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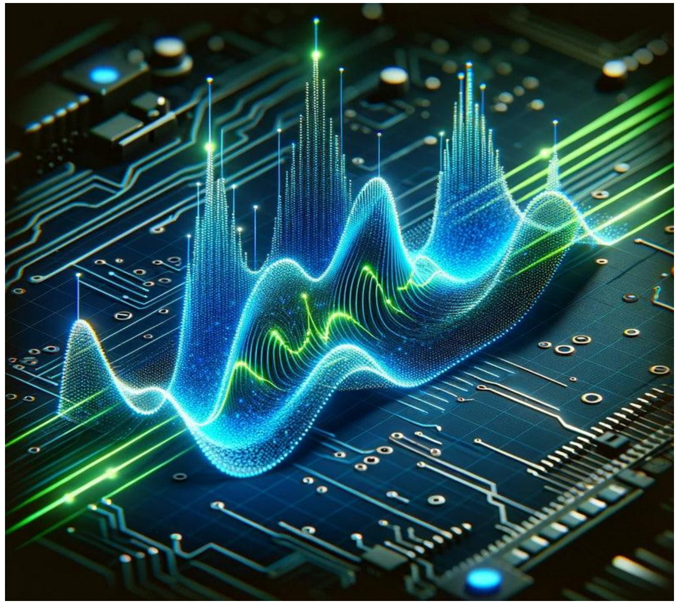
22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING



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22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - CO's

CO1	Illustrate the basics of discrete time signals and systems. [U]
CO2	Interpret the concepts of Discrete and Fast Fourier transform [U]
CO3	Comprehend the architecture of advanced processors. [U]
CO4	Apply the concept of transformation techniques in Discrete Time systems. [AP]
CO5	Design different types of filters using various filter design techniques. [AP]

22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - Modules

1	Signals and Systems
2	Discrete Fourier Transform and Fast Fourier Transform
3	FIR Filters, IIR Filters and Digital Signal Processors

MODULE – I : Signals and Systems

1.1	Generation and Representation of Discrete Time signals
1.2	Classification of signals
1.3	Classification of systems
1.4	Sampling of continuous time signals and aliasing effect

1.3 Classifications of systems

Classification of Discrete time systems:

1. Static & Dynamic system
2. Causal & Non-causal system
3. Linear & Non Linear system
4. Time Variant & Time-invariant system
5. Stable & Unstable system

1.3 Classifications of systems

1. Static & Dynamic system

Static system

- Output depends on present input only.
- Does not have memory

Ex**:** x(n)cos ωn

Dynamic system

- Output depends on past and future inputs.
- Has memory

Ex: y(n)= x(n+a)

1.3 Classifications of systems

Problem1: Find whether the following system is static or dynamic.

$y(n)=x(n)x(n-1)$

Solution

The output y(n) depends on the past input.

Therefore, the system is dynamic.

Problem2: Find whether the following system is static or dynamic.

$y(n)= x^2(n)+x(n)$

Solution

The output y(n) depends on the input at that instant only.

Therefore, the system is static.

1.3 Classifications of systems

Practice Problem: Find whether the following systems are static or dynamic.

i) $y(n)= x(2n)$

ii) $y(n)=x^2(n)$

1.3 Classifications of systems

2. Causal & Non-causal system

Causal system:

- Output of the system depends on
  - ✔ Present
  - ✔ Past inputs

Ex: $y(n)= x(n)+ x(n-a)$

Non causal or anticausal:

- Output of the system depends on future inputs also.

Ex: $y(n)= x(2n)$

- Physically unrealizable
- In pre-recorded signals, future values are available at the time of processing

1.3 Classifications of systems

Past input - x(n-1), x(n-2), x(n-3)......
Present Input - x(n)
Future input - x(n+1), x(n+2), x(n+3).....

Past output - y(n-1), y(n-2), y(n-3)......
Present output - y(n)
Future output - y(n+1), y(n+2), y(n+3).....

Video link

This is video explains the causal and non-causal system

https://drive.google.com/open?id=1Jhr1p_7rAbEUtRvgb59ws4mELEkeY263

1.3 Classifications of systems

Real time example

Information about day

Present

- x(n)- Today

Past

- x(n-1) – Yesterday
- x(n-2) - Day before yesterday

Future

- x(n+1)- Tomorrow
- x(n+2) – Day after tomorrow

1.3 Classifications of systems

Problem 1: Determine if the systems described by the following equations are causal or non-causal.

$$y(n)= x(n)+ \frac{1}{x(n-1)}$$

Solution:

For n=-1, $y(-1)= x(-1)+ \frac{1}{x(-1-1)}$

$$=x(-1)+ \frac{1}{x(-2)}$$

For n=0, $y(0) = x(0)+ \frac{1}{x(-1)}$

For n=1, $y(1) = x(1)+ \frac{1}{x(0)}$

For all values of n, the system output depends on present and past inputs. Therefore, the system is causal.

1.3 Classifications of systems

Problem 2: Determine if the systems described by the following equations are causal or non-causal.

$y(n)= x(n^2)$

Solution:

For n=-1, $y(-1)= x((-1)^2) =x(1)$

For n=0, $y(0)= x((0)^2) =x(0)$

For n=1, $y(1)= x((1)^2) =x(1)$

For all values of n, (except for n=0, n=1), the system output depends on future inputs. Therefore, the system is non causal.

1.3 Classifications of systems

Practice problems: Determine if the systems described by the following equations are causal or non-causal.

i) $y(n)= Ax(n)+ B$

ii) $y(n)= ax(n)+bx(n-1)$

1.3 Classifications of systems

3. Linear & Non Linear system

A System is said to be linear system if it satisfies the superposition principle.

Superposition principle statement

The response of the system to a weighted sum of signals should be equal to the corresponding weighted sum of outputs of the system to each of the individual input signals.

$x_1(n)$, $x_2(n)$- Input

$y_1(n)$, $y_2(n)$- Output or response

$y_1(n) = T\{ x_1(n)\}$

$y_2(n) = T\{ x_2(n)\}$

a_1, a_2 -Weight

The output due to weighted sum of inputs

$$y_3(n)= T\{a_1x_1(n)+a_2x_2(n)\}$$

1.3 Classifications of systems

Weighted sum of outputs

$$y'_3(n)= a_1y_1(n)+a_2y_2(n)$$

$y_3(n)= y_3'(n)$ ------ Linear system

$y_3(n)\neq y_3'(n)$ ------Non Linear system

Video link

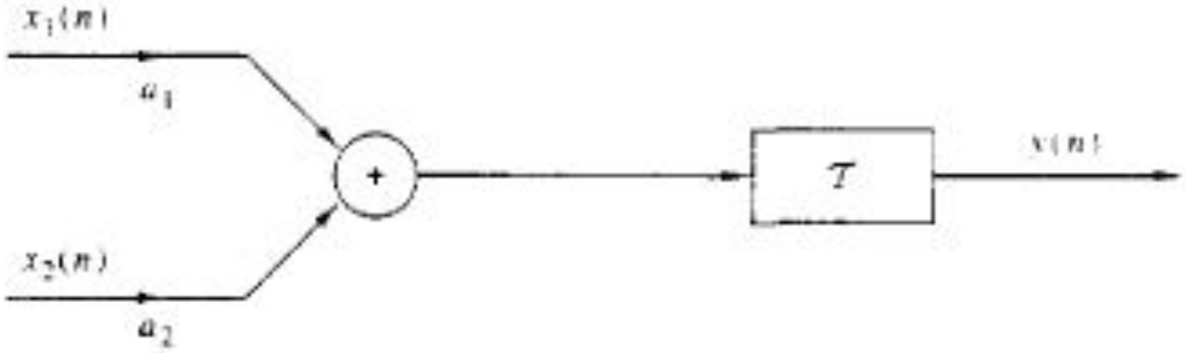
1. This video explains the linear and non-linear system

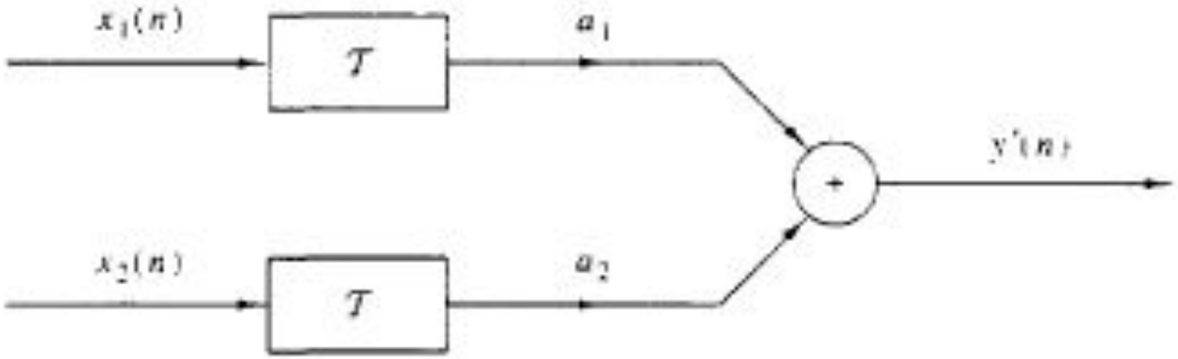
https://drive.google.com/open?id=13ce1-DreayVNYjDDnTKX3vhgfJ3AK6ZH

2. This is an additional NPTEL video which explains the linear and non-linear system with an example problem

https://drive.google.com/open?id=1s9UELIXqrwYJfEOz09Zy4BjhiCPngbBF

1.3 Classifications of systems





1.3 Classifications of systems

Problem 1: Determine if the systems described by the following input-output equations are linear or nonlinear.

$$y(n) = nx(n)$$

Solution

For two input sequences $x_1(n)$ and $x_2(n)$ the corresponding outputs $y_1(n)$ and $y_2(n)$ are

$$y_1(n) = nx_1(n)$$

$$y_2(n) = nx_2(n)$$

The output due to weighted sum of inputs

$$y_3(n) = T\{a_1x_1(n)+a_2x_2(n)\}$$

$$= n\{a_1x_1(n)+a_2x_2(n)\}$$

$$= na_1x_1(n)+na_2x_2(n)$$

1.3 Classifications of systems

Weighted sum of outputs

$$y'_3(n) = a_1y_1(n)+a_2y_2(n)$$
$$= a_1nx_1(n)+a_2nx_2(n)$$

$y_3(n) = y_3'(n)$

The given system is Linear system

1.3 Classifications of systems

Problem 2: Determine if the systems described by the following input-output equations are linear or nonlinear.

$$y(n) = x(n^2)$$

Solution

For two input sequences $x_1(n)$ and $x_2(n)$ the corresponding outputs $y_1(n)$ and $y_2(n)$ are

$y_1(n) = x_1(n^2)$

$y_2(n) = x_2(n^2)$

The output due to weighted sum of inputs

$$y_3(n) = T\{a_1 x_1(n) + a_2 x_2(n)\}$$

$$= a_1\, x_1(n^2) + a_2 x_2(n^2)$$

Weighted sum of outputs

$$y'_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

$$= a_1\, x_1(n^2) + a_2 x_2(n^2)$$

$$\mathbf{y_3(n) = y_3'(n)}$$

The given system is Linear system

1.3 Classifications of systems

Problem 3: Determine if the systems described by the following input-output equations are linear or nonlinear.

$y(n) = x^2(n)$

Solution

For two input sequences $x_1(n)$ and $x_2(n)$ the corresponding outputs $y_1(n)$ and $y_2(n)$ are

$$y_1(n) = x_1^2(n)$$
$$y_2(n) = x_2^2(n)$$

The output due to weighted sum of inputs

$$y_3(n) = T\{a_1x_1(n)+a_2x_2(n)\}$$

$$= [a_1x_1(n) + a_2x_2(n)]^2$$

$$= a_1^2x_1^2(n)+2a_1x_1(n)a_2x_2(n)+a_2^2x_2^2(n)$$

1.3 Classifications of systems

Weighted sum of outputs

$$y'_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

$$= a_1^2 x_1^2(n) + a_2^2 x_2^2(n)$$

$$\mathbf{y_3(n) \neq y_3'(n)}$$

The given system is non-linear system

Practice problems: Determine if the systems described by the following input-output equations are linear or nonlinear.

i) $Ax(n) + B$

ii) $y(n) = e^{x(n)}$

1.3 Classifications of systems

4. Time Variant &Time-invariant system

- If the time shift in the input signal results in corresponding time shift in the output

Let y(n)=H[x(n)]

- If x(n) is delay by time ‘m’ ,

output y(n,m) = H[x(n-m)]

- if output y(n) is delayed by k samples i.e y(n-m)

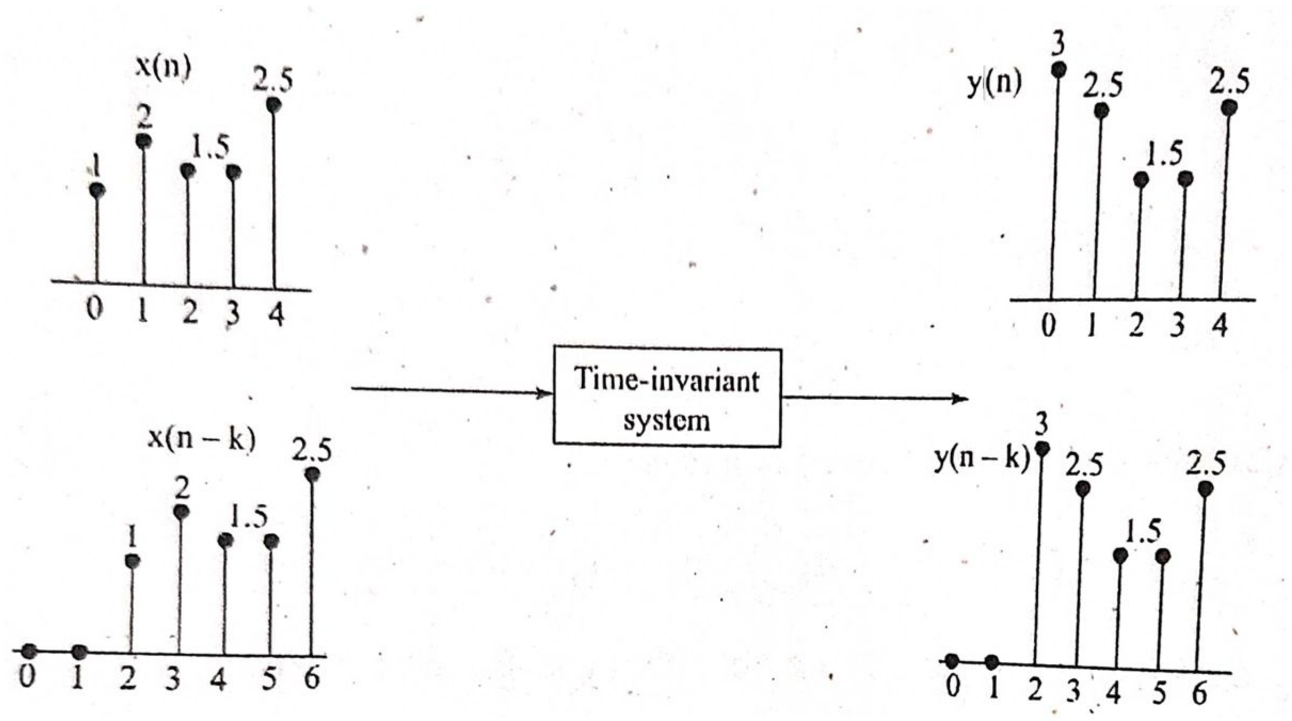
H[x(n-m)]=y(n,m)

- Satisfies the following relationship

y(n, m) = y(n-m) ------ Time invariant system

y(n, m) ≠ y(n-m) ------ Time variant system

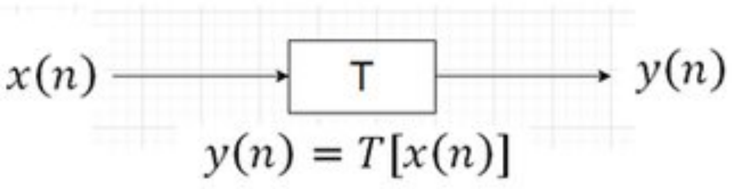
1.3 Classifications of systems

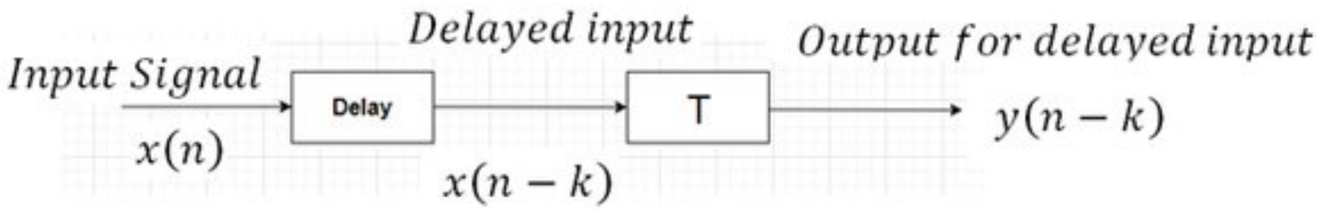


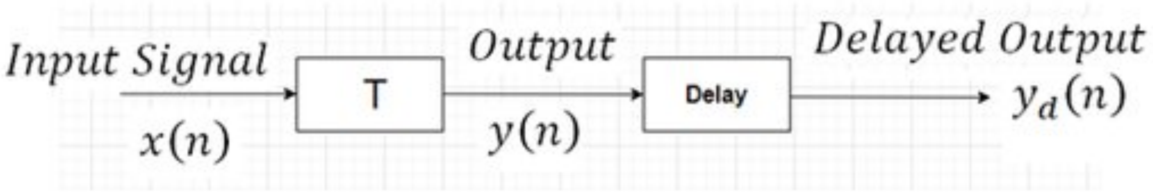
Example of Time invariant (shift invariant) system

1.3 Classifications of systems

Time/Shift Variant and Invariant System







1.3 Classifications of systems

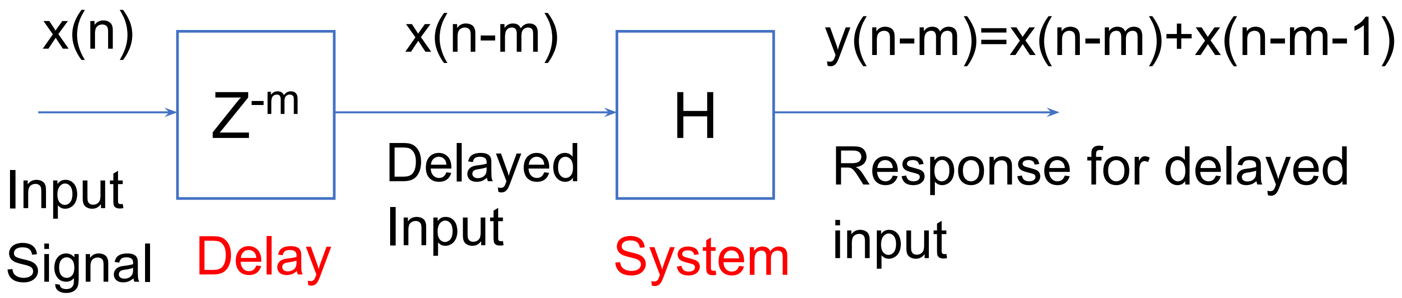
Problem 1: Determine if the system is time invariant or time variant.

$y(n)=x(n)+x(n-1)$

Solution:

Test 1: Response for delayed input

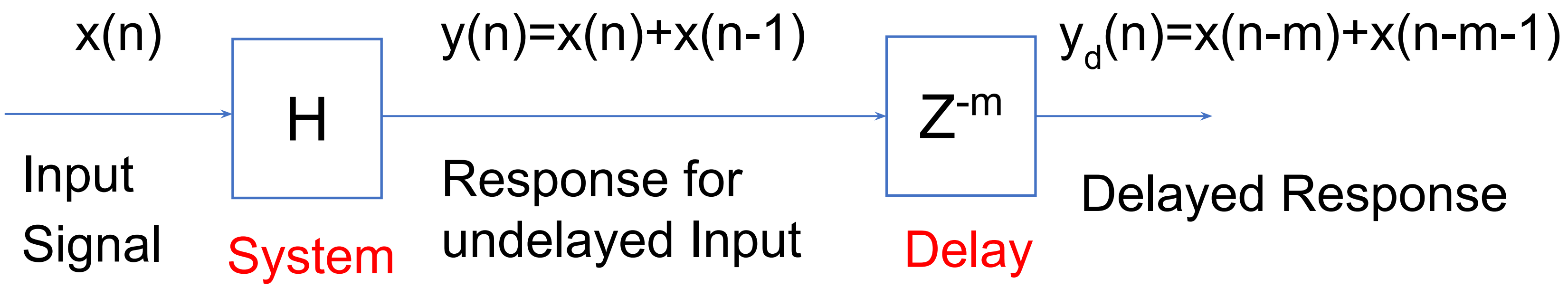
Let, $y(n-m)$=Response for delayed input



1.3 Classifications of systems

Test 2: Delayed Response

Let $y_d(n)$ = Delayed Response



Conclusion:

Here, $y(n-m) = y_d(n)$, therefore the system is time invariant.

1.3 Classifications of systems

Practice Problems : Determine if the system is time invariant or time variant.

i) $y(n)= 2nx(n)$ – Time Variant

ii) $y(n)=x(n)-bx(n-1)$ – Time Invariant

iii) $y(n)=x(n)+B$ - Time Invariant

iv) $y(n)= nx^3(n)$ - Time Variant

1.3 Classifications of systems

5. Stable and Unsatble System

- Stable system
   - Produces bounded output for the bounded input
   - Also called Bounded input-bounded output (BIBO) stable systems.
- Unstable systems
   - Produces unbounded output for the bounded input
- Arbitrary relaxed system
   - Said to be BIBO stable if and only if every bounded input produces a bounded output
- Necessary and sufficient conditions for stability

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

1.3 Classifications of systems

Problem: Test the stability of the system whose impulse response

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

Solution: Necessary and sufficient conditions for stability

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{2}\right)^n u(n)\right|$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$= \frac{1}{1-1/2} = 2 < \infty$$

Hence, the system is stable

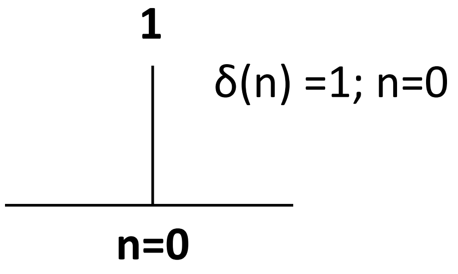
1.3 Classifications of systems

- Problem: Test the stability of the system whose $\boldsymbol{y(n) = \cos x(n)}$

$$y(n) \rightarrow h(n)$$
$$x(n) \rightarrow \delta(n)$$

$$= \sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$= \sum_{n=-\infty}^{\infty} |\cos \delta(n)|$$



n=0; cos δ(0)=cos(1)=0.9

n=1;cos δ(1)=cos(0)=1

n=-1;cos δ(-1)=cos(0)=1

= ....+1+1+1+0.9+1+1+1+.......

$= \infty$

The system is unstable

1.3 Classifications of systems

- Practice problem: Test the stability of the following systems.

1. $y(n) = x(n)e^{n}$
2. $y(n) = a^{x(n)}$