(13) Arithmetico - Geometric Progression :-

$$a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3 + \text{------} a_n b_n$$

Where,

$a_1, a_2, a_3, a_4, \ldots \rightarrow$ A.P.

$b_1, b_2, b_3, b_4, \ldots \rightarrow$ G.P.

Imp

Q. Find the value of :-

$$1 + 2\cdot 2 + 3\cdot 2^2 + 4\cdot 2^3 + \ldots 100\cdot 2^{99}$$

Soln

$$S = 1 + 2\cdot 2 + 3\cdot 2^2 \ldots\ldots + 100\cdot 2^{99}$$

$$2S = 1\cdot 2 + 2\times 2^2 \ldots\ldots 99\cdot 2^{99} + 100\cdot 2^{100}$$

$$- \quad - \quad - \quad - \quad - \quad -$$

$$-S = 1 + 1\cdot 2 + 1\cdot 2^2 + 1\cdot 2^3 + \ldots\ldots 1\cdot 2^{99} - 100\cdot 2^{100}$$

$$-S = 1 + 1\underbrace{(2 + 2^2 + 2^3 + \ldots\ldots 2^{99})}_{\text{Sum of G.P.}} - 100\cdot 2^{100}$$

$$-S = 1 + \left[\frac{2(2^{99} - 1)}{2 - 1}\right] - 100\cdot 2^{100}$$

$$-S = 1 + 1\cdot 2^{100} - 2\cdot 100\cdot 2^{100}$$

$$-S = -1 - 99\cdot 2^{100}$$

$$\boxed{S = 1 + 99\cdot 2^{100}}$$ Ans.

Q. Find the value of :-

$$S = 1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \dots \infty$$

Soln

$$\frac{1}{2}S = 0 + 1 \cdot \frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \dots \infty$$

$$S - \frac{1}{2}S = 1 + \left[1\left(\frac{2}{2^2} + \frac{2}{2^2} + \frac{2}{2^3} + \dots\right)\right]$$

$$\frac{1}{2}S = 1 + \left(\frac{1}{1 - 1/2}\right) \rightarrow \boxed{6} = S$$

(14) Series :-

↳ Sum of first 'n' natural nos =>

$$\frac{n(n+1)}{2} = \sum_{n=1}^{n=\infty} n$$

↳ Sum of squares of first 'n' natural nos. :-

$$\frac{n(n+1)(2n+1)}{6} = \sum n^2$$

↳ Sum of cubes of first 'n' natural nos. :-

$$\left[\frac{n(n+1)}{2}\right]^2 = \sum n^3$$

1

Q. If $a_{nth} = 12n^2 - 6n + 5$ then, find the S_{nth}.

Solⁿ.

$S_{nth} = a_1 + a_2 + a_3 + \ldots \ldots a_n$

$= \sum a_n = \sum (12n^2 - 6n + 5)$

$= \sum 12n^2 - \sum 6n + \sum 5$

$\Rightarrow 12\sum n^2 - \sum 6n + 5\sum 1$

$= 12\left(\frac{n(n+1)(2n+1)}{6}\right) - 6\left(\frac{n(n+1)}{2}\right) + 5n$

Q. find the value of :-

* $(1)^2 + (3)^2 + (5)^2 \ldots\ldots\ldots$ n terms.

Solⁿ. $a_{nth} = (2n-1)^2 = 4n^2 - 1 - 4n$

$S_{nth} = \sum a_{nth} = 4\sum n^2 - 4\sum n + \sum 1$

$= \frac{4n(n+1)(2n+1)}{6} + n - \frac{4n(n+1)}{2}$

• Relation b/w A.M., G.M., H.M. :-

If there exists two positive nos. Then,

A.M. $\geq$ G.M. $\geq$ H.M.

* If all the nos. are equal, then there is equality

Q. Find the minimum value of :-

$\cos^2 x + \sec^2 x$

Sol^n $\dfrac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x \cdot \sec^2 x}$

$\dfrac{\cos^2 x + \sec^2 x}{2} \geq 1$

$\therefore \boxed{\cos^2 x + \sec^2 x \geq 2}$

Q. Find the min^m value of :-

$4^x + 4^{1-x}$

Sol^n So, since, A.M. $\geq$ G.M.

$\Rightarrow \dfrac{4^x + 4^{1-x}}{2} \geq \sqrt{4^x \cdot 4^{1-x}}$

$\Rightarrow \boxed{4^x + 4^{1-x} \geq 4}$

Q. If $a+b+c = 09$ and $a, b, c > 0$. Find the max. value of abc.

Sol^n

Since, $\dfrac{a+b+c}{3} \geq (abc)^{1/3}$

$\Rightarrow 9/3 \geq (abc)^{1/3}$

$\Rightarrow \boxed{(3)^3 \geq abc}$

Imp. ①

Q. If $a+b+c=12$ and $a, b, c > 0$. Find the max. value of $a^2 b^3 c$.

Sol^n.

$$\frac{\cancel{12}^{2}}{\cancel{6}} = \frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+c}{6} \geq \left(\frac{a}{2}\cdot\frac{a}{2}\cdot\frac{b}{3}\times\frac{b}{3}\times\frac{b}{3}\times c\right)^{1/6}$$

$$\Rightarrow (2)^6 \geq \frac{a^2 b^3 c}{4\times 27} \rightarrow \boxed{a^2b^3c \leq 108\times 64}$$

• Shortcut to solve some Special Series :-

Type - I) * $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+ \dots\dots \infty$

Sol^n $T_{n} = \frac{1}{n(n+2)} = \frac{1}{2}\cdot\left[\frac{(n+2)-n}{(n)(n+2)}\right]$

$$= \frac{1}{2}\left[\frac{n+2}{n(n+2)} - \frac{n}{n(n+2)}\right]$$

$$= \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right]$$

Now, $t_n = \frac{1}{2}\left[\frac{1}{n} - \frac{1}{n+2}\right]$

$t_1 = \frac{1}{2}\left[\frac{1}{1} - \cancel{\frac{1}{3}}\right]$ | $t_3 = \frac{1}{2}\left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right]$

$t_2 = \frac{1}{2}\left[\frac{1}{2} - \cancel{\frac{1}{4}}\right]$ | $t_4 = \frac{1}{2}\left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right]$

$$t_{n-1} = \frac{1}{2}\left[\cancel{\frac{1}{n-1}} - \frac{1}{n+1}\right] \quad t_n = \frac{1}{2}\left[\cancel{\frac{1}{n}} - \frac{1}{n+2}\right]$$

$$\Rightarrow \text{Sum} = \left[1 + \frac{1}{2} + \left(-\frac{1}{n+1} - \frac{1}{n+2}\right)\right]$$

Method - 2) Same till,

$$* \quad T_n = \frac{1}{2}\left[\frac{1}{n} - \frac{1}{(n+2)}\right]$$

$$V_n \qquad V_{n+2}$$

$$\left[\frac{1}{1} \quad \frac{1}{2}\right] \qquad \left[\frac{1}{n-1} \quad \frac{1}{n}\right]$$

$$S_{nth} = \frac{1}{2}\left[\frac{1}{1} + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

Q. Find the value of :-

$$* \quad \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \frac{1}{4\cdot5} + \text{------} \; n \text{ terms}$$

Sol^n $\quad t_{nth} = \frac{1}{(n+1)(n+2)}$

$$= \frac{1}{2-1}\left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$t_{nth} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$V_n \qquad V_{n+1}$$

(2)

$$V_r \quad\quad\quad V_{n+2}$$

$$\Rightarrow \boxed{\frac{1}{2} - \frac{1}{n+2}} \rightarrow \underline{\text{Sum}}$$

Q. Find the value of :—

$$\frac{1}{1\cdot 4} + \frac{1}{2\cdot 5} + \frac{1}{3\cdot 6} + \frac{1}{4\cdot 7} + \ldots\ldots n \text{ terms}$$

Sol^n

$$T_{term} = \frac{1}{n(n+3)} = \frac{1}{3}\left[\frac{(n+3)-n}{n(n+3)}\right]$$

$$= \frac{1}{3}\left[\underbrace{\frac{1}{n}}_{V_r} - \frac{1}{n+3}\right] \quad V_{r+3}$$

$$1 \quad 2 \quad 3 \quad \ldots \quad n-2 \quad n-1 \quad n$$

$$\boxed{S_{term} = \frac{1}{3}\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+1} - \frac{1}{n+3}\right]}$$

Type -2) $1\cdot 4\cdot 7 + 4\cdot 7\cdot 10 + 7\cdot 10\cdot 13 + \ldots n$ term

$$T_n = \{(3r-2)(3r+1)(3r+4)\}$$

↳ 'n' wise consecutive term.

$$T_r = \frac{1}{12}\left[(3r-2)(3r+1)(3r+4)(3r+7) - (3r-5)(3r-2)(3r+1)(3r+4)\right]$$

r+1 r+1 r+1 r−1

r−1 r−1

②

$$T_{r^{th}} = \frac{1}{12}\Big[\underbrace{(3r-1)(3r+2)(3r+5)(3r+8)}_{V_r} - \underbrace{(3r-4)(3r-1)(3r+2)(3r+5)}_{V_{r-1}}\Big]$$

1   n

$$\cancel{T_{r^{th}} = \frac{1}{12}\Big[(2)(5)(8)(10) - (3n-4)(3n-1)(3n+2)(3n+5)\Big]}$$

~~Imp~~

Q. Find the value of :-

* $2\cdot5\cdot8 + 5\cdot8\cdot11 + 8\cdot11\cdot14 + \text{------}$ n terms.

$$T_{r^{th}} = \underbrace{(3r-1)}_{r+1}(3r+2)\underbrace{(3r+5)}_{r+1}$$

1 → n

$$T_{r^{th}} = \frac{1}{12}\Big[\underbrace{(3r-1)(3r+2)(3r+5)(3r+8)}_{V_r} - \underbrace{(3r-4)(3r-1)(3r+2)(3r+5)}_{V_{r-1}}\Big]$$

$$\Rightarrow S_{n^{th}} = \frac{1}{12}\Big[(3n-1)(3n+2)(3n+5)(3n+8) - (-1)(2)(5)(8)\Big]$$

~~Imp~~

Q. $2\cdot5 + 5\cdot8 + 8\cdot11 + \text{------}$ n terms.

Soln.

$$T_{r^{th}} = \frac{1}{9}\Big[\underbrace{(3r-1)(3r+2)(3r+5)}_{V_{r+1}} - \underbrace{(3r-4)(3r-1)(3r+2)}_{V_r}\Big]$$

1   n

$$\text{Sum} = \frac{1}{9}\Big[(3n-1)(3n+2)(3n+5) - (-1)(2)(5)\Big]$$

Type-3) $\dfrac{1}{1\cdot4\cdot7} + \dfrac{1}{4\cdot7\cdot10} + \dfrac{1}{7\cdot10\cdot13} + \cdots$

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Sol^n $T_{r^{th}} = \dfrac{1}{(3r-2)(3r+1)(3r+4)}$

$$\Rightarrow T_{r^{th}} = \frac{1}{6}\Bigg[\underbrace{\frac{1}{(3r-2)(3r+1)}}_{V_r} - \underbrace{\frac{1}{(3r+1)(3r+4)}}_{V_{r+1}}\Bigg]$$

$$\Big[\qquad\qquad\Big]_{1}^{n}$$

$$= \frac{1}{6}\left[\frac{1}{1\cdot4} - \frac{1}{(3n+1)(3n+4)}\right]$$

for $S_\infty \rightarrow$

$$\frac{1}{6}\left[\frac{1}{1\cdot4} - \cancel{\frac{1}{(3n+1)(3n+4)}}\right]$$

$$= \boxed{\frac{1}{6}\left[\frac{1}{1\cdot4}\right]}$$