SRI KRISHNA COLLEGE OF TECHNOLOGY

END SEMESTER REGULAR EXAMINATION - APRIL/MAY 2024

21EE602- Principles of Digital signal processing

Answer key.

PART-A

1) Check whether the given system is causal and stable.

$y(n) = 4x(n+2) + 3x(n-2)$ (2 Marks)

Ans:

$y(n)$ is a non-causal system because the system $y(n)$ depends on future and past inputs.

~~y(n) is a~~

2) Is the system describe by the equation $y(t) = x(2t+2)$ Time invariant or not? (2 Marks)

$y_1(t) = x(2t+2)$

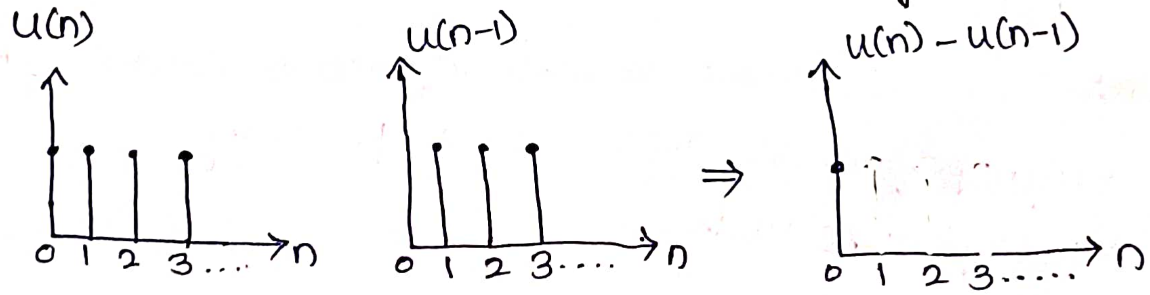
$y_1(t) = x(2t+2-m)$

$y_2(t) = x(2t+m)$

$y_2(t) = x(2t+2-m)$

$y_1(t) = y_2(t)$, so the system is time invariant.

3) Show that $\delta(n) = u(n) - u(n-1)$ graphically. (2 Marks)



4) compute DFT of a discrete time sequence $x(n)=\{1,2,3,4\}$

(2 Marks)

$x(0)=1$; $x(1)=2$; $x(2)=3$; $x(3)=4$

$$X(K)=\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$X(K)=\sum_{n=0}^{3} x(n)\, e^{-j2\pi kn/4} = \sum_{n=0}^{3} x(n)\, e^{-j\pi kn/2}$$

$$X(K)=\{10, -2+2j, -2, -2-2j\}$$

5) The number of points is given by N=128. compute the number of complex multiplications and additions required to perform DFT and FFT.

(2 Marks)

FFT:

The number of additions required = ~~256~~ 512 ($N \log_2 N$)

The number of multiplication required = 256 ($\frac{N}{2} \log_2 N$)

DFT:

Addition = $N(N-1) = 128(128-1) = 128 \times 127 = 16256$

Multiplication = $N^2 = (128)^2 = 16384$

6) Write the difference and similarities between DIT and DIF FFT algorithm.

(2 Marks)

Difference:

In DIT, the input is bit-reversed while the output is in natural order. For DIF, the reverse in time that is input is normal order, while output is bit-reversed.

Similarities:

Both algorithm requires same number of operations to compute DFT.

7) Summarize on the execution unit of TMS320C64XX. (2 Marks)

* High performance digital signal processing.
* Multiple MAC units
* Versatile ALU
* Efficient data movement capabilities.
* Pipeline architecture.

8) Obtain the circular convolution of the following sequences $x(n)=\{1,2,1\}$, $h(n)=\{1,-2,2\}$.

(2 Marks)

$$[y(n)]_{3\times1} = [x(n)]_{3\times3}\,[h(n)]_{3\times1}$$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+(-2)+4 \\ 2-2+2 \\ 1-4+2 \end{bmatrix}$$

$$y(n) = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

9) Define bilinear transformation? List the properties of bilinear transformation.

(2 Marks)

In digital filtering, it is a standard method of mapping the s or analog plane into the z or digital plane.

Properties:

* Analog dc $(s=0)$ maps to digital dc $(z=1)$
* Infinite analog frequency $(s=\alpha)$ maps to the maximum digital frequency $(z=-1)$

10) Distinguish between FIR and IIR filter.

(2 Marks)

FIR Filter	IIR Filter
(i) FIR filter stands for finite impulse response filter.	(i) IIR filter stands for infinite impulse response filter.
(ii) A FIR filter provides impulse responses for a finite duration of time.	(ii) An IIR filter gives impulse responses for an infinite duration of time.
(iii) Feedback system is available in IIR filters.	(iii) Feedback system is not present in FIR filters.

PART-B

11) Determine whether the following system are static, linear, time invariant, causal and stable with proper justifications. (16 Marks)

a) $y(n) = x(t-3) + (3-t)$ [6 Marks]

* The system is Static. $y(n)$ depends on ~~past~~ input $x(t)$.
* Linear system
* Time-invariant system
* causal system

b) $y(n) = x(2n+1)$ [6 Marks]

* Dynamic system
* Linear system
* Time invariant system.
* ~~non~~causal system
* Stable system

c) $y(n) = x(-n)$ [4 Marks]

* dynamic ~~Static~~ system
* Linear System
* Time-~~in~~variant system
* causal system
* Stable system

12) Determine whether the following system are static, linear, time invariant, causal and stable with proper justifications. (16 Marks)

(i) $y(t) = x(t/3)$ [~~5~~ Marks]

* Dynamic system
* Linear system
* Time-invariant system
* Causal system
* Stable

(ii) $y(t) = \int_{-\infty}^{2t} x(t).dt$ [6 Marks]

* dynamic system
* linear system
* Time-Invariant system
* Non-causal system
* Stable system.

(iii) $y(n) = 2x(n-2)$ [10 Marks]

* Dynamic System
* Linear system.
* Time-Invariant System
* causal & stable system

13) a) Find whether the signal is an energy or power signal. [8 Marks]

(i) $x(t) = r(t) - r(t-2)$ [4 Marks]

$r(t)$ is the unit step function, equal to 1 for $t \geq 0$ & 0 for $t < 0$.

To simplify, let's consider the time intervals where $x(t)$ is non-zero.

1) For $0 \leq t < 2$; $x(t) = 1 - 0 = 1$

2) For $2 \leq t < 4$; $x(t) = 1 - 1 = 0$

3) For $t \geq 4$; $x(t) = 0 - 1 = 1$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \cdot dt = \int_0^2 1^2 \cdot dt + \int_4^{\infty} 1^2 \cdot dt = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = 2W$$

Therefore, the signal $x(t) = r(t) - r(t-2)$ is not an energy signal but ~~not~~ it is a power signal.

(ii) $x(n) = (1/3)^n \cdot u(n)$ [4 Marks]

$$E = \sum_{n=0}^{\infty} \left|\left(\frac{1}{3}\right)^n\right|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \left(\frac{1}{9}\right)^0 + \left(\frac{1}{9}\right)^1 + \cdots \left(\frac{1}{9}\right)^{\infty}$$

$E = ~~\infty~~ 1.123$ Joules.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\left(\frac{1}{3}\right)^n\right|^2 = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{3}\right)^{2n}$$

$N \to \infty$, the sum converges to the same value as the energy, resulting in a finite power.

The signal $x(n) = \left(\frac{1}{3}\right)^n \cdot u(n)$ is ~~both~~ an energy signal. ~~and power signal.~~

13) b) compute IDFT for the following sequence $X(K) = \{3, (2+j), 1, (2-j)\}$

[8 Marks]

$X(0) = 3 \; ; X(1) = 2+j \; ; X(2) = 1 \; ; X(3) = 2-j$

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) e^{j2\pi kn/N} \qquad 0 \le n \le N-1$$

$n = 0$

$$x(0) = \frac{1}{4} \sum_{K=0}^{3} X(K) e^{j2\pi (K)(0)/4}$$

$$= \frac{1}{4}[3 + 2 + \cancel{j} + 1 + 2 - \cancel{j}] = \frac{1}{4}[8]$$

$\boxed{x(0) = 2}$ — [2 Marks]

$n = 1 \; ; x(1) = 0$ — [2 Marks]

$n = 2 \; ; x(2) = 0$ — [2 Marks]

$n = 3 \; ; x(3) = 1$ — [2 Marks]

$\boxed{x(n) = \{2, 0, 0, 1\}}$

14) a) check whether the following signals are periodic. If the signal is periodic, find the fundamental period. [8 Marks]

(i) $x(t) = [\cos(2t - \pi/3)]^2$ [4 Marks]

$x(t)$ is aaperiodic signal & the fundamental period is 2π.

(ii) $x(n) = \cos\left(\frac{n}{8} - \pi\right)$ [4 Marks]

the period of the cosine function must be a irrational multiple of 2π, which means $\frac{n}{8} - \pi$ must be a irrational multiple of 2π.

$$\frac{n}{8} - \pi = 2\pi K$$

$$n = 16\pi K + 8\pi$$

n is a multiple of 8π, which means the signal $x(n)$ is aperiodic with a period of 8π.

14) b) compute DFT for the following sequence $x(n) = \{1, -1, 1, -1\}$ [8 Marks]

$x(0) = 1$; $x(1) = -1$; $x(2) = 1$; $x(3) = -1$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$ [2 Marks]

$$= \sum_{n=0}^{3} x(n) e^{-j2\pi kn/4}$$

$$X(K) = \sum_{n=0}^{3} x(n) e^{-j\pi kn/2}$$

$K=0$; $X(0) = 0$

$K=1$; $X(1) = 0$ [4 Marks]

$K=2$; $X(2) = 4$

$K=3$; $X(3) = 0$

$X(K) = \{0, 0, 4, 0\}$ [2 Marks]

15. Evaluate the 8-point DFT for the following sequence using DIT-FFT algorithm. [16 Marks]

$$\text{for } x(n) = \left\{ \begin{array}{ll} 1 & \text{for } 0 \le n \le 7 \\ 0 & \text{for otherwise} \end{array} \right\}$$

Normal order	Bit reversal order
$x(0) = 1$	$x(0) = 1$
$x(1) = 1$	$x(4) = 1$
$x(2) = 1$	$x(2) = 1$
$x(3) = 1$	$x(6) = 1$
$x(4) = 1$	$x(1) = 1$
$x(5) = 1$	$x(5) = 1$
$x(6) = 1$	$x(3) = 1$
$x(7) = 1$	$x(7) = 1$

[3 Marks]

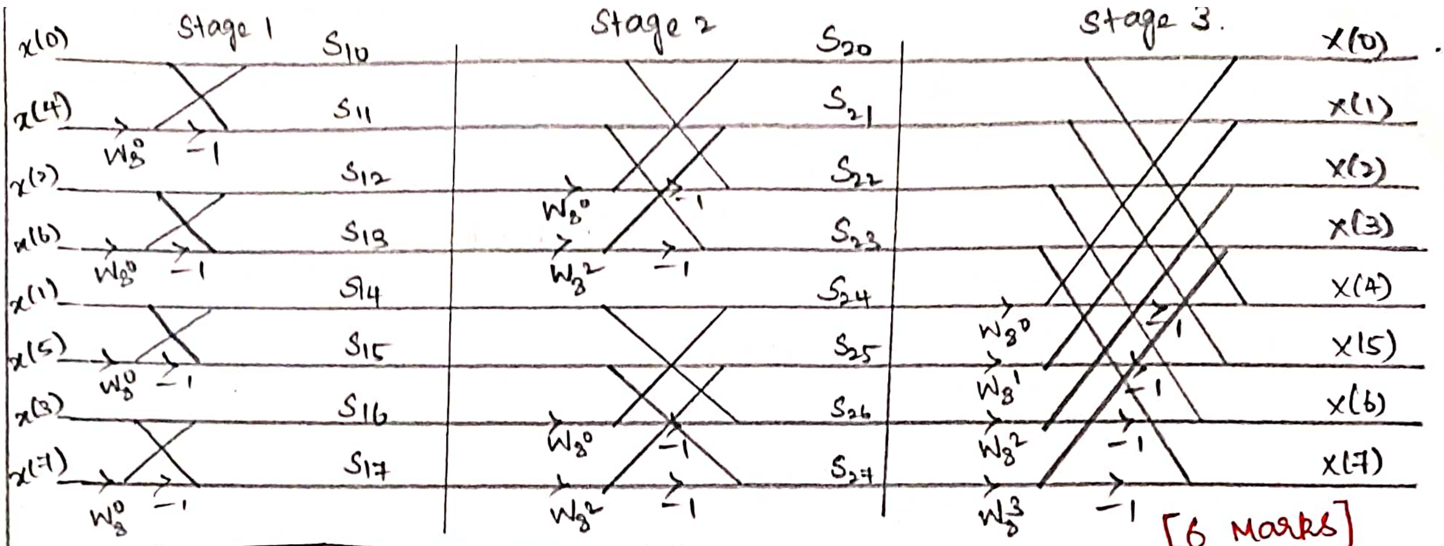
Twiddle factor: [3 Marks]

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$



[6 Marks]

$$\boxed{X(K) = \{8,0,0,0,0,0,0,0\}}$$

Calculation – [4 Marks]

16) Given $X(K) = \{36, -4+j9.656, -4+j4, -4+j1.656, -4, -4-j1.656, -4-j4, -4-j9.656\}$, find $x(n)$, using DIF-FFT algorithm [16 Marks]

Step 1: Take Conjugate of the input sequence [3 Marks]

$X^*(K) = \{36, -4-j9.656, -4-j4, -4-j1.656, -4, -4+j1.656, -4+j4, -4+j9.656\}$

Step 2: Twiddle factor [3 Marks]

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$.

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

Step 3: Butterfly diagram – [6 Marks]

Stage 1: 8-point

Stage 2: 4 point.

Stage 3: 2 point

Calculation – [4 Marks]

$x(n) = \{1,2,3,4,5,6,7,8\}$

$\boxed{\cancel{x(n) = \{4, -4+j1.656, -4+j4, -4+j9.656, -4, -4-j9.656, -4-j4, -4-j1.656\}}}$

17) Design an ideal high pass filter with a frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \pi/4 \le |\omega| \le \pi \\ 0 & \text{for } |\omega| \le \pi/4 \end{cases}$$ [16 Marks]

Find the values of h(n) using hamming window for N=11

The window sequence for N=11 is given by. (4 Marks)

$$W_H(n) = 0.54 + 0.46 \cos \frac{\pi n}{5} \quad \text{for } -5 \le n \le 5$$

$$0 \quad \text{otherwise.}$$

$W_H(0) = 1$

$W_H(-1) = W_H(1) = 0.912$

$W_H(-2) = W_H(2) = 0.682$

$W_H(-3) = W_H(3) = 0.398$

$W_H(-4) = W_H(4) = 0.1678$

$W_H(-5) = W_H(5) = 0.08$

Filter co-efficient using Hamming window sequence are (4 Marks)

$$h(n) = h_d(n) \cdot W_H(n)$$

Transfer function of the filter (4 Marks)

$$H(z) = h(0) = \sum_{n=1}^{5} \left[h(n)(z^{-n} + z^{n})\right]$$

Transfer function of the realizable filter is (4 Marks)

$$H'(z) = z^{-5} H(z)$$

18) Design a digital Butterworth filter to meet the constraints.

$0.8 \le H(e^{j\omega}) \le 1$ for $0 \le \omega \le 0.2\pi$ [16 Marks]

$0 \le H(e^{j\omega}) \le 0.2$ for $0.6\pi \le \omega \le \pi$

by using impulse invariance method and assume sampling period T = 1 sec

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \; ; \; \varepsilon = 0.75 \, , \quad \frac{1}{\sqrt{1+\lambda^2}} = 0.2 \; ; \; \lambda = 4.899$$

(2 Marks)

1) TO find N where $N = 1.71$ (3 Marks)

2) Roundoff it to Next higher integer value ; $N=2$

3) Frame the transfer function (4 Marks)

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

4) TO find $\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}}$ (4 Marks)

5) Find $H_a(s)$ by substituting Ω_c value. (3 Marks)

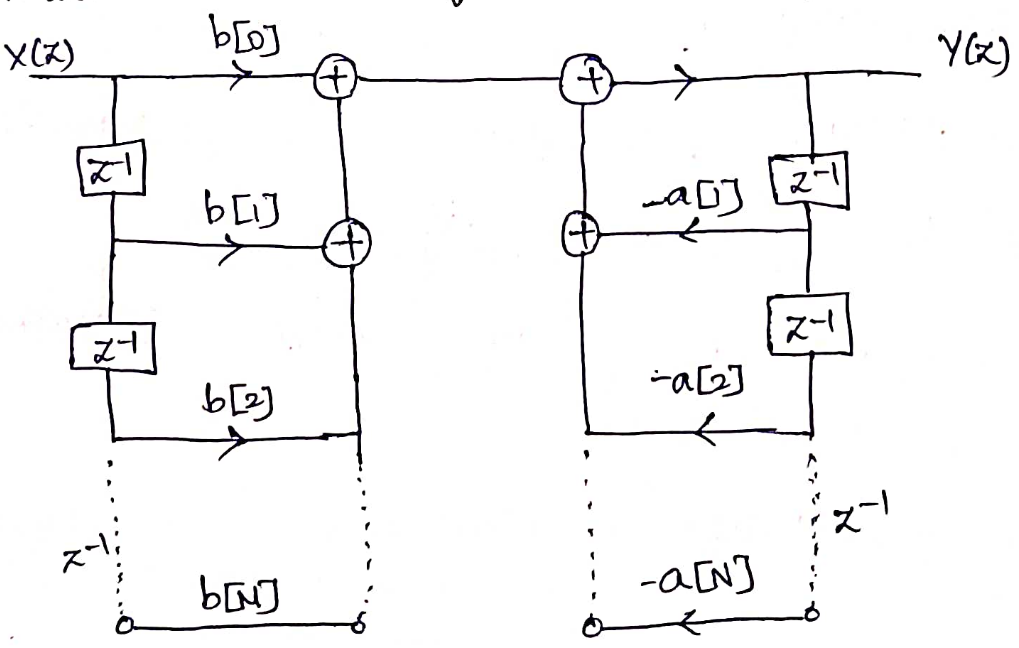
6) Find $H(z)$

19) a) Elaborate on the pipelining facility of TMS320C63XX processor.

→ Introduction to pipelining concept (3 Marks)

→ Different methods of pipelining facility of TMS320C63XX processor - Explanation (10 Marks)

19) b) Draw the direct form structure of IIR filters



(4 Marks)

Explanation — (4 Marks)

20) a) With a neat function block diagram, elaborate in detail about any one of the latest DSP architectures. [8 Marks]

1) Introduction - [2 marks]

2) Diagram of any latest DSP architecture (3 Marks)

3) Explanation - (3 Marks)

20) b) Elaborate on the design procedure of FIR filters by windows

1) $H_d(\omega)$

2) $h_d(n) = \frac{1}{2\pi} \int H_d(\omega) e^{j\omega n} \, d\omega$

3) $h(n) = h_d(n) \cdot w(n)$; $0 \le n \le N-1$

4) $H(z)$

Each point carriers - 2 Marks

13/04/2024
Faculty Incharge

HOD/EEE 13/04/24

Principal 13/04/24