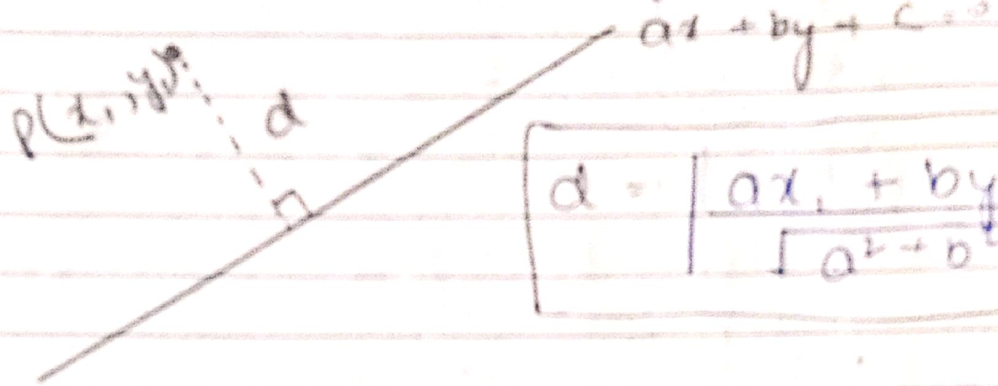
(13) Length of $\perp$ar :-

Distance of a point $P(x_1, y_1)$ from a st. line $ax + by + c = 0$



$$\boxed{d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|}$$

Q. Find points on x-axis which are 4 units apart from line $4x + 3y = 12$

Sol^n Let the point be $(h, 0) \longrightarrow$ x-axis

$$\left| \frac{4h + 3(0) - 12}{\sqrt{4^2 + 3^2}} \right| = 4$$

$4h - 12 = 20$; $\boxed{h = 8} \rightarrow \boxed{(8, 0)}$

$4h - 12 = -20$; $\boxed{h = -2} \rightarrow \boxed{(-2, 0)}$

Q. Find the Area of an Equilateral Δ, whose one vertice's coordinate is $(2, 1)$ and Eqn of base of Δ is $x + y - 2 = 0$

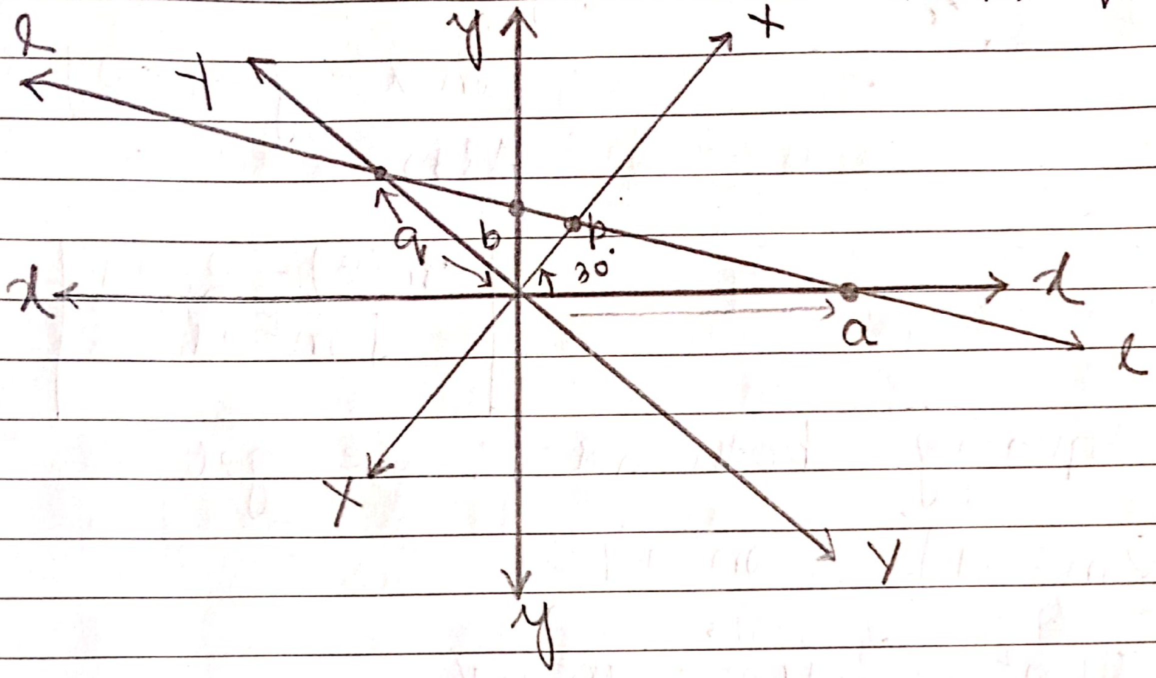
Sol^n

$$\text{Area of } \Delta = \frac{\sqrt{3}}{4}$$

Q. An Axis has been rotated by 30°.

		X'	Y'
$x-y$	$\rightarrow$	a	b
$X-Y$	$\rightarrow$	p	q

find Relat^n b/w a; b; p; q.



Here, $\boxed{\dfrac{x}{a} + \dfrac{y}{b} = 1}$ and $\boxed{\dfrac{X}{p} + \dfrac{Y}{q} = 1}$

Now, Distance of these lines from the origin must be equal.

So,

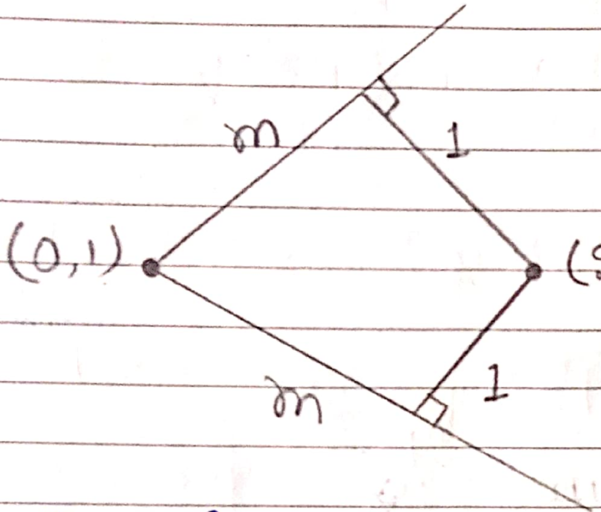
$$\left|\frac{0/a + 0/b - 1}{\sqrt{(1/a)^2 + (1/b)^2}}\right| = \left|\frac{0/p + 0/q - 1}{\sqrt{\left(\frac{1}{p}\right)^2 + \left(\frac{1}{q}\right)^2}}\right|$$

$$\Rightarrow \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = \sqrt{\frac{1}{p^2} + \frac{1}{q^2}}$$

$$\Rightarrow \boxed{\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}}$$

★ Q. If a line is passing from $P(0,1)$. Then, find eqn. of line such that $\perp$ar distance of this line from $(2,2)$ is 1 unit.

Soln



$$\boxed{y-1 = m(x-0)}$$

$$\boxed{mx - y + 1 = 0}$$

Also,

$$\left| \frac{m(2) - 2 + 1}{\sqrt{m^2+1}} \right| = 1$$

On Squaring both sides; we get:-

$$\Rightarrow (2m-1)^2 = m^2+1$$

$$\Rightarrow 4m^2 + \cancel{1} - 4m = m^2 + \cancel{1}$$

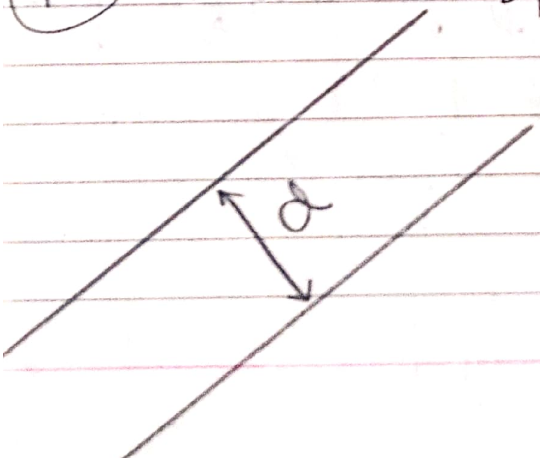
$$\Rightarrow 3m^2 - 4m = 0$$

$$m(3m-4) = 0$$

$\therefore$ $\boxed{m=0}$ or $\boxed{m = 4/3}$

$\downarrow$ $\boxed{y=1}$ $\quad$ $\downarrow$ $\boxed{y = \frac{4}{3}x + 1}$

(14) Distance b/w Parallel lines:-



$$ax + by + C_1 = 0$$

$$ax + by + C_2 = 0$$

$$\boxed{d = \left| \frac{C_1 - C_2}{\sqrt{a^2+b^2}} \right|}$$

Q15. Find Parallel Lines at a Distance of 4 Units from $5x - 12y + 26 = 0$

Sol^n. Let $5x - 12y + C = 0$ be the parallel line.

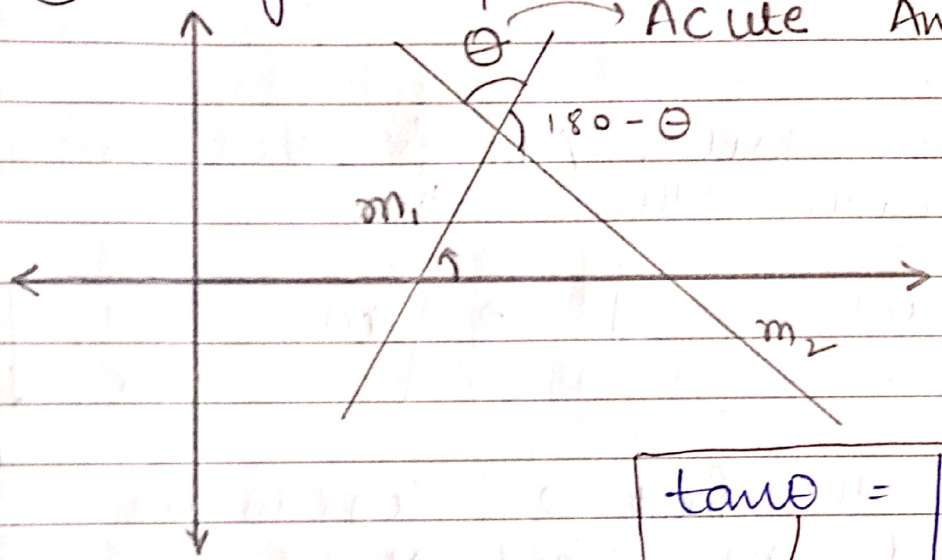
$$\left| \frac{C - 26}{\sqrt{5^2 + 12^2}} \right| = 4$$

$\frac{C-26}{13} = 4 \Rightarrow \boxed{C = 78}$

$\frac{C-26}{13} = -4 \Rightarrow \boxed{C = -26}$

(15) Angle b/w two lines :-

θ → Acute Angle



$$\boxed{\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right|}$$

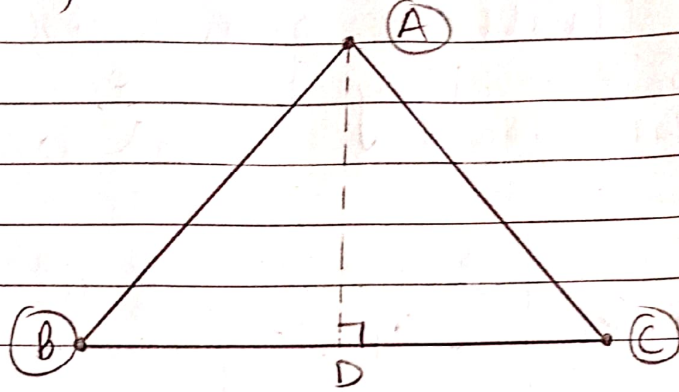
Acute angle $\qquad m_1 * m_2 \neq 1$

Case - I) $\theta = 0^\circ \Rightarrow \boxed{m_1 = m_2} \Rightarrow$ || lines

Case - II) $\theta = 90^\circ \Rightarrow 1 + m_1 * m_2 = 0$

$\Rightarrow \boxed{m_1 \cdot m_2 = -1} \Rightarrow \perp$ar lines.

(16) Eqn. Of Altitudes :-



for the Equat^n Of AD altitude :-

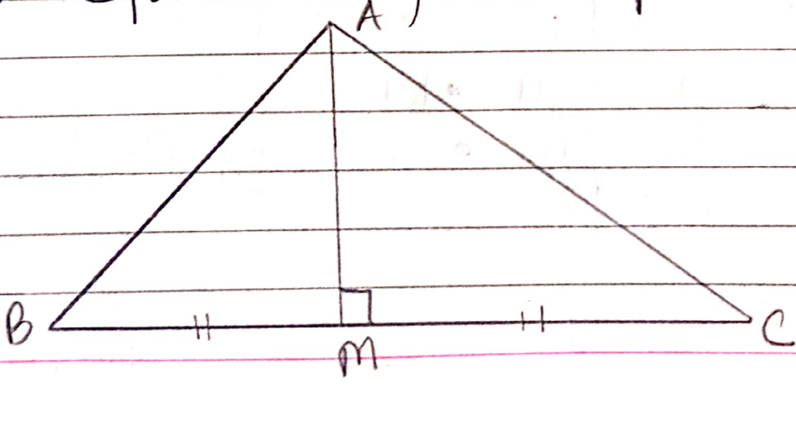
(A) Find the slope of BC line either from eqn. of BC or coordinates B; C.

(B) As we know that, AD & BC are $\perp$ to each other, then

$$m_{AD} \cdot m_{BC} = -1 \Rightarrow \boxed{m_{AD} = \frac{-1}{m_{BC}}}$$

(C) Now, with the help of coordinates of A and m_{AD}, by applying slope-point form; we get the equat^n of Altitude AD.

(17) Equat^n Of Perpendicular Bisector :-



(A) Find the coordinates of point M.

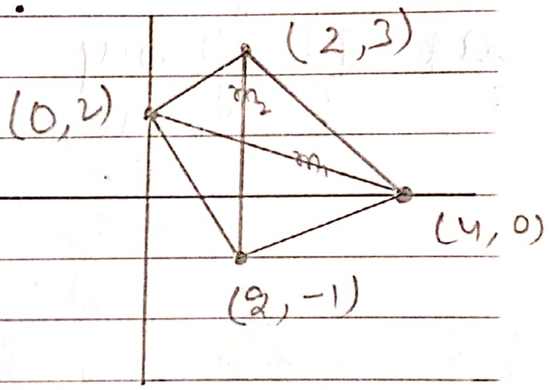
(B) Since, AM and BC are $\perp^{ar}$ to each other, then,

$$m_{AM} \times m_{BC} = -1 \Rightarrow \boxed{m_{AM} = \frac{-1}{m_{BC}}}$$

(C) Now, with the help of M (coordinat-es) and m_{AM} $\longrightarrow$ We get the eqn. of AM.

*Q. If a quadrilateral given with coordinates as (2, -1); (0, 2); (2, 3) and (4, 0). Then, find the Angle b/w the Diagonals.

Sol^n. Here, $m_1 = \frac{-1}{2}$



and $m_2 = \infty$

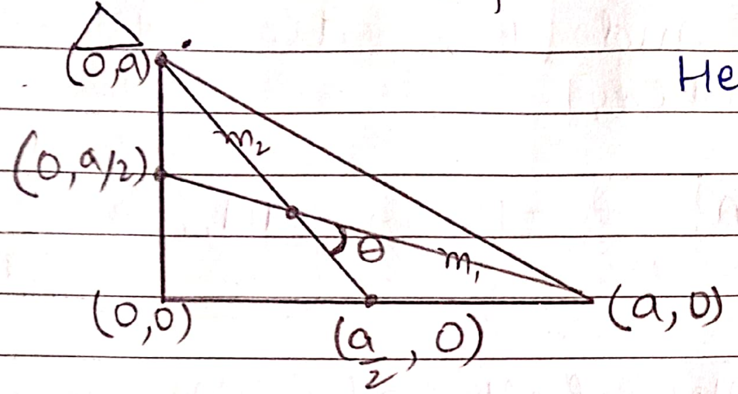
$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{m_1}{m_2} - 1}{\frac{1}{m_2} + m_1}\right| = \left|\frac{0 - 1}{0 + \left(-\frac{1}{2}\right)}\right|$$

$$\therefore \boxed{\tan\theta = 2}$$

Q. Find the Acute angle formed b/w Medians of Isosceles Right Angled $\triangle$

Sol^n



Here, $m_1 = \frac{a}{-a/2} = \boxed{-2}$

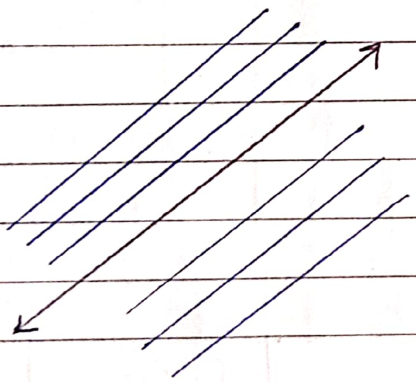
$m_2 = \frac{a/2}{-a} = \frac{-1}{2}$

$\therefore \tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right| = \boxed{\frac{3}{4}}$

(18) Concept of family of lines :-

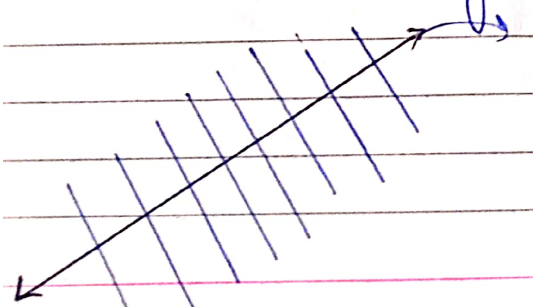
↳ group with similiar Properties.

(Case-I) Group of lines parallel to another lines.



$ax + by + C_1 = 0$
$ax + by + C_2 = 0$
$ax + by + C_3 = 0$
$ax + by + C_4 = 0$
⋮

(Case-II) Group of lines perpendicular to a single line.



$ax + by + c = 0$

↳ $\boxed{m_1 = \frac{-a}{b}}$

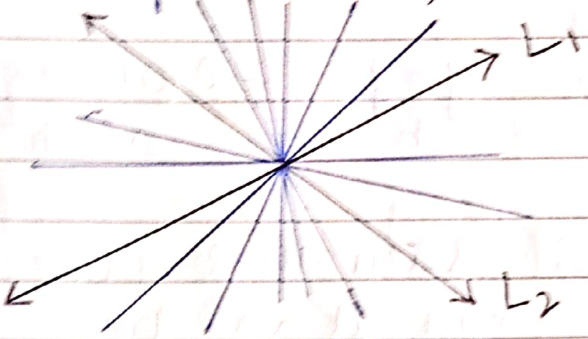
$\because$ Both lines are $\perp^{ar}$

$\therefore m_1 \cdot m_2 = -1$

Then, $\boxed{m_2 = \frac{b}{a}}$

$\therefore$ Equat^n of group of lines perpendicular to another line :- $bx - ay + $

(Case - III) Group of lines passing through the point of intersect^n of two lines.



$\therefore$ Eqn. of line passing through intersection of two lines =

$$\boxed{L_1 + \lambda L_2 = 0}$$

Q. Two lines intersecting each other ~~[illegible]~~ are as followed:- $2x + 5y - 8 = 0$ & $3x - 4y - 35 = 0$. Find the eqn. of line passing through $(2, -9)$?

Sol^n Here, for eqn. of line passing through $(2, -9)$ can be found as

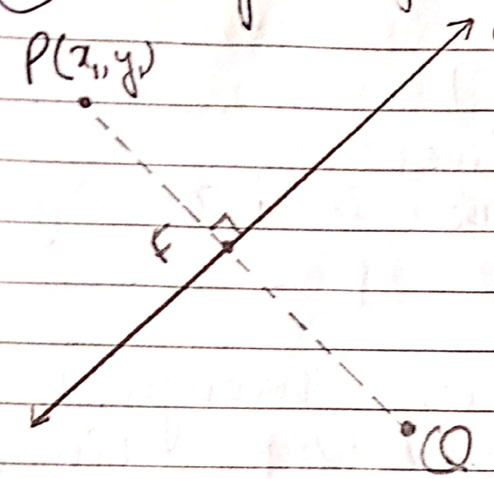
$\boxed{L_1 + \lambda L_2 = 0} \Rightarrow (2x + 5y - 8) + \lambda(3x - 4y - 35) = 0$

$\Rightarrow \boxed{\lambda = 7}$

$\therefore$ Eqn. of line $= 2x + 5y - 8 + 7(3x - 4y - 35) = 0$

$= \boxed{x - y = 11}$

(19) Image of a point and foot of Lar.



Foot of Lar :-

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$$

Image of a Point :-

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

Q. Given a point P (2,3) such that find the foot of Lar from ~~it~~ a line $2x+7y-1=0$.

Soln.

Foot of Lar $\Rightarrow \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$

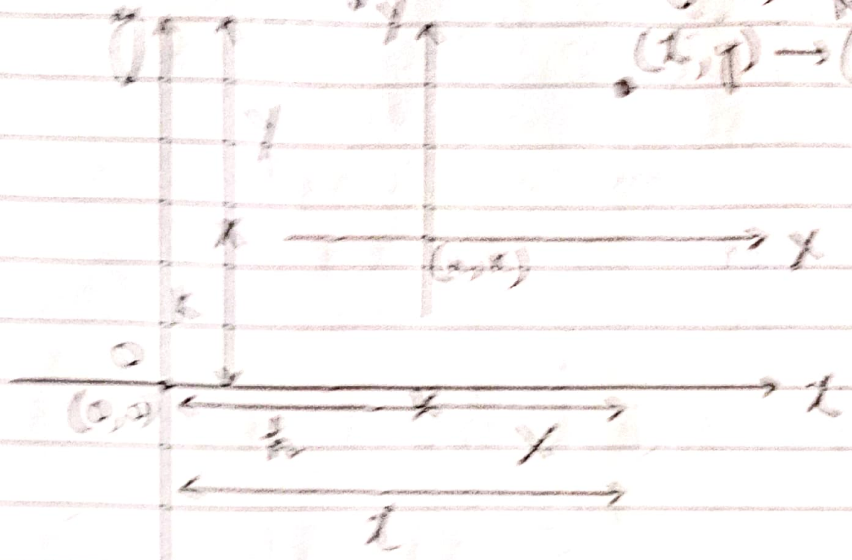
$$\Rightarrow \frac{x-2}{2} = \frac{y-3}{7} = \frac{-(2(2)+7(3)-1)}{2^2+7^2}$$

$$\Rightarrow x = \frac{58}{53} \quad \& \quad y = -\frac{9}{53}$$

(20) Shifting of Origin (Translation) :-

↳ No Rotation of Axis

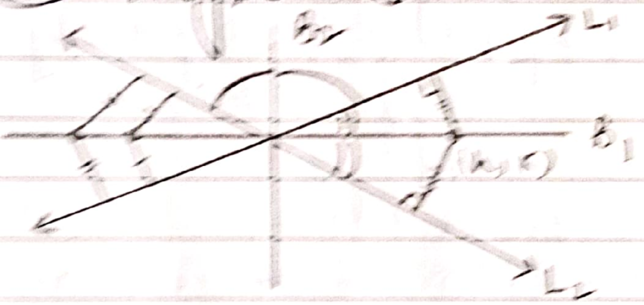
$(x, y) \rightarrow (X, Y)$



$$\boxed{\begin{aligned} x &= h + X \\ y &= k + Y \end{aligned}}$$

(21) Angle Bisector :-

Locus of Points equidistant from L_1 & L_2 ⇒ Angle Bisector



where,

$L_1 = a_1x + b_1y + c_1 = 0$

$L_2 = a_2x + b_2y + c_2 = 0$

Eqn of Bisectors ⇒

$$\left|\frac{a_1h + b_1k + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{a_2h + b_2k_2 + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

↓

$$\boxed{\frac{L_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{L_2}{\sqrt{a_2^2 + b_2^2}}}$$

→ B_1

→ B_2

Q. Find the two Bisectors formed by the intersection of two lines such as $x + y + 1 = 0$ & $2x - 3y + 5 = 0$.

Soln.

Two Bisector can be formed with the help of eqn. :-

$$\Rightarrow \frac{L_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{L_2}{\sqrt{a_2^2 + b_2^2}}$$

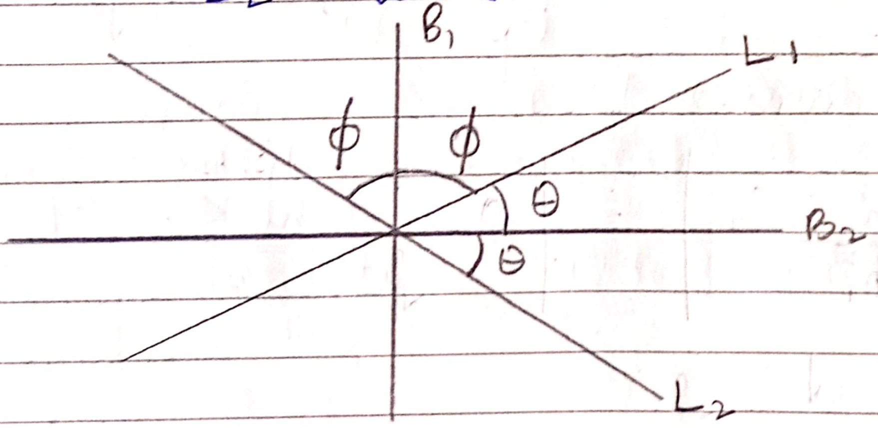
$$\Rightarrow \frac{x + y + 1}{\sqrt{1+1}} = \pm \frac{2x - 3y + 5}{\sqrt{4+9}}$$

$$\Rightarrow \sqrt{13}\,(x + y + 1) = \pm\left[(\sqrt{2})(2x - 3y + 5)\right]$$

B_1

B_2

- B_1 and B_2 are bisectors of L_1 and L_2 lines.



Here,

$2\theta + 2\phi = 180°$

$\theta + \phi = 90°$

$\therefore \boxed{B_1 \perp B_2}$

Whereas, L_1 & L_2 are not bisectors of B_1 and B_2.

↳ Only possible when $\theta = \phi = 45°$

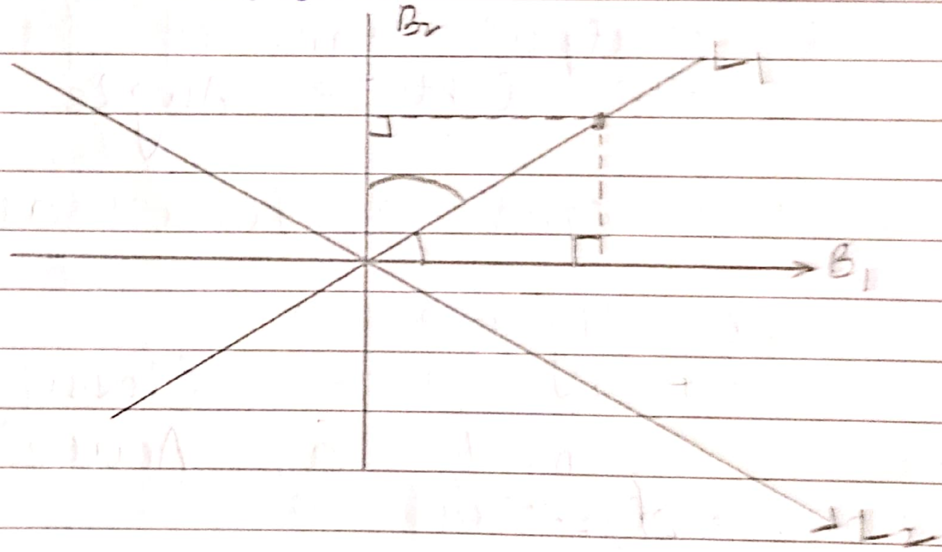
• Methods to find or Define bisectors B_1 and B_2 as Acute / Obtuse angle Bisector :-

M-1) Take a random point P on line L_1.

↳ Then, find the distance of P from bisector B_1 and B_2.

↳ Then, compare the distance of P from B_1 and B_2.

↳ Thus, Classify the nearer one as Acute Bisector and the other as Obtuse Bisector.



Method-2). Find the Angle b/w L_1 & B_1 and L_1 B_2.

If Angle b/w them is less than or greater than 45° → Then classify the angles as Acute / Obtuse

M-3) Given the equatⁿ of lines as:-

$$a_1 x + b_1 y + c_1 = 0$$
$$a_2 x + b_2 y + c_2 = 0$$

↳ Then, find the two bisectors B_1 & B_2 through,

$$\frac{L_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{L_2}{\sqrt{a_2^2 + b_2^2}}$$

↳ Now, Calculate $\boxed{a_1 \cdot a_2 + b_1 \cdot b_2}$.

↳ The output sign of ↑ → gives us the Obtuse Angle Bisectors

Q. Given two eqns of lines as,

$$3x + 4y + 2 = 0$$
$$4x + 3y - 7 = 0$$

, Classify the two bisectors formed as Acute & Obtuse angle bisectors.

Solⁿ. The two bisectors formed are,

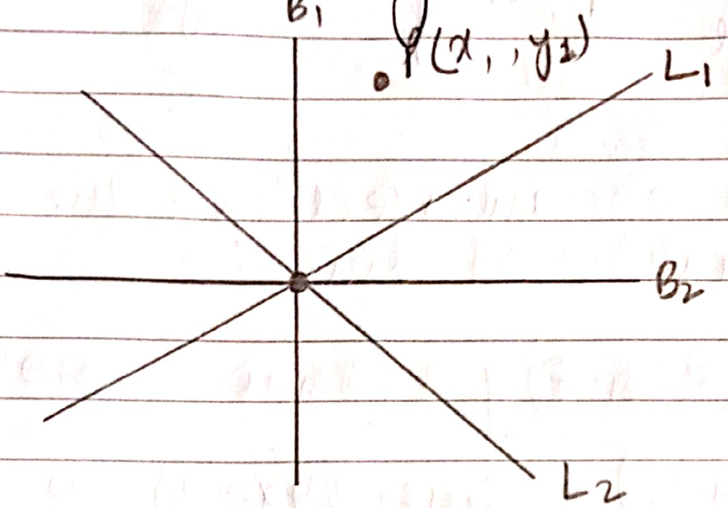
$$x - y - 9 = 0 \quad ; \quad 7x + 7y - 5 = 0$$

Also, $a_1 \cdot a_2 + b_1 b_2 = 24 \rightarrow (+)$

$\therefore$ $B_1 = x - y - 9 = 0 \rightarrow$ Obtuse

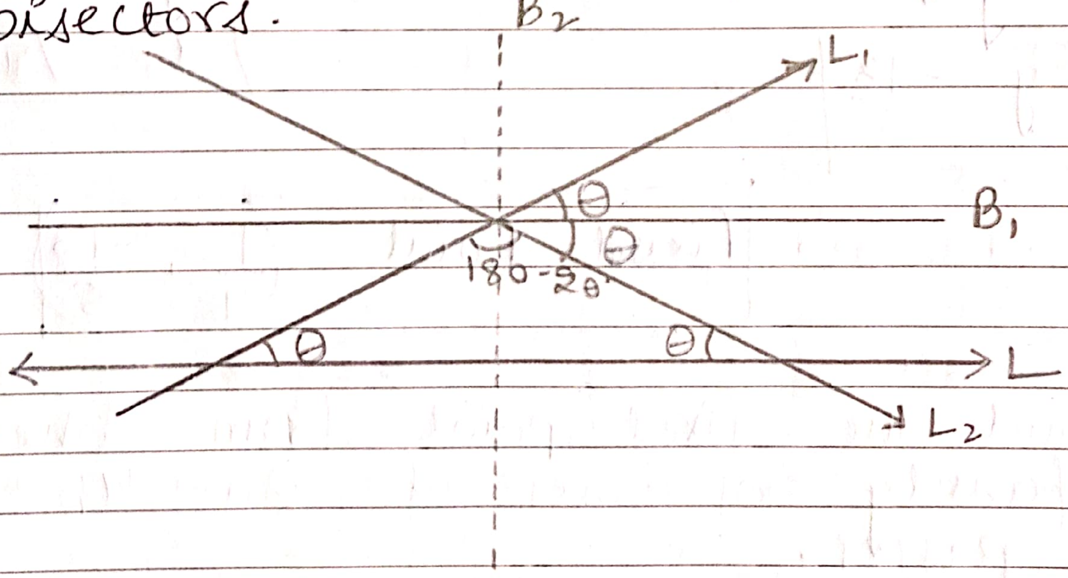
$B_2 = 7x + 7y - 5 = 0 \rightarrow$ Acute

- Equation Of Bisector Of the Region containning point P :-



→ Calculate
$(L_1(P)) \cdot (L_2(P))$
↓
$(a_1x_1 + b_1y_1 + c_1)$
$(a_2x_2 + b_2y_2 + c_2)$
↓
Sign (⊕; ⊖)
↓
Same Sign of Bisector

- If a line L makes equal Angles with L_1 & L_2 then it will be parallel to one of the Angle bisectors.



- Lines passing through a fixed point
  ↳ Family of lines passing through a single point.

Q. Find the point of intersectⁿ of the following equatⁿ of line :-

$(\sin\theta + 4\cos\theta)x + (2\sin\theta)y + (3\sin\theta - \cos\theta)$

Solⁿ $\sin\theta(x+2y+3) + \cos\theta(4x-1) = 0$

$\Rightarrow (x+2y+3) + \cot\theta(4x-1) = 0$

$[L_1 + \lambda L_2 = 0]$

L_1: $(x+2y+3=0)$ $\rightarrow$ $\boxed{y = \frac{-13}{8}}$

L_2: $(4x-1=0)$ $\rightarrow$ $\boxed{x = 1/4}$

$\therefore$ $\boxed{\text{Fixed Point} = \left(\frac{1}{4}, \frac{-13}{8}\right)}$

Q. Find the fixed point from where the family of lines of $3a+4b+7c=0$ passes.

Solⁿ Here, $3a+4b+7c=0$

$\Rightarrow \frac{3a}{7} + \frac{4b}{7} + c = 0$

On comparing with, $ax+by+c=0$; we get

$\boxed{\text{Fixed Point} = \frac{3}{7}, \frac{4}{7}}$