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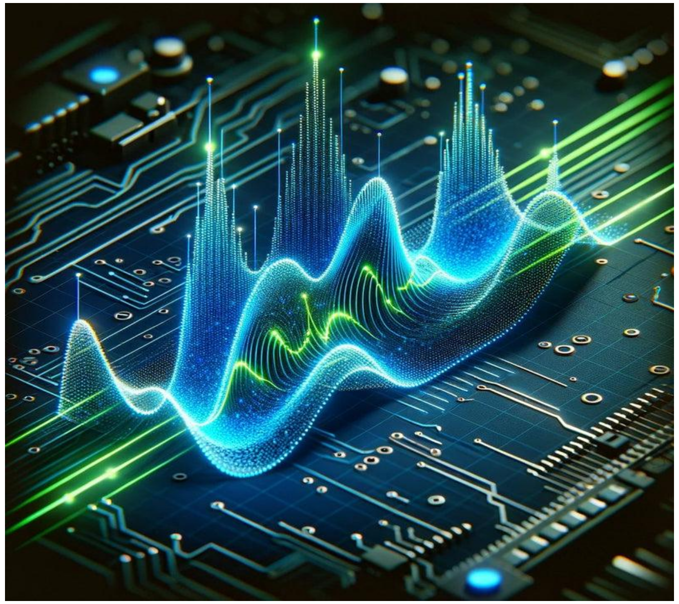
22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING



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22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - CO's

CO1	Illustrate the basics of discrete time signals and systems. [U]
CO2	Interpret the concepts of Discrete and Fast Fourier transform [U]
CO3	Comprehend the architecture of advanced processors. [U]
CO4	Apply the concept of transformation techniques in Discrete Time systems. [AP]
CO5	Design different types of filters using various filter design techniques. [AP]

22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - Modules

1	Signals and Systems
2	Discrete Fourier Transform and Fast Fourier Transform
3	FIR Filters, IIR Filters and Digital Signal Processors

MODULE – II : Discrete Fourier Transform and Fast Fourier Transform

2.1	Discrete Fourier Transform (DFT)
2.2	Circular convolution of two sequence using DFT
2.3	DFT properties
2.4	Fast Fourier Transform (FFT)

2.1 Discrete Fourier Transform (DFT)

- The DFT is used to convert a finite discrete time sequence x(n) to an N-point frequency domain sequence denoted by X(k),.
- The N-point DFT of a finite duration sequence x(n) of length L

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nk}{N}} \qquad k = 0, 1, \ldots, N-1$$

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2.1 Discrete Fourier Transform (DFT)

- Consider finite duration signal

$$\{x(n)\} \qquad n = 0 \ldots N-1$$

- Its z-transform is

$$X(z) = \sum_{n=0}^{N-1} x(n).z^{-n}$$

- Evaluate at points on z-plane as

$$X(k) = X(z)\big|_{z_k} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

- We can evaluate N independent points

DFT ⟹ $$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

2.1 Discrete Fourier Transform (DFT)

Analysis Equation

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

Synthesis Equation

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W^{nk}$$

Where,

$$W = e^{-j2\pi/N}$$

2.1 Discrete Fourier Transform (DFT)

The Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j(2\pi/N)kn}$$

The DFT pair can also be written as

$$X[k] \xleftrightarrow{\text{DFT}} x[n]$$

2.1 Discrete Fourier Transform (DFT)

4 Point DFT:

- Write the Expression for DFT of N Point Sequence
- Substitute the value of n and Elaborate the series
- Determine X(K) in terms of exponential values
- Substitute the value of k in X(K), then find X(0), X(1), X(2), X(3).

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}} \qquad k = 0, 1, \ldots, N-1$$

2.1 Discrete Fourier Transform (DFT)

Solved Problems- 4 Point DFT

1. Find the 4-point DFT of *x*(*n*) = {0, 1, 2, 3}

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}} \qquad k = 0, 1, \ldots, N-1$$

$N = 4,$

$$\therefore X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi nk}{2}} \qquad k = 0, 1, 2, 3$$

2.1 Discrete Fourier Transform (DFT)

$k = 0,$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{0}$$
$$= x(0) + x(1) + x(2) + x(3)$$
$$= 0 + 1 + 2 + 3 = 6$$

$k = 1,$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi n}{2}}$$
$$= x(0) + x(1)\,(-j) + x(2)\,(-1) + x(3)\,(j)$$
$$= 0 + (1)\,(-j) + 2(-1) + 3(j)$$
$$= -2 + 2j$$

2.1 Discrete Fourier Transform (DFT)

$k = 2,$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$
$$= x(0) + x(1)\, e^{-j\pi} + x(2) e^{-2j\pi} + x(3)\, e^{-3j\pi}$$
$$= 0 + (1)\ (-1) + 2\,(1) + 3(-1) = -2$$

$k = 3,$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-\frac{3j\pi n}{2}}$$
$$= x(0) + x(1)\, e^{-\frac{3j\pi}{2}} + x(2) e^{-3j\pi} + x(3)\, e^{-\frac{9j\pi n}{2}}$$
$$= 0 + (1)\ (j) + (2)(-1) + (3)\ (-j)$$
$$= -2 - 2j$$

$$\therefore X(k) = \{6, -2+2j, -2, -2-2j\}$$

2.1 Discrete Fourier Transform (DFT)

8 Point DFT

- Write the Expression for DFT of N Point Sequence
- Substitute the value of n and Elaborate the series
- Determine X(K) in terms of exponential values
- Substitute the value of k in X(K), then find X(0), X(1), X(2), X(3). X(4), X(5), X(6), X(7)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}} \qquad k = 0, 1, \ldots, N-1$$

2.1 Discrete Fourier Transform (DFT)

Compute 8 Point DFT of the sequence

Determine the 8-point DFT of the sequence $x(n) = \{0,0,1,1,1,0,0,0\}$.

Solution:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi nk}{N}} \qquad k = 0, 1, \ldots, N-1$$

For $k=0$,

$$x(0) = \sum_{n=0}^{7} x(n) e^{0}$$

$$= 0+0+1+1+1+0+0+0 = 3$$

2.1 Discrete Fourier Transform (DFT)

For k=1,

$$x(0)=\sum_{n=0}^{7}x(n)\,\mathrm{e}^{-j\left(\frac{\pi}{4}\right)n}$$

$$=x(0)+x(1)\mathrm{e}^{-j\left(\frac{\pi}{4}\right)}+x(2)\mathrm{e}^{-j\left(\frac{\pi}{2}\right)}+x(3)\mathrm{e}^{-j\left(\frac{3\pi}{4}\right)}$$

$$+x(4)\mathrm{e}^{-j\pi}+x(5)\mathrm{e}^{-j\frac{5\pi}{4}}+x(6)\mathrm{e}^{-j\frac{3\pi}{2}}+x(7)\mathrm{e}^{-j\frac{7\pi}{4}}$$

$$=-1.707-1.707j$$

For k= 2,

$$x(2)=\sum_{n=0}^{7}x(n)\,\mathrm{e}^{-j\left(\frac{\pi}{2}\right)n}$$

$$=x(2)\mathrm{e}^{-j\pi}+x(3)\mathrm{e}^{-j\frac{3\pi}{2}}+x(4)\mathrm{e}^{-j2\pi}$$

$$=-1+j+1=j$$

2.1 Discrete Fourier Transform (DFT)

For k= 3,

$$x(3)=\sum_{n=0}^{7}x(n)\,\mathrm{e}^{-j\left(\frac{3\pi}{4}\right)n}$$

$$=x(2)\mathrm{e}^{-j\frac{3\pi}{2}}+x(3)\mathrm{e}^{-j\frac{9\pi}{4}}+x(4)\mathrm{e}^{-j3\pi}$$

$$=j+0.707-0.707j-1$$

$$=-0.293+0.293j$$

For $k=4$,

$$x(4)=\sum_{n=0}^{7}x(n)\,\mathrm{e}^{-j\pi n}$$

$$=x(2)\mathrm{e}^{-j2\pi}+x(3)\mathrm{e}^{-j3\pi}+x(4)\mathrm{e}^{-j4\pi}$$

$$=1-1+1=1$$

For $k=5$,

$$x(5)=\sum_{n=0}^{7}x(n)\,\mathrm{e}^{-j\frac{5\pi}{4}n}$$

$$=x(2)\mathrm{e}^{-j\frac{5\pi}{2}}+x(3)\mathrm{e}^{-j\frac{15\pi}{4}}+x(4)\mathrm{e}^{-j5\pi}$$

$$=-j+\mathbf{0}.707+0.707j-1$$

$$=-0.293-0.293j$$

2.1 Discrete Fourier Transform (DFT)

For $k = 6$,

$$x(6) = \sum_{n=0}^{7} x(n)\,e^{-j\frac{3\pi}{4}n}$$

$$= x(2)e^{-j3\pi} + x(3)e^{-j\frac{9\pi}{2}} + x(4)e^{-j6\pi}$$

$$= -1 - j + \mathbf{1} = -j$$

For $k = 7$,

$$x(7) = \sum_{n=0}^{7} x(n)\,e^{-j\frac{7\pi}{4}n}$$

$$= x(2)e^{-j\frac{7\pi}{2}} + x(3)e^{-j\frac{21\pi}{4}} + x(4)e^{-j7\pi}$$

$$= j - 0.707 + 0.707j - 1$$

$$= -1.707 + 1.707j$$

$$x(k) = \{3,\ -1.707 + 1.707j,\ j,\ -0.293 + 0.293j,\ 1,\ -0.293 - 0.293j,\ -j,\ -\mathbf{1}.707 + 1.707j\}$$

2.1 Discrete Fourier Transform (DFT)

Practice Problems- DFT

1. Find 4-point DFT of the following sequences

(a) x(n)={1,-1,0,0}

(b) x(n)={1,1,-2,-2}

2. Find 8-point DFT of the following sequences

(a) x(n)={1,1,1,1,0,0,0,0}

(b) x(n)={1,2,1,2}

DFT Computation using Matlab

Watch this video:

https://drive.google.com/file/d/1igprzFKRzyfZrXPIL2_TZG0Yi78vXtlz/view?usp=sharing