Some Important Questions.

Q. If sum of the roots of the quad. equat^n, $ax^2 + bx + c = 0$ is eq. to sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in :-

A). A.P. B). G.P. C). H.P. D). None

Sol^n

Given :-

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$$

$$\Rightarrow \boxed{\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} = \alpha + \beta}$$

$$\Rightarrow \frac{-b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{b^2 - 2ac}{c^2} \Rightarrow 2a^2c = ab^2 + bc^2$$

$$\Rightarrow -bc^2 = ab^2 - 2ac^2 \Rightarrow \boxed{\frac{2a}{b} = \frac{b}{c} + \frac{c}{a}}$$

$\therefore$ These three are in H.P.

Trick :- $\frac{a}{b} = \frac{c}{d} \rightarrow \frac{a-c}{b-d} = \frac{a}{b} = \frac{c}{d}$

Q. Given four positive numbers in A.P. If 5, 6, 9, and 15 were added resp. to these nos., we get a G.P. then which of the following holds?

(A) The common ratio of G.P. is 3/2.
(B) Common ratio of G.P. is 2/3.
(C) Comon diff. of the A.P. is 3/2.
(D) Common difference of the A.P. is 2/3.

Sol^n. Let the 4 nos. be A_1, A_2, A_3, A_4.

Then, $(a+5), (a+d+6), (a+2d+9), (a+3d+15)$ are in G.P.

Then, $r = \frac{a+d+6}{a+5} = \frac{a+2d+9}{a+d+6}$

$r = \boxed{\frac{3+d}{d+1} = \frac{d+6}{d+3}} \rightarrow \boxed{d=3}$

$\boxed{r = 3/2}$

Q. Consider an A.P. with first term 'a' and the common diff. 'd'. Let S_K denote the sum of the first K terms. Let $\frac{S_{Kx}}{S_x}$ is independent of x, then,

A). $a = d/2$
B) $a = d$
c) $a = 2d$
D) none

Solⁿ. $\frac{S_{kx}}{S_x} = \frac{\frac{kx}{2}[2a + (kx-1)d]}{\frac{x}{2}[2a + (x-1)d]}$

$\Rightarrow \left[\frac{(2a-d) + kxd}{(2a-d) + xd}\right] \cdot k$

$\Rightarrow$ for $\frac{S_{kx}}{S_x}$ to be independent of x

$\boxed{a = d/2}$

$\Rightarrow \left[\frac{kxd}{xd}\right] \cdot k \rightarrow \underline{\underline{k^2}}$

Q. Consider the A.P. $a_1, a_2, a_3, \ldots$
the G.P. $b_1, b_2, b_3, b_4, \ldots$

Such that $a_1 = b_1$; $a_9 = b_9$ and $\sum_{r=1}^{9} a_r = 369$

then,

A). $b_6 = 27$ B). $B_7 = 27$

C). $b_8 = 81$ D). $b_9 = 18$

Solⁿ Given that; $\boxed{a = 1}$ & $\boxed{b_1 = 1}$

Also, $a + 8d = ar^8$

$1 + 8d = r^8$

$81 = r^8 \rightarrow \boxed{r = \sqrt{3}}$

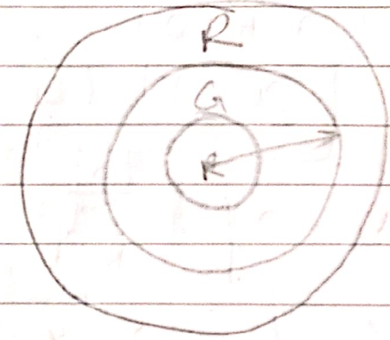
$\frac{9}{2}[2 + 8d] = 369 \rightarrow \boxed{d = 10}$

$\therefore \boxed{b_7 = 27}$

Q. Concentric circles of radii 1, 2, 3, ---- 100 cm are drawn. The interior of the smallest circle is coloured red and the annular regions are coloured alternately green & red, so that no two adjacent regions are of the same colour. The total area of the green region in sq. cm is equal to :-

A). 1000π B). 5050π
C). 4950π D). 5151π

Sol^n



$\Rightarrow \pi(2^2 - 1^2) + \pi(4^2 - 3^2) + \text{-----} \pi(100^2 - 99^2)$

$\Rightarrow \pi(3) + \pi(7) + \pi(11) + \text{-----} \pi(199)$

↳ A.P.

$\therefore$ Sum of green regions $= 5050\pi$

Q. For an increasing A.P. $a_1, a_2, a_3, \text{----} a_n$ if $a_1 + a_2 + a_3 = -12$ and $a_1 \cdot a_3 \cdot a_5 = 80$ then which of the following does not hold?

A). $a_1 = -10$ B). $a_2 = -1$
C). $a_3 = -4$ D) $a_5 = 2$

$\underline{Sol^n}$ Given that :-

$\Rightarrow (a)+(a+2d)+(a+4d) = -12$

$\Rightarrow 3a+6d = -12$ — (1)

$a+2d = -4$

$\boxed{2d = -4-a}$ — (1)

Also,

$(a)(a+2d)(a+4d) = 80$

$a^2+4ad+20=0$

$a^2-8a-2a^2+20=0$

$a(a+10)-2(a+10a)=0$

$\boxed{(a=2)\ (a=-10)=0}$

Q. The sum of $\sum_{k=1}^{\infty} \frac{2^{k+2}}{3^k}$ equal to

$\underline{Sol^n}$ $\boxed{a = \frac{2^3}{3}} \rightarrow \boxed{r = \frac{2}{3}}$

$\therefore S_\infty = \frac{\frac{2^3}{\cancel{3}}}{\frac{3-2}{\cancel{3}}} \rightarrow \boxed{8}$

Q. The sum of $5\sum_{n=1}^{\infty} \frac{2^{n+2}}{4^{n-2}}$ is equal to

A). 1372 B). 440

C). 320 D). 388

$\underline{Sol^n}$ $5\left[\frac{2^{n+2}}{2^{2n-4}}\right] \Rightarrow 5\left[\sum \frac{1}{2^{n-6}}\right]$

$\Rightarrow 5\left[\sum 2^{6-n}\right] \rightarrow 2^6 \times 5\left[\sum \frac{1}{2^n}\right]$

Now, $S_\infty = 64 \times 5 \times \left(\frac{1/2}{1-1/2}\right) = \boxed{320}$

Q. The sum $\sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$ is equal to :-

A). 1 B). 3/4

C). 4/3 D). none.

Solⁿ

$$\sum_{r=2}^{\infty} \frac{1}{(r-1)(r+1)} \Rightarrow \sum_{r=2}^{\infty} \frac{1}{2}\left[\frac{(r+1)-(r-1)}{(r+1)(r-1)}\right]$$

$$\boxed{3/4}$$

Q. If a, b, c are in H.P., then a, a − c, a − b are in :-
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Solⁿ

$\because$ a, b, c $\longrightarrow$ H.P. $\longrightarrow$ $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ $\rightarrow$ A.P.

$$\boxed{b = \frac{2ac}{a+c}}$$

Now, $a, a-c, a-b \Rightarrow a, a-c, \frac{a^2-ac}{a+c}$

Thus, Since, $\frac{1}{a} + \frac{a+c}{a^2-ac} = \frac{2}{a-c} \Longrightarrow \boxed{\text{H.P.}}$