmonic Progression :-

If a_1, a_2, a_3, a_4, a_5 ------ is a series and if their reciprocal are in A.P. then, they in H.P.

i.e.

$$a_1, a_2, a_3, a_4 \text{ ....} \rightarrow \text{H.P.}$$

But $\rightarrow$ $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}, \text{ ....} \rightarrow$ A.P.

$\uparrow$

H.P.

Q. Find the n^{th} term for the series:-

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \text{ ....}$$

Soln. R $\rightarrow$ 2, 5, 8, ---- $\rightarrow$ A.P.

$$\boxed{a_{n^{th}} \rightarrow 3n-1}$$

$\therefore$ N^{th} term of H.P. $\rightarrow \frac{1}{3n-1}$

Q. If 3^{rd}, 6^{th} and last term of H.P. are $\frac{1}{3}, \frac{1}{5}, \frac{3}{203}$, find the no. of terms

Soln. $\because$ 3, 5, $\frac{203}{3}$ are in A.P.

$\therefore$

$$a + 5d = 5$$
$$a + 2d = 3 \longrightarrow d = 2/3$$

Then, $a_{n^{th}} = \frac{203}{\cancel{3}} = \frac{5}{\cancel{3}} + (n-1)\frac{2}{\cancel{3}}$

$$\boxed{a = 5/3} \longrightarrow \boxed{n = 100 \text{ terms}}$$

Imp

Q. If m^{th} terms of H.P is n and n^{th} is m. find $(m+n)^{th}$ term

Sol^n m^{th} term of A.P. $= \frac{1}{n} = a + (m-1)d$

n^{th} term of A.P. $= \frac{1}{m} = a + (n-1)d$

(1) - (2)

$d = 1/mn$

Then, $\cancel{\frac{1}{n}} = a + \cancel{\frac{1}{n}} - \frac{1}{mn} \rightarrow a = \frac{1}{mn}$

$\therefore a_{m+n} = a + (mn-1)\frac{1}{mn}$

$= \cancel{\frac{1}{mn}} + \left(\frac{m+n}{mn}\right) \cancel{-\frac{1}{mn}}$

$\therefore$ H.P.'s $(m+n)^{th}$ term $= \frac{mn}{m+n}$

☆

Q. If a_1, a_2, a_3, a_4, a_5 ...... a_{nm} are in H.P.

Then, Prove that $a_1a_2 + a_2a_3 +$ ---- $a_{n-1} \cdot a_n = a_1 \cdot a_n (n-1)$

Sol^n $\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots\ldots \frac{1}{a_n} \rightarrow$ A.P.

$d = \frac{1}{a_2} - \frac{1}{a_1} = \frac{a_1 - a_2}{a_1a_2} \Rightarrow a_1a_2 = \frac{a_1 - a_2}{d}$

$$\Rightarrow \frac{a_1 - \cancel{a_2}}{d} + \frac{\cancel{a_2} - \cancel{a_3}}{d} + \frac{\cancel{a_3} - \cancel{a_4}}{d} + \cdots \cdots \frac{\cancel{a_{n-1}} - a_n}{d}$$

$$\Rightarrow \frac{a_1 - a_n}{d} = (n-1)\, a_1 a_n$$

(10) Arithmetic Mean (A.M.)

→ If a, b, c, are in A.P. then b is called A.M. of a & c.

where, $\boxed{\frac{a+c}{2} = b}$

Q:- * If there exists a series such as, 5, p_1, p_2, p_3, p_4, —— p_n, 41 Then, find the value of p, if

$$\frac{A_3}{A_{p+1}} = \frac{2}{5}$$

Sol^n $41 = 5 + (p+2-1)d$

$$\Rightarrow \boxed{d = \frac{36}{p+1}}$$

$$\Rightarrow \frac{A_3}{A_{p-1}} = \frac{a_4}{a_p} = \frac{5 + 3\left(\frac{36}{p+1}\right)}{5 + (p-1)\frac{36}{(p+1)}} = \frac{2}{3}$$

$$\Rightarrow p = 11$$

Q. If $x, y, z \rightarrow$ A.P.
and,
a_1 is the A.M. b/w x & y
a_2 is the A.M. b/w y and z.
Than, find the A.M. b/w a_1 and a_2.

Sol^n The series made is as follows

$$x, a_1, y, a_2, z$$

$\therefore$ A.M. b/w a_1 and a_2 = y

• Arithmetic Mean of 'n' numbers :-

$$\frac{x_1 + x_2 + x_3 + x_4 \dots x_n}{n} = \text{A.M.}$$

(II) Geometric Mean :-

Single G.M. b/w a and b :-

$$\underbrace{a, G, b}_{\text{G.P.}}$$

$$\because \frac{G}{a} = \frac{b}{G} \Rightarrow \boxed{G^2 = ab}$$

$$\boxed{G = \sqrt{ab}}$$

Case - I). $a, b \rightarrow$ +ve both

G.M. = $\sqrt{ab}$

Case – II). $a, b \longrightarrow$ both –ve.
↳ G.M. $= -\sqrt{ab}$

Case – III). $a, b \rightarrow$ One +ve & One –ve
↳ G.M. → Not Defined.

- G.M. of 'n' nos. $= (a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdots a_n)^{1/n}$
  For Example :–
  ↳ G.M. of 3 nos. → $(abc)^{1/3}$
  ↳ G.M. of 7 nos. → $(abcdefg)^{1/7}$
  ↳ G.M. of 2 nos. → $(ab)^{1/2}$

- Insertion of 'n' terms b/w two numbers :–
  The insertion of 'n' terms b/w 2 nos. will finally turn the Series into G.P. again.

*Q. Insert five G.M. b/w 576 and 9.

Sol^n The final G.P. formed by inserting 5 G.M. will be:–

$576, G_1, G_2, G_3, G_4, G_5, 9.$

$\therefore \quad a_7 = 9 = 576 \times (r)^{7-1}$

$= \frac{1}{64} = (r)^6 \Rightarrow r = 1/2$

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Q. If $\dfrac{a^{n+1}+b^{n+1}}{a^{n}+b^{n}}$ is G.M. b/w a & b

Then, find the value of n.

Sol^n.

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$$

$$\Rightarrow a^{n+1}+b^{n+1} = a^{1/2}\cdot b^{1/2}(a^n+b^n)$$

$$\Rightarrow a^{n+1}+b^{n+1} = a^{n+1/2}\cdot b^{1/2} + a^{1/2}\cdot b^{n+1/2}$$

$$\Rightarrow a^{n+1} - a^{n+1/2}\cdot b^{1/2} = a^{1/2}\cdot b^{n+1/2} - b^{n+1}$$

$$\Rightarrow a^{n+1/2}\cancel{(a^{1/2}-b^{1/2})} = b^{n+1/2}\cancel{(a^{1/2}-b^{1/2})}$$

$$\Rightarrow a^{n+1/2} \div b^{n+1/2} = 0_1$$

$$\Rightarrow (a\div b)^{n+1/2} = (a\div b)^0$$

$$\Rightarrow n+1/2 = 0 \Rightarrow n = -1/2$$

Q. Find 2 +ve nos. whose difference is 12 and AM. exceeds G.M. by 2.

Sol^n

$$a-b=12 \quad \text{---}(1)$$

$$\frac{a+b}{2} - \sqrt{ab} = 2 \quad \text{---}(2)$$

From (1) $\rightarrow$ $\boxed{a = b+12}$ $\rightarrow$ (2)

$$\frac{b+12+b}{2} - 2 = \sqrt{b(b+12)}$$

$\Rightarrow b+4 = \sqrt{b(b+12)}$

$\Rightarrow (b+4)^2 = b^2 + 12b$

$\Rightarrow 4b = 16$

A Imp $\Rightarrow$ $b=4$ $\longrightarrow$ $a=16$ Ans

Q. * x is A.M. and y, z are 2 G.M. b/w two positive nos.; Then Prove that $\frac{y^3+z^3}{xyz} = ??$

Soln. $x = \frac{a+b}{2}$ | $y = ar = a \cdot \frac{b^{1/3}}{a^{1/3}} = a^{2/3} \cdot b^{1/3}$

$z = ar^2 = a\frac{b^{2/3}}{a^{2/3}} = a^{1/3} \cdot b^{2/3}$

$\therefore \boxed{yz = ab}$

$\Rightarrow$ Now, $\frac{y^3+z^3}{xyz} = \frac{a^2b + ab^2}{\frac{a^2b+ab^2}{2}} = \boxed{2}$ Ans

$$\left[xyz = \left(\frac{a+b}{2}\right)(ab)\right]$$

(12) Harmonic Mean :-

If a, H.M., b $\longrightarrow$ H.P.

Then, $\frac{1}{a}, \frac{1}{H.M.}, \frac{1}{b}$ $\longrightarrow$ A.P.

$\frac{2}{H.M} = \frac{1}{a} + \frac{1}{b} \Rightarrow \boxed{H.M. = \frac{2ab}{a+b}}$

- Insert the harmonic mean of $a_1, a_2, a_3, a_4 ------ a_n$ b/w two nos.

$$\Rightarrow \frac{1}{H} = \frac{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} ------ \frac{1}{a_n}}{n}$$

$\downarrow$

Reciprocal

- Insert 'n' H.M. b/w two nos. :-

$a, H_1, H_2, H_3, b \rightarrow$ H.P.

$\downarrow$

$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, 1/b \rightarrow$ A.P.

Q. If $a, b, c \rightarrow$ H.P.
then, find the value of

$$\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right) = ??$$

Sol^n :- $\frac{2}{b} = \frac{1}{a} + \frac{1}{c} \Rightarrow \boxed{\frac{1}{c} = 2/b - \frac{1}{a}}$

$$\therefore \left(\frac{1}{b} + \frac{2}{b} - \frac{1}{a} - \frac{1}{a}\right)\left(\frac{2}{b} - \frac{1}{b}\right)$$

$$\Rightarrow \left(\frac{3}{b} - \frac{2}{a}\right)\left(\frac{1}{b}\right) \Rightarrow \boxed{\frac{3}{b^2} - \frac{2}{ab}}$$