circular convolution:

Condition:

(i) Total number of terms $n = n_1 = n_2$

(ii) $\Sigma y(n) = \Sigma x(n) * \Sigma h(n)$

Two methods:

(i) Concentric circle method.

(ii) Matrix Method.

Problem:

1) Find the circular convolution of two finite sequences $x(n) = \{1, 2, 3\}$ $h(n) = \{1, 1\}$

Solution:

(i) Concentric circle Methods:

$$n_1 = 3 \; ; \; n_2 = 2$$

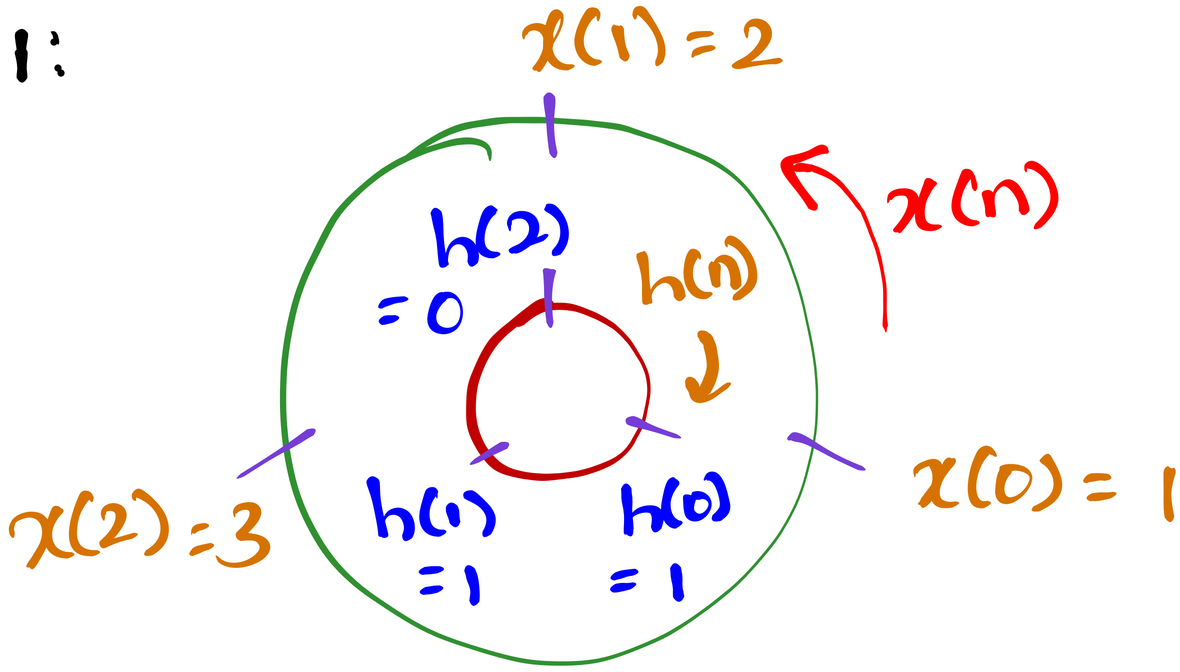
$$n = n_1 = n_2$$

$$x(n) = \{1, 2, 3\}$$

$$h(n) = \{1, 1, 0\}$$

Adding zeros at the end of the sequence $\Rightarrow$ zero Padding.

Step 1:

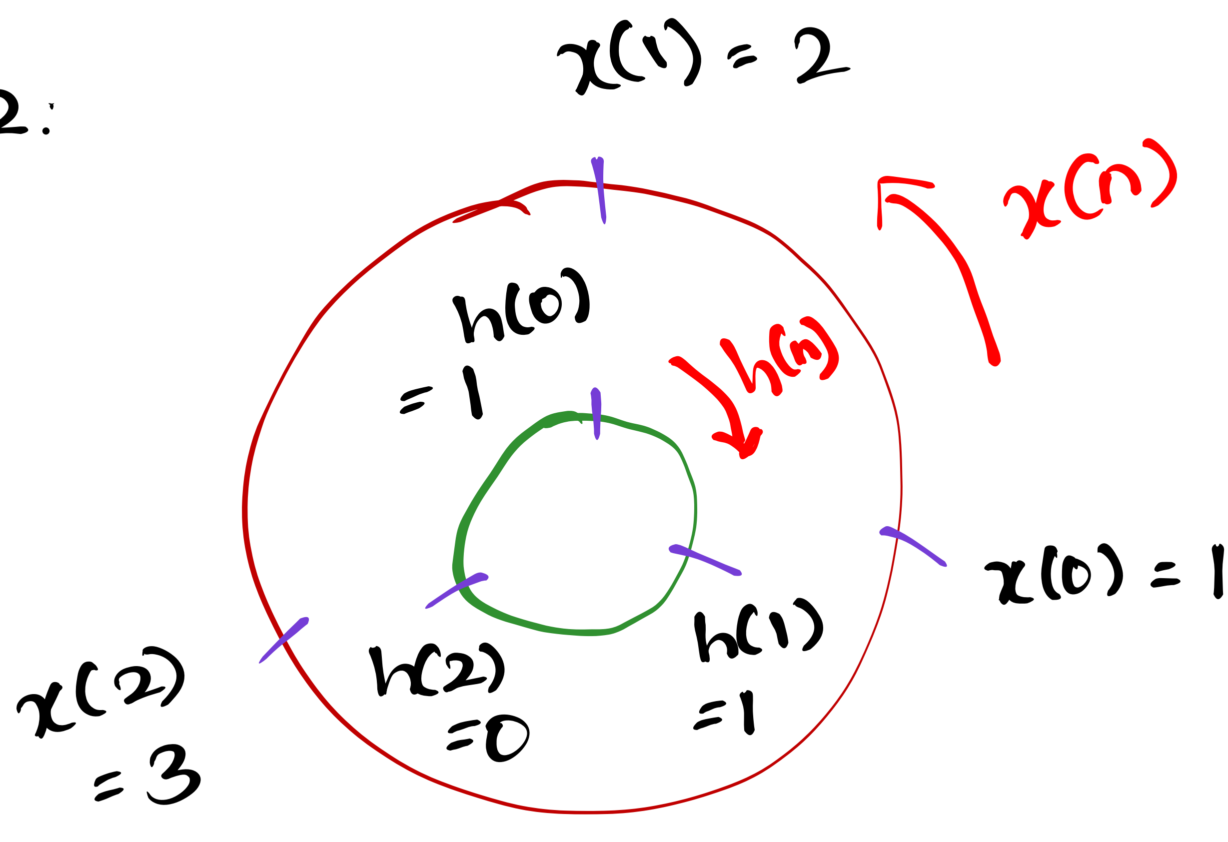


$$y(0) = \{(1\times1) + (2\times0) + (3\times1)\}$$

$$y(0) = \{1+3\}$$

$$\boxed{y(0) = 4}$$

Step 2:

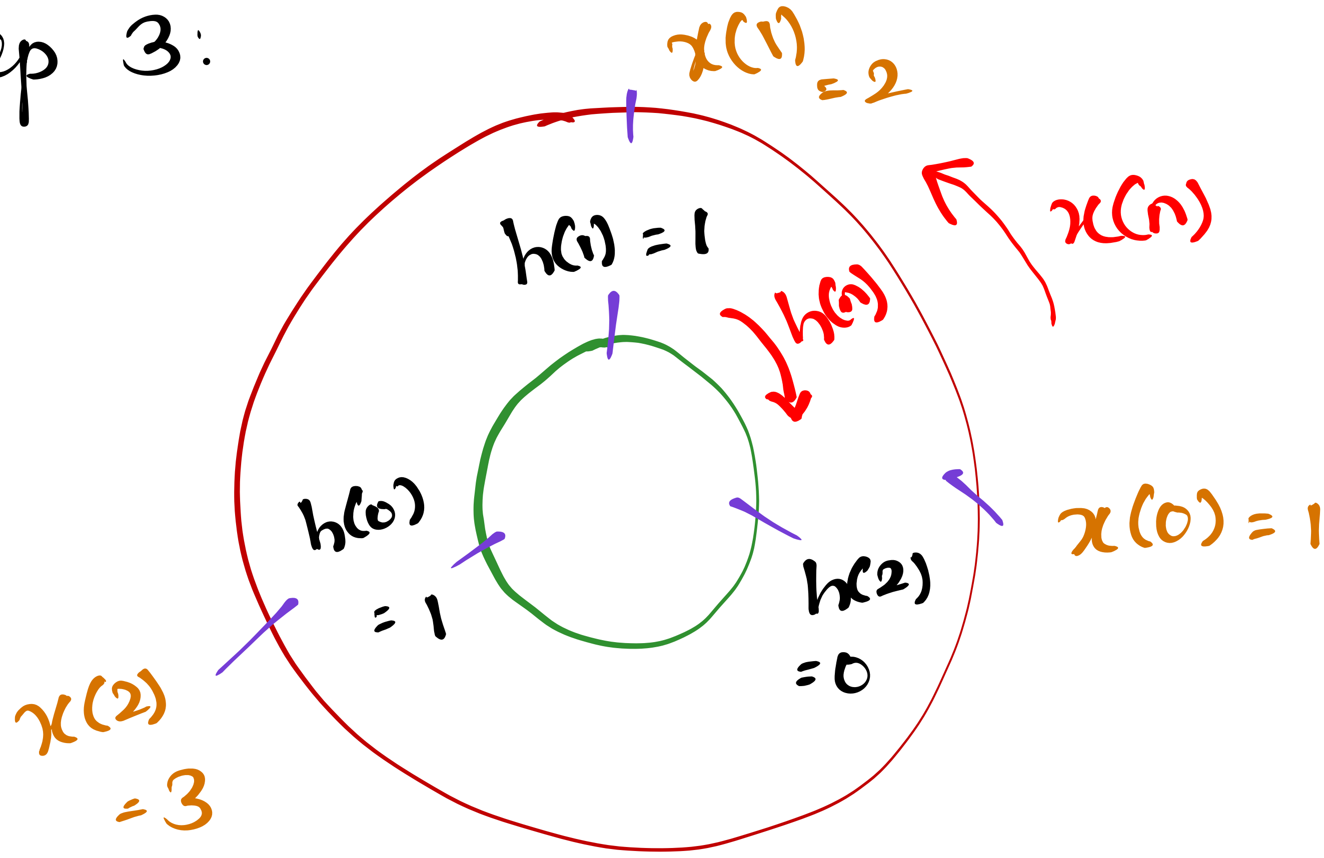


$$y(1) = \{(1\times1)+(2\times1)+(3\times0)\}$$

$$= \{1+2\}$$

$$\boxed{y(1) = \{3\}}$$

Step 3:



$$y(2) = \{(1\times 0) + (2\times 1) + (3\times 1)\}$$

$$= \{2+3\}$$

$$y(2) = \{5\}$$

$$\boxed{y(n) = \{4, 3, 5\}}$$

(ii) Matrix method:

$$[y(n)]_{3\times 1} = [x(n)]_{3\times 3}\,[h(n)]_{3\times 1}$$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times1)+(3\times1)+(2\times0) \\ (2\times1)+(1\times1)+(1\times0) \\ (3\times1)+(2\times1)+(3\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 1+3+0 \\ 2+1+0 \\ 3+2+0 \end{bmatrix}$$

$$\boxed{y(n) = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}}$$

2) Find the circular convolution of two sequences $x(n) = \{1, 2, 2, 1\}$, $h(n) = \{-1, 2, 3, 4\}$

Solution:

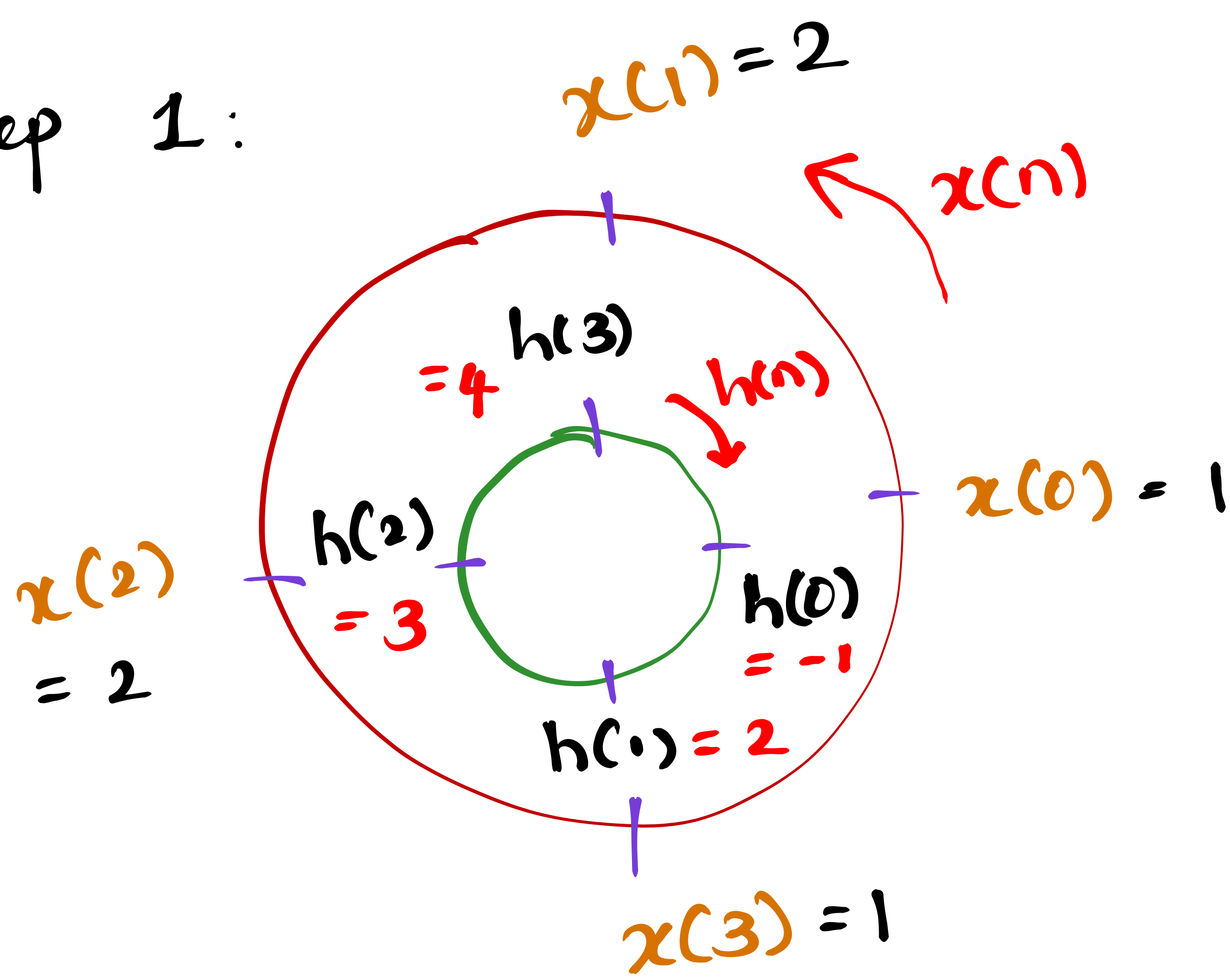
(i) Concentric circle method:

$n_1 = 4$

$n_2 = 4$

$n = n_1 = n_2$

Step 1:

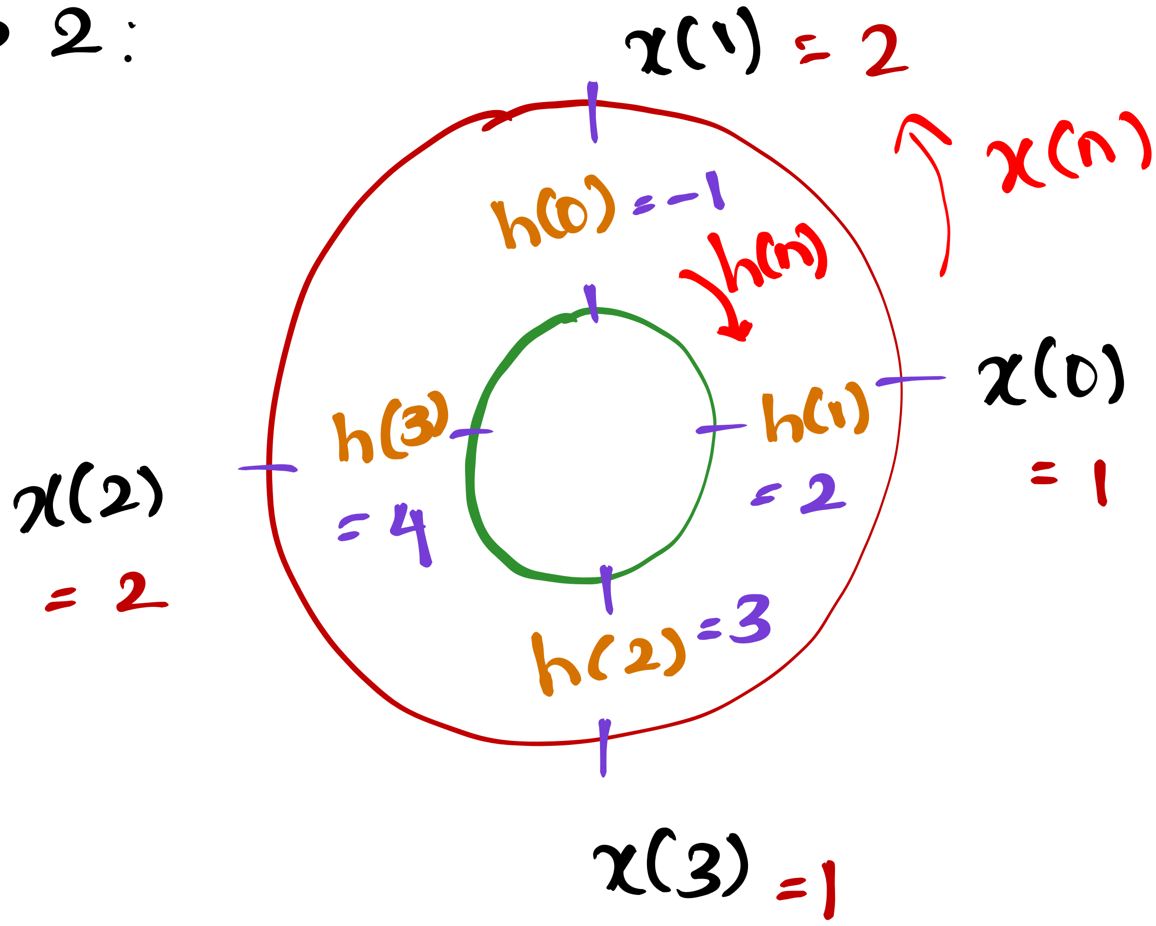


$$y(0) = \{(1 \times -1) + (2 \times 4) + (2 \times 3) + (2 \times 1)\}$$

$$= \{-1 + 8 + 6 + 2\}$$

$$\boxed{y(0) = \{15\}}$$

Step 2:

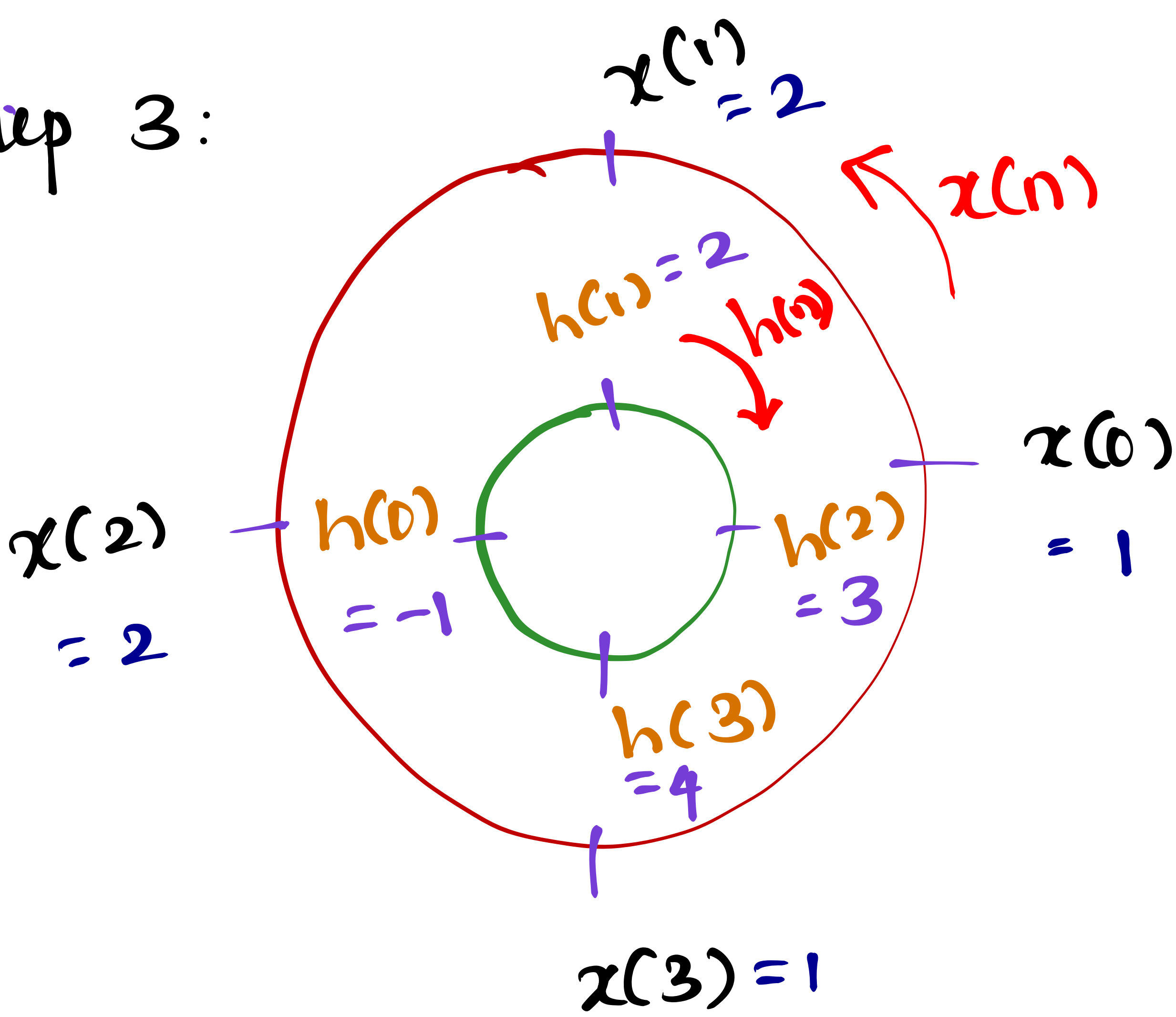


$$y(1) = \{(1\times 2) + (2\times -1) + (2\times 4) + (3\times 1)\}$$

$$= \{2 + (-2) + 8 + 3\}$$

$$\boxed{y(1) = \{11\}}$$

Step 3:

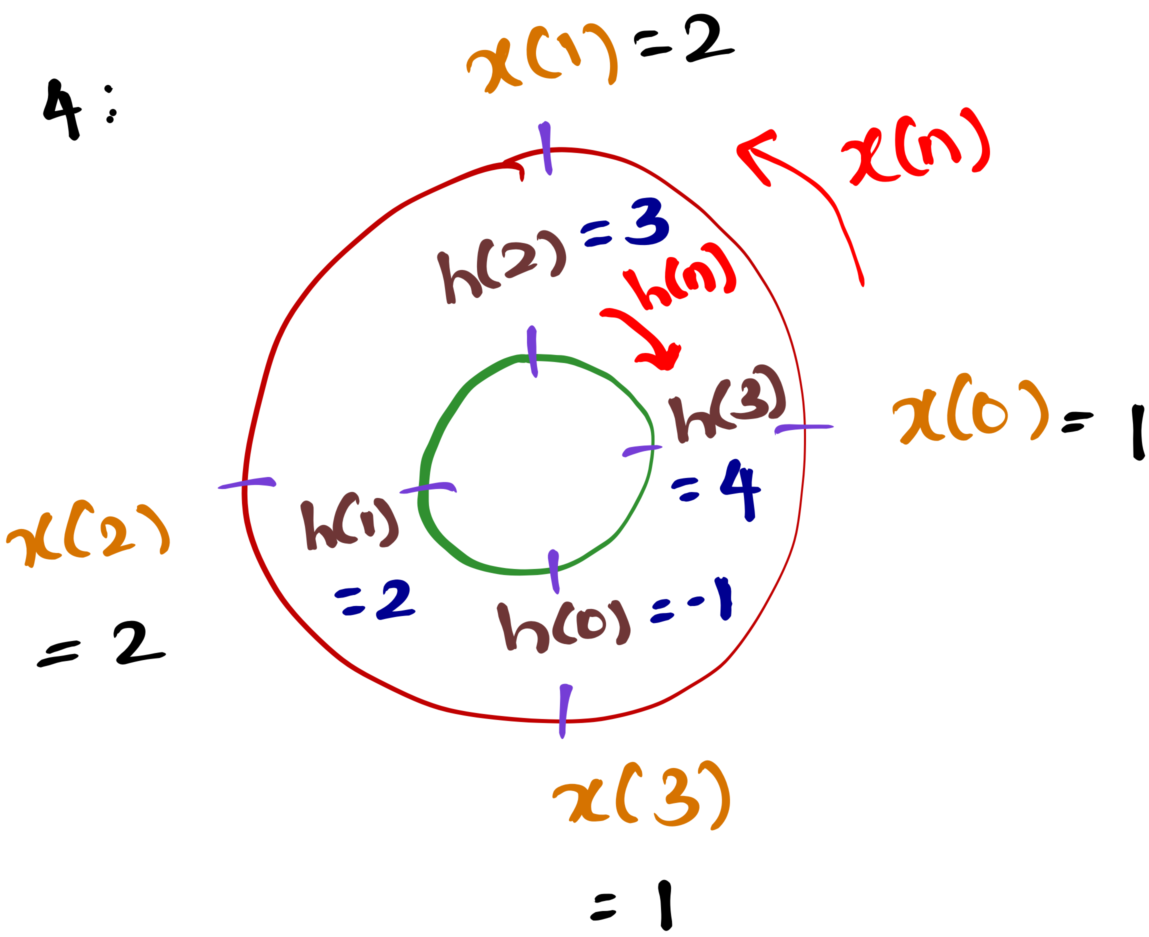


$$y(2) = \{(1\times3) + (2\times2) + (2\times-1) + (4\times1)\}$$

$$= \{3 + 4 + (-2) + 4\}$$

$$\boxed{y(2) = \{9\}}$$

Step 4:



$$y(3) = \{(1\times4) + (2\times3) + (2\times2) + (1\times-1)\}$$

$$= \{4 + 6 + 4 - 1\}$$

$$\boxed{y(3) = \{13\}}$$

$$y(n) = \{y(0), y(1), y(2), y(3)\}$$

$$\boxed{y(n) = \{15, 11, 9, 13\}}$$

(ii) Matrix Method:

$$[y(n)]_{4\times1} = [x(n)]_{4\times4}\,[h(n)]_{4\times1}$$

$$= \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times-1)+(1\times2)+(2\times3)+(2\times4) \\ (2\times-1)+(1\times2)+(1\times3)+(2\times4) \\ (2\times-1)+(2\times2)+(1\times3)+(1\times4) \\ (1\times-1)+(2\times2)+(2\times3)+(1\times4) \end{bmatrix}$$

$$= \begin{bmatrix} -1+2+6+8 \\ -2+2+3+8 \\ -2+4+3+4 \\ -1+4+6+4 \end{bmatrix}$$

$$y(n) = \begin{bmatrix} 15 \\ 11 \\ 9 \\ 13 \end{bmatrix}$$