Arithmetic Progression (Important Questions)

Q1. Show that $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$ are in AP.

$a_1 = (a-b)^2$
$a_2 = (a^2+b^2)$
$a_3 = (a+b)^2$

$a_2 = a_1 + d$
$a^2+b^2 - (a-b)^2 = d$
$a^2+b^2 - (a^2+b^2 - \cancel{+}2ab) = d$
$a^2+b^2 - a^2 - b^2 + 2ab = d$
$d_1 = \cancel{+}\,2ab$

$a_3 = a_2 + d$
$(a+b)^2 = (a^2+b^2) + d$
$a^2+b^2+2ab - a^2 \cancel{+} - b^2 = d$
$d_2 = 2ab$
$\because d_1 = d_2 = 2ab$

$\therefore$ The terms $(a-b)^2$, (a^2+b^2) and $(a+b)^2$ are in ~~ap~~ A.P.

Q2. The sum of 1st 3 terms of an AP is 42 and the product of the 1st and 3rd term is 52. find the 1st term and common difference.

let the 1st 3 terms of AP be $a-d, a, a+d$

Acc to question

$$a-d+a+a+d = 42$$
$$3a = 42$$
$$a = 14$$

$$(a-d)(a+d) = 52$$
$$a^2 - d^2 = 52$$
$$14^2 - d^2 = 52$$
$$196 - 52 = d^2$$
$$144 = d^2$$
$$d = \sqrt{144}$$
$$\boxed{d = \pm 12}$$

if $d = +12$

$$a_1 = a-d = 14-12$$
$$\boxed{a_1 = 2}$$

if $d = -12$

$$a_1 = a-d = 14+12$$
$$\boxed{a_1 = 26}$$

Q.3 Find the 16th term of the AP 7, 11, 15, 19...
Also, find the sum of the first 6 term.

$$a_{16} = a + (n-1)d$$
$$= \cancel{a}\,7 + (16-1)d$$
$$= 7 + 15 \times 4$$
$$= 7 + 60$$
$$= 67$$

$$d = 11 - 7$$
$$= 4$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{\cancel{16}\,6}{2}[2(7) + (6-1)d]$$
$$= 3[14 + 5d]$$
$$= 3[14 + 20]$$
$$= 3 \times 34$$
$$= 102$$

$\boxed{16\text{th term of AP} = 67}$

$\boxed{\text{Sum of 1st 6 terms of AP} = 102}$

Q4. The first term of an AP is 5, the last term is 50 and their sum is 440. Find

i) The number of terms   ii) common difference

$a = 5$

$a_n = 50$

$a_n = a + (n-1)d$

$50 = 5 + (n-1)d$

$(n-1)d = 45$

$S_n = 440$

$S_n = \frac{n}{2}[2a + (n-1)d] = 440$

$\Rightarrow \frac{n}{2}[2(5) + 45] = 440$

$\Rightarrow \frac{n}{2}[45 + 10] = 440$

$\Rightarrow \frac{n}{2} \times 55 = 440$

$n = \frac{\cancel{440}^{\cancel{40}\ 8}}{\cancel{55}_{5}} \times 2 = 16$

$\boxed{\text{i) no. of terms} = 16}$

$(n-1)d = 45$

$(16-1)d = 45$

$15d = 45$

$d = \frac{\cancel{45}\ 3}{\cancel{15}}$

$\boxed{\text{ii) common difference} = 3}$

Q 5: An AP has 3 as its first term. The sum of the first 8 terms is twice the sum of the first 5 terms. Find the common difference of the AP.

$a = 3$

$S_8 = 2 \times S_5$

$\frac{n}{2}[2a + (n-1)d] = 2 \times \frac{5}{2}[2a + (n-1)d]$

$\frac{8}{2}[2(3) + (8-1)d] = 5[2(3) + (5-1)d]$

$4[6 + 7d] = 5[6 + 4d]$

$24 + 28d = 30 + 20d$

$28d - 20d = 30 - 24$

$d8 = 6$

$d = \frac{\cancel{6}\,3}{\cancel{8}\,4}$

$d = \frac{3}{4}$

$\boxed{\text{common difference} = \frac{3}{4}}$

Q6. If the pth term of an AP is $\frac{1}{q}$ and the qth term is $\frac{1}{p}$ then show that pqth term is 1.

$$a_p = \frac{1}{q} \quad \text{and} \quad a_q = \frac{1}{p}$$

$$a_n = a + (n-1)d$$

$$\frac{1}{q} = a + (p-1)d \quad \text{---(i)}$$

$$\frac{1}{p} = a + (q-1)d \quad \text{---(ii)}$$

On subtracting Eq (ii) from Eq (i)

$$\frac{1}{q} - \frac{1}{p} = pd - d - qd + d$$

$$\left(\frac{p-q}{pq}\right) = d(p-q)$$

$$d = \frac{1}{pq}$$

On putting $d = \frac{1}{pq}$

$$\frac{1}{q} = a + (p-1)\cdot\frac{1}{pq}$$

$$a = \frac{1}{pq}$$

$$a_{pq} = a + (pq - 1)\cdot d$$
$$a_{pq} = \frac{1}{pq} + (pq-1)\frac{1}{pq}$$

$$a_{pq} = \frac{1}{pq} + 1 - \frac{1}{pq} = 1$$

$$\boxed{a_{pq} = 1}$$