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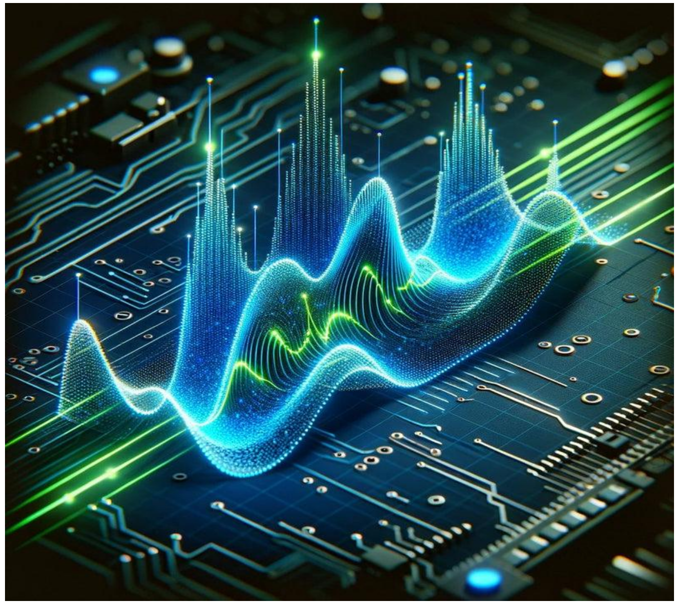
22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING



Ms.A.Elakya
Assistant Professor/EEE

22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - CO's

CO1	Illustrate the basics of discrete time signals and systems. [U]
CO2	Interpret the concepts of Discrete and Fast Fourier transform [U]
CO3	Comprehend the architecture of advanced processors. [U]
CO4	Apply the concept of transformation techniques in Discrete Time systems. [AP]
CO5	Design different types of filters using various filter design techniques. [AP]

22EE603 – PRINCIPLES OF DIGITAL SIGNAL PROCESSING - Modules

1	Signals and Systems
2	Discrete Fourier Transform and Fast Fourier Transform
3	FIR Filters, IIR Filters and Digital Signal Processors

MODULE – III : FIR Filters, IIR Filters and Digital Signal Processors

3.1	Design of FIR filters
3.2	**Design of IIR filters**
3.3	Architecture of TMS320C64xx processor

3.2 Design of IIR filters

3.2.1 Introduction:

- IIR filters are of recursive type, whereby the present output sample depends on the present input, past input samples and output samples.

- The impulse response h(n) for a realizable filter is

$$h(n)=0 \quad \text{for } n \leq 0$$

and for stability it must satisfy the condition

$$\sum_{n=0}^{\infty} |h(n)| < \infty$$

IIR digital filter have the transfer function of the form

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

3.2 Design of IIR filters

3.2.2 Types of IIR Filter:

There are two methods to design IIR filter. They are

(i) Butterworth Filter

(ii) Chebyshev filter

3.2.3 Design of IIR filter from analog filters:

There are two methods for digitizing the analog filter into a digital filter. They are

3.2.3.1 Bilinear Transformation

3.2.3.2 Impulse Invariant Transformation

3.2 Design of IIR filters

3.2.3.1 Bilinear Transformation

Steps to design filter using bilinear transform technique

Step 1: From the given specifications, find prewarping analog frequencies using formula

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

Step 2: Using the analog frequencies find H(s) of analog filter.

Step 3: Select the sampling rate of the digital filter, call it T seconds per sample.

Step 4: Substitute $s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$ into the transfer function found in step 2.

3.2 Design of IIR filters

Problem on Bilinear Transformation:

1. **Apply bilinear transformation to H(s)=** $\frac{2}{(s+1)(s+2)}$ **when T=1 sec and find H(z).**

Solution:

$$\text{Given } H(s) = \frac{2}{(s+1)(s+2)}$$

$$\text{Substitute } s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \text{ in } H(s) \text{ to get } H(z)$$

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{2}{(s+1)(s+2)}\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

3.2 Design of IIR filters

Given $T = 1$ sec

$$H(z) = \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}}$$

$$= \frac{2(1+z^{-1})^2}{(3-z^{-1})(4)}$$

$$= \frac{(1+z^{-1})^2}{6-2z^{-1}}$$

$$= \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$

Watch this video
https://www.youtube.com/watch?v=CwuOD5fn5X8

3.2.3.2 Impulse Invariant Transformation

Steps to design filter using bilinear transform technique

Step 1: From the given specifications, find $H_a(s)$, the transfer function of an analog filter.

Step 2: Select the sampling rate of the digital filter, T seconds per sample.

Step 3: Express the analog filter transfer function as the sum of single – pole filters.

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}$$

3.2 Design of IIR filters

Step 4: Compute the Z-Transform of the digital filter by using the formula.

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

For high sampling rates use

$$H(z) = \sum_{k=1}^{N} \frac{T c_k}{1 - e^{p_k T} z^{-1}}$$

Watch this video
https://www.youtube.com/watch?v=_IFsxNkw7Hc

3.2 Design of IIR filters

Problem on Impulse Invariance Method:

1. **For the analog transfer function H(s)=** $\frac{2}{(s+1)(s+2)}$ **determine H(z) using impulse invariance method. Assume T=1 sec.**

Solution:

Given $H(s) = \frac{2}{(s+1)(s+2)}$

Using partial fraction we can write

$$H(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$H(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$= \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

$$A = (s+1)\frac{2}{(s+1)(s+2)}\Big|_{s=-1} = 2$$

$$B = (s+2)\frac{2}{(s+1)(s+2)}\Big|_{s=-2} = -2$$

Using impulse invariance technique we have, if

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{s-p_k} \quad \text{then} \quad H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_k T} z^{-1}}$$

3.2 Design of IIR filters

i.e., $(s - p_k)$ is transformed to $1 - e^{p_k T} z^{-1}$

There are two poles $p_1 = -1$ and $p_2 = -2$. So

$$H(z) = \frac{2}{1 - e^{-T} z^{-1}} - \frac{2}{1 - e^{-2T} z^{-1}}$$

For $T = 1$ sec

$$\begin{aligned} H(z) &= \frac{2}{1 - e^{-1} z^{-1}} - \frac{2}{1 - e^{-2} z^{-1}} \\ &= \frac{2}{1 - 0.3678 z^{-1}} - \frac{2}{1 - 0.1353 z^{-1}} \\ H(z) &= \frac{0.465 z^{-1}}{1 - 0.503 z^{-1} + 0.04976 z^{-2}} \end{aligned}$$

3.2 Design of IIR filters

(i) Butterworth Filter:

Steps to design an analog butterworth lowpass filter

Step 1: From the given specifications find the order of the filter N.

Step 2: Round off it to the next higher integer.

Step 3: Find the transfer function H(s) for Ω_c = 1 rad/sec for the value of N.

Step 4: Calculate the value of cut-off frequency Ω_c.

Step 5: Find the transfer function $H_a(s)$ for the above value of Ω_c by substituting s -> $\frac{s}{\Omega_c}$ in H(s).

3.2 Design of IIR filters

Problems on Butterworth Filter:

1. **Design an analog Butterworth filter that has a -2 dB passband attenuation at a frequency of 20 rad/sec and atleast -10 dB stopband attenuation at 30 rad/sec.**

Solution:

Given $\alpha_p = 2\,\text{dB}; \quad \Omega_p = 20\,\text{rad/sec}$

$\alpha_s = 10\,\text{dB}; \quad \Omega_s = 30\,\text{rad/sec}$

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

3.2 Design of IIR filters

$$\geq \frac{\log \sqrt{\frac{10-1}{10^{0.2}-1}}}{\log \frac{30}{20}}$$

$$\geq 3.37$$

Rounding off N to the next highest integer we get

$$N = 4$$

The normalized lowpass Butterworth filter for $N = 4$ can be found from table 5.1 as

$$H(s) = \frac{1}{(s^2 + 0.76537s + 1)(s^2 + 1.8477s + 1)}$$

From Eq. (5.31) we have

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}} = \frac{20}{(10^{0.2} - 1)^{1/8}} = 21.3868$$

3.2 Design of IIR filters

The transfer function for $\Omega_c = 21.3868$ can be obtained by substituting

$$s \rightarrow \frac{s}{21.3868} \text{ in } H(s)$$

$$\text{i.e.,} \quad H(s) = \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 0.76537\left(\frac{s}{21.3868}\right) + 1}$$

$$\times \frac{1}{\left(\frac{s}{21.3868}\right)^2 + 1.8477\left(\frac{s}{21.3868}\right) + 1}$$

$$= \frac{0.20921 \times 10^6}{(s^2 + 16.3686s + 457.394)(s^2 + 39.5176s + 457.394)}$$

Watch this video

https://www.youtube.com/watch?v=3yyp5JRqNXs