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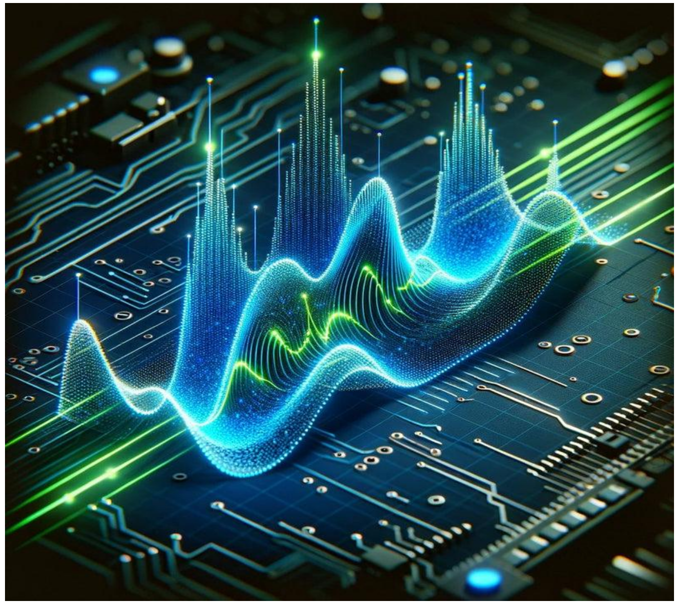
22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING



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22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - CO’s

CO1	Illustrate the basics of discrete time signals and systems. [U]
CO2	Interpret the concepts of Discrete and Fast Fourier transform [U]
CO3	Comprehend the architecture of advanced processors. [U]
CO4	Apply the concept of transformation techniques in Discrete Time systems. [AP]
CO5	Design different types of filters using various filter design techniques. [AP]

22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - Modules

1	Signals and Systems
2	Discrete Fourier Transform and Fast Fourier Transform
3	FIR Filters, IIR Filters and Digital Signal Processors

MODULE – II : Discrete Fourier Transform and Fast Fourier Transform

2.1	Discrete Fourier Transform (DFT)
2.2	Circular convolution of two sequence using DFT
2.3	DFT properties
2.4	Fast Fourier Transform (FFT)

2.3 DFT Properties

DFT Properties:

1. Periodicity Property
2. Linearity Property
3. Circular Frequency Shift
4. Complex Conjugate Property
5. Circular Correlation
6. Time Reversal of a sequence
7. Multiplication of two sequences

2.3 DFT Properties

- Periodicity

$$x(n) \overset{DFT}{\leftrightarrow} X(k)$$

- if

$$x(n+N) = x(n)$$

- Then

$$X(k+N) = X(k)$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(k+N)n} = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n} e^{-j\frac{2\pi}{N}kn}$$

$$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} = X(k) \quad since\ e^{-j2\pi n} = 1$$

- Hence we note that X(k+N) = X(k)
- This tells us that DFT is periodic with period N. This is known as the cyclical property of DFT.

2.3 DFT Properties

- Linearity

$$x_1(n) \overset{DFT}{\leftrightarrow} X_1(k)$$

$$x_2(n) \overset{DFT}{\leftrightarrow} X_2(k)$$

$$ax_1(n) + bx_2(n) \overset{DFT}{\leftrightarrow} aX_1(k) + bX_2(k)$$

Proof

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} ax_1(n) + bx_2(n) W_N^{kn} \\ &= \sum_{n=0}^{N-1} ax_1(n) W_N^{kn} + \sum_{n=0}^{N-1} bx_2(n) W_N^{kn} \\ &= aX_1(k) + bX_2(k) \end{aligned}$$

2.3 DFT Properties

Circular Frequency Shift

$$x(n) \overset{DFT}{\leftrightarrow} X(k)$$

- then

$$x(n)e^{j2\pi n/N} \overset{DFT}{\leftrightarrow} X((k-l))_N$$

- Shifting the frequency components of DFT circularly is equivalent to multiplying the time domain sequence by $e^{j2\pi n/N}$

Complex Conjugate Properties

$$x(n) \overset{DFT}{\leftrightarrow} X(k)$$

then

$$x^*(n) \overset{DFT}{\leftrightarrow} X^*((-k))_N = X^*(N-k)$$

and

$$x^*((-n))_N = x^*(N-k) \overset{DFT}{\leftrightarrow} X^*(k)$$

2.3 DFT Properties

Circular Correlation

$$x(n) \overset{\text{DFT}}{\underset{\text{N}}{\longleftrightarrow}} X(k)$$

$$y(n) \overset{\text{DFT}}{\underset{\text{N}}{\longleftrightarrow}} Y(k)$$

$$\tilde{r}_{xy}(l) \overset{\text{DFT}}{\underset{\text{N}}{\longleftrightarrow}} = \tilde{R}_{xy}(k) = X(k)Y^*(k)$$

- where $r_{xy}(l)$ is the circular cross correlation which is given as

$$\tilde{r}_{xy}(l) = \sum_{n=0}^{N-1} x(n)y^*((n-l))_N$$

Multiplication of DFT one sequence and conjugate DFT of another sequence is equivalent to circular cross correlation of these two sequences in time domain

2.3 DFT Properties

Time Reversal of a sequence

$$x(n) \overset{DFT}{\Leftrightarrow} X(k)$$

$$x((-n))_N = x(N-n) = \overset{DFT}{\leftrightarrow} X((-k))_N = X(N-k)$$

Changing the index from n to m=N-n we get

$$\begin{aligned} DFT[x(N-m)] &= \sum_{m=0}^{N-1} x(m)e^{-j2\pi k(N-m)/N} \\ &= \sum_{m=0}^{N-1} x(m)e^{j2\pi km/N} \\ &= \sum_{m=0}^{N-1} x(m)e^{-j2\pi m(N-k)/N} \quad \boxed{\because e^{j2\pi k} = 1 \text{ for } k = 0, 1, 2, \ldots}. \\ &= X(N-k). \end{aligned}$$

2.3 DFT Properties

Multiplication Of Two Sequences

$$x(n) \xleftrightarrow[N]{\text{DFT}} X(k)$$

$$y(n) \xleftrightarrow[N]{\text{DFT}} Y(k)$$

Then

$$x(n)y(n) \xleftrightarrow[N]{\text{DFT}} \frac{1}{N} X(k) \textcircled{N} Y(k)$$

Multiplication of two sequences in time domain is equivalent to circular convolution of their DFTs in frequency domain