T.I.P. ⇒ Homogeneous eqn. in 'n' degree in x; y will represent 'n' st. lines passing through origin.

(19) Pair Of Straight Lines :-

(A) ↳ Homogeneous eqn. in x & y is
$ax^2 + 2hxy + by^2 = 0$
↳ 2^{nd} degree eqn. in x & y

So, $ax^2 + 2hxy + by^2 = 0$ can be solved as :-

$\Rightarrow \quad a + 2h\left(\frac{y}{x}\right) + \frac{by^2}{x^2} = 0$

$\Rightarrow \quad a + 2hm + bm^2 = 0 \qquad \left[\text{Let } \frac{y}{x} = m\right]$

$\Rightarrow \quad m = \frac{-2h \pm \sqrt{(2h)^2 - 4ab}}{2(b)} \qquad [D > 0]$

$= \frac{-h \pm \sqrt{h^2 - ab}}{b} \quad < \begin{matrix} m_1 \\ m_2 \end{matrix}$

$\therefore \quad \frac{y}{x} = m_1 \quad \& \quad \frac{y}{x} = m_2$

$\Rightarrow \quad \boxed{y = m_1 x} \quad \& \quad \boxed{y = m_2 x}$

Passes from Origin

↓

$ax^2 + 2hxy + by^2 = 0$

Note :- for two distinct lines, $h^2 > ab$
But if $h^2 = ab$ $[D = 0]$ ⇒ Coincident lines are formed due to coincident roots.

↳ For finding angles b/w the st. lines:-

M-1) $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 \cdot m_2}\right|$

M-2) $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{|a+b|}$

where, if $h^2 = ab \rightarrow \tan\theta = 0$

Or, $a + b = 0 \Rightarrow$ $\perp$ar lines passing from origin.

↳ Angle Bisectors of P.O.S.L. :-

M-1) $\dfrac{L_1}{\sqrt{a^2+b^2}} = \pm \dfrac{L_2}{\sqrt{a^2+b^2}}$ (Basic Method)

M-2) $\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$ (Direct Method)

Q. Find the pair of st. lines ~~is~~ formed by the eqn. $x^2 - 6xy + 8y^2 = 0$

Sol^n $1 - 6(y/x) + 8(y^2/x^2) = 0$

$\Rightarrow 8m^2 - 6m + 1 = 0 \Rightarrow (2m-1)(4m-1) = 0$

$\Rightarrow \boxed{m = 1/2}$ & $\boxed{m = 1/4}$

$\therefore$ Lines formed $\Rightarrow \boxed{y = \frac{1}{2}x}$

& $\boxed{y = \dfrac{x}{4}}$

↳ Sum and Product of Slopes of Lines:

$$\Rightarrow ax^2 + 2hxy + by^2 = 0$$

$$\Rightarrow y^2 + \frac{2h}{b}\cdot xy + \frac{a}{b}x^2 = 0$$

$$\Rightarrow (y - m_1x)(y - m_2x) = 0$$

$$\Rightarrow y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$$

$$\boxed{m_1 + m_2 = \frac{-2h}{b}} \qquad \boxed{m_1 \cdot m_2 = \frac{a}{b}}$$

Q. Find the coordinates of centroid of Δ formed by the lines $2x - 3y + 4 = 0$; $12x^2 - 20xy + 7y^2 = 0$.

Sol^n

$$\boxed{y = 2x} \qquad \boxed{7y = 6x}$$

∴ Coordinates of Δ formed $\Rightarrow$

$A(1, 2)$; $B(7, 6)$; $C(0,0)$

∴ Coordinates of Centroid :-

$$\boxed{G\left(\frac{8}{3}, \frac{8}{3}\right)}$$

Q. Find the relat^n b/w n; a; h; b from the eqn. $ax^2 + 2hxy + by^2 = 0$ if $m_1 = n m_2$

Sol^n. $ax^2 + 2hxy + by^2 = 0$ → m_1, m_2

As we know that;

$m_1 + m_2 = \frac{-2h}{b}$ & $m_1 \cdot m_2 = \frac{a}{b}$

$\Rightarrow nm_2 + m_2 = \frac{-2h}{b}$

$\Rightarrow m_2(n+1) = \frac{-2h}{b}$

$\Rightarrow \boxed{m_2 = \frac{-2h}{b(n+1)}}$

$(nm_2)(m_2) = \frac{a}{b}$

$\Rightarrow \boxed{m_2^2 = \frac{a}{nb}}$

$\therefore \frac{4h^2}{b^{\cancel{2}}(n+1)^2} = \frac{a}{n\cancel{b}}$

$\Rightarrow \boxed{4h^2 n = ab(n+1)^2}$

Q. Find the relat^n b/w a; b; c; d from the eqn. $ax^3 + bx^2y + cxy^2 + dy^3 = 0$, if 2 out of the 3 lines are ⊥ar.

Sol^n $ax^3 + bx^2y + cxy^2 + dy^3 = 0$

$\Rightarrow dm^3 + cm^2 + bm + a = 0$ → m_1, m_2, m_3

Here, $m_1 + m_2 + m_3 = -\frac{c}{d}$

$m_1 m_2 + m_2 \cdot m_3 + m_3 \cdot m_1 = \frac{b}{a}$

$m_1 \cdot m_2 \cdot m_3 = \frac{-a}{d}$

T.I.P. ⇒ Eqn. of two lines passing through Origin & ⊥ar to each other ⇒

$$\boxed{x^2 + 2hxy - y^2 = 0}$$

So, Let m_1 & m_2 be the two lines which are perpendicular

$\therefore \boxed{m_1 m_2 = -1}$ ⇒ $m_1 \cdot m_2 \cdot m_3 = \frac{-a}{d}$

$\Rightarrow \boxed{m_3 \Rightarrow \frac{a}{d}}$

Also, $dm_3^3 + cm_3^2 + bm_3 + a = 0$

$\Rightarrow d\left(\frac{a}{d}\right)^3 + c\left(\frac{a}{d}\right)^2 + b\left(\frac{a}{d}\right) + a = 0$

$\Rightarrow \boxed{a^2 + ac + bd + d^2 = 0}$

Note :- from the Eqn. ;-

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h} \text{ ; where } \boxed{a = b}$$

$\Rightarrow x^2 - y^2 = 0 \Rightarrow \boxed{y = \pm x}$

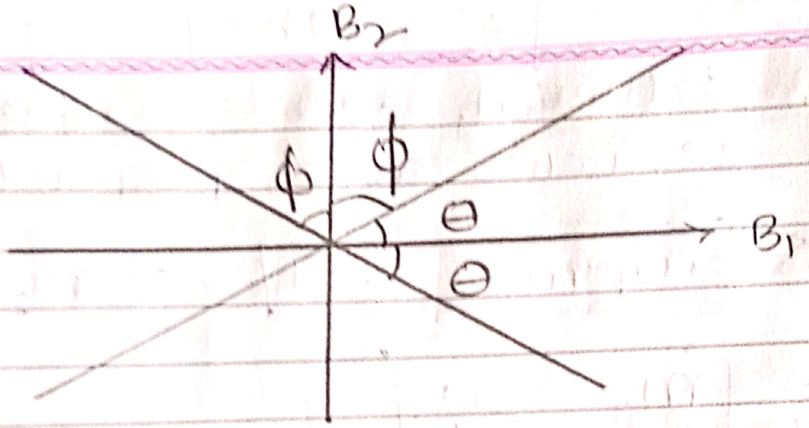
Here, lines will be mirror images of each other about ; $\boxed{y = x}$ or $\boxed{y = -x}$

Note :- Again, from the eqn. :-

Let $\boxed{h = 0}$ ⇒ $xy = 0$

Then, 'x'-axis & 'y'-axis ⇒ Angle Bisector

⇒ Lines will be equally inclined to x-axis or y-axis.



Q.

(B) 2^{nd} Degree ; Non- Homogenous eqn in x & y :-

$\hookrightarrow$ $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2gfh - af^2 - bg^2 - ch^2$$

→ If $\Delta = 0$ → Pair of st. lines passing through any point

→ If $h^2 > ab$ → Distinct lines

→ If $h = ab$ → Coincident lines

$\hookrightarrow$ Sum ; Products of Roots & Angle Bisector:-

$m_1 + m_2 = \dfrac{-2h}{b}$	$\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{\lvert a+b \rvert}$
& $m_1 \cdot m_2 = \dfrac{a}{b}$	

Q. Find out the P.O.I. of the lines formed by the equatn $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$

M-1) $6x^2 + (13y + 8)x + 6y^2 + 7y + 2 = 0$

$$\Rightarrow x = \frac{-(13y+8) \pm \sqrt{(13y+8)^2 - 4(6)(6y^2+7y+}}{2(6)}$$

$$\Rightarrow x = \frac{-(13y+8) \pm \sqrt{(5y+4)^2}}{12}$$

$$\boxed{3x + 2y + 1 = 0} \qquad \boxed{2x + 3y + 2 = 0}$$

M-2) By Partial Differentiation :-

↳ $12x + 13y + 0 + 8 + 0 + 0 = 0$

$\Rightarrow 12x + 13y + 8 = 0$

↳ $0 + 13x + 12y + 0 + 7 + 0 = 0$

$\Rightarrow 13x + 12y + 7 = 0$

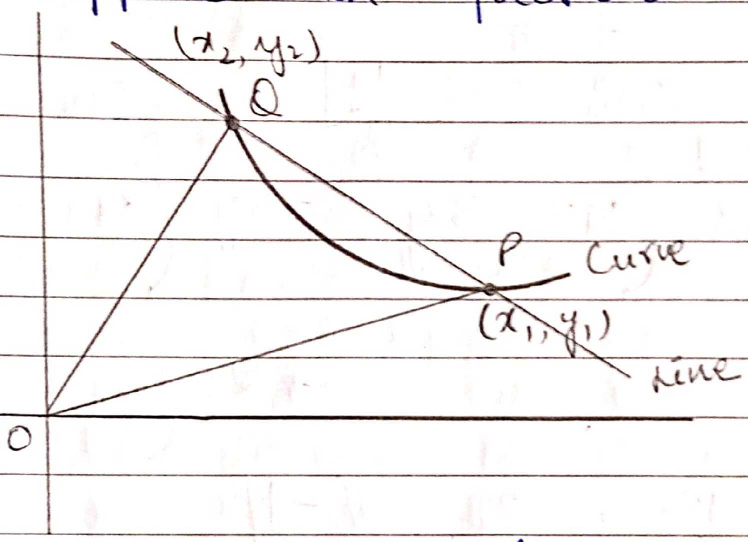
→ Not actual eqn but P.O.I. can be found

Q4) Concurrent Lines :-

(Q4) Homogenisation :-

↳ Method of Converting a non-homogeneous eqn. of line (like, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$) into a homogenous eqn. form (like $lx + my + n = 0$)

↳ Applied in questions like :-



→ for finding eqn. of lines OP & OQ.

↳ Methodology for Conversion :-

S-1) $lx + my = -n \rightarrow \boxed{\dfrac{lx + my}{-n} = 1}$

S-2) $ax^2 + 2hxy + by^2 + 2g(1)x + 2fy(1) + c(1)^2 = 0$

S-3) $ax^2 + 2hxy + by^2 + 2gx\left(\dfrac{lx + my}{-n}\right) + 2fy\left(\dfrac{lx + my}{-n}\right) + c\left(\dfrac{lx + my}{-n}\right)^2$

→ $Ax^2 + By^2 + Cxy = 0$

(2) (2) (2)

Q1 What is the angles made by the lines joining the origin to the P.O.I. of the line and the curve.

Given :- Eqn. of line :- $4x - 3y = 10$

Eqn. of curve :- $x^2 + y^2 + 3x - 6y$:

Soln. Here, $4x - 3y = 10 \Rightarrow \boxed{\frac{4x - 3y}{10} = 1}$

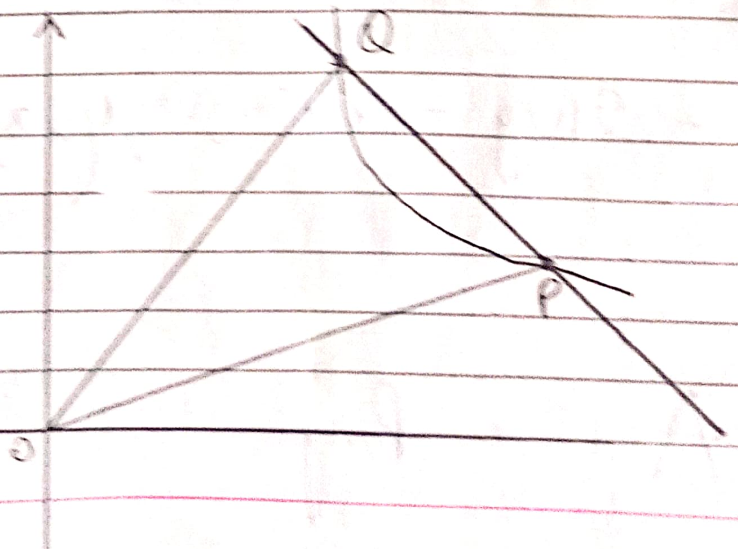
Substituting value of $\boxed{1}$ in curve's eqn.

$$\Rightarrow x^2 + y^2 + 3x\left(\frac{4x-3y}{10}\right) - 6y\left(\frac{4x-3y}{10}\right) + 20\left(\frac{4x-3y}{10}\right)^2 = 0$$

Here, $a = 1 + \frac{12}{20} + \frac{32}{10} = \boxed{-1}$

& $b = 1 + \frac{(6 \times 3)}{10} - \frac{26 \times 9}{100} = \boxed{1}$

$\therefore \boxed{a + b = 0} \Rightarrow \boxed{\theta = 90^\circ}$



Q. The st. line $ax + by = 1$ make with a circle a chord which subtends a right angle at the origin.

Given :- Eqn. of circle :- $px^2 + 2axy + qy^2 = r$

Find the relatⁿ b/w $a ; b ; b ; q ; r$:-

Solⁿ.

$$px^2 + 2axy + qy^2 - r(ax + by)^2 = 0$$

To get the angle at Origin as $90°$:-

Coefficient of x^2 $y^2 = 0$

$\Rightarrow$ $p - ra^2 + q - rb^2 = 0$ $\Rightarrow$ $\boxed{p + q = r(a^2 + b^2)}$

Q. The straight lines $x + ky = 1$ bisects a curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$

The line joining the point of intersection from Origin, makes equal angle with x-axis. Then, find the value of 'K'.

Solⁿ

$$5(x^2) + 12xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$$

$\Rightarrow$ $12 + 4K - 2 + 3(2K) = 0$ $\Rightarrow$ $\boxed{K = -1}$

Q. If curve $C_1 = \lambda x^2 + 4y^2 - 2xy - 9x + 3 = 0$ and curve $C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1 = 0$ get intersect and subtend a right angle at the origin. Then, find the value of λ.

Solⁿ. Here,

$$3C_2 = 6x^2 + 9y^2 - 12xy + 9x - 3 = 0$$

On Adding $C_1 + 3C_2 = 0$; we get :-

$$(\lambda+6)x^2 + 13y^2 - 14xy = 0$$

$\because$ $\boxed{\theta = 90^\circ}$ at the origin ; $\boxed{a+b=0}$

$\Rightarrow \lambda + 6 + 13 = 0 \Rightarrow \boxed{\lambda = -19}$

Q. A St. line $kx - 2y = 1$ intersects a curve $2x^2 + 4y^2 - 4xy + 2x - 3y = 0$
The line joining the P.O.I. from Origin are mirror images of each other about $y = x$; $y = -x$
Then, find the value of K.

Solⁿ.

$$2x^2 + 4y^2 - 4xy + 2x(kx - 2y) - 3y(kx - 2y) = 0$$

$\because$ $\boxed{a - b = 0} \longrightarrow \boxed{y = x}$

$\therefore$ $a = 2 + 2K$ & $b = 4 + 6 = 10$

$\therefore$ $a - b = 2 + 2K - 10 = 0$

$\Rightarrow \boxed{K = 4}$