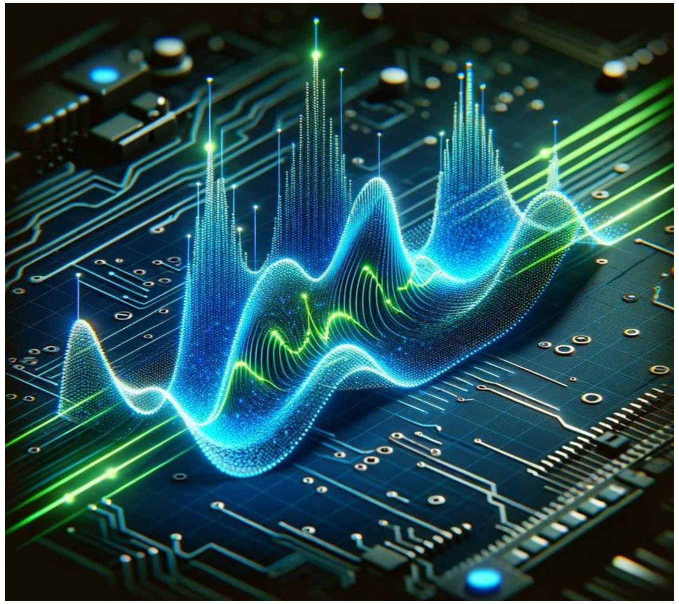
22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING



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22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - CO’s

CO1	Illustrate the basics of discrete time signals and systems. [U]
CO2	Interpret the concepts of Discrete and Fast Fourier transform [U]
CO3	Comprehend the architecture of advanced processors. [U]
CO4	Apply the concept of transformation techniques in Discrete Time systems. [AP]
CO5	Design different types of filters using various filter design techniques. [AP]

22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - Modules

1	Signals and Systems
2	Discrete Fourier Transform and Fast Fourier Transform
3	FIR Filters, IIR Filters and Digital Signal Processors

MODULE – I : Signals and Systems

1.1	Generation and Representation of Discrete Time signals
1.2	Classification of signals
1.3	Classification of systems
1.4	Sampling of continuous time signals and aliasing effect

1.1 Generation and Representation of Discrete Time signals

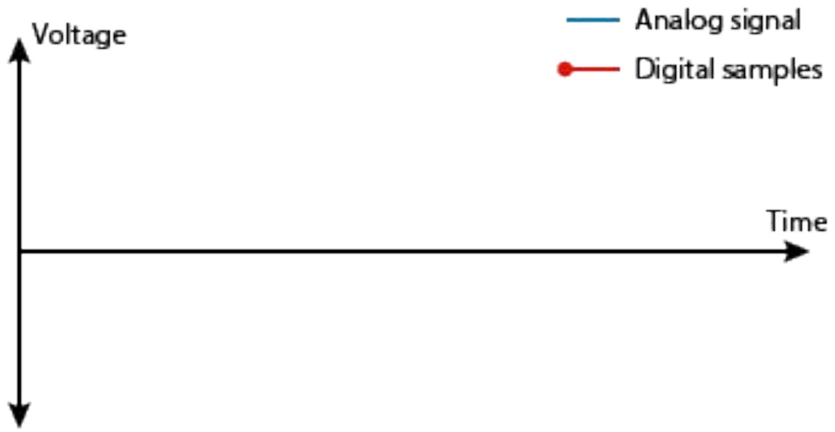
Introduction:

Signal:

A Physical quantity that varies with time, space or any other independent variables.

Eg. $s_1(t)= 5t$

$s(x,y)=3x+2xy+10y^2$



1.1 Generation and Representation of Discrete Time signals

Signal Processing:

- Modify a signal to extract / enhance / rearrange the information.
- Converting continuously changing waveforms(Analog) into a series of discrete levels(Digital).

1.1 Generation and Representation of Discrete Time signals

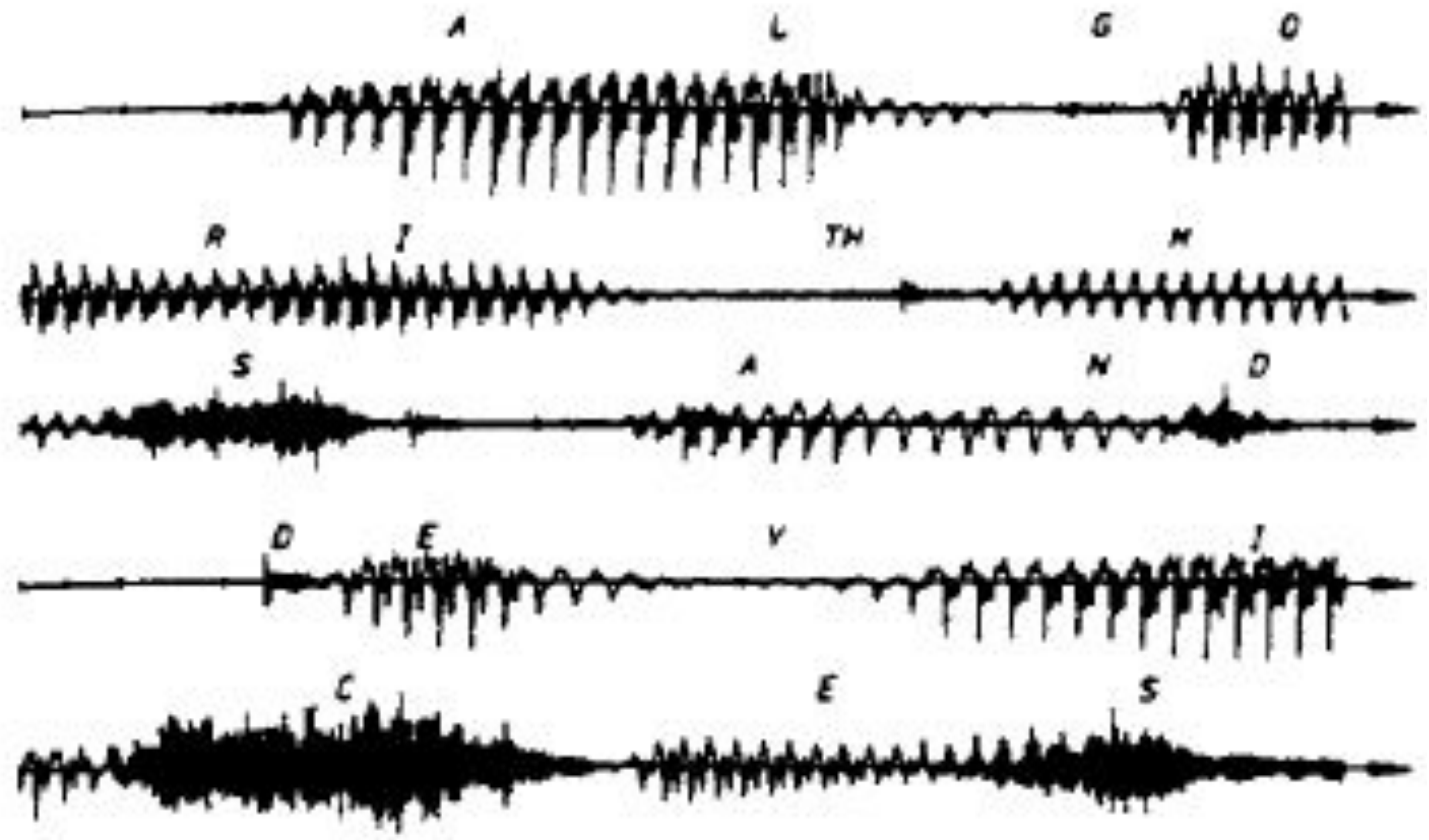
Natural signals

- Electro cardiogram (ECG)
- Electroencephalogram (EEG)
- Speech signal

Characteristics of signal

1. Amplitude
2. Shape
3. Phase
4. Frequency content of signal

1.1 Generation and Representation of Discrete Time signals



Example of a speech signal

1.1 Generation and Representation of Discrete Time signals

- Digital signal processing

 An area of science and engineering

 Provides proper solution for all signal processing problems

- What is a System?

 Process input signals to produce output signals.

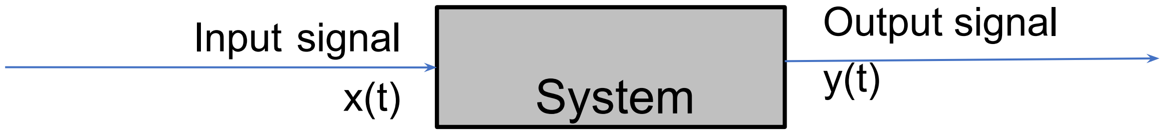
- **Examples:**

   Electric Circuit

   CD player

   Communication system

How is a System Represented?



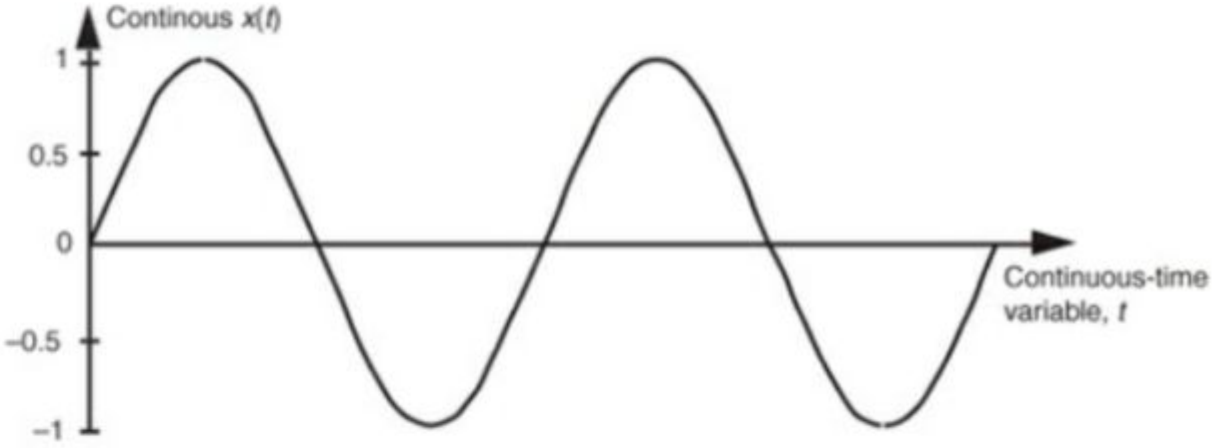
1.1 Generation and Representation of Discrete Time signals

Classification of signals

1. Continuous time signal
2. Discrete time signal

Continuous time signal

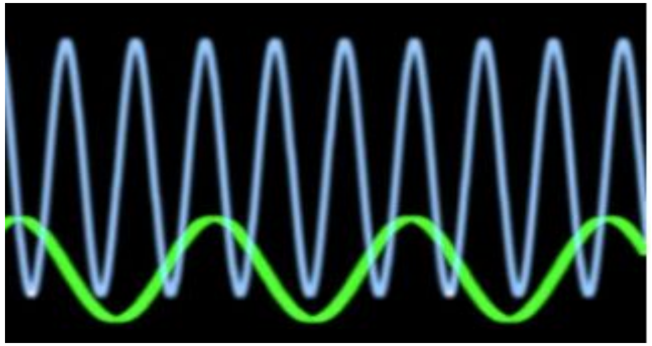
- The Signals that are defined for every instant of time are known as continuous-time signals. They are denoted by x(t).



1.1 Generation and Representation of Discrete Time signals

Real time examples of continuous time signal

Electrical signal

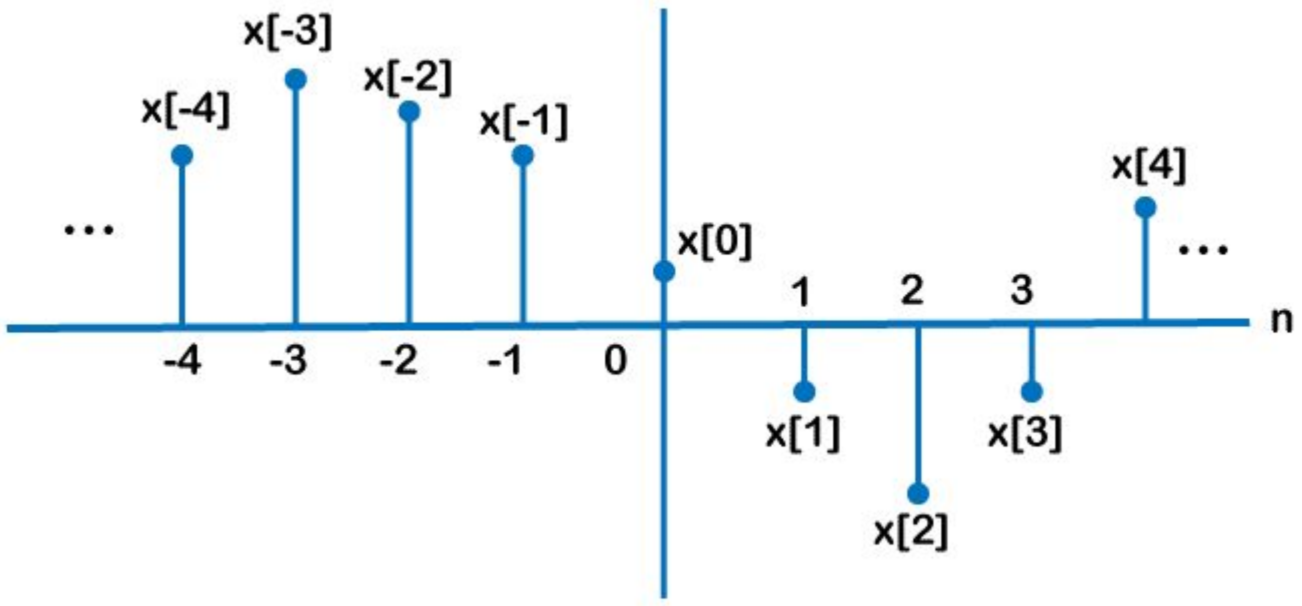


ECG signal

1.1 Generation and Representation of Discrete Time signals

Discrete time signal

- The Signals that are defined at discrete instants of time are known as discrete-time signals. The discrete-time signals are continuous in amplitude and discrete in time. They are denoted by x(n) where n is the independent variable in time domain.



1.1 Generation and Representation of Discrete Time signals

Real time example for sample

Continuous

Sample

1.1 Generation and Representation of Discrete Time signals

Real time example for sample

x(n)- run

n- ball index

x(n)- Student name

n- Roll no

1.1 Generation and Representation of Discrete Time signals

Real time examples - Continuous to digital

Analog watch

Digital watch

1.1 Generation and Representation of Discrete Time signals

Representation of Discrete-time signals:

A discrete time signal may be represented by any one of the following four ways −

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

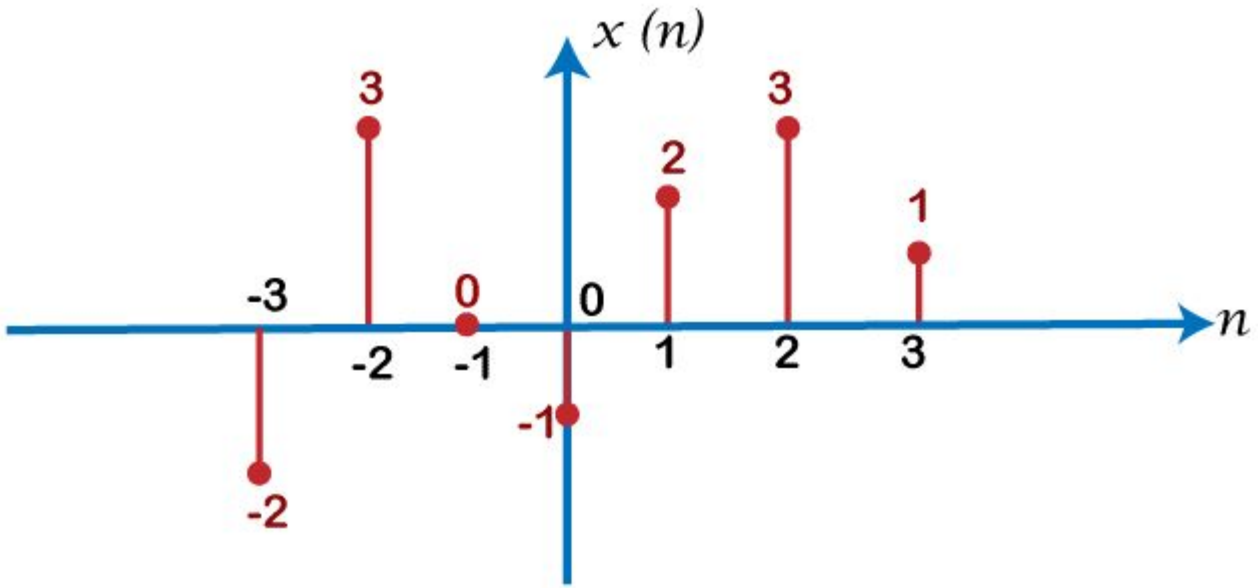
1.1 Generation and Representation of Discrete Time signals

1. Graphical representation:

Consider a discrete time signal x(n) with the values,

$$x(-3) = -2,\ x(-2) = 3,\ x(-1) = 0,\ x(0) = -1,\ x(1) = 2,\ x(2) = 3,\ x(3) = 1$$

We can represent the discrete-time signal graphically in the following way:



1.1 Generation and Representation of Discrete Time signals

2. Functional representation:

In the functional representation of discrete time signals, the magnitude of the signal is written against the values of n. Therefore, the above discrete time signal x(n) can be represented using functional representation as given below.

x[n] = {-2 for n = -3,
3 for n = -2,
0 for n = -1,
-1 for n = 0,
2 for n = 1,
3 for n = 2,
1 for n = 3}

1.1 Generation and Representation of Discrete Time signals

3. Tabular representation:

In case of a tabular representation of discrete-time signal, we use the table to represent the sampling instant n and the magnitude of discrete-time signal at the corresponding sampling instant. In the following way, we can represent the above discrete time signal x(n) with the help of a tabular form like this:

n	-3	-2	-1	0	1	2	3
x[n]	-2	3	0	-1	2	3	1

1.1 Generation and Representation of Discrete Time signals

4. Sequence representation:

In the form of sequence representation, we can represent the discrete-time signal x[n] in the following way:

Finite-duration sequence:

$$x[n] = \{-2, 3, 0, -1, 2, 3, \underset{\uparrow}{1}\}$$

Here, The arrow mask (↑) is used to indicate the term corresponding to n = 0. If there is a case in which sequence representation of a discrete-time signal does not contain any arrow, then the first term of this sequence will correspond to n = 0.

1.1 Generation and Representation of Discrete Time signals

Infinite-duration sequence

Time origin (n = 0) indicated by the symbol (↑)

$$x(n) = \{\ldots 0, \underset{\uparrow}{0}, 1, 4, 1, 0, 0, \ldots\}$$

Sequence x(n), which is zero for n < 0,

$$x(n) = \{\underset{\uparrow}{0}, 1, 4, 1, 0, 0, \ldots\}$$

1.1 Generation and Representation of Discrete Time signals

Elementary Discrete -Time Signals:

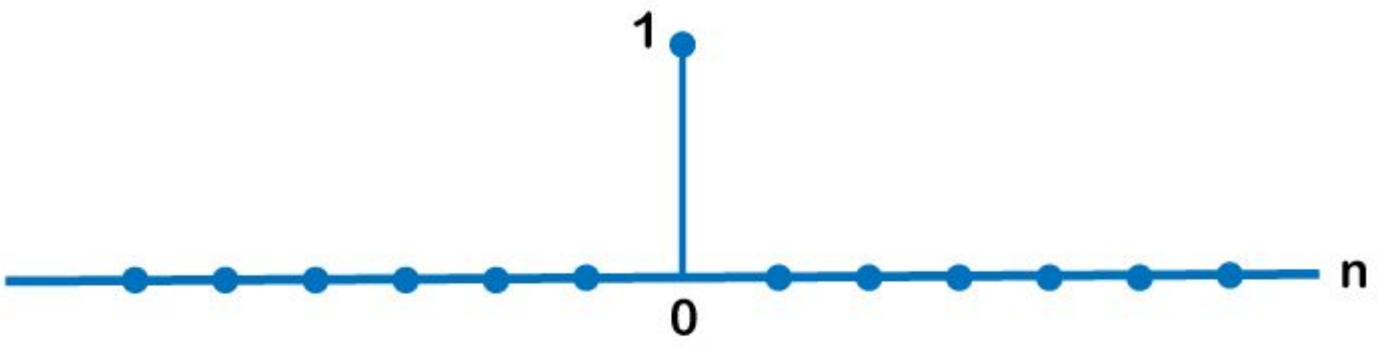
1. Unit impulse or sample signal
2. Unit Step signal
3. Unit ramp signal
4. Unit parabolic signal
5. Rectangular signal
6. Triangular signal
7. Exponential signal

1.1 Generation and Representation of Discrete Time signals

1. Unit impulse or sample signal:

The discrete-time unit impulse signal is denoted by δ(n) and is defined as −

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0. \end{cases}$$

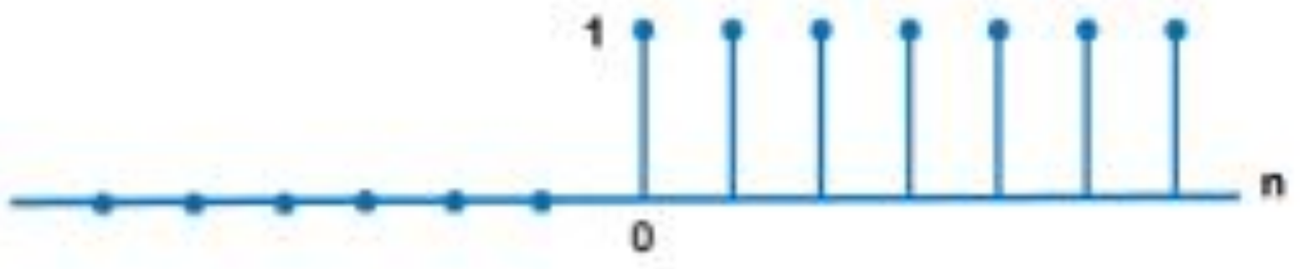


1.1 Generation and Representation of Discrete Time signals

2. Unit Step signal:

The discrete-time unit step signal is denoted by u(n) and is defined as −

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0. \end{cases}$$



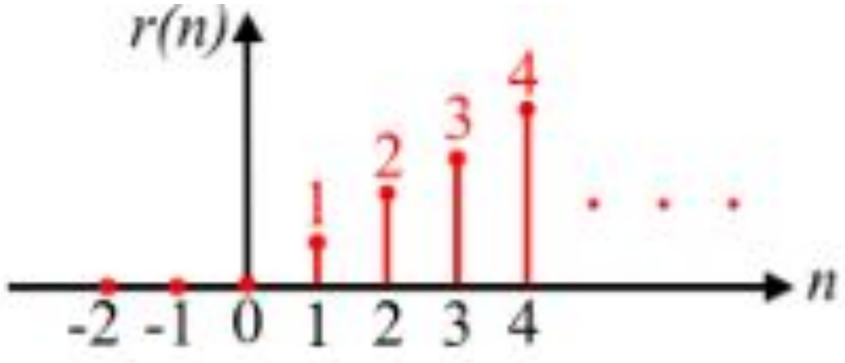
1.1 Generation and Representation of Discrete Time signals

3. Unit ramp signal:

The **discrete time unit ramp signal** is that function which starts from n = 0 and increases linearly. It is denoted by r(n). It is signal whose amplitude varies linearly with time n. **Mathematically**, the discrete time unit ramp sequence is defined as −

$$r(n) = \{ n \;\; for\, n \geq 0 \;\; 0 \;\; for\, n < 0$$

$$r(n) = n\, u(n)$$

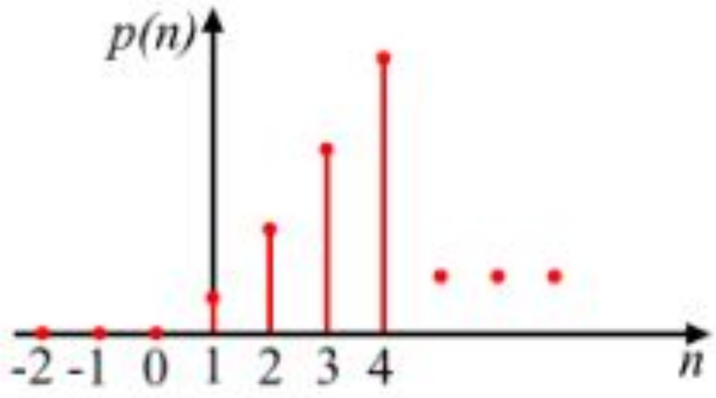


4. Unit parabolic signal:

The discrete-time unit parabolic sequence is a unit parabolic signal which is defined only at discrete instants of positive time n. It is denoted by p(n). **Mathematically**, p(n) is given as,

$$p(n) = \begin{cases} \frac{n^2}{2} & for\ n \geq 0 \\ 0 & for\ n < 0 \end{cases}$$

$$p(n) = \frac{n^2}{2} u(n)$$



1.1 Generation and Representation of Discrete Time signals

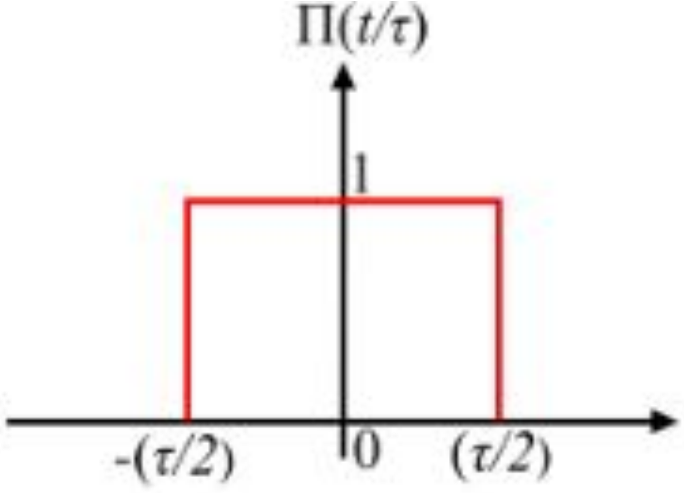
5. Rectangular signal

A signal that produces a rectangular shaped pulse with a width of τ (where τ = 1 for the unit rectangular function) centred at t = 0 is known as **rectangular signal**. The rectangular signal pulse also has a height of 1. **Mathematically**, the unit rectangular signal is defined as,

$$\Pi\left(\frac{t}{\tau}\right) = \left\{ 1 \; for \; |t| \leq \left(\frac{\tau}{2}\right) \; 0 \; otherwise \right.$$

1.1 Generation and Representation of Discrete Time signals

The rectangular signal is also known as the **unit pulse**, gate function or **normalised boxcar function**. Also, the rectangular function is an even function of time. The graphical representation of a rectangular pulse signal is shown in Figure



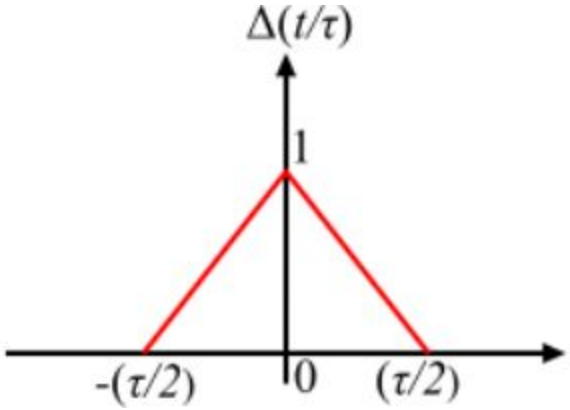
1.1 Generation and Representation of Discrete Time signals

6. Triangular signal

A function whose graph takes the shape of a triangle is known as **triangular signal**. The triangular signal is also known as **hat function** or **tent function**. Mathematically, the unit triangular pulse signal Δ(t/τ) is defined as,

$$\Delta\left(\frac{t}{\tau}\right) = \left\{1 - \left(\frac{2|t|}{\tau}\right)\ for\ |t| < \left(\frac{\tau}{2}\right)\ 0\ for\ |t| > \left(\frac{\tau}{2}\right)\right.$$

The triangular signal is also an even function of time. The graphical representation of a triangular signal is shown in Figure



1.1 Generation and Representation of Discrete Time signals

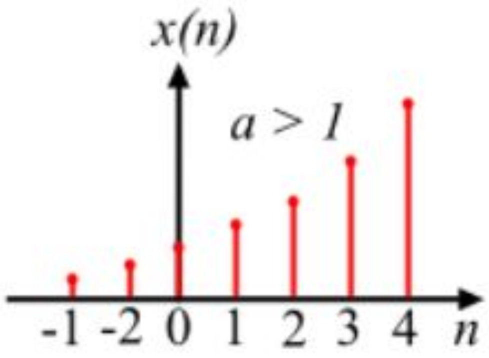
7. Exponential signal

A real exponential signal which is define at discrete instants of time is called a discrete-time real exponential signal or sequence. A discrete-time real exponential sequence is defined as

$$x(n) = a^n \quad \text{for all } n$$

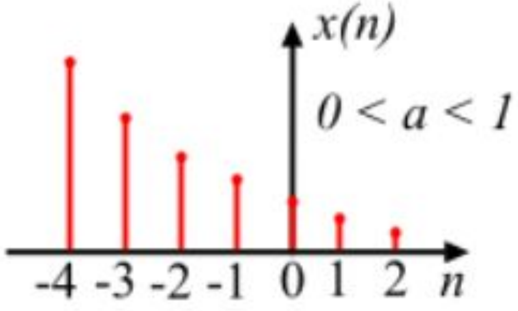
Depending upon the value of the discrete time real exponential signal may be of following type −

- When a > 1, the exponential sequence x(n) grows exponentially.

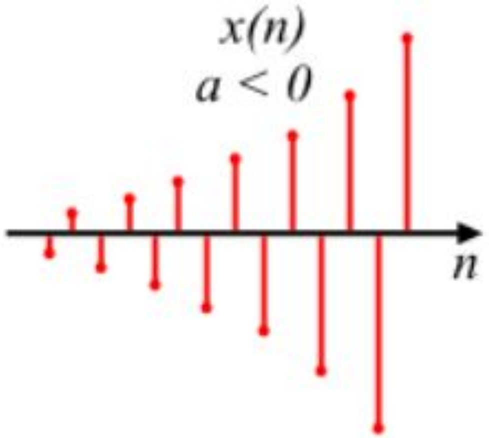


1.1 Generation and Representation of Discrete Time signals

- When 0 < a < 1, the exponential signal x(n) decays exponentially.



- When a < 0, the exponential sequence x(n) takes alternating signs



1.1 Generation and Representation of Discrete Time signals

Basic Properties of signals:

1. Time Shifting
2. Time Scaling
3. Time Inversion or Time Folding
4. Amplitude scaling
5. Signal addition and multiplication

1.1 Generation and Representation of Discrete Time signals

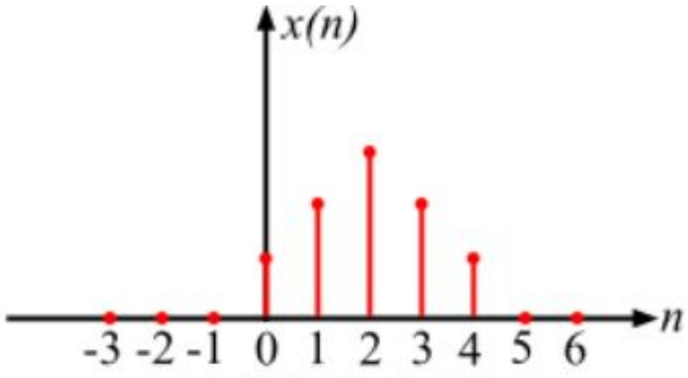
1. Time Shifting:

The time shifting operation of a discrete time signal x(n) is represented as

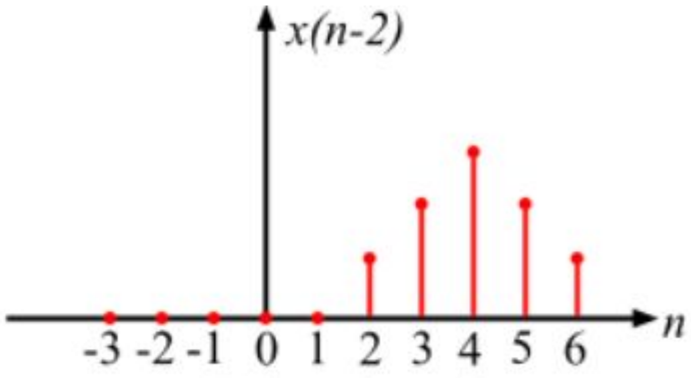
$$y(n) = x(n - n_0)$$

This equation shows that the signal y(n) can be obtained by time-shifting the signal x(n) by n_0 units. If the value of n_0 is positive, then the shift of the signal is to the right and the resulting signal is the delayed version. Whereas, if the value of n_0 is negative, then the shift of the signal is to the left and it is the time advanced version of the signal.

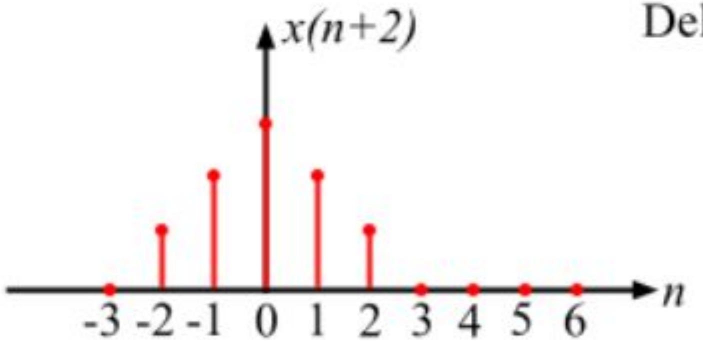
1.1 Generation and Representation of Discrete Time signals



Original signal



Delayed version



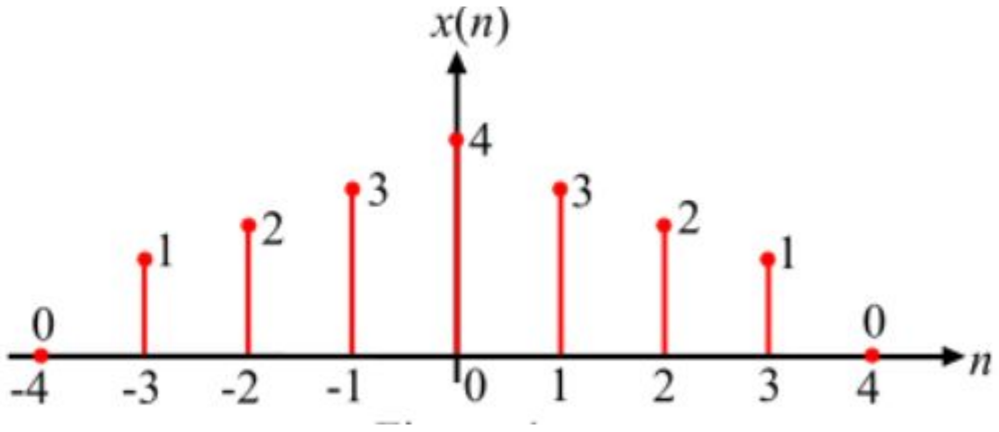
Advanced version

1.1 Generation and Representation of Discrete Time signals

2. Time Scaling:

The process of multiplying a constant to the time axis of a signal is known as **time scaling of the signal**. The time scaling for the discrete sequence can be defined as,

$$x(n) \rightarrow y(n) = x(kn)$$



Case I – If k = 2, then

$$x(n) \rightarrow y(n) = x(2n)$$

Here, k > 1, thus the signal is compressed in time. We can plot the time compressed signal y(n) by substituting different values of n as shown in Figure

1.1 Generation and Representation of Discrete Time signals

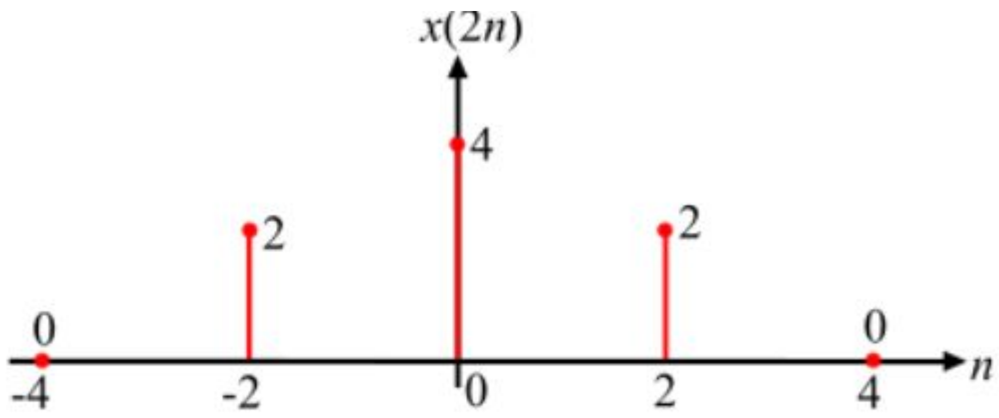
$n = 0 \rightarrow y(0) = x(0) = 4$

$n = (-1) \rightarrow y(-1) = x(-2) = 2$

$n = (-2) \rightarrow y(-2) = x(-4) = 0$

$n = 1 \rightarrow y(1) = x(2) = 2$

$n = 2 \rightarrow y(2) = x(4) = 0$



Therefore, to plot the signal x(2n), we have to skip the odd numbered samples in the signal x(n).

1.1 Generation and Representation of Discrete Time signals

Case II – If k = (1/2), then

$$x(n) \rightarrow y(n) = x(n/2)$$

Since k < 1, hence the signal is expanded by a factor 2. We can plot the time expanded signal y(n) by substituting different values of n as follows −

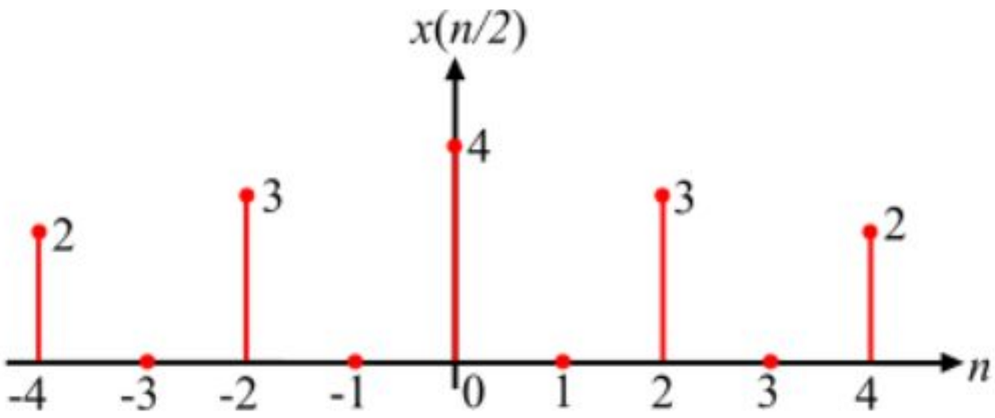
$n = 0 \rightarrow y(0) = x(0) = 4$

$n = 2 \rightarrow y(2) = x(1) = 3$

$n = 4 \rightarrow y(4) = x(2) = 2$

$n = (-2) \rightarrow y(-2) = x(-1) = 3$

$n = (-4) \rightarrow y(-4) = x(-2) = 2$



In this case, all odd components in x(n/2) are zero because the signal x(n) does not have any value in between the sampling instants. Figure shows the plot of the signal y(n)=x(n/2)

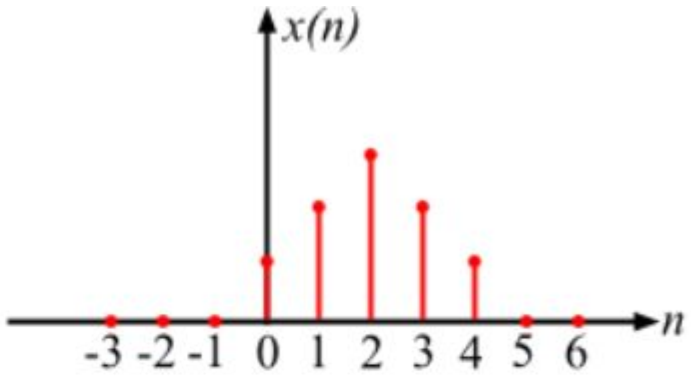
1.1 Generation and Representation of Discrete Time signals

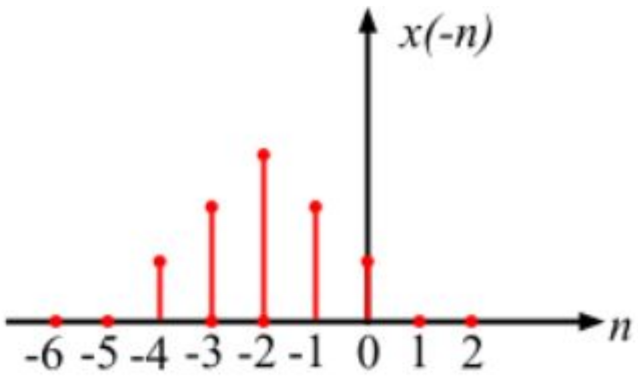
3. Time Inversion or Time Folding:

For a discrete time sequence x(n), the time reversal is given by,

$$y(n) = x(-n)$$

An arbitrary discrete-time signal x(n) and its time reversal x(-n) are shown in Figure





4. Amplitude scaling:

The process of rescaling the amplitude of a signal, i.e., the amplitude of the signal is either amplified or attenuated, is known as **amplitude scaling**. In the amplitude scaling operation on signals, the shape of the resulting signal remains the as that of the original signal but the amplitude is altered (i.e., increased or decreased).

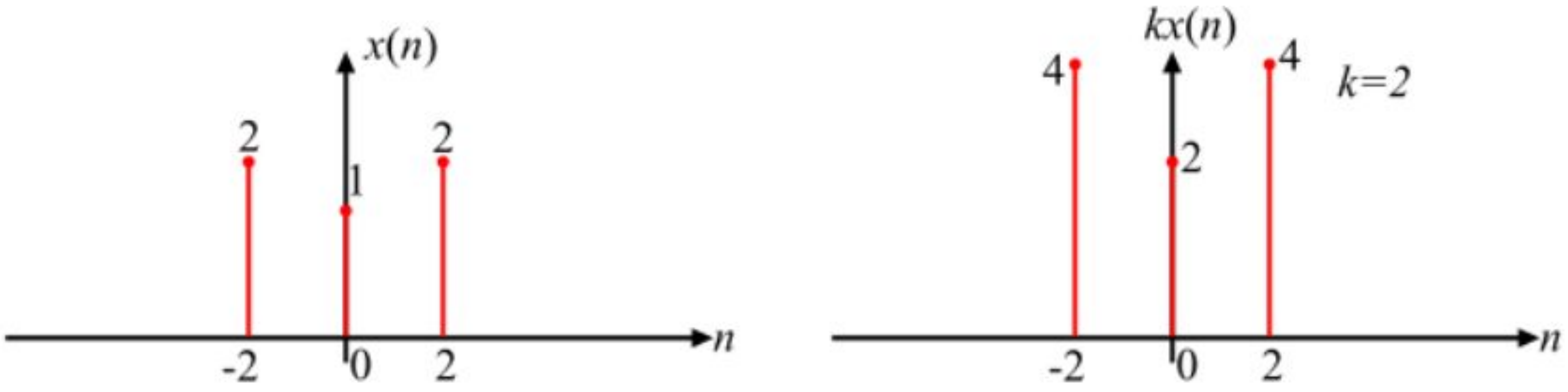
The amplitude scaling of a discrete time sequence x(n) is defined as,

$$y(n) = k\,x(n)$$

Where, k is a constant. If $k > 1$, the scaling is called **amplification of the signal**, while if $k < 1$, the scaling is called **attenuation of the signal**.

1.1 Generation and Representation of Discrete Time signals

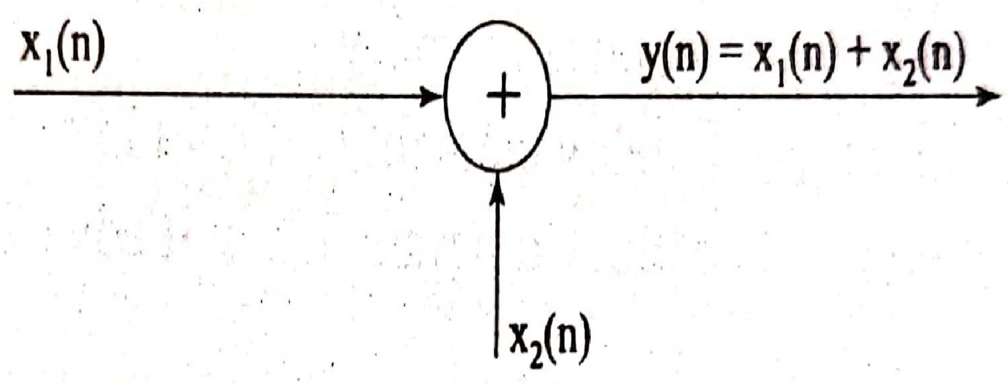
An arbitrary discrete time sequence x(n) and its scaled version y(n) are shown in Figure



5. Signal addition and multiplication:

Signal addition

- Signals are added using an adder



For Example, if

$x_1(n)=\{1,2,3,4\}$ and

$x_2(n)=\{4,3,2,1\}$

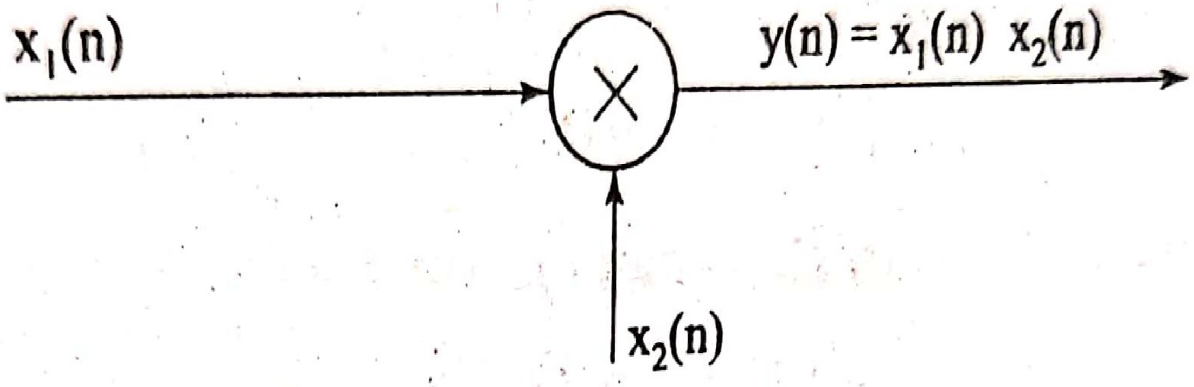
Then, $x_1(n)+ x_2(n) = \{1+4, 2+3, 3+2, 4+1\}$

$=\{5,5,5,5\}$

1.1 Generation and Representation of Discrete Time signals

Signal Multiplication

- Signals are multiplied using a multiplier.



For Example, if

$x_1(n)$={-1,2,-3,-2} and

$x_2(n)$={1,-1,-2,1}

Then, $x_1(n)$. $x_2(n)$ = {-1.1, 2.-1, -3.-2, -2.1}

={-1, -2, 6, -2}