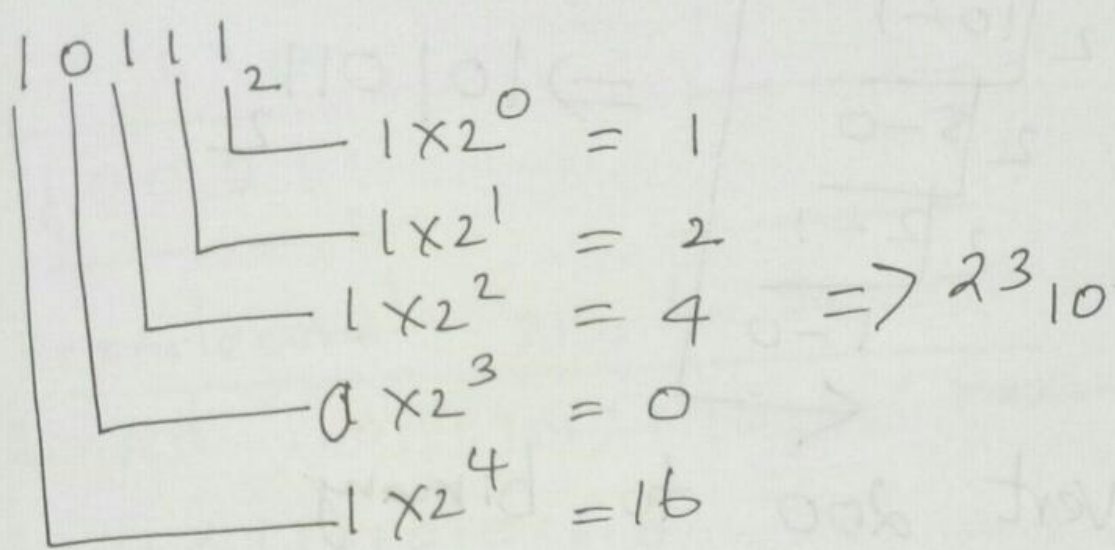
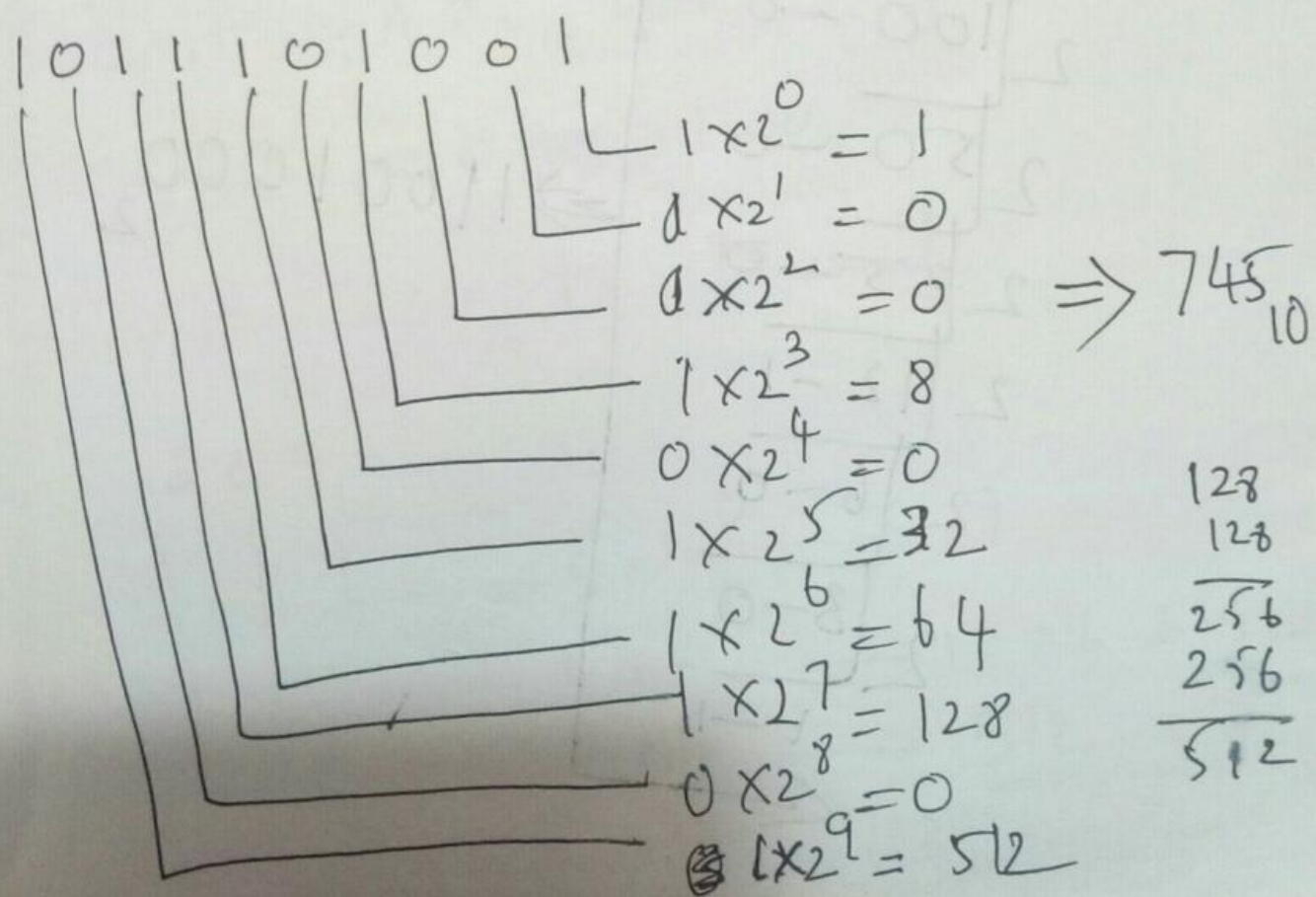


Number systems

Convert 10111_2 to decimal

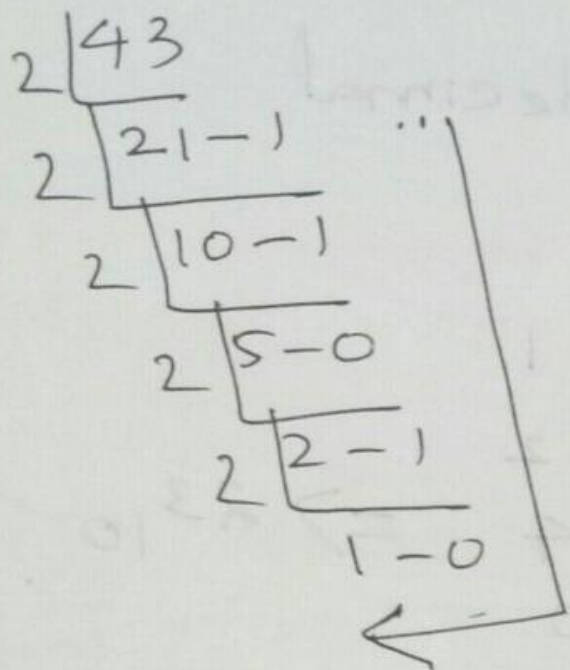


2) Convert 1011101001_2



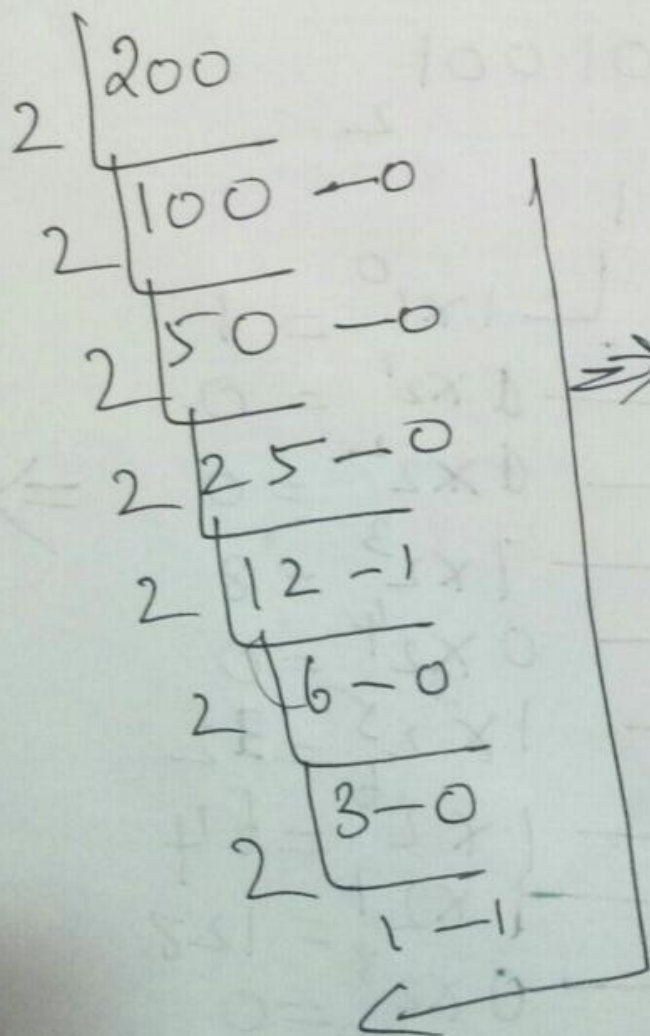
Convert 43_{10} to binary

(Successive division method)



$\Rightarrow 101011_2$

Convert 200_{10} to binary



$\Rightarrow 11001000_2$

Binary Addition

Add $1011_2 + 110_2$

$$\begin{array}{r} 1011 \\ 110 \\ \hline 10001_2 \end{array}$$

Binary subtraction

sub $100_2 - 01_2$

$$\begin{array}{r} 100 \\ - 01 \\ \hline 11_2 \end{array}$$

Hexadecimal Number System:

Convert 1101110101110_2 to hexadecimal

1101110101110_2

3 7 A E \Rightarrow 37AE

Convert $4A8C_{16}$ to binary

0100101010001100_2

Convert $2C9_{16}$ to Decimal

$2C9_{16}$

$$\begin{array}{l} 9 \times 16^0 \rightarrow 9 \\ C \times 16^1 \rightarrow 12 \times 16 = 192 \\ 2 \times 16^2 \rightarrow 512 \end{array}$$

Result = 713_{10}

Convert Decimal to hexadecimal

$$423_{10} \quad \begin{array}{r} 16 \overline{) 423} \\ \underline{26} \\ 10 \end{array} \Rightarrow 1A7_{16}$$

Introduction to Boolean Algebra & logic gates:

1) Associative law: $(a * b) * c = a * (b * c)$
for all $a, b, c \in S$ set.

2) Commutative law: $a * b = b * a$ for all $a, b \in S$

3) Identity element: $I * a = a * I$
 $I =$ either 0 or 1

4) Inverse: $a * a^{-1} = 0$

5) Distributive law:

$$a * (b + c) = (a * b) + (a * c)$$

$$a + (b * c) = (a + b) * (a + c)$$

Axiomatic definition of Boolean Algebra

In 1854 George boole developed an algebraic system now called Boolean Algebra.

In 1938 C.E. Shannon introduced 2 Variable or Value boolean algebras called switching algebras, it represent the properties of bistable electrical switching circuits.

Formal definition of boolean algebras one can use E.V. huntington in 1904.

Boolean Algebra is an algebraic structure defined by a set of elements together with 2 binary operators.

Difference between Boolean Algebra & ordinary algebras

- 1) Distributive law suit for Boolean Algebra not for ordinary Algebra

2) Boolean Algebra don't have Subtraction & division operations.

3) Compliment operator available in boolean not in ordinary Algebras.

4) Boolean Algebra constitute with 0 and 1 only.

Basic theorem and properties of

Boolean Algebra:-

$$a + 0 = a$$

$$a + \bar{a} = 1$$

$$a + a = a$$

$$a + 1 = 1$$

$$\overline{\bar{a}} = a \text{ Involution Theorem}$$

$$x + y = y + x$$

commutative

$$a + (b + c) = (a + b) + c$$

Associate

$$a \cdot (b + c) = ab + ac$$

Distributive

$$a \cdot 0 = 0$$

$$a \cdot \bar{a} = 0$$

$$a \cdot a = a$$

$$a \cdot 1 = a$$

$$a \cdot 0 = 0$$

$$xy = yx$$

$$a(bc) = (ab)c$$

$$a + bc = (a + b)(a + c)$$

Demorgan's law

$$\overline{(a+b)} = \bar{a} \cdot \bar{b}, \quad \overline{ab} = \bar{a} + \bar{b}$$

Absorption law

$$x + xy = x, \quad x(x+y) = x$$

Consensus theorem

$$ab + \bar{a}c + bc = ab + \bar{a}c$$

$$(a+b) \cdot (\bar{a}+c) (b+c) = (a+b) (\bar{a}+c)$$

Simplify the following function to a minimum number of literals.

$$\textcircled{1} x(\bar{x}+y) = x\bar{x} + xy = 0 + xy = xy$$

$$\textcircled{2} x + \bar{x}y = (x + \bar{x}) \cdot (x + y) = 1 \cdot (x + y) = (x + y)$$

$$\begin{aligned} \textcircled{3} (x+y)(x+\bar{y}) &= x \cdot x + x\bar{y} + yx + y \cdot \bar{y} \\ &= x + x\bar{y} + xy + 0 \\ &= x + xy + x\bar{y} \\ &= x + x\bar{y} = x \end{aligned}$$

$$(4) xy + \bar{x}z + yz$$

$$\Rightarrow xy + \bar{x}z + yz(x + \bar{x})$$

$$\Rightarrow xy + \bar{x}z + xyx + \bar{x}yz$$

$$\rightarrow xy[1+z] + \bar{x}z[1+y]$$

$$\Rightarrow xy + \bar{x}z$$

$$(5) (x+y)(\bar{x}+z)(y+z) \neq (x+y)(\bar{x}+z)$$

by duality function ref sum (4)

find the complement of the functions

$$(1) F = \bar{x}y\bar{z} + \bar{x}\bar{y}z$$

$$F_c = \overline{\bar{x}y\bar{z} + \bar{x}\bar{y}z}$$

Apply De Morgan's law

$$F_c = \overline{\bar{x}y\bar{z}} \cdot \overline{\bar{x}\bar{y}z}$$

$$= (\overline{\bar{x}} + \overline{y} + \overline{\bar{z}}) (\overline{\bar{x}} + \overline{\bar{y}} + \overline{z})$$

$$= (x + \bar{y} + z) \cdot (x + \bar{y} + \bar{z})$$

$$(2) F = x(\bar{y}\bar{z} + yz)$$

$$F_c = x(\bar{y}\bar{z} + yz)$$

$$= \bar{x} + (\bar{y}\bar{z} + yz)$$

$$= \bar{x} + [\bar{y}\bar{z} \cdot yz]$$

$$= \bar{x} + [(y+z) \cdot (\bar{y} + \bar{z})]$$

$$= \bar{x} + [0 + y\bar{z} + z\bar{y} + 0]$$

$$= \bar{x} + y\bar{z} + \bar{y}z$$

Canonical \rightarrow standard forms:-

Sum of Product (SOP): Minterms - AND gate

Product of Sum (POS): Maxterms - OR gate

Sum of Minterms (Sum of Product Term)

$$\text{Ex: } F = A + \bar{B}C$$

Find out the minterms:

$$F = A + \bar{B}C = A(B + \bar{B})(C + \bar{C}) + \bar{B}C(A + \bar{A})$$

$$\Rightarrow (AB + A\bar{B})(C + \bar{C}) + \bar{B}C(A + \bar{A})$$

$$ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{B}CA + \bar{B}C\bar{A}$$

$$ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$$

$$111 + 110 + 101 + 100 + \bar{A}\bar{B}C$$

$$101 + 001$$

$$7 + 6 + 5 + 4 + 5 + 1$$

$$F(A, B, C) = \sum_m (1, 4, 5, 6, 7)$$

Truth Table : $F = A + \bar{B}C$

A	B	\bar{B}	C	F	
0	0	1	0	0	0
0	0	1	1	1	1
0	1	0	0	0	2
0	1	0	1	0	3
1	0	1	0	1	4
1	0	1	1	1	5
1	1	0	0	0	6
1	1	0	1	1	7

Product of Max Term (Product of Sum)

Express the boolean function

$F = xy + \bar{x}z$ as a product of maxterms

$$= (x + \bar{x})(y + \bar{x})(xy + z)$$

$$= (x + \bar{x})(y + \bar{x})(x + z)(y + z)$$

$$= (\bar{x} + y)(x + z)(y + z)$$

$$\bar{x} + y = \bar{x} + y + z \bar{z}$$

$$= (\bar{x} + y + z) (\bar{x} + y + \bar{z})$$

$$x + z + y \bar{y} = (x + z + y) (x + z + \bar{y})$$

$$y + z + x \bar{x} = (y + z + x) (y + z + \bar{x})$$

$$F = (\bar{x} + y + z) (\bar{x} + y + \bar{z}) (x + y + z) (x + y + \bar{z}) (y + z + x) (y + z + \bar{x})$$

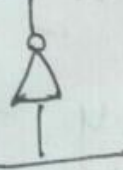
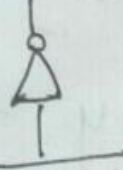
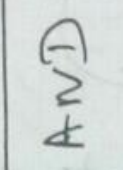

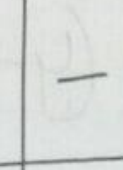
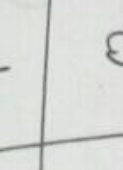
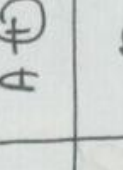
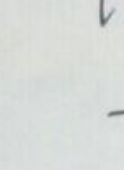
$$F = 100, 101, 000, 010$$

$$F = M_4, M_5, M_0, M_2$$

$$F(x, y, z) = \prod (0, 2, 4, 5)$$

Digital Logic Gates

A	B	\bar{A}	\bar{B}	AB	A+B	\overline{AB}	$\overline{A+B}$	$A \oplus B$	$\overline{A \oplus B}$
0	0	1	1	0	0	1	1	0	1
0	1	1	0	0	1	1	0	1	0
1	0	0	1	0	1	1	0	1	0
1	1	0	0	1	1	0	0	0	1

InV	InV	AND	OR	NAND	NOR	XOR	XNOR
							

Simplify the following Boolean expression to a minimum no of literals.

$$1) xy + x\bar{y} \Rightarrow x(y + \bar{y}) \Rightarrow x$$

$$\Rightarrow (y+x)(x+\bar{y})$$

$$\rightarrow (x+x)(y+x)(x+\bar{y})(y+\bar{y})$$

$$\Rightarrow x(x+y)(x+\bar{y})$$

$$2) (x+y)(x+\bar{y})$$

$$\Rightarrow xx + xy + x\bar{y} + y\bar{y}$$

$$\Rightarrow x + xy + x\bar{y} + 0$$

$$\Rightarrow x[1 + y + \bar{y}] + 0$$

$$\Rightarrow x$$

$$3) xyz + \bar{x}y + xy\bar{z}$$

$$\Rightarrow xy(z + \bar{z}) + \bar{x}y$$

$$\Rightarrow xy + \bar{x}y \Rightarrow y(x + \bar{x}) \Rightarrow y$$

$$\textcircled{4} (\overline{A+B}) \overline{A+B}$$

$$\Rightarrow (\overline{A+B}) (\overline{A+B})$$

$$\Rightarrow \overline{A} \cdot \overline{B} \cdot \overline{A} \cdot \overline{B}$$

$$\Rightarrow 0$$

$$\textcircled{5} xy\bar{z} + x\bar{y}z + x\bar{y}z + x\bar{y}\bar{z}$$

$$\Rightarrow xy(\bar{z}+z) + x\bar{y}(z+\bar{z})$$

$$\Rightarrow xy + x\bar{y} \Rightarrow y(x+\bar{x}) \Rightarrow y$$

$$\textcircled{6} (x+y+\bar{z})(\bar{x}+\bar{y}+z)$$

$$\Rightarrow x\bar{x} + x\bar{y} + xz + y\bar{x} + y\bar{y} + yz + \bar{z}\bar{x} + \bar{z}\bar{y} + \bar{z}z$$

$$\Rightarrow 0 + x\bar{y} + xz + \bar{x}y + 0 + yz + \bar{z}\bar{x} +$$

$$\Rightarrow x\bar{y} + xz + \bar{x}y + yz + \bar{z}\bar{x} + \bar{z}\bar{y} + 0$$

$$\textcircled{7} xy + x\bar{y} + \bar{x}y$$

$$= x(y+\bar{y}) + \bar{x}y = x + \bar{x}y = (x+\bar{x})y$$

$$= \underline{\underline{x+y}}$$

$$\textcircled{8} ABC + \bar{A}B + AB\bar{C} + AC$$

$$\Rightarrow AB(C + \bar{C}) + \bar{A}B + AC$$

$$\Rightarrow B(A + \bar{A}) + AC$$

$$\Rightarrow B + AC \Rightarrow (A + B)(B + C)$$

$$\textcircled{9} \bar{x}yz + xz + \bar{x}z$$

$$\Rightarrow \bar{x}z(y + 1) + xz$$

$$\Rightarrow \bar{x}z + xz \Rightarrow z$$

$$\textcircled{10} (\bar{x} + y)(\bar{x} + \bar{y})$$

$$(\bar{x} \cdot \bar{y})(\bar{x} + \bar{y}) = \bar{x}\bar{y}$$

$$\bar{x}\bar{y}\bar{x} + \bar{x}\bar{y}\bar{y}$$

$$\bar{x}\bar{y} + \bar{x}\bar{y} \Rightarrow \bar{x}\bar{y}$$

$$\textcircled{11} xy + x(wz + w\bar{z})$$

$$\Rightarrow xy + xw$$

$$\Rightarrow x(y + w)$$

$$(12) (\overline{B}C + \overline{A}D)(\overline{A}\overline{B} + C\overline{D})$$

$$\Rightarrow \overline{B}C\overline{A}\overline{B} + \overline{B}C\overline{D} + \overline{A}D\overline{A}\overline{B} +$$

$$\Rightarrow 0 + 0 + 0 + 0$$

$$\Rightarrow 0$$

$$(13) (x + \overline{y} + \overline{z})(\overline{x} + \overline{z})$$

$$\Rightarrow 0 + \overline{x}\overline{y} + \overline{x}\overline{z} + \overline{z} + \overline{z}\overline{y} + \overline{z}x$$

$$\Rightarrow 0 + \overline{x}\overline{y} + \overline{x}\overline{z} + \overline{z}\overline{y} + \overline{z}x$$

$$\Rightarrow \overline{x}\overline{y} + \overline{x}\overline{z} + \overline{z}$$

$$\Rightarrow \overline{x}\overline{y} + \overline{z}$$

Reduce the following Boolean expressions to the indicated number of literals.

$$(1) \overline{A}\overline{C} + ABC + A\overline{C} + A\overline{B} \quad (2 \text{ literals})$$

$$= \overline{C}(\overline{A} + A) + ABC + A\overline{B}$$

$$\Rightarrow \overline{C} + ABC + A\overline{B}\overline{C} + A\overline{B}C$$

$$\Rightarrow \overline{C} + AC + A\overline{B}\overline{C} \Rightarrow \overline{C}\overline{A} + \overline{C}A + AC + A\overline{B}\overline{C}$$

Use boolean algebra to simplify the function to a minimum no of literals

$$F = x\bar{y}z + \bar{x}\bar{y}z + \bar{w}xy + w\bar{x}y + wxy$$

$$= (x + \bar{x})\bar{y}z + xy + w\bar{x}y$$

$$\Rightarrow \bar{y}z + xy + \bar{x}yw$$

$$\Rightarrow \bar{y}z + y[x + \bar{x}w]$$

$$\Rightarrow \bar{y}z + y[(x + \bar{x})(x + w)]$$

$$\Rightarrow \bar{y}z + y(x + w)$$

Map Method (or) Karnaugh Map (or) K-Map

(Simple, straight forwarded method.

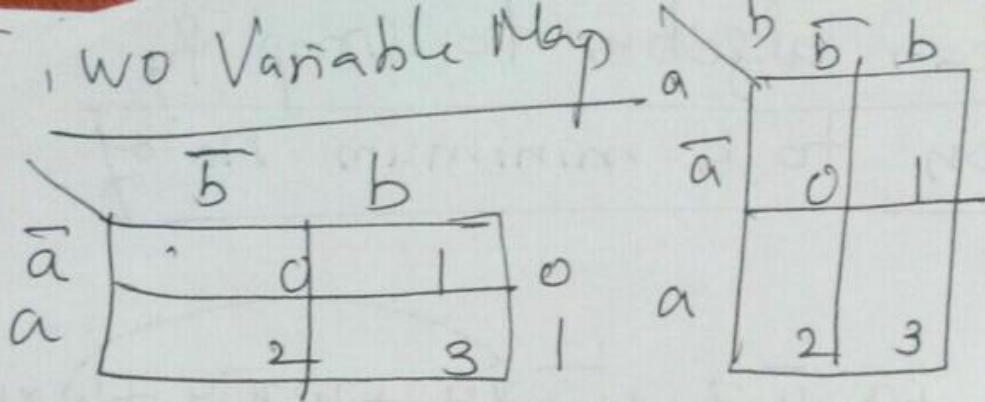
$$2 \text{ inputs} \Rightarrow 2^n = 0, 1, 2, 3$$

$$3 \text{ inputs} = 2^3 = 0, 1, 2, 3, 4, 5, 6, 7.$$

$$4 \text{ inputs} = 2^4 = 0 \text{ to } 15$$

$$5 \text{ inputs} = 2^5 = 0 \text{ to } 32$$

Two Variable Map

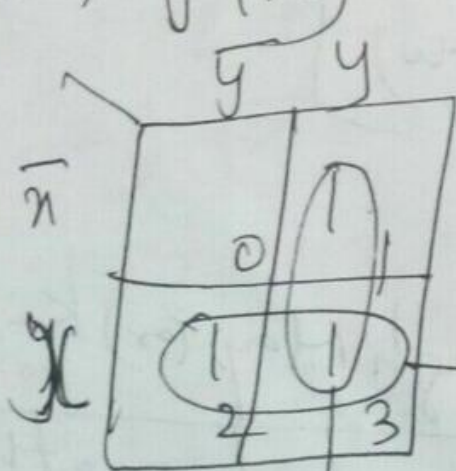


$$F = \bar{x}y + x\bar{y} + xy$$

$$= y[\bar{x} + x] + x\bar{y}$$

$$= y + x\bar{y} \Rightarrow (y + x)(y + \bar{y})$$

$$\Rightarrow (y + x)$$



$$F = \bar{x}y + x\bar{y} + xy$$

$$= 01, 10, 11$$

$$\Rightarrow x + y$$

Simplify the boolean function

① $F(x, y, z) = \Sigma(2, 3, 4, 5)$

	$\bar{y}z$	$\bar{y}\bar{z}$	yz	$y\bar{z}$
\bar{x}	0	1	1	1
x	1	1	7	6

$\bar{x}y$ (circled in original)
 $x\bar{y}$ (circled in original)
 00, 01, 11, 10

$$F(x, y, z) = \bar{x}y + x\bar{y}$$

$x\bar{y}$ (circled in original)

$$(2) F(x, y, z) = \sum (3, 4, 6, 7)$$

	$y\bar{z}$	$\bar{y}\bar{z}$	yz	$y\bar{z}$
\bar{x}	0	1	1	2
x	1	5	1	6

$x\bar{z}$ (circled in original)
 yz (circled in original) $\Rightarrow yz + x\bar{z}$

$$F(x, y, z) = \sum (0, 2, 4, 5, 6) = \bar{z} + x\bar{y}$$

	$y\bar{z}$	$\bar{y}\bar{z}$	yz	$y\bar{z}$
\bar{x}	1	0	1	1
x	1	4	5	6

$\bar{y}\bar{z}$ (circled in original)
 $x\bar{y}$ (circled in original)
 $\bar{y}\bar{z} + x\bar{y}$

$$F(a, b, c) = \sum(1, 2, 3, 5, 7) = c + \bar{a}b$$

	hc	$\bar{b}\bar{c}$	$\bar{b}c$	bc	$b\bar{c}$
\bar{a}		0	1	3	2
a		4	5	7	6

$\bar{a}b$

↓

Four Variable Map

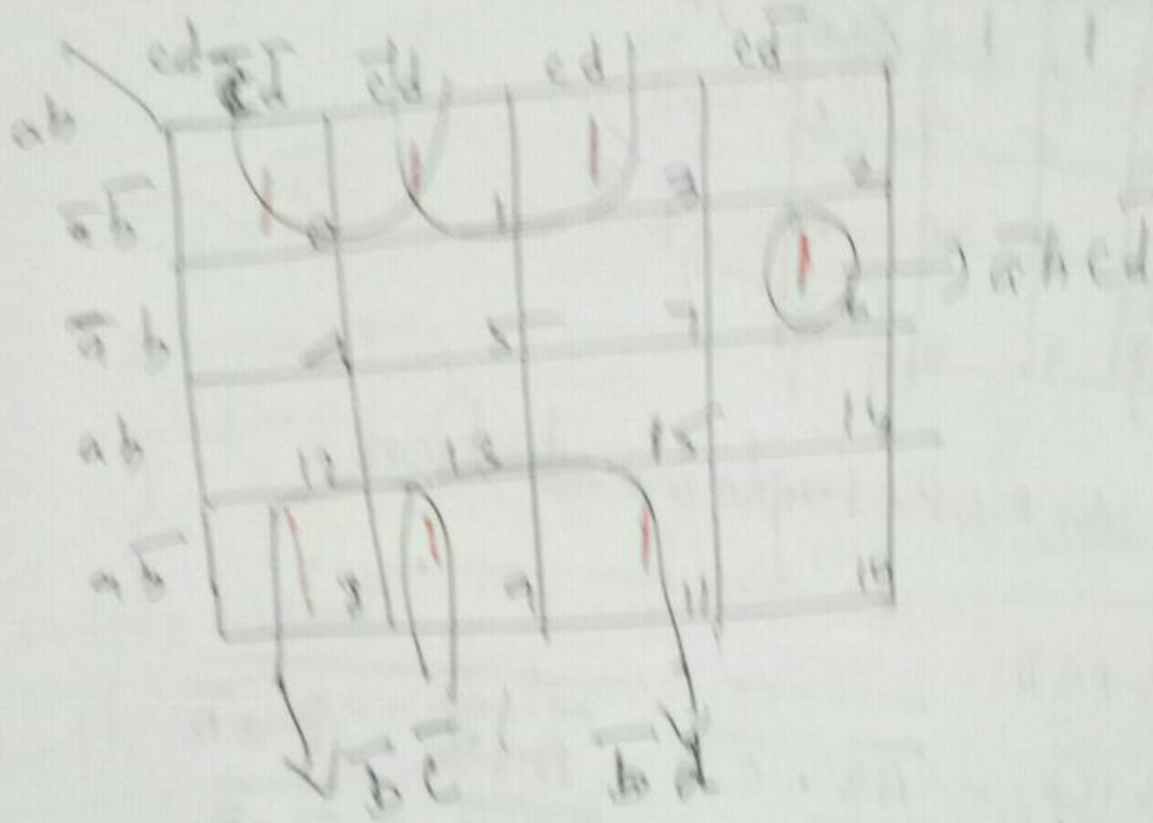
$$F(w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

$wx \backslash yz$				
$\bar{w}\bar{x}$	0	1	3	2
$\bar{w}x$	4	5	7	6
wx	12	13	15	14
$w\bar{x}$	8	9	11	10
	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz

$$\Rightarrow \bar{y} + \bar{w}z + xz$$

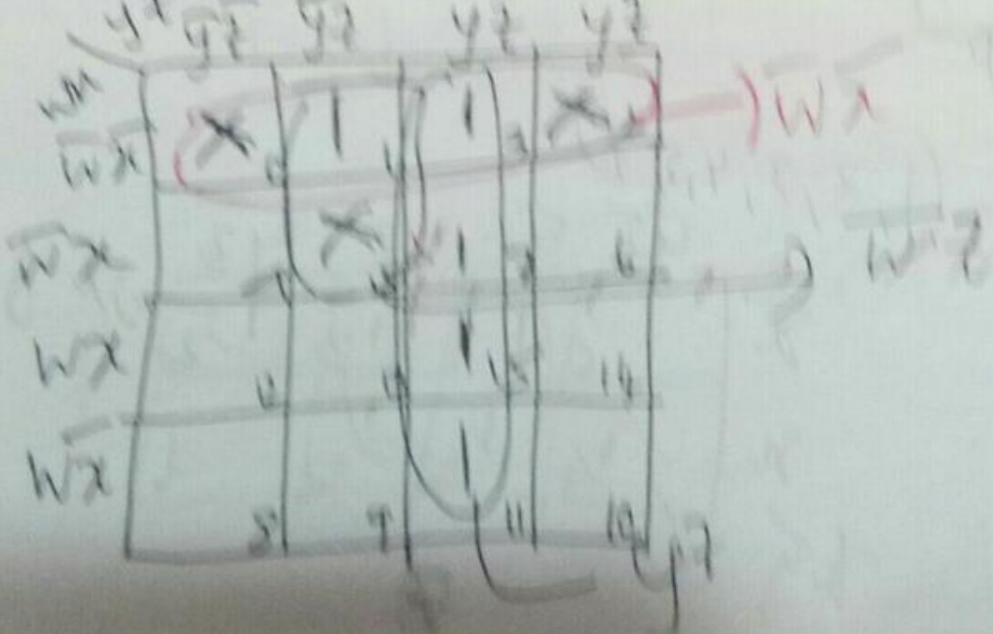
Simplify the boolean function

$$F = a\bar{b}c + \bar{b}cd + a\bar{b}cd + a\bar{b}\bar{c}$$



Simplify the Boolean function

$$F(w,x,y,z) = \sum(1, 3, 7, 11, 15) + d(0, 2, 6)$$



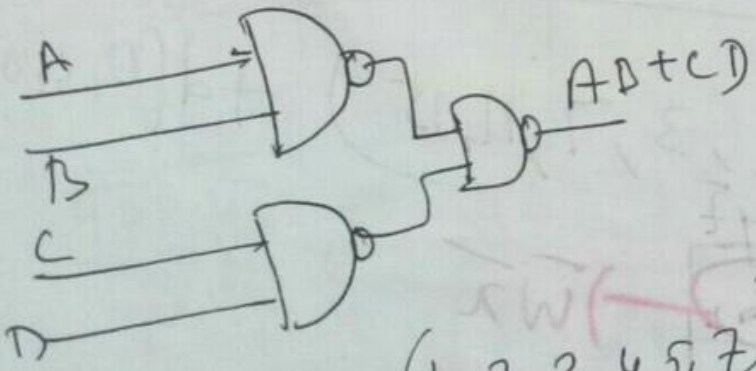
$$F(w, x, y, z) = \sum(1, 3, 10) + \sum_d(0, 2, 8, 12)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
$\bar{w}\bar{x}$	0	1	3	2
$\bar{w}x$	4	5	7	6
$w\bar{x}$	8	9	15	14
wx	10	11	13	12

NAND * NOR implementation

$$F = AB + CD$$

$$F = \overline{\overline{AB + CD}} = \overline{\overline{AB} \cdot \overline{CD}} = \overline{\overline{A+B} \cdot \overline{C+D}}$$



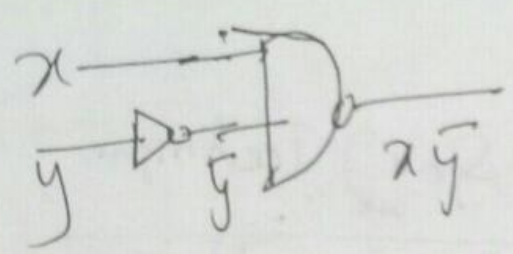
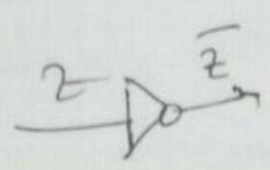
$$F(x, y, z) = \sum(1, 2, 3, 4, 5, 7)$$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	yz
\bar{x}	0	1	3	2
x	4	5	7	6

$$F(x, y, z) = z + x\bar{y} + \bar{x}y$$

$$\overline{z + x\bar{y} + \bar{x}y} = \bar{z} \overline{x\bar{y}} \overline{\bar{x}y}$$

$$= \bar{z} + \overline{x\bar{y}} + \overline{\bar{x}y}$$

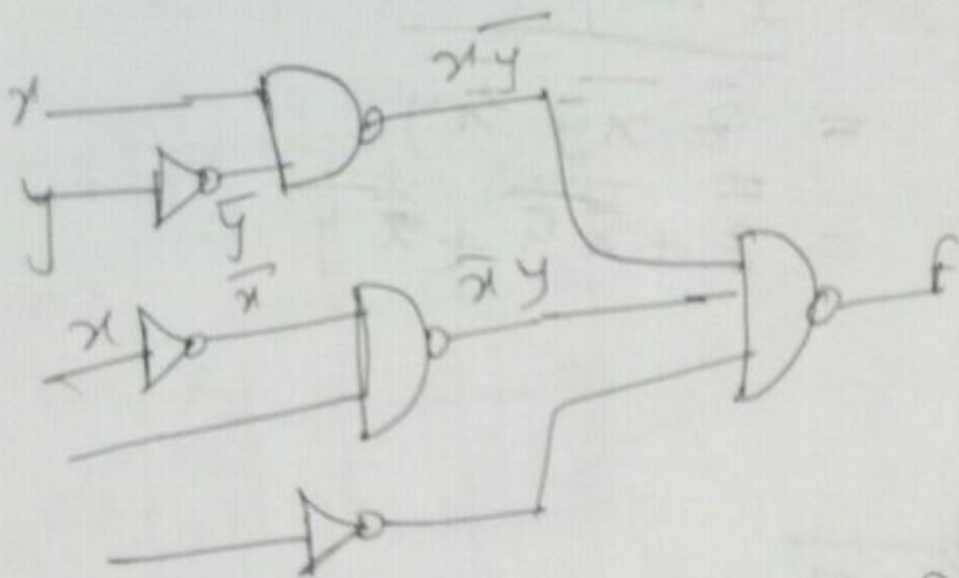


Implement the following Boolean function
with NAND gates

$$F(x, y, z) = \sum m(1, 2, 3, 4, 5, 7)$$

	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$y\bar{z}$	
x			1	1	1) $\bar{x}y$
\bar{x}	0	1	1	1	1	
x	1	1	1	1	1	
	4	5	6	7	0	

$$F = \overline{\bar{x}\bar{y}} + \bar{x}y + z = \overline{\bar{x}\bar{y}} \cdot \overline{\bar{x}y} + z$$



Quine Mc-cluskey (QM) Technique

For increased no of Variable

QM method can be adopted.

QM method is more effective

since a digital computer can be

used to simplify the given booleans
Switching function.

K map is graphical method

QM method is a tabular method.

The basic QM cluskey procedure

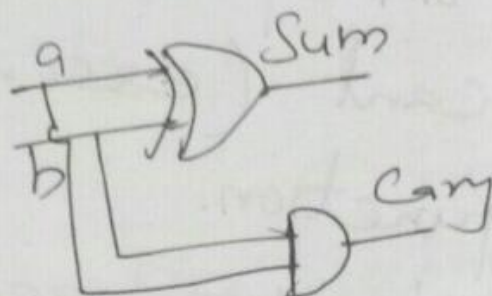
is as follows

- ① Find the prime implicants of the function
- ② Construct the prime implicant table and find the essential prime implicant (essential rows) of the function.
- ③ Include the essential prime implicants in the minimum sum.
- ④ After all essential prime implicants are deleted from the Prime implicant table, determine the dominated rows & dominated columns in the table, delete all dominated rows and dominating columns and find the (if any) essential prime implicants.
- ⑤ Repeat Steps ③ & ④ as many times as they are applicable until a minimal cover of the function is found.

Implementation of Combinational Logic Ckts using HA/FA using Verilog HDL

HA \rightarrow Half Adder

a	b	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



Create level modelling

```
module halfadder (a, b, sum, carry);
```

```
input a, b; output sum, carry;
```

```
xor (sum, a, b);
```

```
and (carry, a, b); end module
```

Data flow modelling

```
assign sum = a ^ b;
```

```
assign carry = a & b;
```

Behavioral

```
reg sum, carry;  
always @ (a, b)
```

```
begin
```

```
sum = a ^ b;
```

```
carry = a & b;
```

```
end  
end module
```

Full Adder

	a	b	c	Sum	Carry
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1

Sum

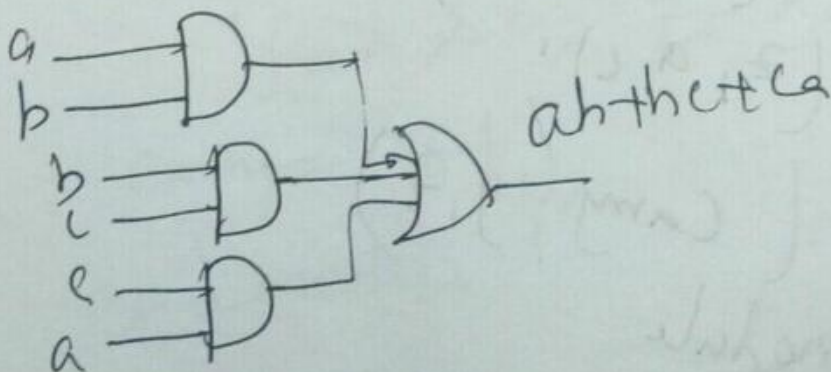
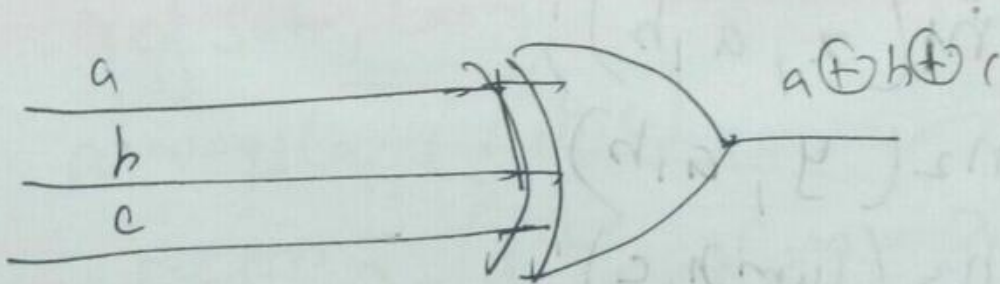
a	$\bar{b}\bar{c}$	$\bar{b}c$	$b\bar{c}$	bc
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1

a	$\bar{b}\bar{c}$	$\bar{b}c$	$b\bar{c}$	bc
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1

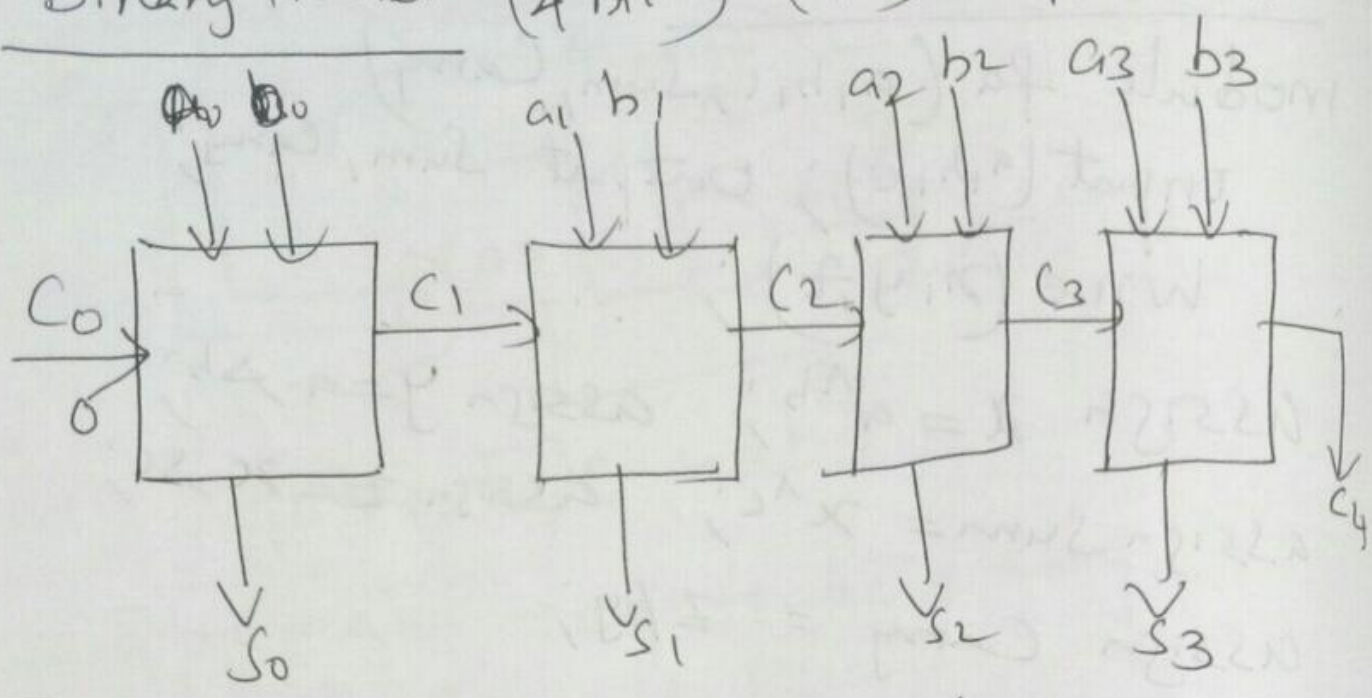
ac bc ab
 $ab + bc + ca = \text{Carry}$

$$\text{Sum} = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$$

$$= a \oplus b \oplus c$$



Binary Adder (4 bit) (RPA) ripple carry adder



$A = 1011$ $B = 0011$

$a = 1011$

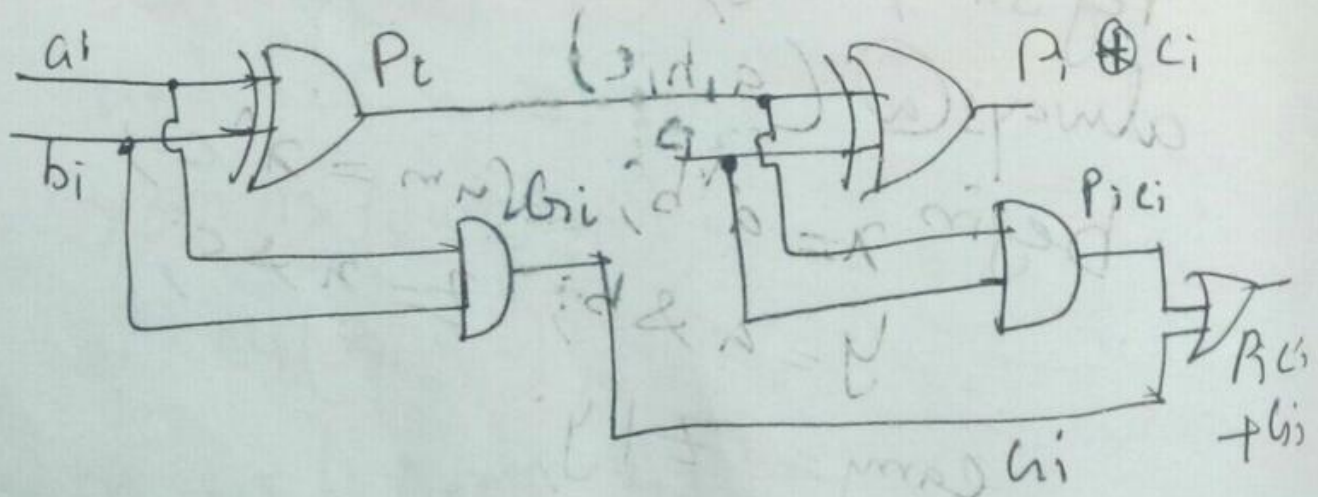
$b = 0011$

$S = \underline{1110}$

$C = 0011$

At $C_0 = 0$

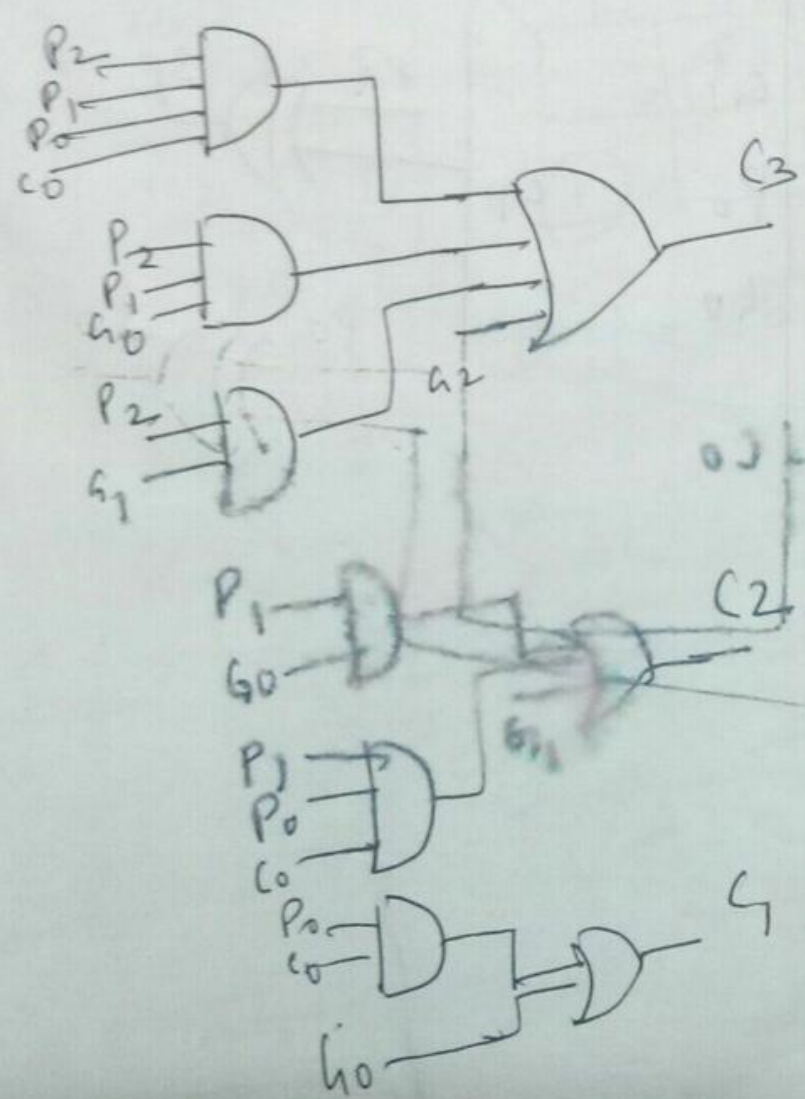
$a = \text{augend}$
 $b = \text{addend}$



$C_0 = \text{input carry}$ Carry lookahead generator
 $C_1 = G_0 + P_0 C_0 ; C_2 = G_1 + P_1 C_1$
 $= G_1 + P_1 (G_0 + P_0 C_0)$

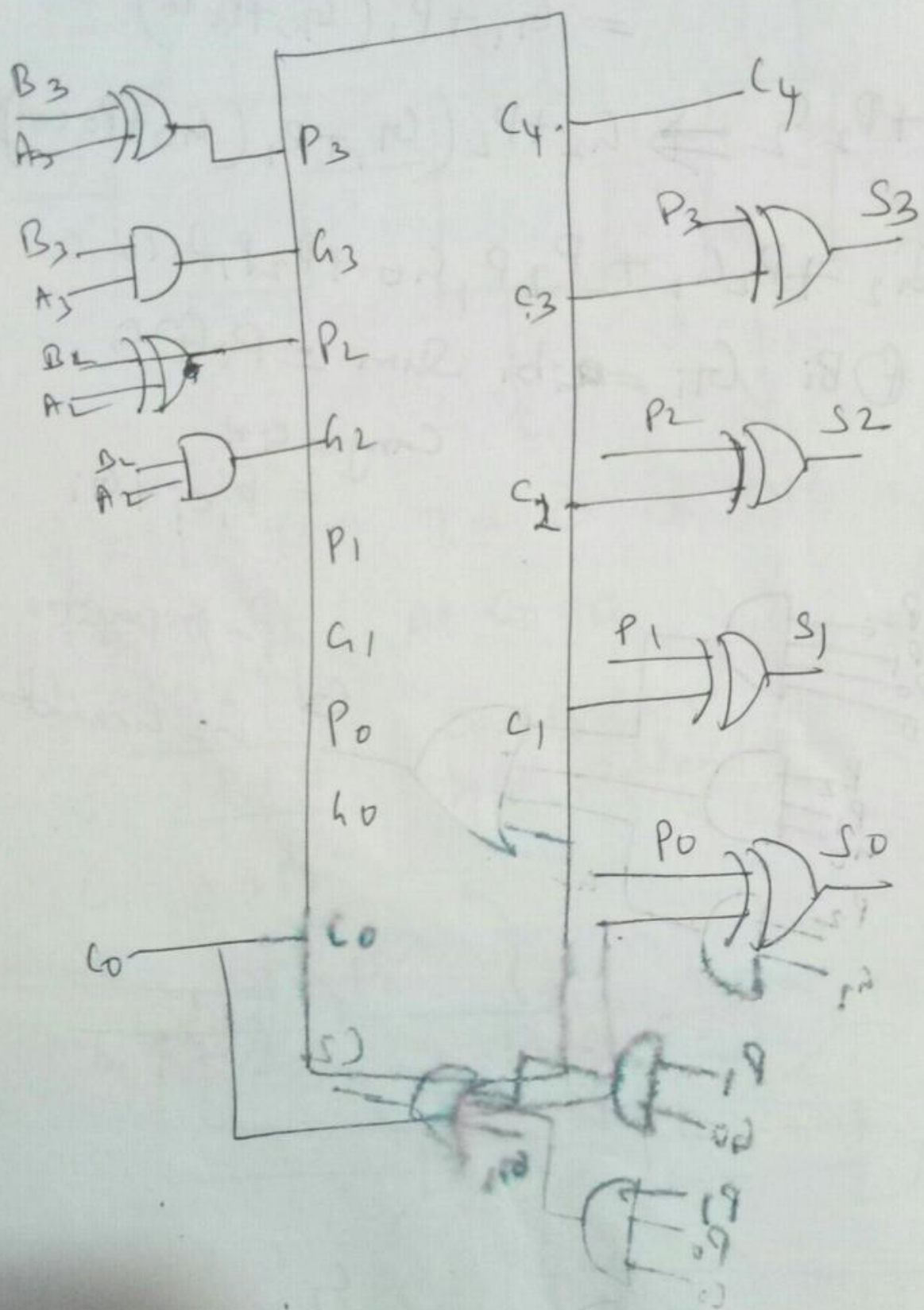
$C_3 = G_2 + P_2 C_2 \Rightarrow G_2 + P_2 (G_1 + P_1 (G_0 + P_0 C_0))$
 $= G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0$

$P_i = A_i \oplus B_i$ $G_i = a_i b_i$ Sum = $P_i \oplus C_i$
 Carry = $C_{i+1} = P_i C_i + G_i$



P-propagator
G-generator

Four-bit adder with carry look ahead



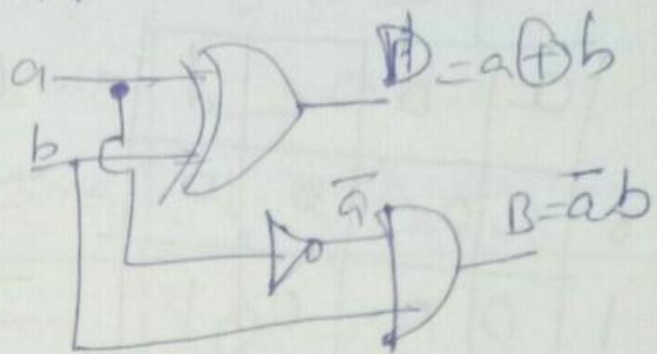
Half Subtractor

a	b	B	D
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

$$\text{Borrow} = B = \bar{a}b$$

$$\bar{a}b + a\bar{b} = a \oplus b$$

difference = $a \oplus b$



Full Subtractor

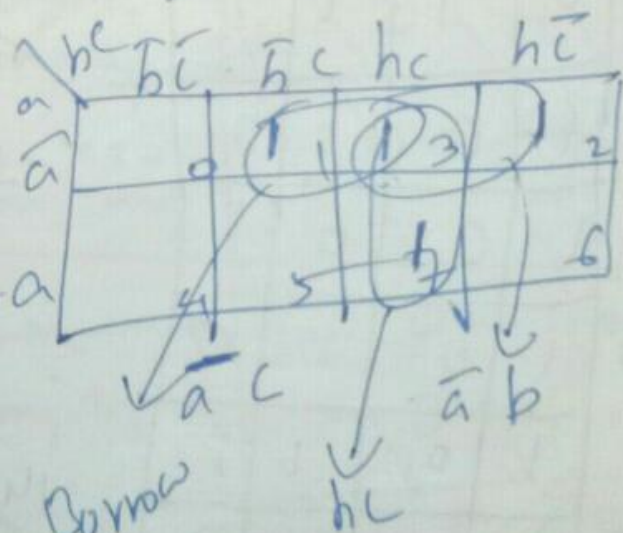
LSB		MSB		
a	b	c	D	B
0	0	0	0	0
1	0	0	1	1
2	0	1	1	1
3	0	1	0	1
4	1	0	1	0
5	1	0	0	0
6	1	1	0	0
7	1	1	1	1

Difference

a	b	c	D	B
0	0	0	0	0
0	1	0	1	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	0

$$\text{Diff} = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}c + ab\bar{c}$$

$$= a \oplus b \oplus c$$



$$\text{Borrow} = \bar{a}c + \bar{a}b + bc$$

Quine Mc cluskey method ^(tabular method)

$$F(a,b,c,d) = \sum_{m=0}^{15} (0, 1, 3, 7, 8, 9, 11, 15)$$

Step ①

	a	b	c	d
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1

Step ②

Group	min terms	Binary added
0	m ₀	0 0 0 0
1	m ₁ m ₃	0 0 0 1 1 0 0 0
2	m ₃ m ₅	0 0 1 1 1 0 0 1
3	m ₇ m ₁₁	0 1 1 1 1 0 1 1
4	m ₁₅	1 1 1 1

	a	b	c	d
14	1	1	1	0
15	1	1	1	1

Step 3

Group	(Decomposed) minterms with patches	First Reduction
0	$m_0 \Delta m_1$ $m_0 \Delta m_2$	$000 -$ -000
1	$m_1 \Delta m_3$ $m_1 \Delta m_2$ $m_2 \Delta m_3$	$00-1$ -001 $100-$
2	$m_2 \Delta m_7$ $m_2 \Delta m_1$ $m_3 \Delta m_1$	$0-11$ -011 $10-1$
3	$m_7 \Delta m_5$ $m_5 \Delta m_1$	-111 $1-11$

Step 4

0

$m_0 \Delta m_1$
 $m_2 \Delta m_3$
 $m_0 - m_2 -$
 $m_1 - m_3$

$a \quad b \quad c \quad d$
 $-00- \left. \begin{array}{l} \overline{b} \overline{c} \\ \overline{c} \end{array} \right\}$
 $-00-$

1

$m_1 - m_3 - m_2 - m_7$
 $m_1 - m_2 - m_3 - m_7$

$-0-1 \left. \begin{array}{l} \overline{b} d \\ \overline{c} d \end{array} \right\}$
 $-0-1$

2

$3, 7, 1$

$m_2 - m_7 - m_1 - m_5$
 $m_3 - m_4 - m_2 - m_7$

$- - 11 \left. \begin{array}{l} \overline{b} c d \\ \overline{c} d \end{array} \right\}$
 $- - 11$

Prime Implicant Step 3

Prime Implicant	Individual is matched Pair	minterms									
		0	1	3	7	8	9	11			
$\bar{b}\bar{c}$	0, 1, 8, 9	\otimes	x			\otimes	x				
$\bar{b}d$	1, 3, 9, 11		x	x					x	x	
cd	3, 7, 11, 15	x		x	\otimes		\otimes	x			

$$F = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}\bar{b}c\bar{d} + \bar{a}b\bar{c}\bar{d} + \bar{a}bc\bar{d}$$

$$F = \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}b\bar{c}\bar{d} + \bar{a}bc\bar{d} + \bar{a}bcd$$

$$= \bar{b}\bar{c}\bar{d} + bcd$$

$$F = \bar{b}\bar{c} + cd$$

Minimization using Tabular method

$$F(a,b,c,d) = \sum_m (6,7,8,9) + d(10,11,12,13,14,15)$$

Step 1

Dec	a	b	c	d
6	0	1	1	0
7	0	1	1	1
8	0	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Step 2

Group	minterm	Binary Representation
0	—	—
1	m ₈	1000
2	m ₆ m ₉ m ₁₀ m ₁₂	0110 1001 1010 1100
3	m ₇ m ₁₁ m ₁₃ m ₁₄	0111 1011 1101 1110
4	m ₁₅	1111

Step III

First Reduction

Group	matched Pairs	First Reduction
1	$m_8 - m_9$	1 0 0 3
	$m_8 - m_{10}$	1 0 - 0
	$m_8 - m_{12}$	1 - 0 0
2	$m_6 - m_7$	0 1 1 -
	$m_6 - m_{14}$	- 1 1 0
	$m_9 - m_{11}$	1 0 + 1
	$m_9 - m_{13}$	1 - 0 1
	$m_{10} - m_{11}$	1 0 1 -
	$m_{12} - m_{14}$	1 1 - 0
	$m_{12} - m_{13}$	1 1 0 -
3	$m_7 - m_{15}$	- 1 1 1 1 1 0
	$m_{14} - m_{15}$	1 1 -
	$m_{13} - m_{15}$	1 1 - 1
	$m_{11} - m_{15}$	1 - 1 1

Step IV Group	Matched pair	Π^2 Reduction
------------------	--------------	----------------------

1	$m_3 - m_9 - m_{10} - m_{11}$ ✓ $m_3 - m_5 - m_{12} - m_{13}$ ✓ $m_3 - m_{10} - m_9 - m_{11}$ ✓ $m_3 - m_{10} - m_{12} - m_{14}$ $m_3 - m_{12} - m_9 - m_{13}$ $m_3 - m_{12}$	$10 - - -$ $1 - 0 -$ $10 - - -$ $1 - - - 0$ $1 - 0 -$
---	--	---

2	$m_6 - m_7 - m_{14} - m_{15}$ * $m_6 - m_{14} - m_7 - m_{15}$ $m_9 - m_{11} - m_{13} - m_{15}$ $m_9 - m_{13} - m_{11} - m_{15}$ $m_{10} - m_{11} - m_{14} - m_{15}$ $m_{12} - m_{14} - m_{13} - m_{15}$ ✓ $m_{12} - m_{13} - m_{14} - m_{15}$	$11 -$ $11 -$ $1 - - - 1$ $1 - - - 1$ $1 - 1 -$ $11 - -$ $11 - -$
---	---	---

Steps Prime Implicant	Individual terms	minterms			
		6	7	8	9
	8, 9, 10, 11, 12 13, 14, 15			X	X
	6, 7, 9, 10,				

3

m17/18 Reduce the following Boolean expression

$$(1) AA + BB = A + B$$

$$(2) A\bar{A} + BBB = B$$

$$(3) BC + B\bar{B}C = BC$$

$$(4) WWX + WXX + WWX\bar{X} = \underline{WX}$$

$$(5) A + \bar{A}B + AB = A + B$$

$$(6) A\bar{B} + \bar{A}B + AB + \bar{A}\bar{B}$$

$$A(\bar{B} + B) + \bar{A}(B + \bar{B}) = A + \bar{A} = 1$$

$$(7) A\bar{B}D + A\bar{B}\bar{D} + \bar{B}\bar{D} = A\bar{B}D + \bar{B}\bar{D}$$

$$\bar{B}(AD + \bar{D}) = \bar{B}(A + \bar{D})(D + \bar{D}) = \bar{B}(A + \bar{D})$$

$$(8) W\bar{X}(W + Y) + WY(\bar{W} + \bar{X})$$

$$\Rightarrow W\bar{X} + W\bar{X}Y + \cancel{WY} + WY\bar{X}$$

$$\Rightarrow W\bar{X} + WY\bar{X} \Rightarrow W\bar{X}$$

$$(9) (AB + C)(AB + D)$$

$$\cancel{(A+C)} \cancel{(A+B)} AB + ABD + ABC + CD$$

$$\Rightarrow AB(1 + D) + ABC + CD$$

$$\Rightarrow AB(1 + C) + CD \Rightarrow \underline{AB + CD}$$

$$(10) A + \bar{B}C(A + \bar{B}C)$$

$$A + \bar{B}C(A + B + \bar{C}) = A + \bar{B}CA + 0 + 0$$

$$= \underline{A}$$

$$(11) C(\overline{ABC} + A\overline{BC})$$

$$C[\overline{A} + \overline{B} + \overline{C} + A\overline{B}C]$$

$$C\overline{A} + C\overline{B} + A\overline{B}C$$

$$C[\overline{A} + \overline{B} + A\overline{B}]$$

$$C[\overline{A} + \overline{B}(1+A)] = C[\overline{A} + \overline{B}]$$

$$(12) \overline{ABC + \overline{A}\overline{B} + BC}$$

$$\Rightarrow \overline{ABC} \cdot \overline{ABC + \overline{A}\overline{B} + BC}$$

$$(ABC + \overline{A}\overline{B})\overline{BC} = 0$$

$$(13) A[B + C(\overline{AB} + \overline{AC})]$$

$$\Rightarrow A[B + C(\overline{A}\overline{B} \cdot \overline{A}\overline{C})]$$

$$\Rightarrow A[B + C[(\overline{A} + \overline{B})(\overline{A} + \overline{C})]]$$

$$\Rightarrow A[B + C[\overline{A} + \overline{A}\overline{C} + \overline{A}\overline{B} + \overline{B}\overline{C}]]$$

$$\Rightarrow A[B + C\overline{A} + 0 + \overline{A}\overline{B} + 0]$$

$$\Rightarrow A[B + C\overline{A} + \overline{A}\overline{B}] \Rightarrow AB$$

$$(12) \overline{ABC + \bar{A}\bar{B} + BC} = \bar{A}\bar{B}$$

$$\Rightarrow \overline{ABC \cdot \bar{A}\bar{B} + BC}$$

$$\Rightarrow (\bar{A} + \bar{B} + \bar{C})(A + B) + BC$$

$$\Rightarrow 0 + A\bar{B} + A\bar{C} + \bar{A}B + 0 + B\bar{C} + BC$$

$$\Rightarrow A\bar{B} + A\bar{C} + \bar{A}B + B\bar{C} + BC$$

$$\Rightarrow A\bar{B} + A\bar{C} + \bar{A}B + B$$

$$\Rightarrow A\bar{B} + A\bar{C} + B = \overline{A(B+C)} + B$$

$$\Rightarrow \overline{A\bar{B} \cdot A\bar{C} \cdot B} \Rightarrow \bar{A}B \cdot \bar{A}C \cdot \bar{B} = 0$$

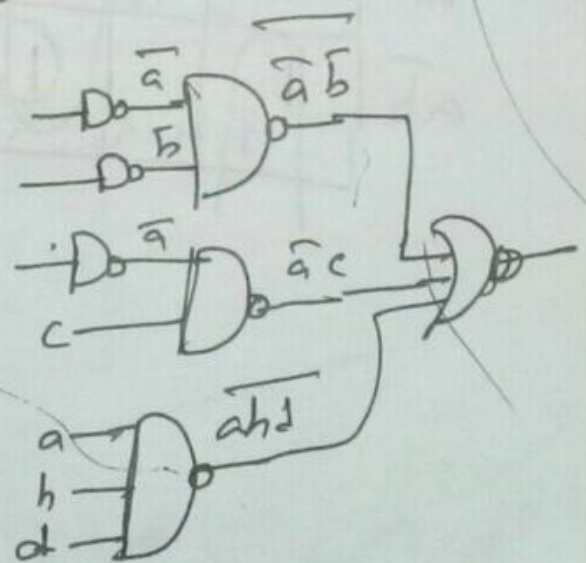
Reduce the expression $F = \sum_m (0, 1, 2, 3, 6, 7, 13, 15)$ by mapping & simplify in NAND logic

	$\bar{c}\bar{d}$	$\bar{c}d$	$c\bar{d}$	cd
$\bar{a}\bar{b}$	1	0	1	3
$\bar{a}b$	4	5	7	6
ab	12	13	15	14
$\bar{a}\bar{b}$	8	9	11	10

$$F = \bar{a}\bar{b} + \bar{a}c + abd$$

$$= \bar{a}\bar{b} + \bar{a}c + abd \text{ (NAND)}$$

$$= \overline{\bar{a}\bar{b} \cdot \bar{a}c \cdot abd}$$



Reduce $F = \sum_m(0, 2, 3, 4, 5, 7)$ Using Mapping

	hc	$\bar{b}c$	$\bar{b}\bar{c}$	bc	hc
a	1		1	1	2
\bar{a}	1				
a	1	1	1		6

$\bar{b}\bar{c} + ac + \bar{a}b$
both is correct

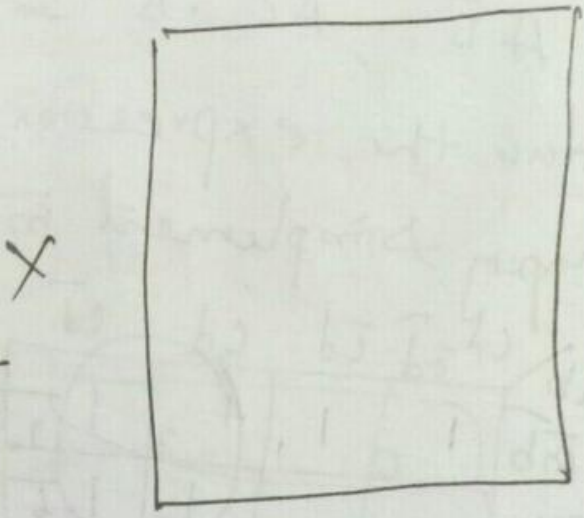
	hc	$\bar{b}c$	$\bar{b}\bar{c}$	bc	hc
\bar{a}	1			1	2
a	1	1		1	6

$a\bar{b} + bc + \bar{a}\bar{c}$

Reduce by mapping & complement is SOP NAND logic

$N = \sum_m(0, 2, 3, 6, 7, 8, 10, 11, 12, 15)$

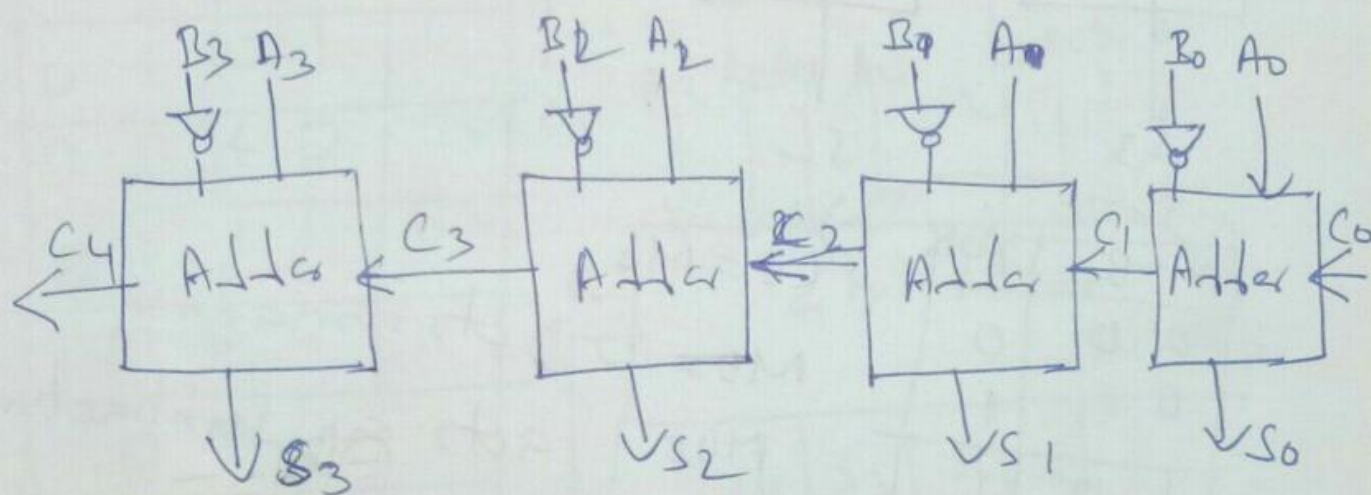
	cd	$\bar{c}d$	$\bar{c}\bar{d}$	cd	$\bar{c}d$
a			1	1	2
\bar{a}	1				
a			1	1	6
\bar{a}	1	1			
a	1			1	14
\bar{a}	1			1	10



$$\begin{array}{r} 1111 - 1111 \\ 0111 \quad \underline{1000} \\ \hline \quad \quad \quad 111 \end{array}$$

Solve the expression by QM & verify with mapping $f = \sum m(0, 2, 3, 6, 7, 8, 9, 10, 13)$
 $= \overline{B} \overline{D} + \overline{A} C + A \overline{C} D$

4 bit binary subtractor



$$\begin{array}{r}
 A = 15 = 1111 \quad 15 \\
 B = 07 = 0111 \quad -7 \\
 \hline
 1000
 \end{array}$$

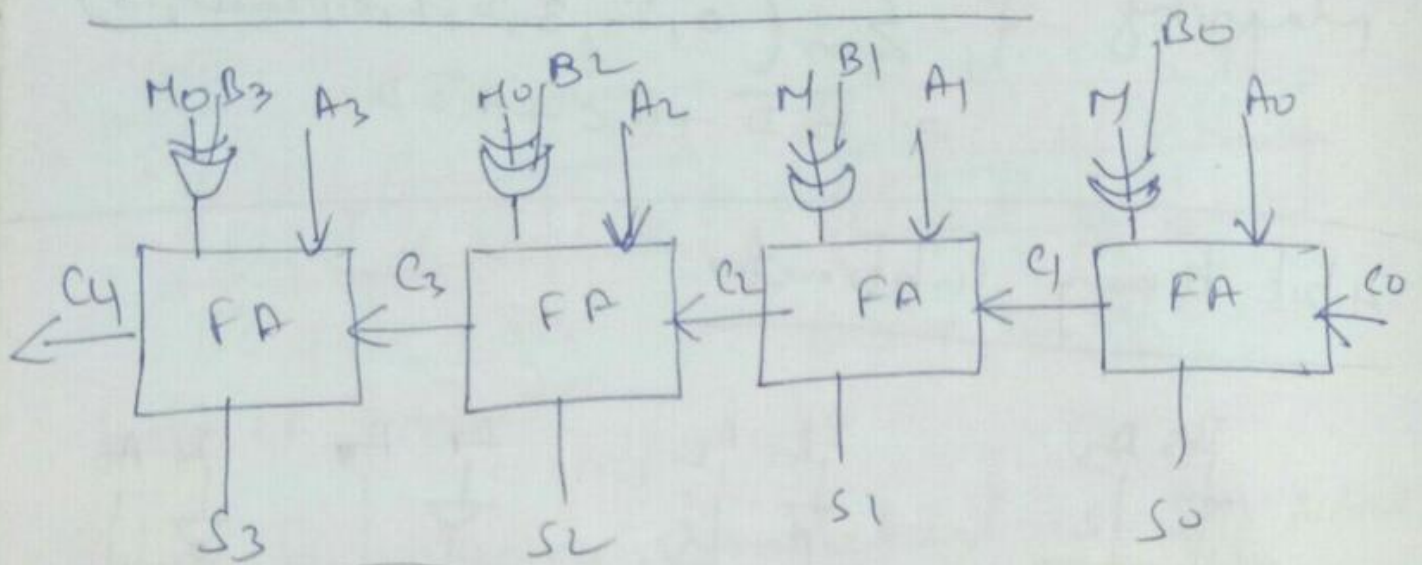
$$\begin{array}{r}
 1111 \quad 15 \\
 1000 \quad 2 \\
 \hline
 10111 \quad 17 \\
 \hline
 1101 \quad 13
 \end{array}$$

$$\begin{array}{r}
 1111 \\
 1101 \\
 \hline
 1100 \\
 \hline
 1
 \end{array}$$

For 4 bit
 Subtractor $C_0 = 1$

Carry Look
 Ahead
 Adder - Carry Generate
 & Propagate

4 bit 1121 Adder & Subtractor

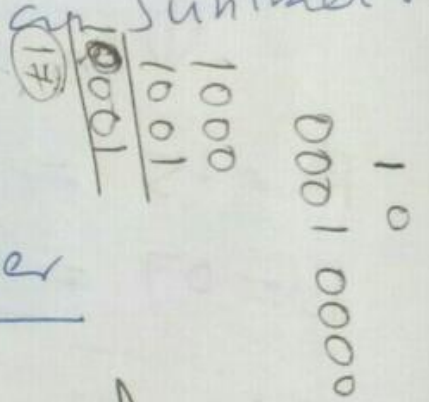


A	B0	XOR
0	0	0
0	1	1
1	0	1
1	1	0

$C_0 = M_0$

$M_0 = 0$ acts as adder

$M_0 = 1$ acts as subtractor



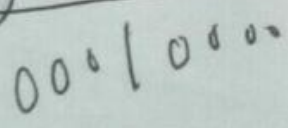
Sexial Adder

BCD Adder

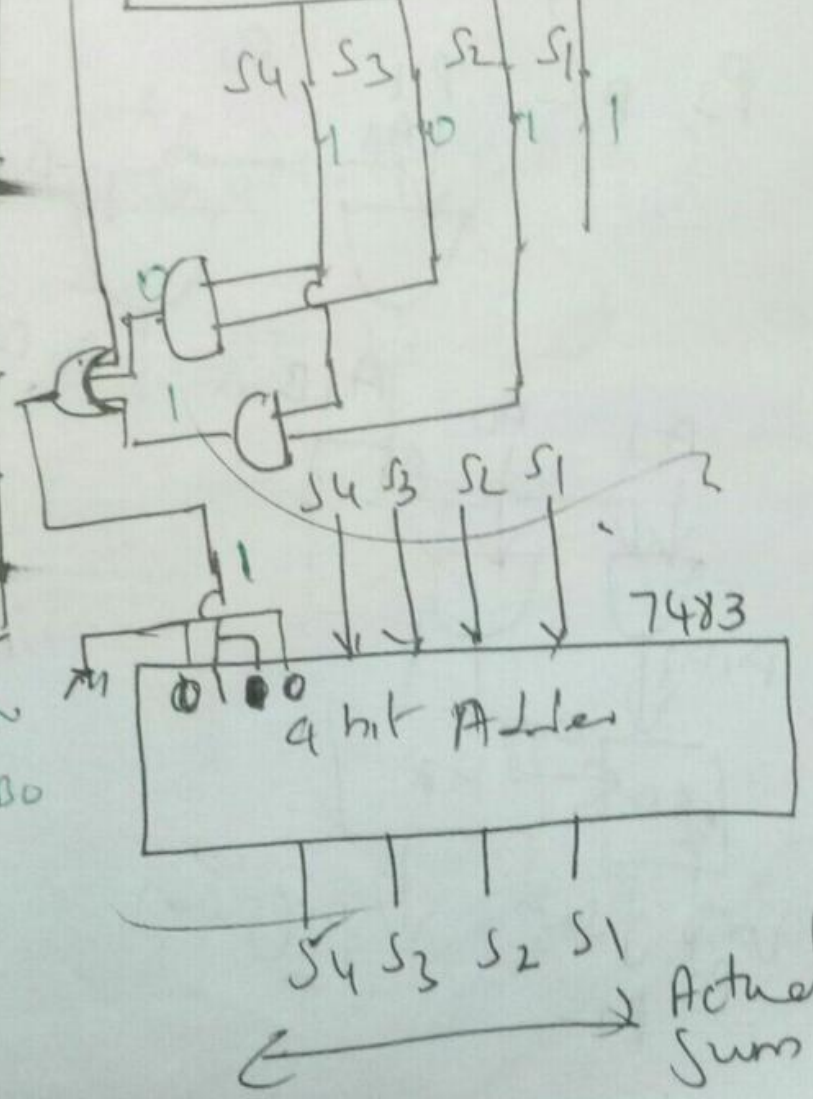
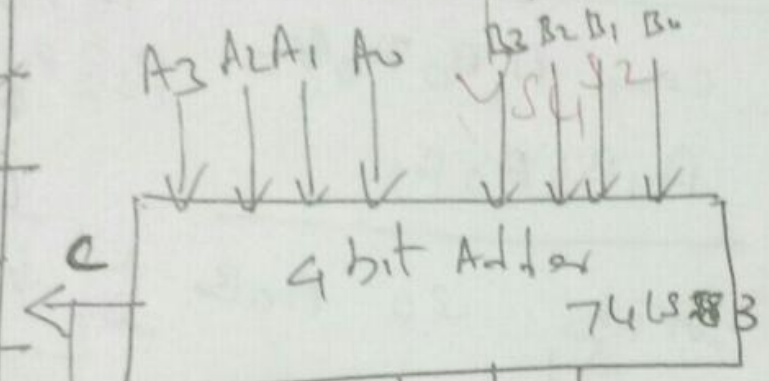
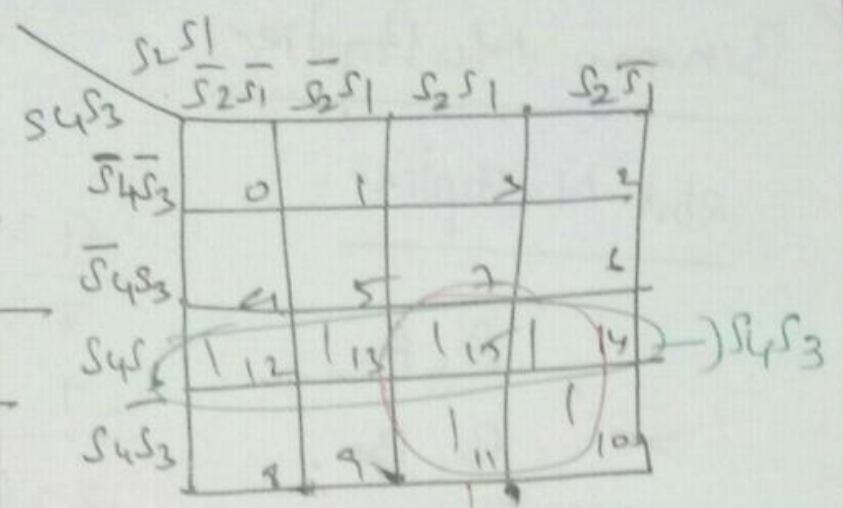
- 0000
- 0001
- 0010
- 0011
- 0100
- 0101
- 0110
- 0111
- 1000
- 1001

- ① 1010
- ② 1011
- ③ 1100
- ④ 1101
- ⑤ 1110
- ⑥ 1111
- ⑩ 40000

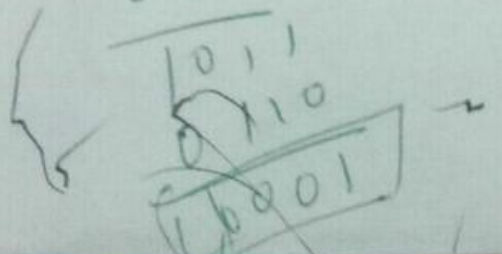
corrections required
Each decimal value is coded by binary



	S_4	S_3	S_2	S_1	Y
0	0	0	0	0	Q
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1



0110 $A_3A_2A_1A_0$
 0101 $B_3B_2B_1B_0$



Binary Multiplier

43
32

2bit Multiplier

A₁ A₀

B₁ B₀

Carry A₁ B₀ A₀ B₀

A₁ B₁ A₀ B₁

A₁ B₁ S₁ S₀ A₀ B₀

43x32

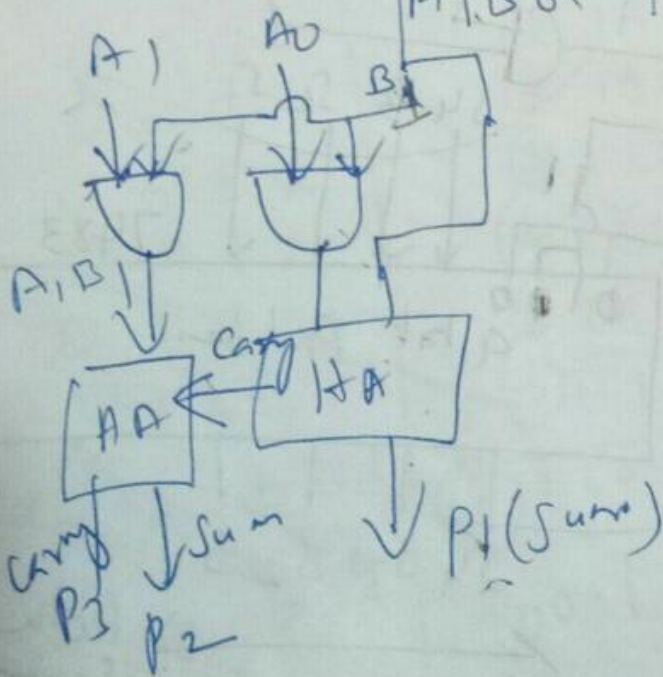
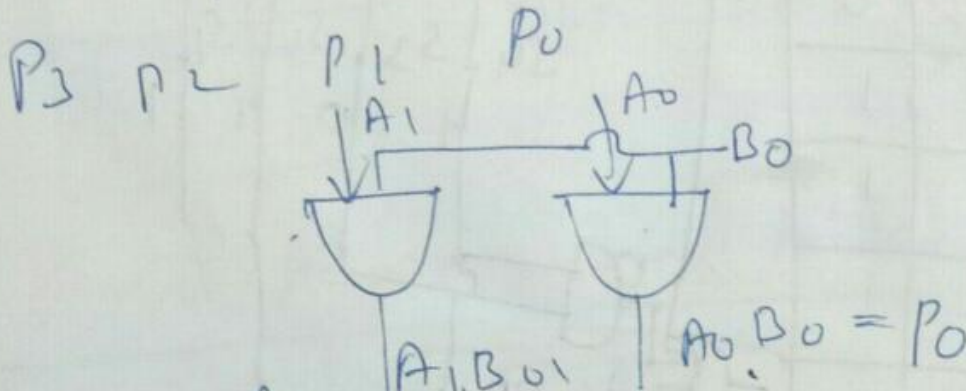
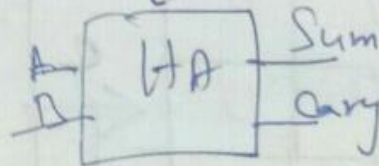
86

129

1376

P₃ P₂ P₁ P₀ A₀ B₁
A₁ B₁

Carry S₁ S₀ A₀ B₀



Magnitude Comparator

Binary division

$$\begin{array}{r} 10011.1 \\ \hline 100 \overline{) 1001110} \\ \underline{100} \\ 000111 \\ \underline{100} \\ 0110 \\ \underline{100} \\ 0100 \\ \underline{-100} \\ 0 \end{array}$$

Introduction to Mux (Multi plexer)

It is a combinational ckt that selects binary information from one of many input lines and directs it to o/p line.

It is simply called data selector.

Advantages :- 1) Reduce no of wires

2) Reduces circuit complexity & cost

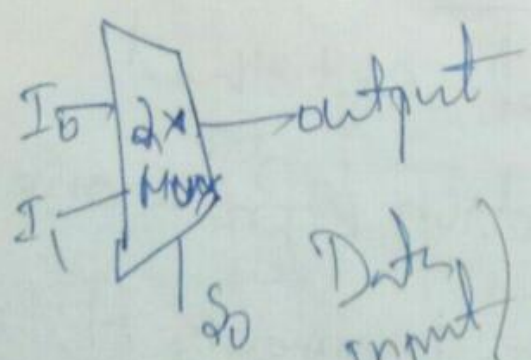
3) Implementation of Various circuit using MUX.

$n = \text{no of inputs} = 2^m \rightarrow \text{no of selection lines.}$

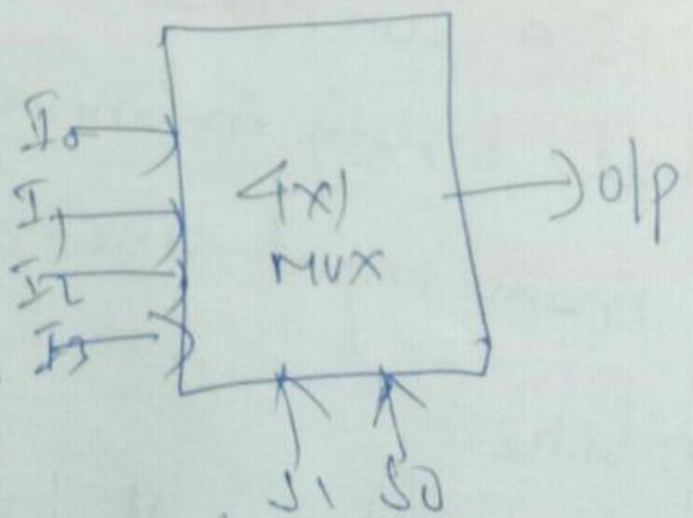
$$m = \log_2 n$$

if $n=4$; $m = \log_2 4 = 2$.
2x1 Multiplexer

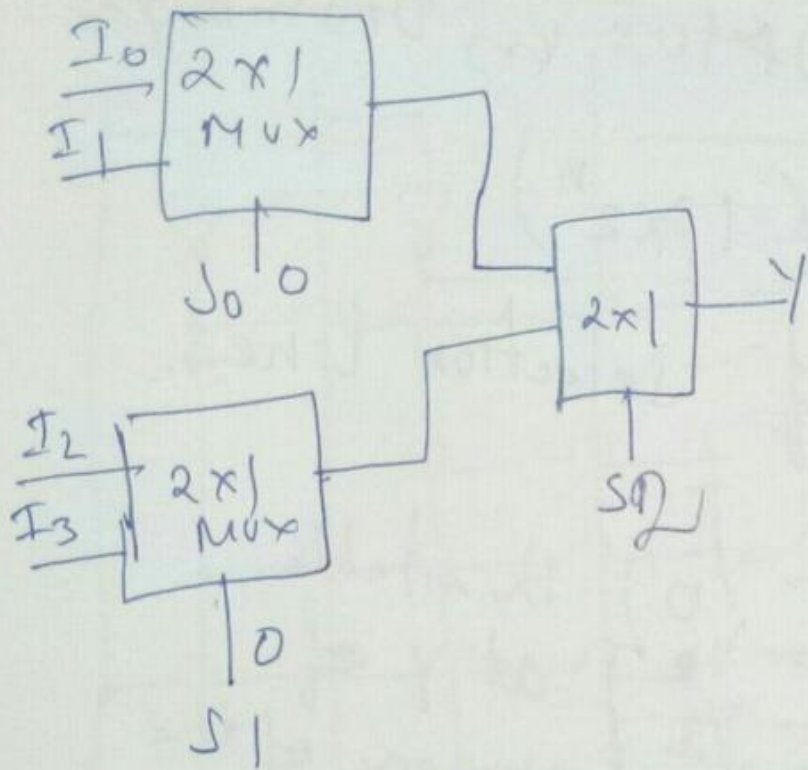
4x1 MUX



Data input lines



Selection lines



$S_1 S_0$	Y
0 0	I_0
0 1	I_1
1 0	I_2
1 1	I_3

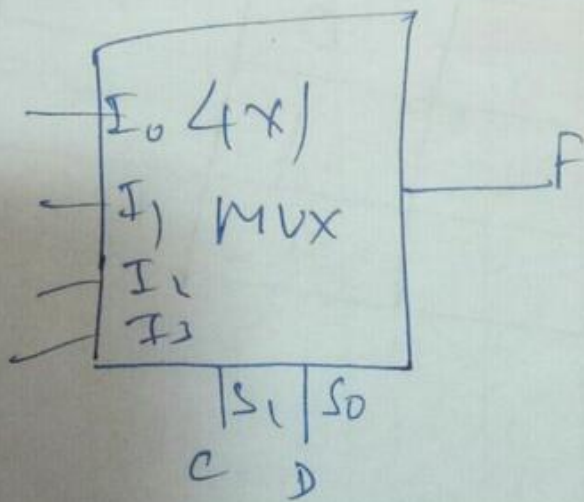
Implementation using Multiplexers

$$F(a,b,c,d) = \sum (1, 3, 4, 11, 12, 13, 14, 15)$$

Using 4x1 MUX with a, b as input Variable and c, d as selection Variables.

	I_0	I_1	I_2	I_3
$\bar{a} \bar{b}$	0	①	2	③
$\bar{a} b$	④	5	6	7
$a \bar{b}$	8	9	10	⑪
$a b$	⑫	⑬	⑭	⑮

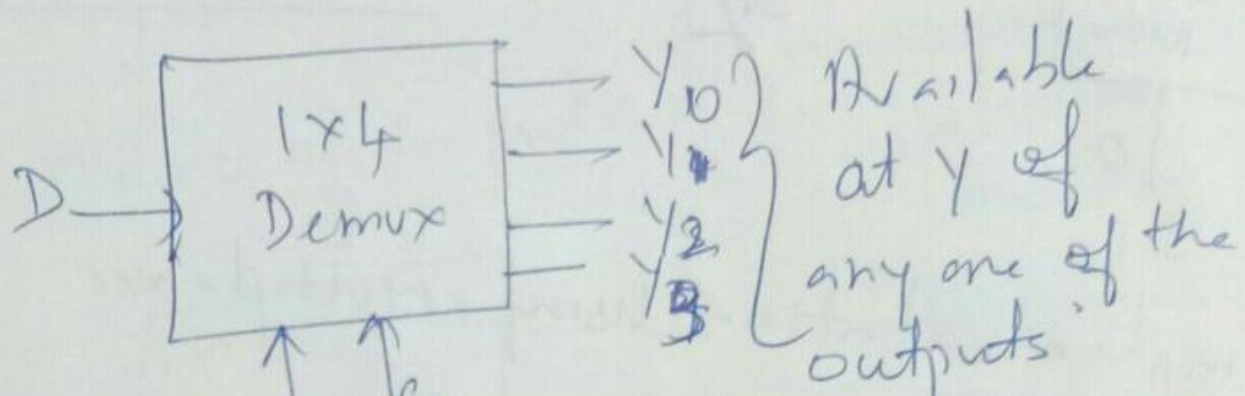
$b \quad \bar{b} \quad a \quad \bar{a}$



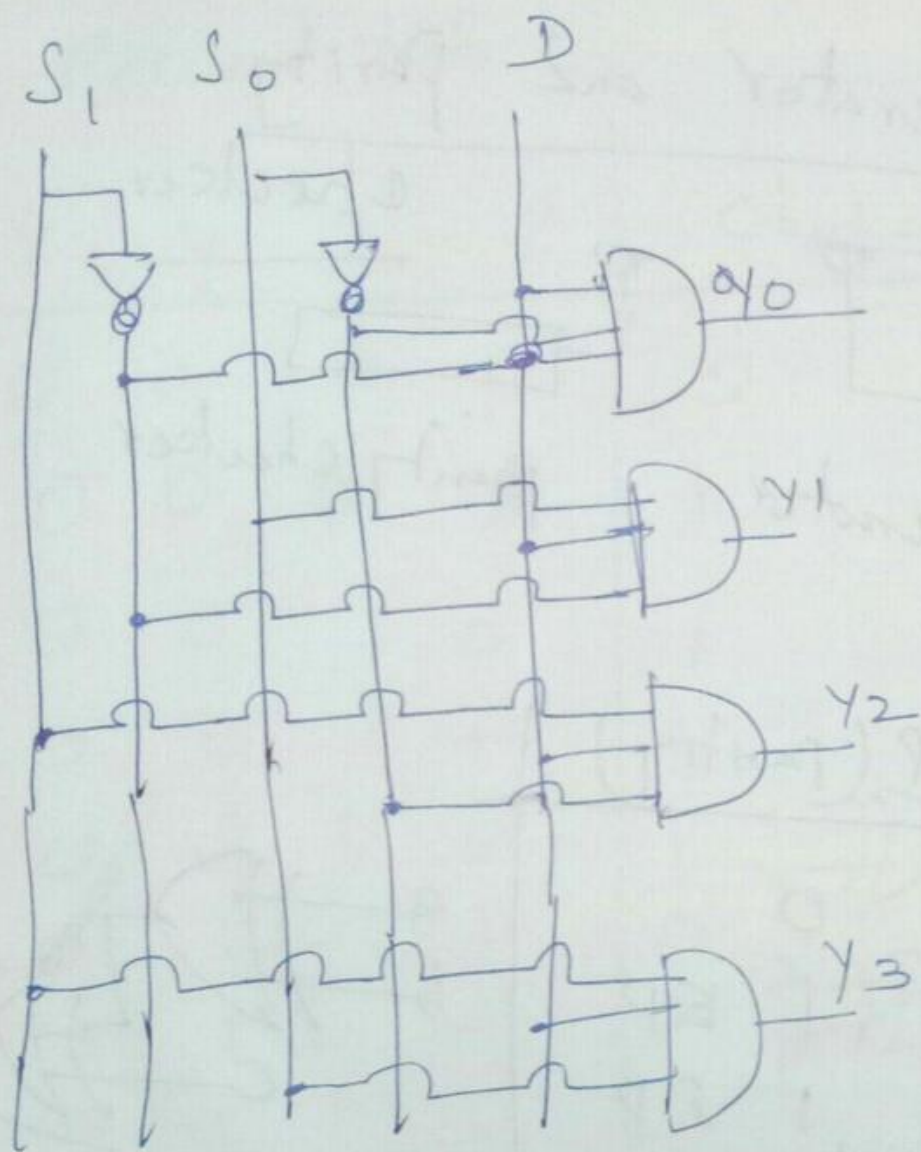
Demultiplexer : Demux or Distributer

One to Many (1×2^n)

$n \rightarrow$ no of selection lines.



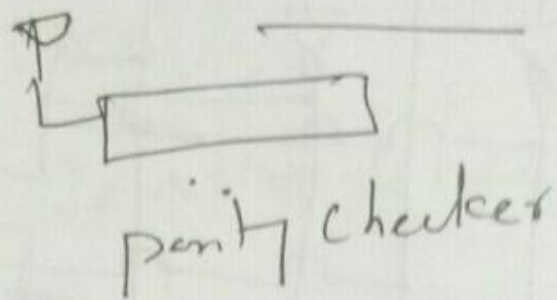
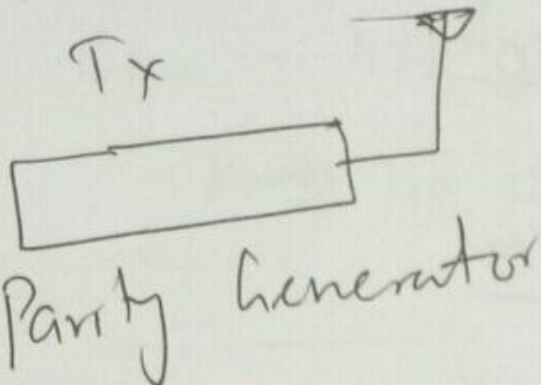
Input D	Selection lines		outputs			
	S ₁	S ₀	Y ₀	Y ₁	Y ₂	Y ₃
D	0	0	D	0	0	0
D	0	1	0	D	0	0
D	1	0	0	0	D	0
D	1	1	0	0	0	D



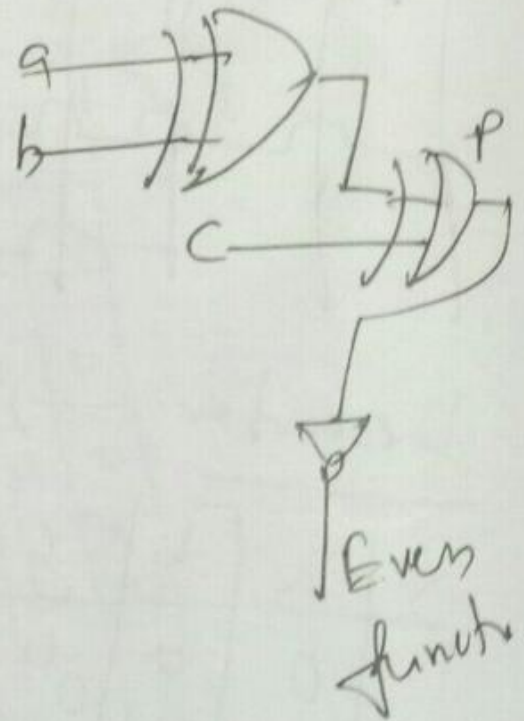
Decoder \rightarrow 3 to 8 decoder. (Full Adder - 8Fs)

D	a	b	c	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	0	1	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0	0
1	0	1	1	0	0	0	1	0	0	0
1	1	0	0	0	0	0	0	1	0	0
1	1	0	1	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1
1	1	1	1	0	0	0	0	0	0	1

Even Parity generator and Parity checker

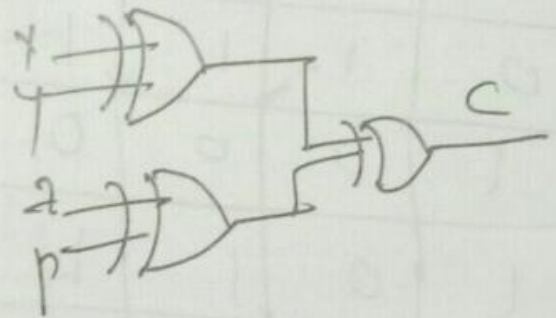


message			P (parity)
a	b	c	
0	0	0	0
0	0	1	1 EP
0	1	0	1 FP
0	1	1	0
1	0	0	0
1	1	0	0
1	1	1	1



Received bits

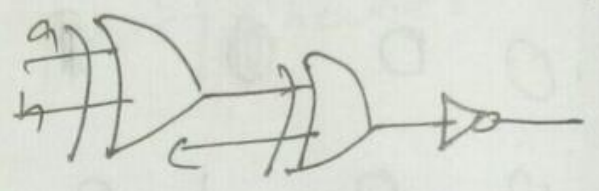
X	Y	Z	P	Checkes
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	1	0	1
1	1	1	1	0



When $C = 0$
 no error
 When $C = 1$
 error.

Odd Parity Generator

message bit			Parity bit (P)
a	b	c	P
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



odd parity checker

a	h	c	p	checker
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Binary to Gray

~~1000~~ → Binary

1100 → Gray

Gray to Binary

1000 → Gray

1111 → Binary

1010
1111

10

1111

1000

1111
1010

code converter

Binary to Graycode (4 bit)

(4) -

Decimal No	Binary				Gray			
	a	b	c	d	G ₁	G ₂	G ₃	G ₄
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	0	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	0	0	0
9	1	0	0	1	1	0	0	1
10	1	0	1	0	1	1	1	1
11	1	0	1	1	1	1	0	1
12	1	1	0	0	1	0	1	0
13	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	0

=

= 0 d

+d)

= +C

b) +

b + C)

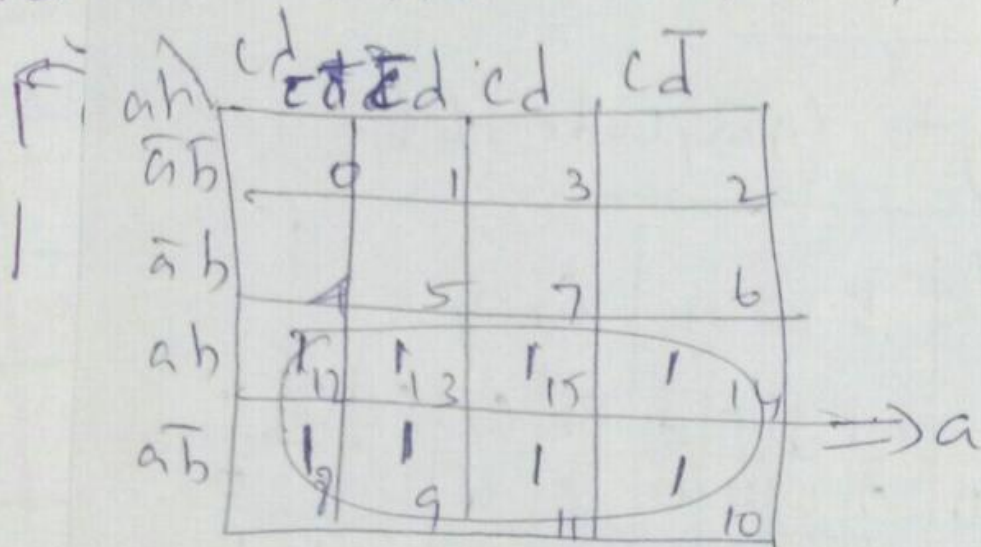
(b + c)

b + d

00 01 11 (4) +)

Binari

$$G_1 = \sum m(8, 9, 10, 11, 12, 13, 14, 15)$$



$$G_2 = \sum m(4, 5, 6, 7, 8, 9, 10, 11)$$

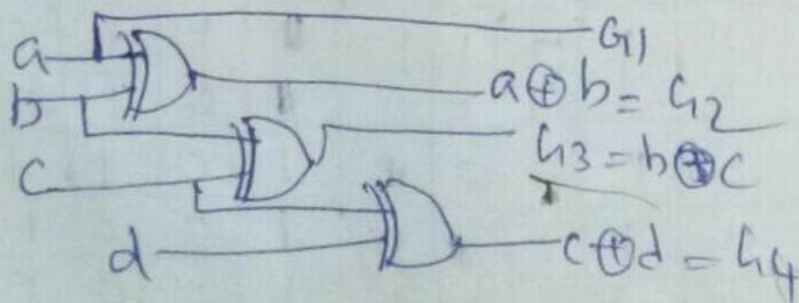
$$= \bar{a}b + a\bar{b} = A \oplus B$$

$$G_3 = \sum m(2, 3, 4, 5, 12, 13, 10, 11)$$

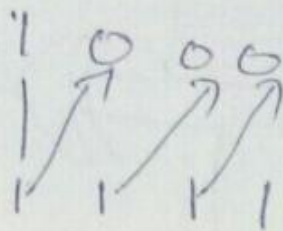
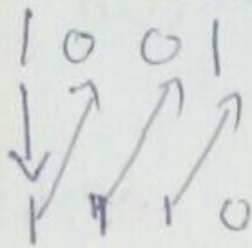
$$= c\bar{b} + \bar{c}b = b \oplus c$$

$$G_4 = \sum m(1, 2, 5, 6, 9, 10, 13, 14)$$

$$G_4 = \bar{c}d + c\bar{d} = c \oplus d$$



Gray to binary



K. May $\Rightarrow A_m = A, A \oplus B,$

$$B = \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$$

$$= A(\bar{B}\bar{C} + B\bar{C}) + \bar{A}(B\bar{C} + \bar{B}C)$$

$$= A(\overline{B\bar{C} + \bar{B}C}) + \bar{A}(B\bar{C} + \bar{B}C)$$

$$= A(\overline{B \oplus C}) + \bar{A}(B \oplus C)$$

$$= A \oplus B \oplus C$$

$$B = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D +$$

$$A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

$$= A \oplus B \oplus C \oplus D$$

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

Diagram showing a 4x4 grid of cells. The top-left cell is labeled '0'. The top-right cell is labeled '3'. The middle-left cell is labeled '4'. The middle-right cell is labeled '7'. The bottom-left cell is labeled '8'. The bottom-right cell is labeled '11'. Each cell contains a '1' in a box. Arrows labeled 'ah' and 'cd' point to the top-left and top-right cells respectively.

, (e) +

c

\bar{c}, \bar{d}

(+d)

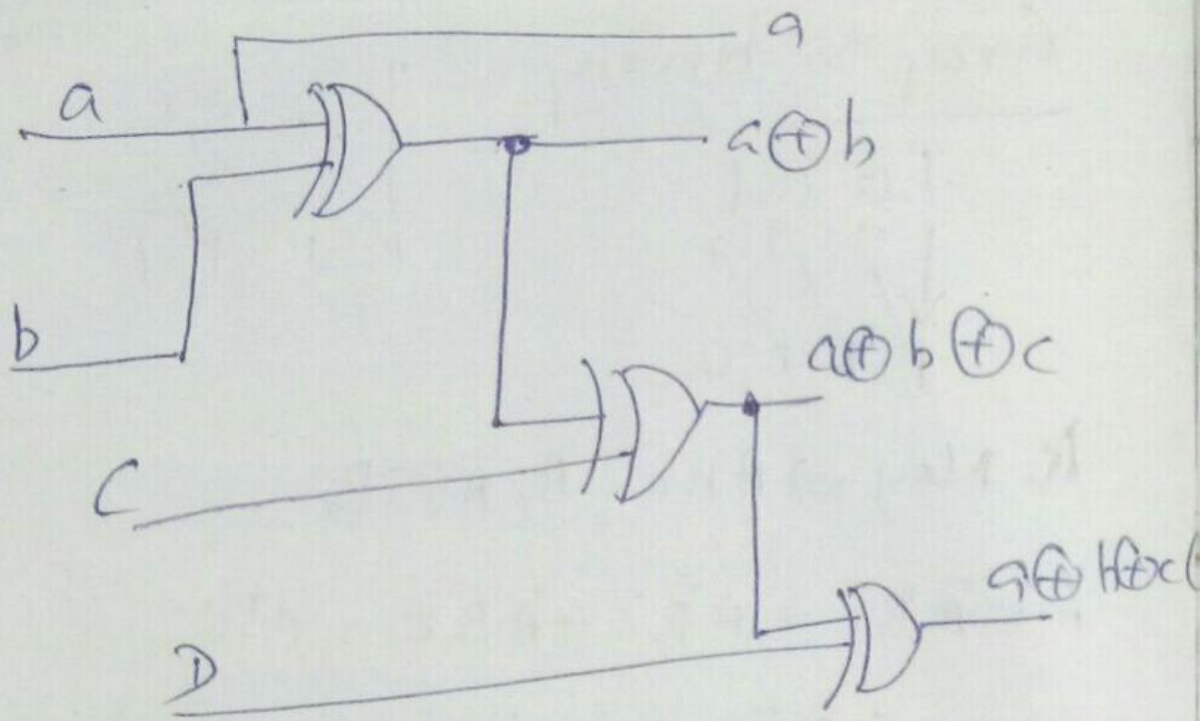
c + c

(+b) +

(b + c)

(b + c)

(b + c)



B, C, D (421) → Excess 3 code

$$D = \bar{z}$$

$$B = \bar{x}\bar{y} + \bar{x}\bar{z} + xy\bar{z}$$

$$C = y \oplus \bar{z}$$

$$A = wx + wyz$$

$$(c+d)(\bar{c}+d) \oplus (x+b)(\bar{x}+b)$$

$$d \oplus (b+c)(\bar{c}+d)(\bar{c}+d)$$

$$z = \bar{D}$$

$$y = cd + \bar{c}\bar{d}$$

A, B, C, D 2/4

w, x, y, z 0/4

$$w = a + bc + bd$$

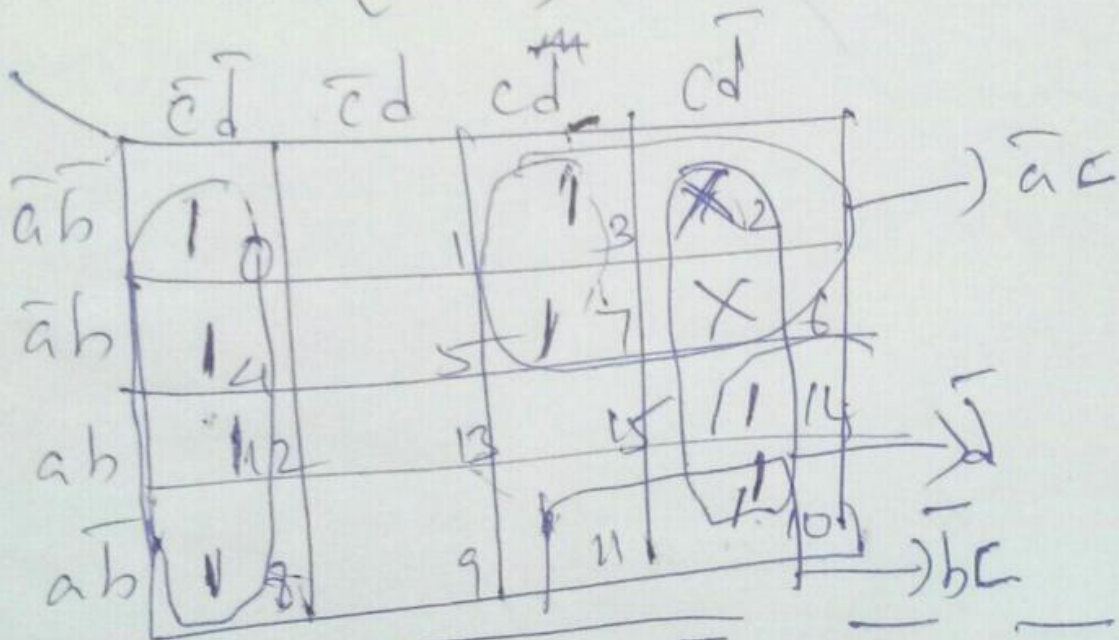
$$x = \bar{b}c + \bar{b}d + b\bar{c}d$$

upto 9, 1

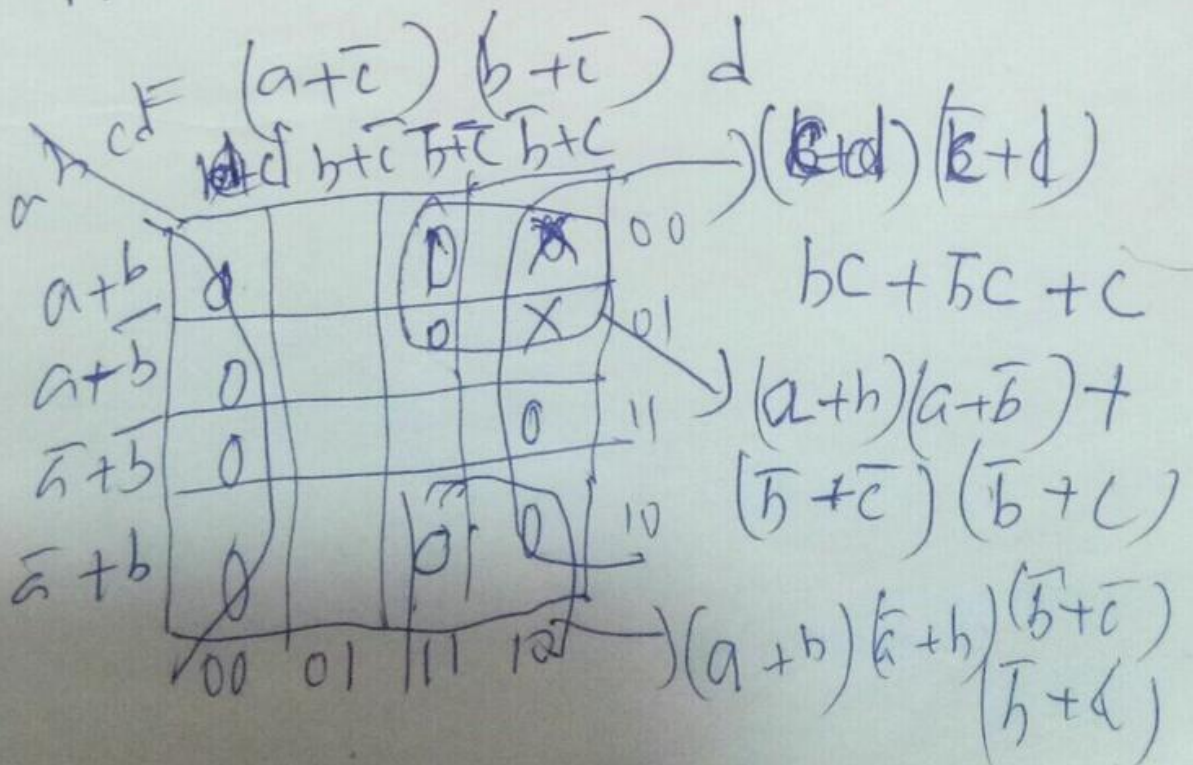
10, 11, 12, 13, 14, 15 xxxxx

Realize / implement the logic function
 Using NAND gates specified by the
 function

$$f = \prod_M (0, 3, 4, 7, 8, 10, 12, 14) + \prod d (2, 6)$$



$$f = \overline{a}c + \overline{b}c + d = \overline{a}c \cdot \overline{b}c \cdot \overline{d}$$



Unit II

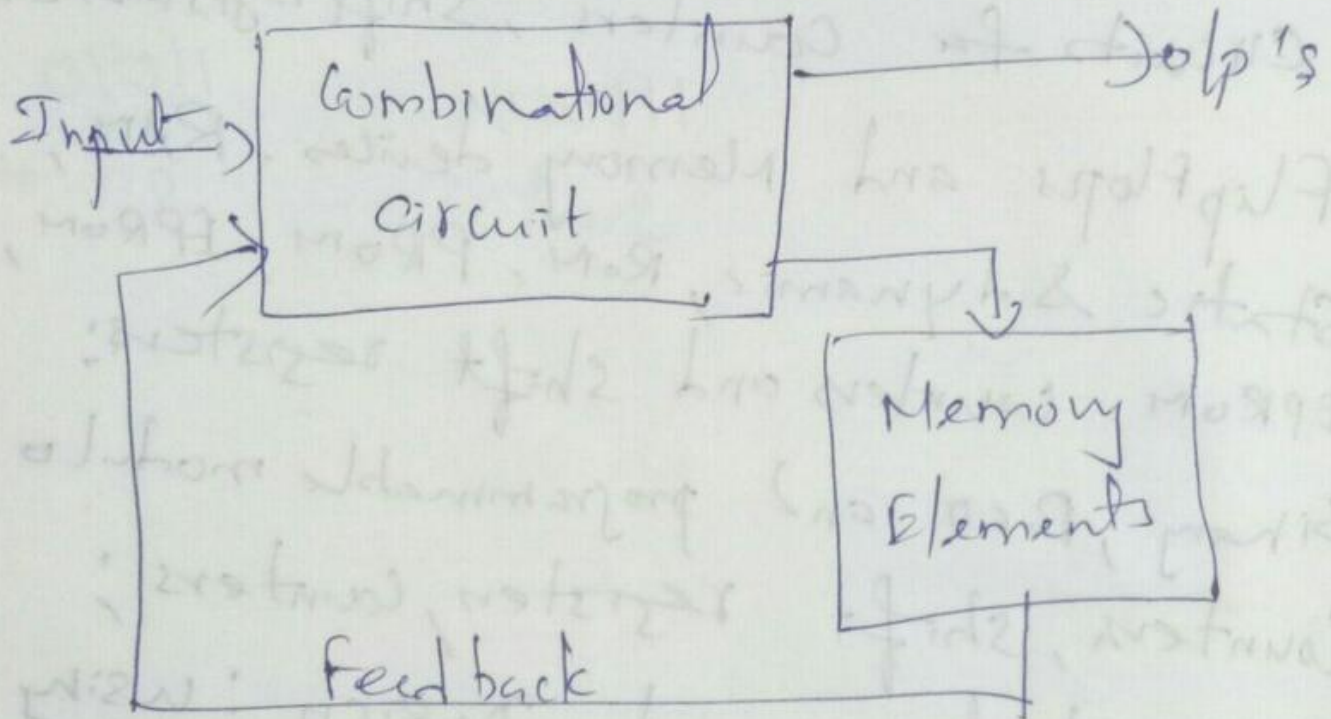
CO2 → Apply the concept of sequential circuits for counters, shift registers etc

Flipflops and Memory devices: RAM, Static & dynamic, ROM, PROM, EPROM, EEPROM, counters and shift registers:

Binary, BCD and programmable modulo counters, shift registers, counters;

Sequential circuit Design: using Mealy and Moore model.

Block Diagram of sequential circuits



In sequential, o/p's depends present input as well as previous output (or previous state of the circuit).

O/p depends input as well as clock pulse.

Sequential { Synchronous clk
Asynchronous clk

A synchronous sequential circuits is a system whose behaviour can be defined from the knowledge of its signal at discrete instant of time.

The behaviour of asynchronous circuit depends upon the input signals at any instant of time and the order in which input change.

The storage elements (memory) used in clocked sequential circuits are called flip flops.

Flip Flop: It is a binary storage device capable of storing one bit of information. Its stable state the output of a flip flop is either 0 or 1.

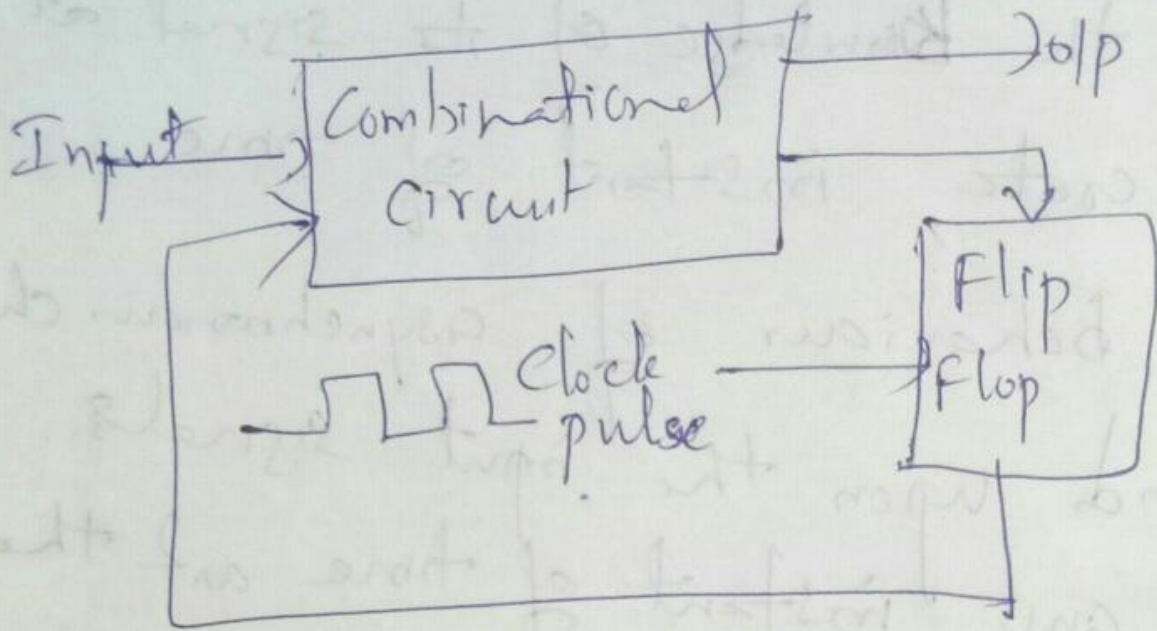
A Synchronous sequential circuits is a system whose behaviour can be defined from the knowledge of its signal at discrete instant of time.

The behaviour of asynchronous circuit depends upon the input signals at any instant of time and the order in which input change.

The storage elements (memory) used in clocked sequential ckt are called flip flops.

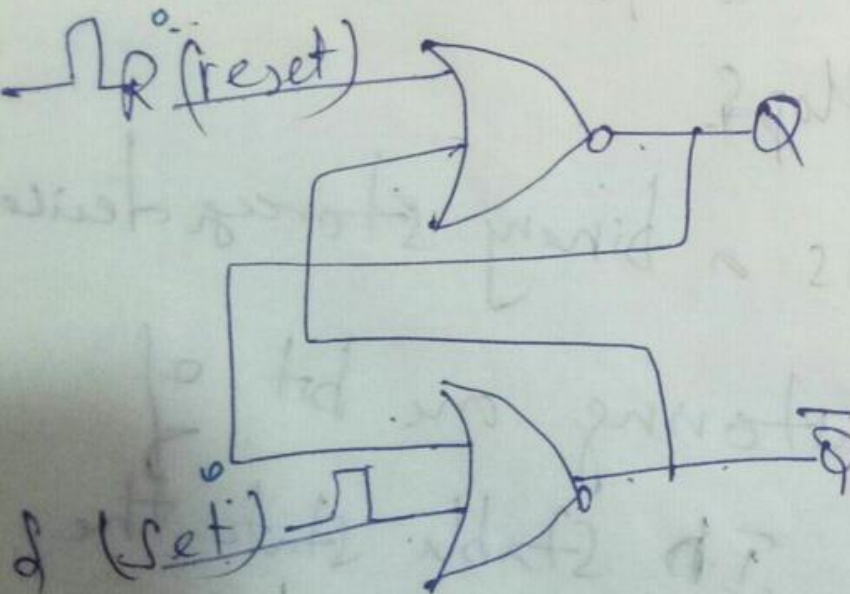
Flip Flop: It is a binary storage device capable of storing one bit of information. Its stable state the the op of a flip flop is either 0 or 1.

Synchronous clocked Sequential circuit



Storage Elements: (Latches)

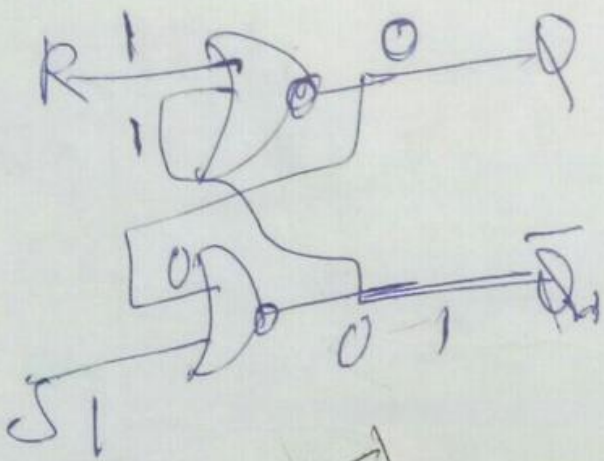
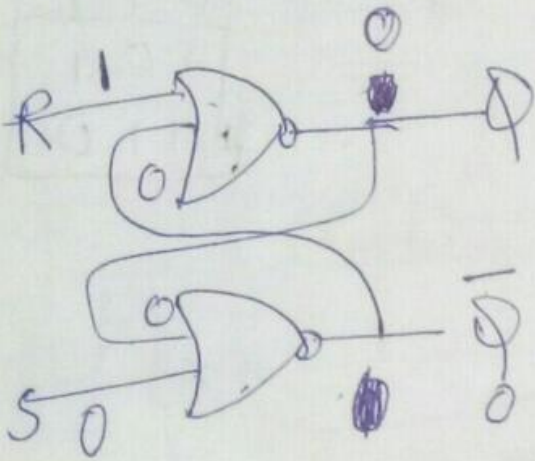
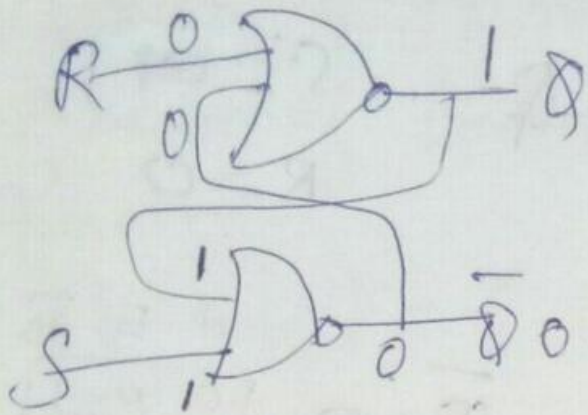
SR Latch with NOR gate



Function Table

S	R	Q	Q̄
0	0	0	0
0	1	0	1
1	0	1	0
1	1	0	0

Per
bidden



A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

Function Table

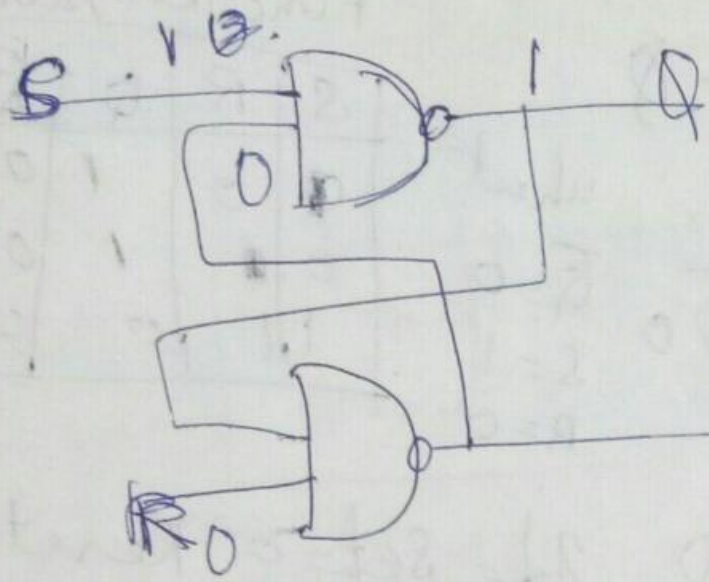
S	R	Q	\bar{Q}
0	0	1	0
0	1	1	0
1	1	0	0

when
 $\bar{Q} = 0$
 $S = 1$
 $R = 0$

If Set = 0 & Reset = 1
 $\bar{Q} = 0$ (previous state)
 Current state $Q_1 = 1$

If Set = 1 & Reset = 1
 prev. state $\bar{Q}_1 = 1$

Current state $Q_1 = Q_0 = 0$
 It is forbidden state,
 undefined state,
 metastable state.



$S = 1$
 $R = 0$

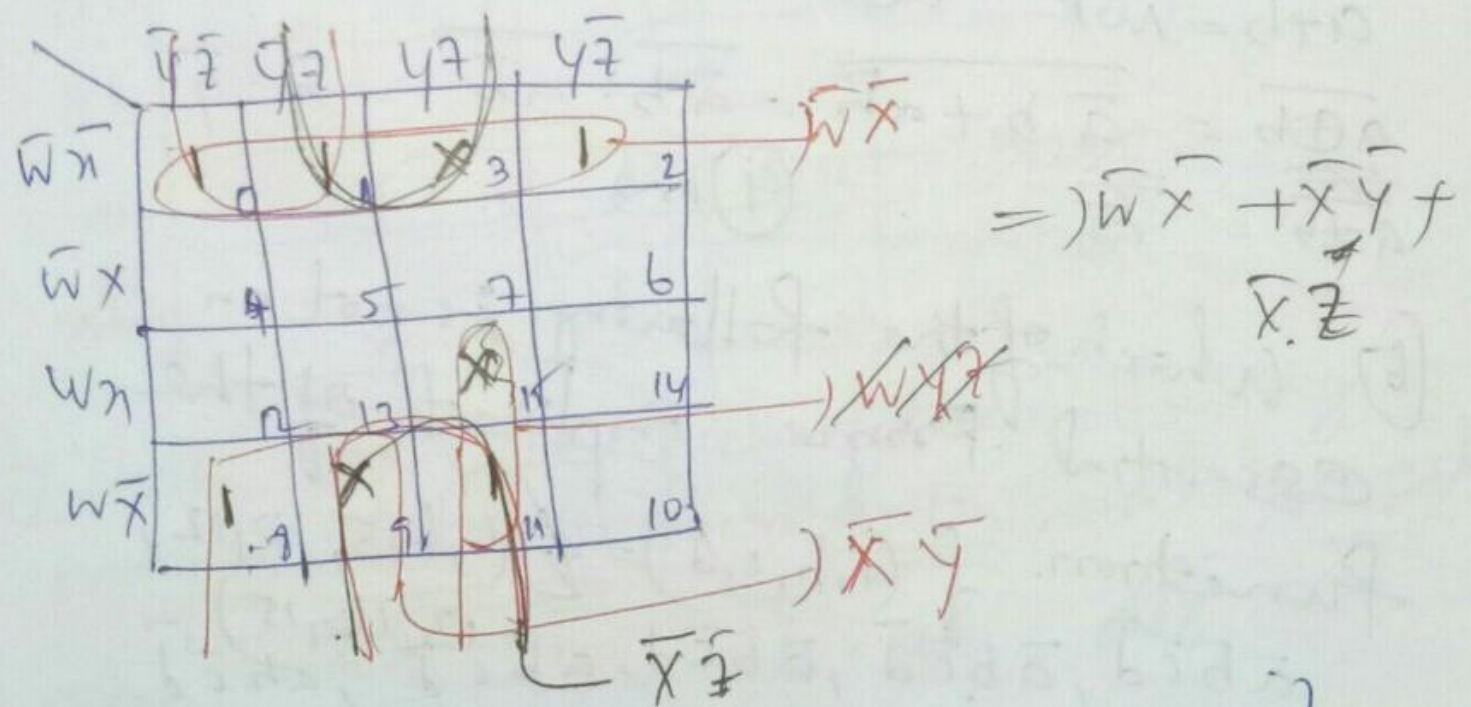
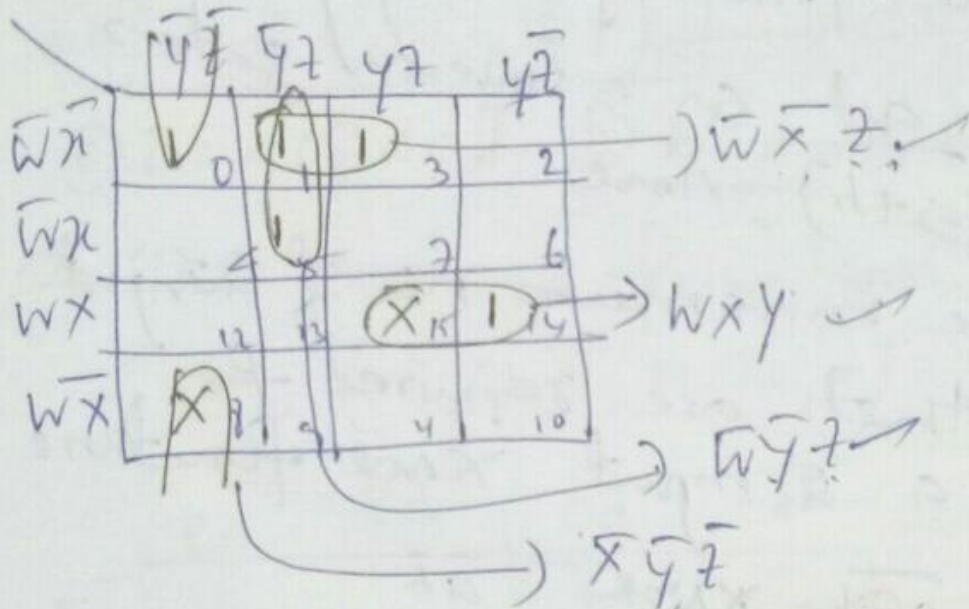
A	B	\overline{AB}
0	0	1
0	1	1
1	0	1
1	1	0

$\overline{Q} = 0$

Current state $Q_1 = 0$
 Next state $Q_2 = 1$
 Next state $Q_3 = 1$
 Next state $Q_4 = 0$
 Next state $Q_5 = 0$
 Next state $Q_6 = 0$
 Next state $Q_7 = 0$
 Next state $Q_8 = 0$
 Next state $Q_9 = 0$
 Next state $Q_{10} = 0$
 Next state $Q_{11} = 0$
 Next state $Q_{12} = 0$
 Next state $Q_{13} = 0$
 Next state $Q_{14} = 0$
 Next state $Q_{15} = 0$

① Which of the following represents the sum of products of expression for the function.

$$f(w, x, y, z) = \sum (0, 1, 3, 5, 14) + \sum_d (8, 15)$$



$$f(w, x, y, z) = \sum (0, 1, 2, 8, 11) + \sum_d (3, 9, 15)$$

③ Which of the following represents the complement of the function $x+yz$

$$\overline{x+yz} = \bar{x} \cdot (\bar{y} + \bar{z})$$

④ Which of the following 2 input logic functions can act as a universal gate?

$a \oplus b, ab, (a+b), \text{none}$

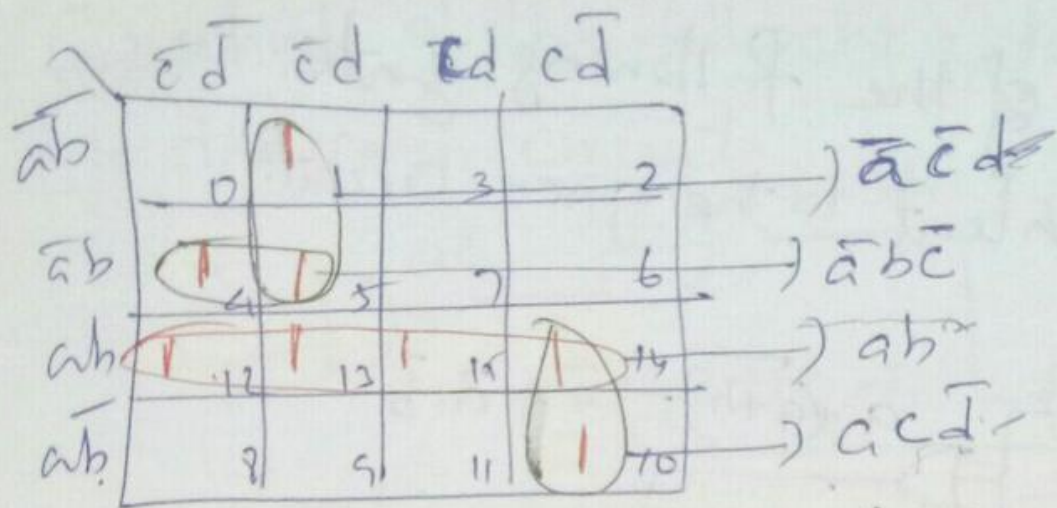
⑤ What is the minimum no of 2 input NOR gates that are required for implementing a 2 input XOR function

$$a+b = \text{NOR} \quad a \oplus b = \text{XNOR} \quad \bar{a}\bar{b}$$

$$\overline{a \oplus b} = \overline{\bar{a}b + a\bar{b}} = \overline{\bar{a}b} \cdot \overline{a\bar{b}} = \bar{\bar{a}b} \cdot \bar{a\bar{b}} = \bar{\bar{a}} \cdot \bar{\bar{b}} = a \cdot b$$

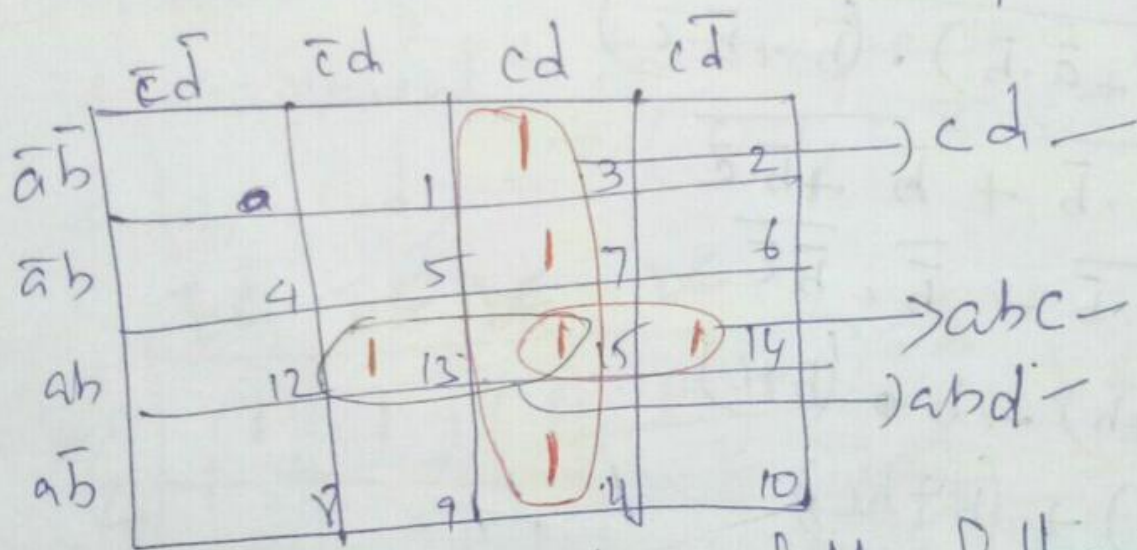
$$\overline{a+b} = \bar{a}\bar{b} \quad (4) \text{ ans}$$

⑥ Which of the following is not an essential prime implicant of the function. $f(a,b,c,d) = \sum (1, 4, 5, 10, 12, \bar{a}\bar{b}\bar{c}\bar{d}, \bar{a}b\bar{c}\bar{d}, \bar{a}b\bar{c}d, a\bar{b}\bar{c}d, a\bar{b}c\bar{d}, a\bar{b}cd)$



- (a) $\bar{a}\bar{c}d$ (b) acd (c) $b\bar{c}d$ (d) ab

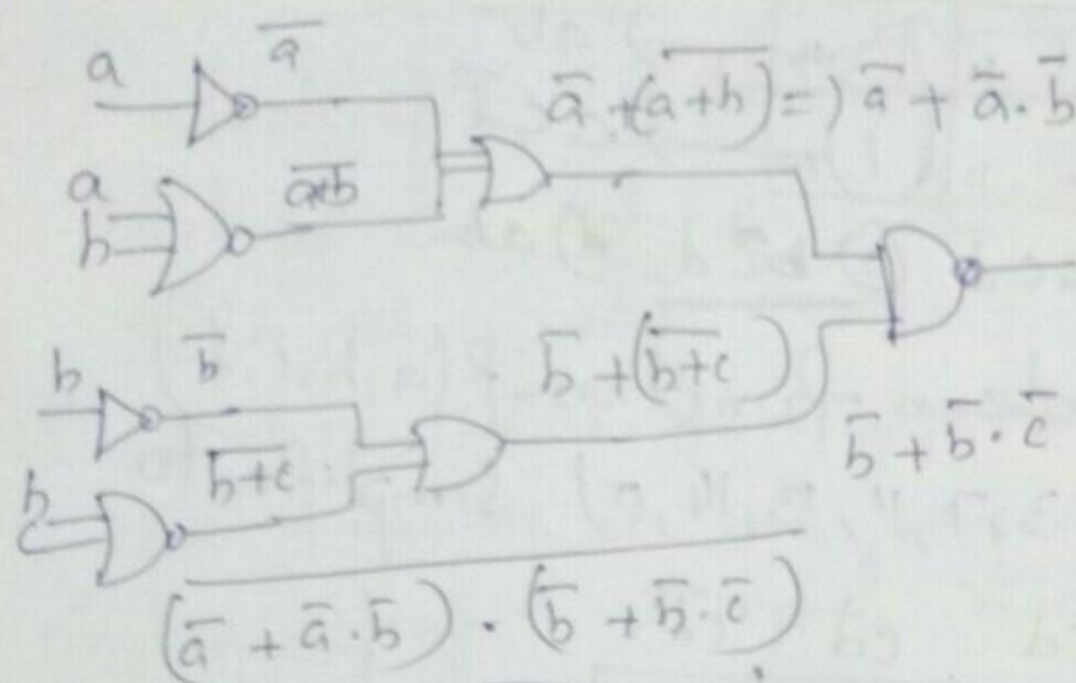
This Boolean function $f(a, b, c, d) = \sum m(3, 7, 11, 13, 14, 15)$ simplifies to



With XOR gate which of the following may be taken to form a universal logic gate

$$\begin{aligned}
 a \oplus b &= \overline{\overline{a}b} + \overline{a\bar{b}} = \overline{\overline{a}b} \cdot \overline{a\bar{b}} = \overline{\overline{a}b \cdot a\bar{b}} \\
 &= \overline{\overline{a}b} \oplus \overline{a\bar{b}} = \overline{\overline{a}b \cdot a\bar{b}} = a \oplus b
 \end{aligned}$$

9) Which of the following gate is equivalent to the given circuit.



$$\overline{a} + \overline{a} \cdot \overline{b} + \overline{b} + \overline{b} \cdot c$$

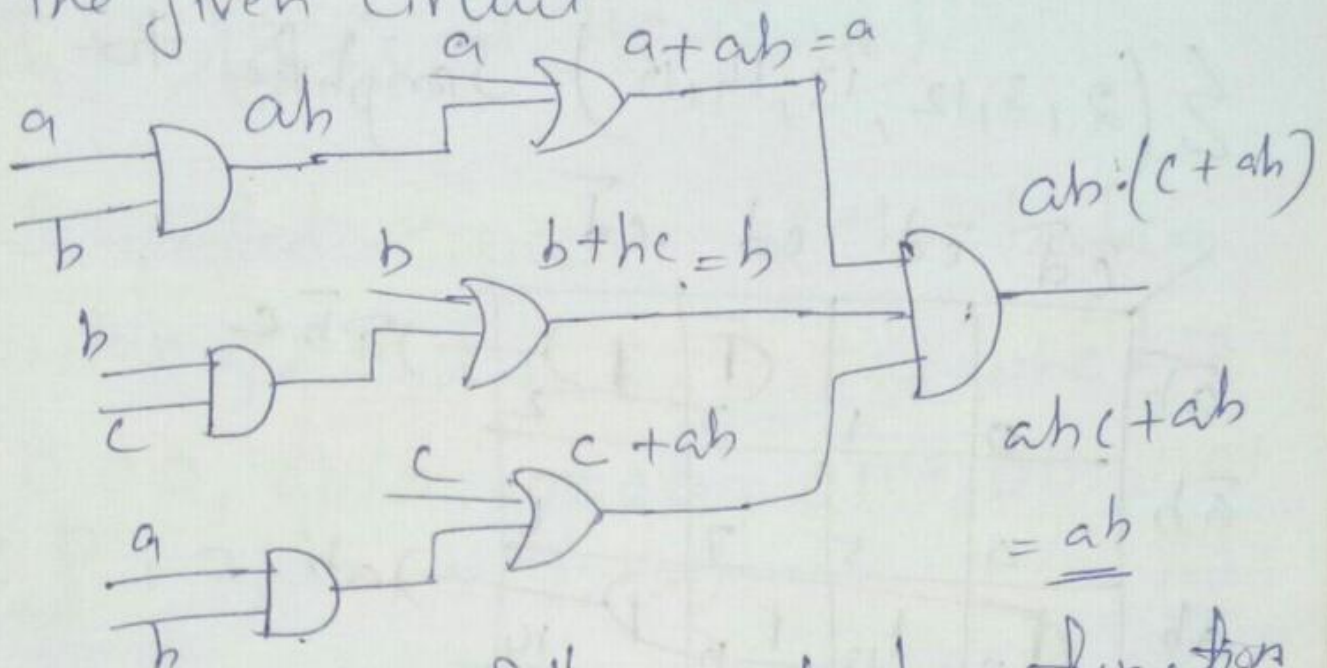
$$\overline{a} \cdot \overline{a} \cdot \overline{b} + \overline{b} \cdot \overline{b} \cdot c$$

$$a \cdot (a + b) + b \cdot (b + c)$$

$$(a + ab) + (b + bc)$$

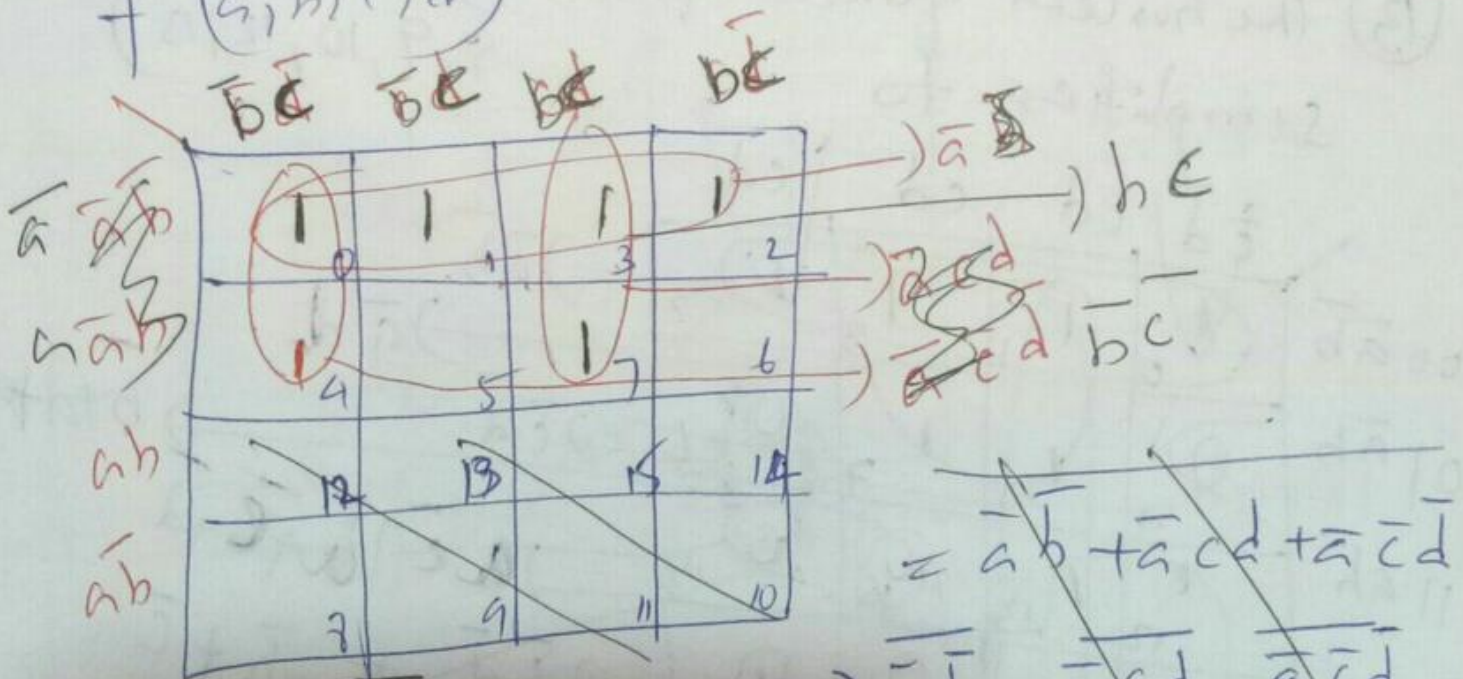
$$\underline{a + b} \rightarrow \text{OR}$$

10) which of the following gate is equivalent to the given circuit.



11) which of the following boolean functions is equivalent to the expression

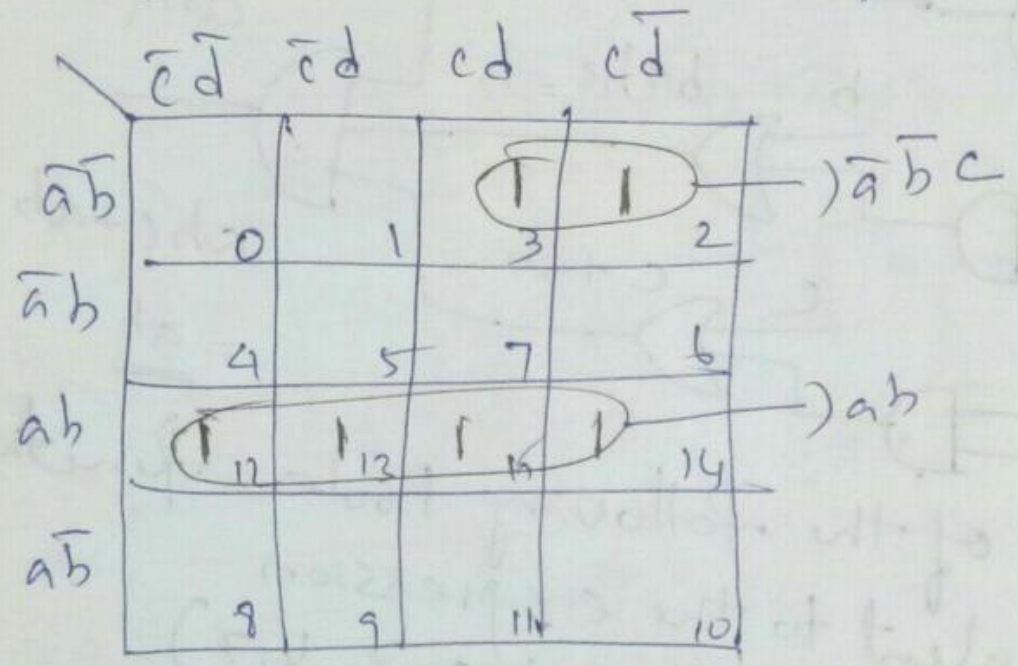
$$f(a, b, c, d) = \Pi(0, 1, 2, 3, 4, 7)$$



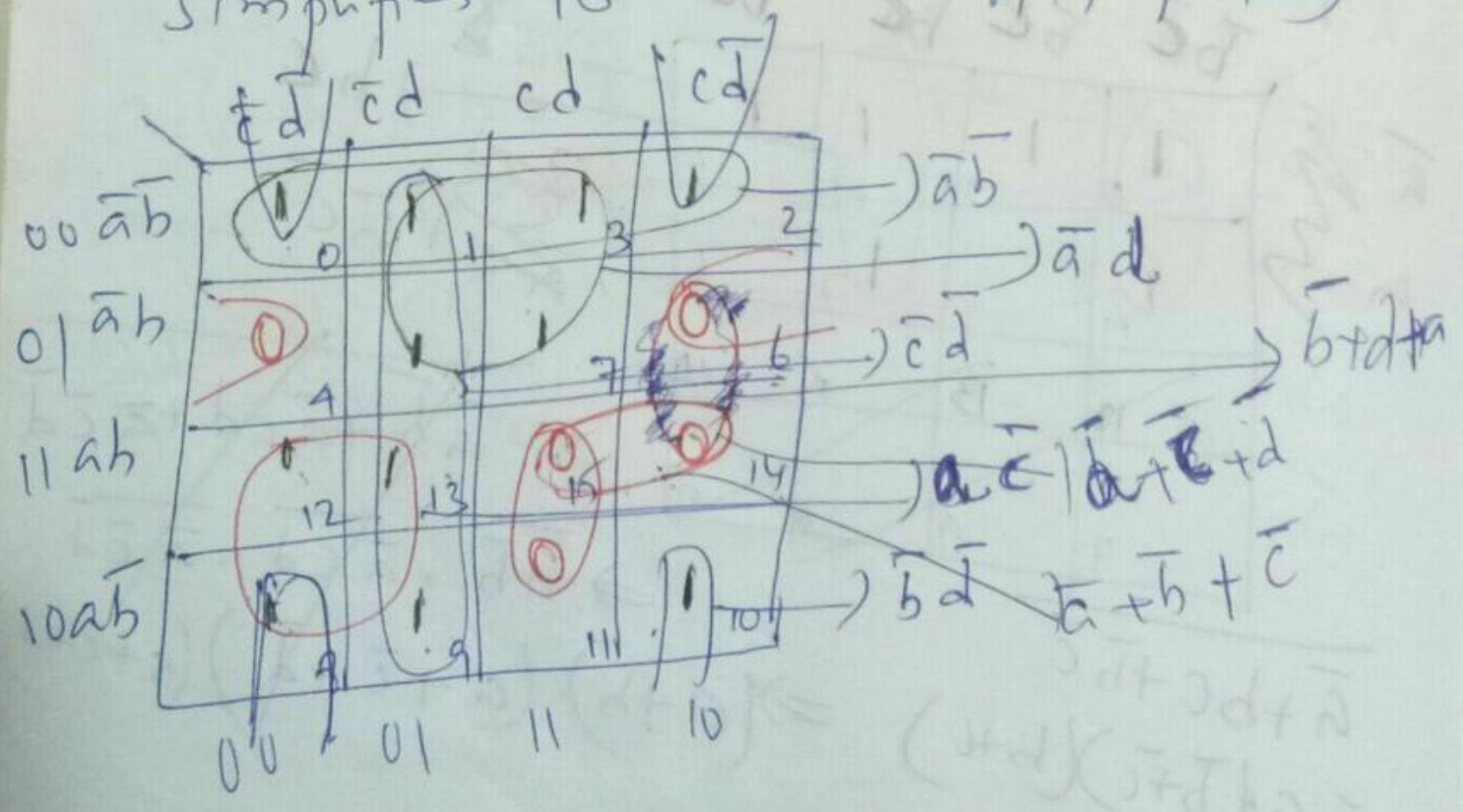
$$\begin{aligned}
 & \bar{a} + bc + \bar{b}\bar{c} \\
 & = a(\bar{b} + \bar{c})(b+c) \Rightarrow (a+\bar{b})(a+\bar{c}+d)(a+c+d)
 \end{aligned}$$

(12) The boolean function $f(a,b,c,d) =$

$\Sigma(2, 3, 12, 13, 14, 15)$ Simplifies to



(13) The boolean function $f(a,b,c,d) = \Sigma(0, 1, 2, 3, 5, 7, 8, 9, 10, 12, 13)$ simplifies to



$$\bar{a}\bar{b} + \bar{a}d + \bar{c}\bar{d} + a\bar{c} + b\bar{d}$$

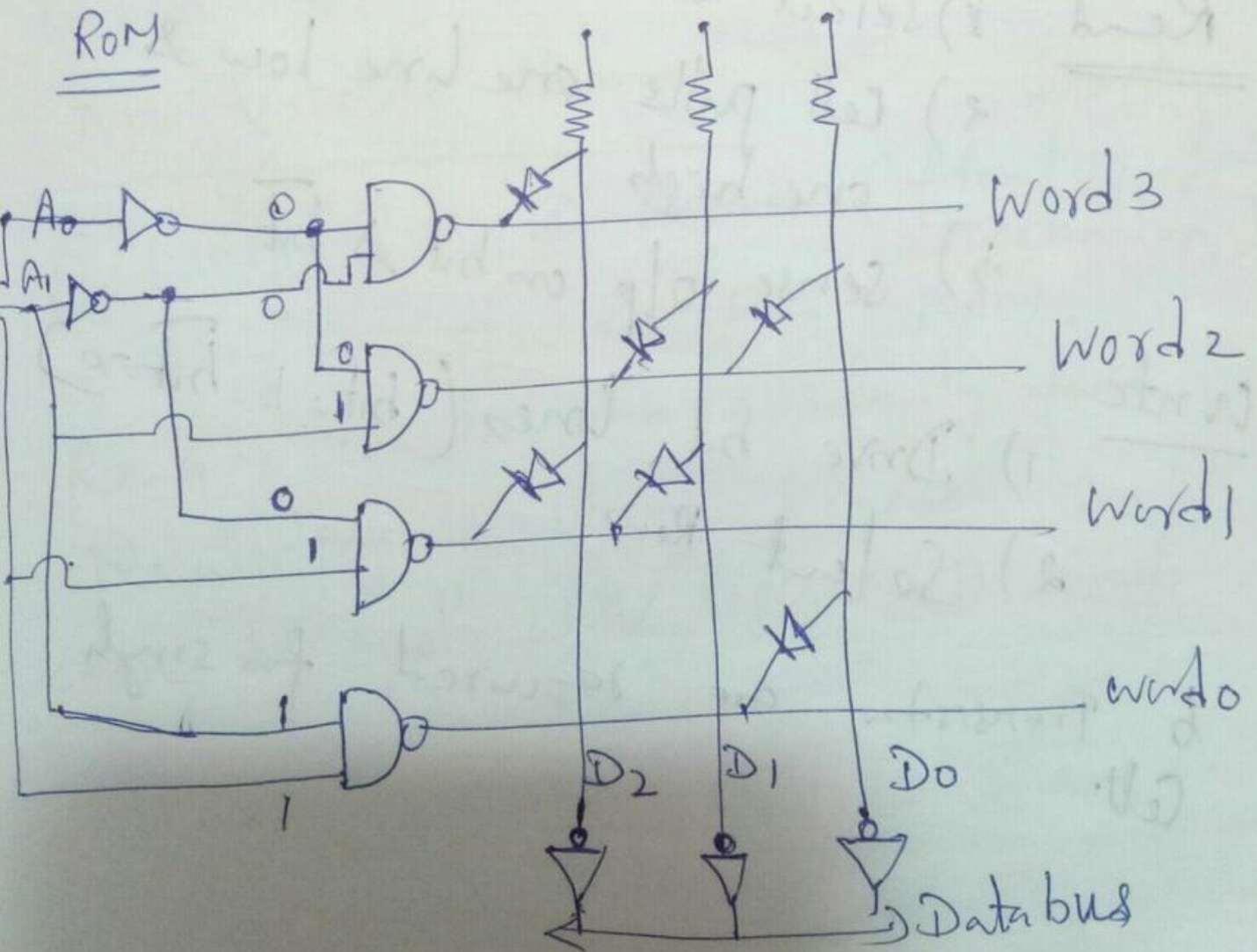
$$(\bar{a} + \bar{b} + \bar{c})(\bar{a} + \bar{c} + \bar{d})(a + \bar{b} + d)$$

(14) Simplified expression of the boolean function $f = \bar{a}\bar{b}\bar{d} + \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}bd + a\bar{b}\bar{c}\bar{d}$ is
 0000, 0010, 0000, 0101, 0111, 1101
 $\Sigma_m(0, 2, 4, 5, 7, 13)$

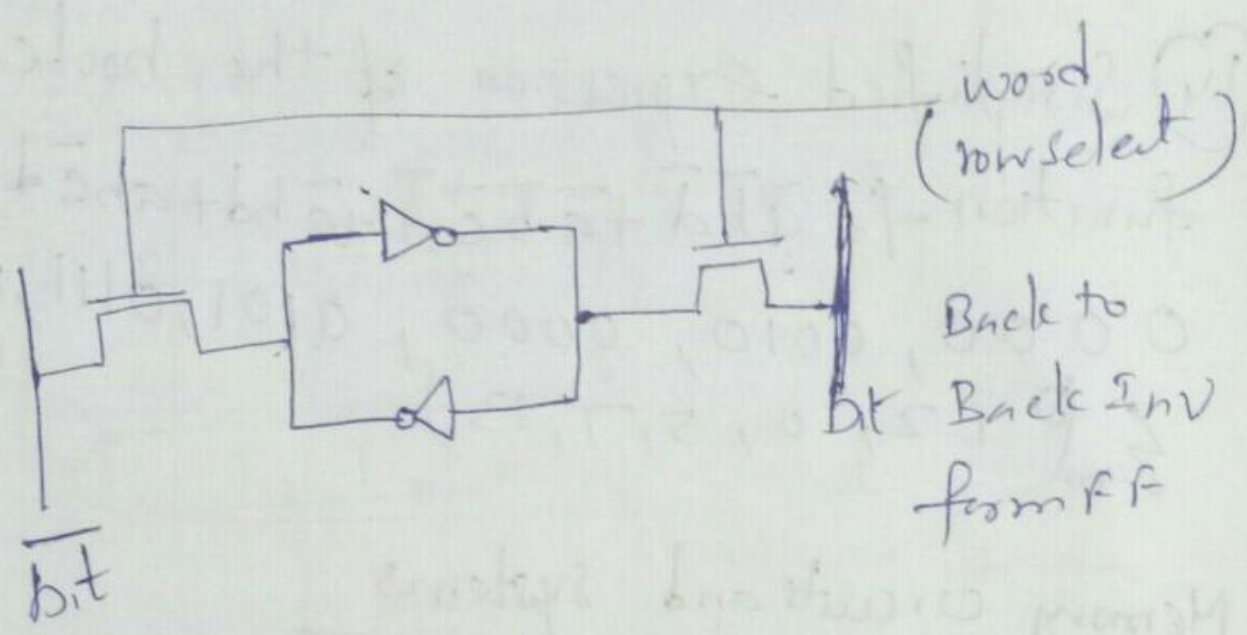
1010
 1011
 1100
 1101

Memory circuits and systems

ROM



Basic static RAM Cell (Random Access Memory)

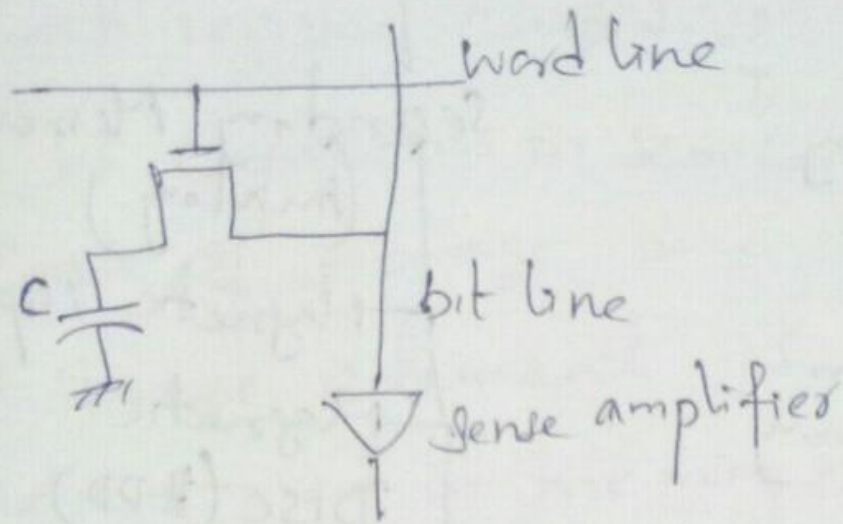


- Read
- 1) Select Row
 - 2) Cell pulls one line low & one high
 - 3) Sense o/p on bit & $\bar{\text{bit}}$

- Write
- 1) Drive bit lines ($\text{bit}=1$ $\bar{\text{bit}}=0$)
 - 2) Select Row

6 Transistors are required for single cell.

DRAM :- Dynamic RAM



SRAM cells exhibit high speed / poor density

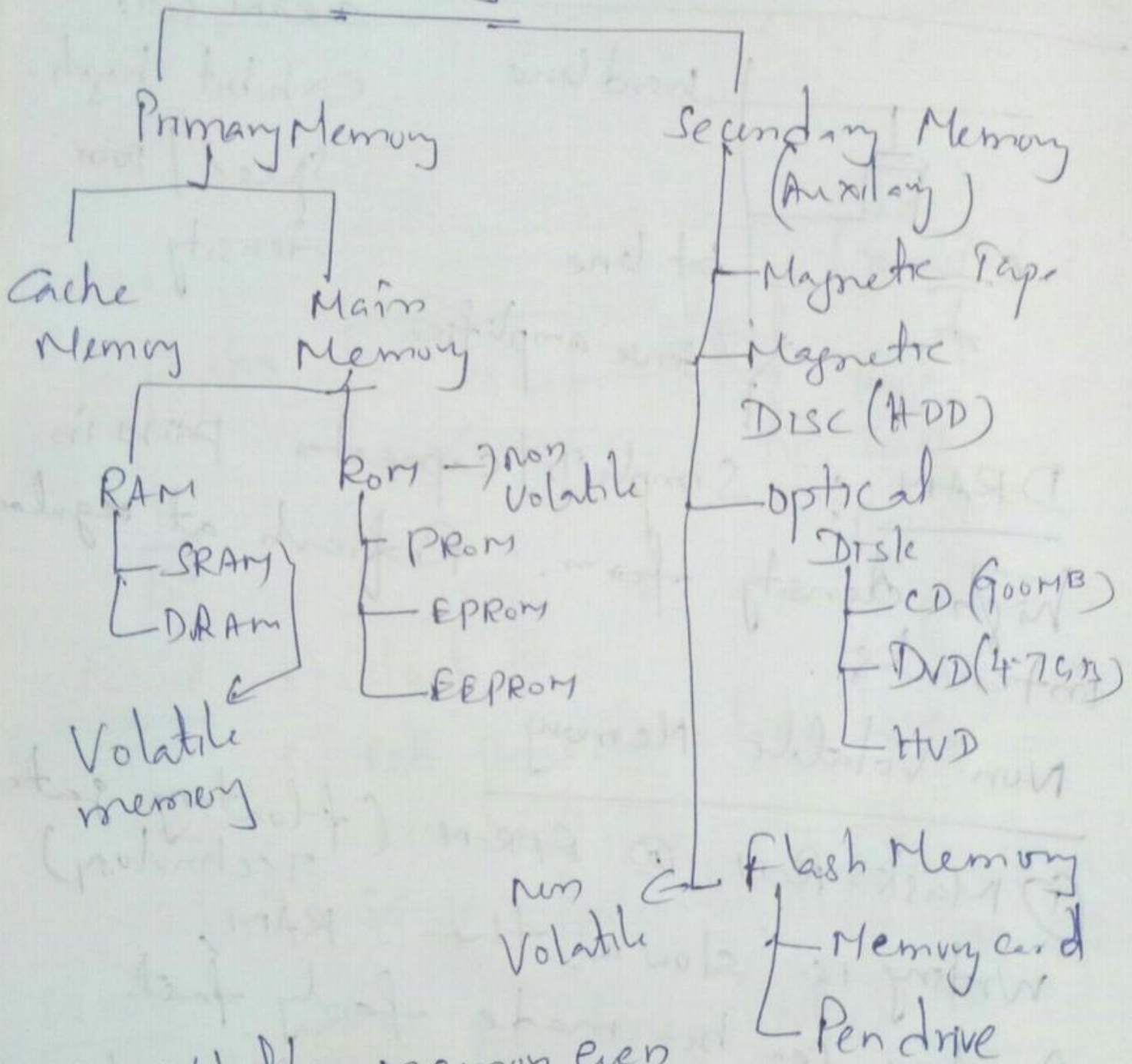
DRAM :- Simple T_1/C pairs in high density form. Refresh at regular intervals

Non Volatile Memory

① Mask ROM ② EPROM (floating gate technology)

writing is slower than RAM, Reading can be made fairly fast. Rewriting is slow. Erasable can be done by apply UV light ③ Flash

Memory

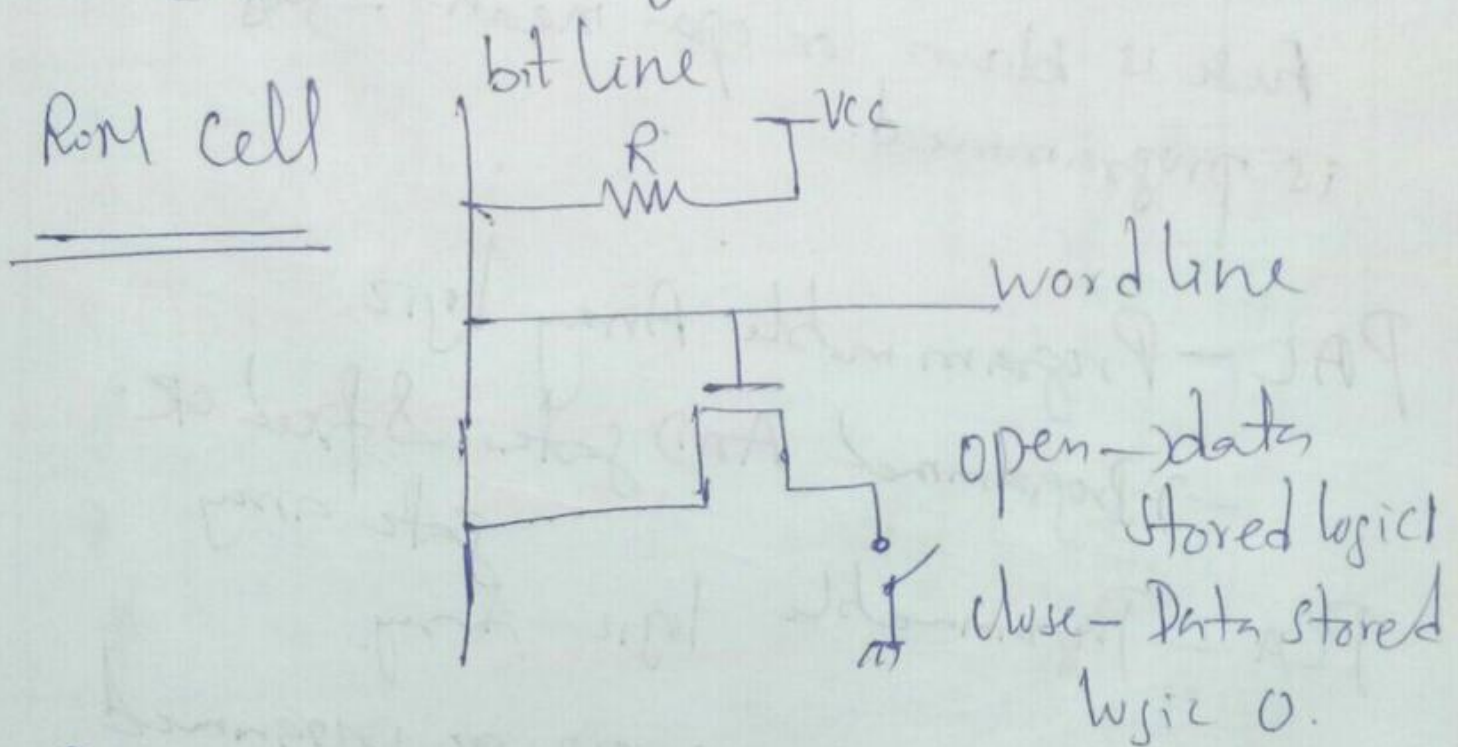


Volatile memory

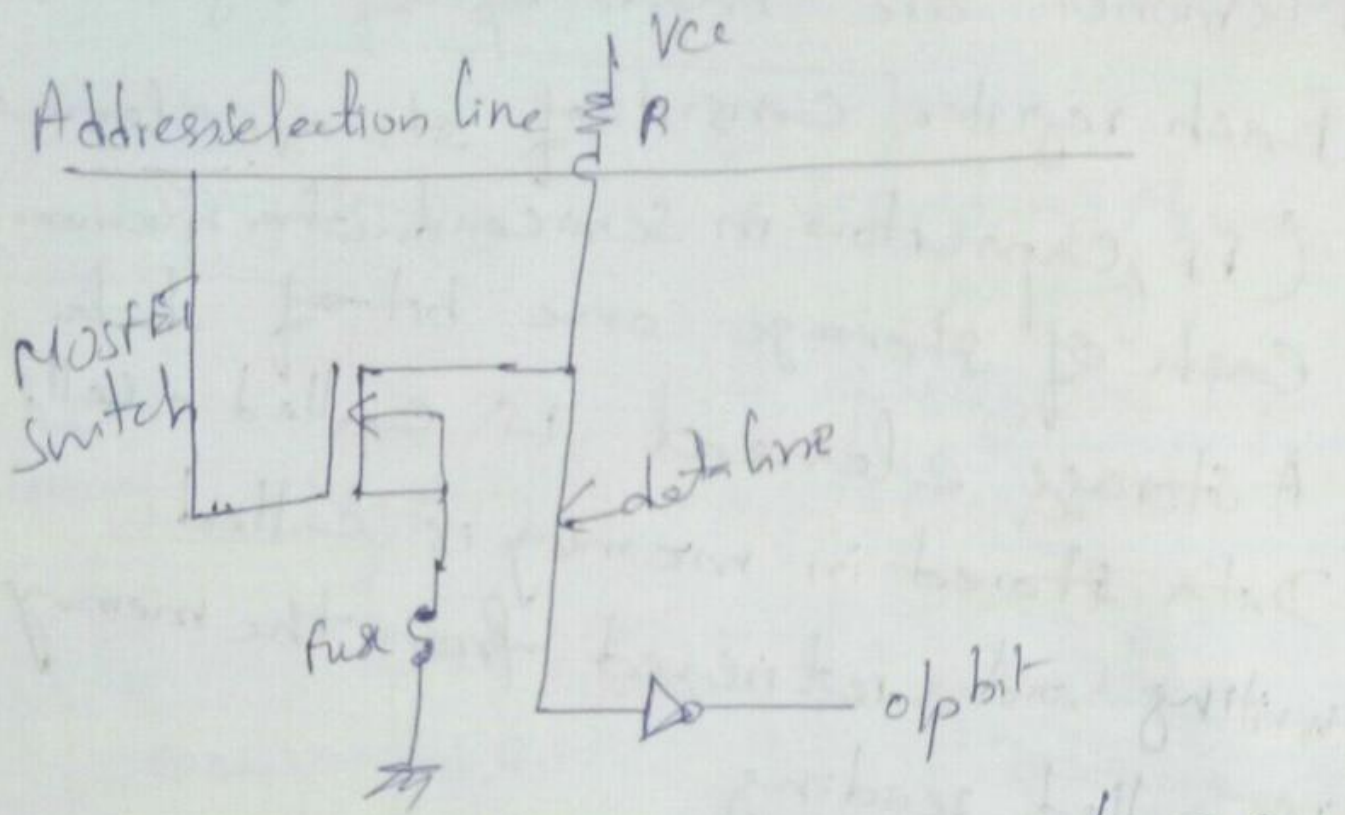
non Volatile

Non Volatile memory even if power is turned off can hold the data. But Volatile memory can hold data during power only.

Memories are made up of registers.
 Each register consists of storage elements
 (FF, capacitors in semiconductor memories)
 Each of storage one bit of data.
 A storage element is called a cell.
 Data stored in memory is called
 writing and retrieved from the memory
 is called reading



PROM: Programmable ROM



fuse is blown or open means \rightarrow logic 0
is programmed.

PAL - Programmable Array Logic

\rightarrow Programmable AND gates & fixed OR gate array

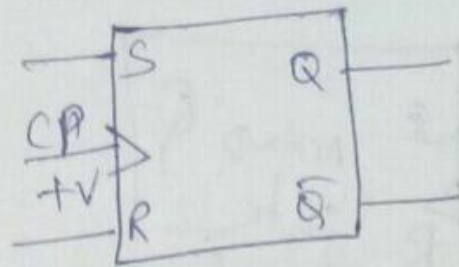
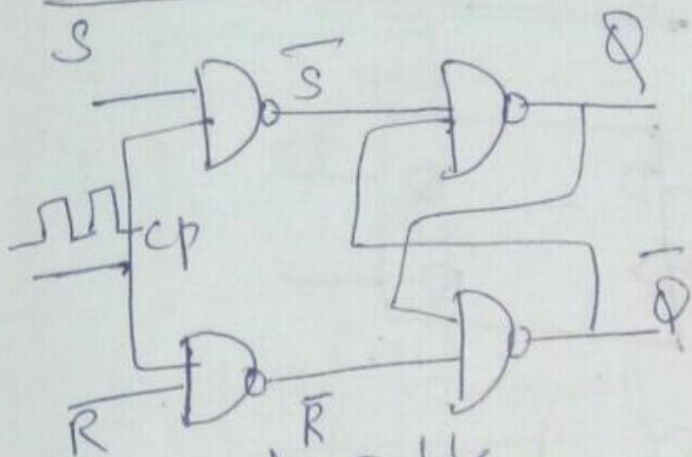
PLA - Programmable Logic Array

Both AND and OR are programmed

PROM \rightarrow Fixed AND array constructed in decoder and programmable OR

SR clocked flip

logic symbol



Truth Table

CP	S	R	Q _n	Q _{n+1}	state
↑	0	0	0	0	No change
↑	0	0	1	1	
↑	0	1	0	0	Reset
↑	0	1	1	0	
↑	1	0	0	1	set
↑	1	0	1	1	
↑	1	1	0	X	Inter mediate
↑	1	1	1	X	

SR	Q _n = 0	Q _n = 1
00	0	1
01	0	0
11	X	X
10	1	0

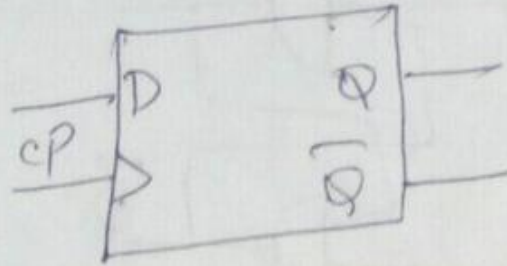
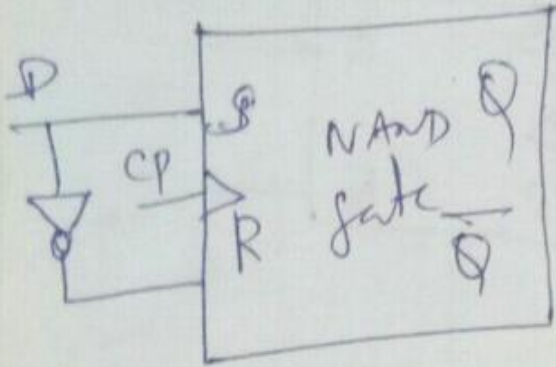
characteristic equation $\Rightarrow S + \bar{R} Q_n$

Q _n	SR	00	01	11	10
0	00	0	1	0	0
1	01	0	0	1	0
1	10	1	0	X	X
1	11	X	X	X	X

CP	S	R	Q _{n+1} NS
En	X	X	NC
0	0	0	NC
1	0	0	Q = 0 Rst
1	0	1	Q = 1 set
1	1	1	Intermediate

D Flip Flop

Logic Symbol



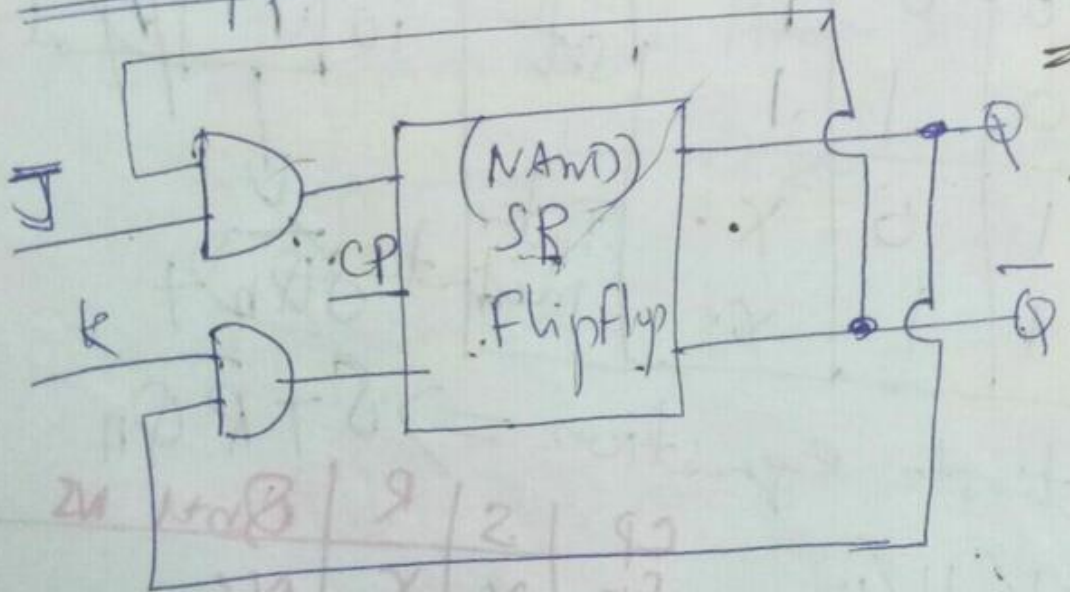
Truth table

CP	D	Q_{n+1}
1	0	0
1	1	1
0	X	Q_n

$$Q_{n+1} = D$$

$$\bar{Q}_{n+1} = \bar{D}$$

JK Flip Flop



$$Q_{n+1} = \overline{JQ} + KQ$$

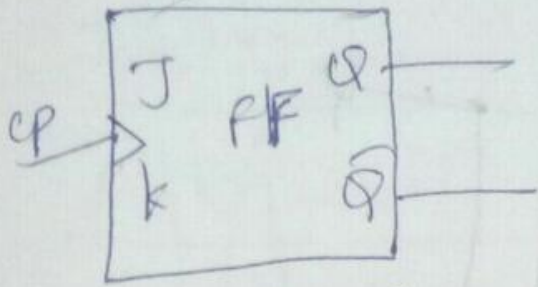
$$= |X\bar{1} + 1X|$$

$$= 0 + 1$$

$$= 1$$

CP	J	K	Q_{n+1}
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	\bar{Q}_n
0	X	X	Q_n

Logic Symbol

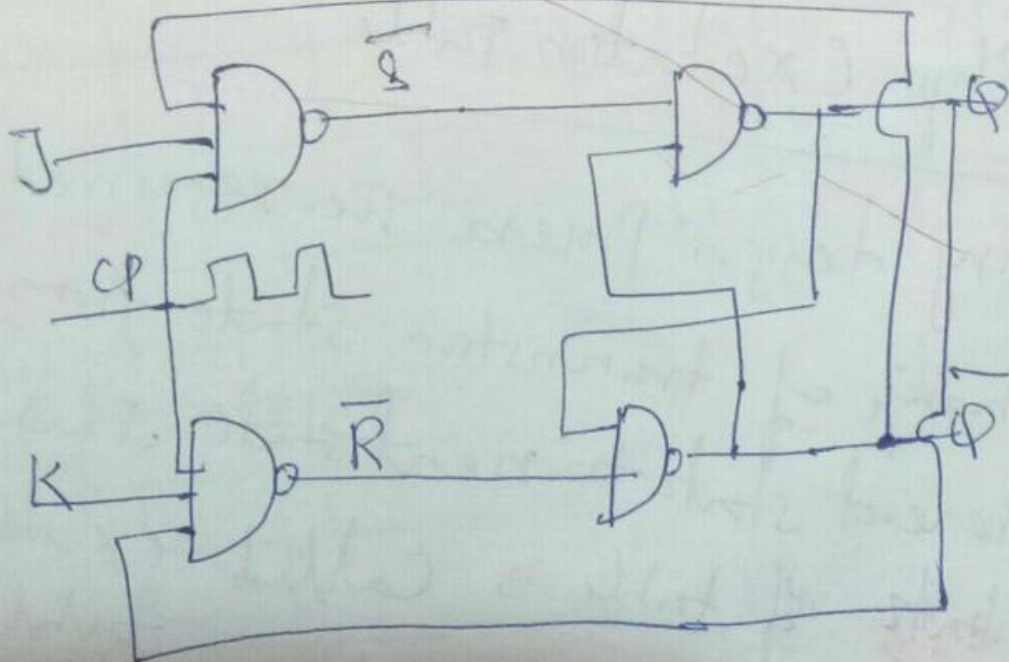


Q_n	J	K	Q_{n+1}
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

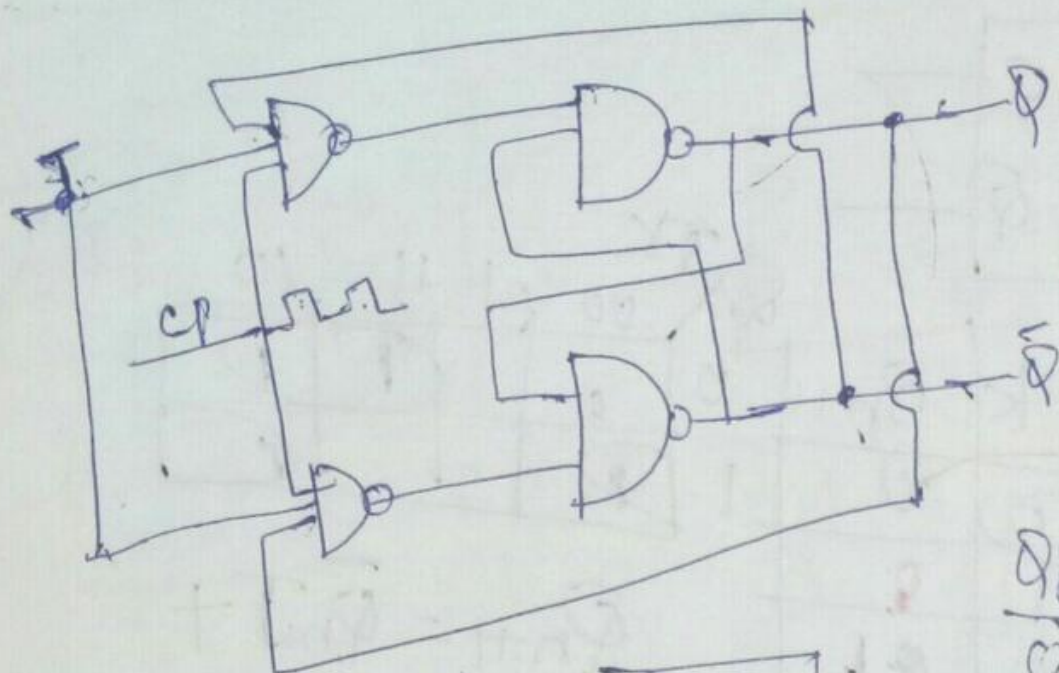
Q_n JK	00	01	11	10
0	0	1	1	1
1	0	0	0	0

$$Q_{n+1} = \bar{Q}_n J + Q_n \bar{K}$$

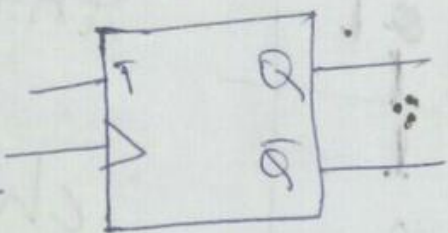
characteristic equation.



T Flip Flop



Logic symbol



Q_n	T	Q_{n+1}
0	0	0
0	1	1
1	0	1
1	1	0

Characteristic Equation: $Q_{n+1} = T\bar{Q}_n + \bar{T}Q_n$

Flip Flop Excitation Table

During design process we required change of transition state from present state to next state. This state of table is called excitation table.

SR Flip Flop

Truth Table

S	R	Q_{n+1}
0	0	Q_n
1	0	1
0	1	0

Excitation Table

Q_n	Q_{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

D Flip Flop Excitation Table

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

T FF excitation Table

Q_n	Q_{n+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

JK FF Excitation Table

Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

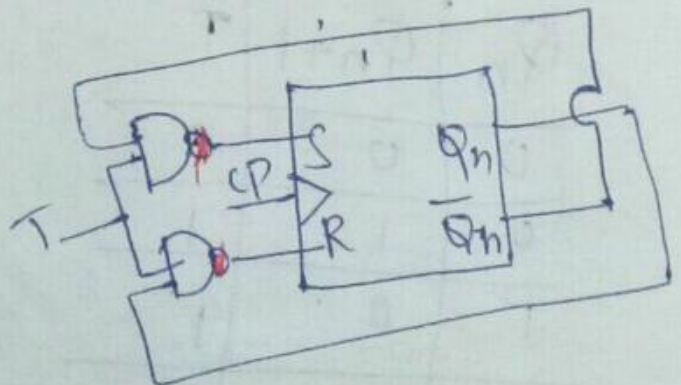
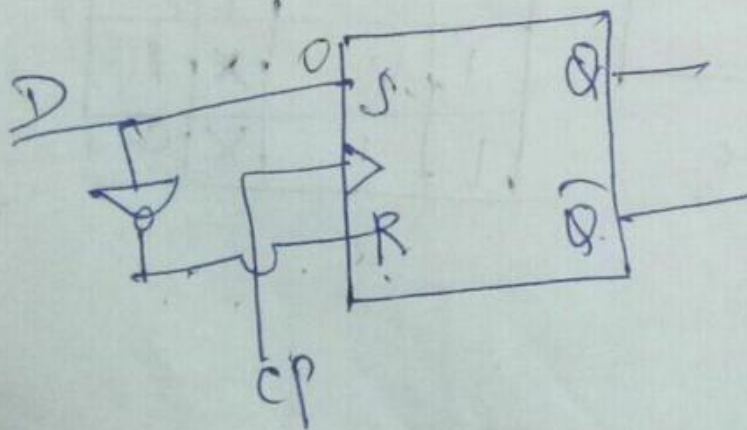
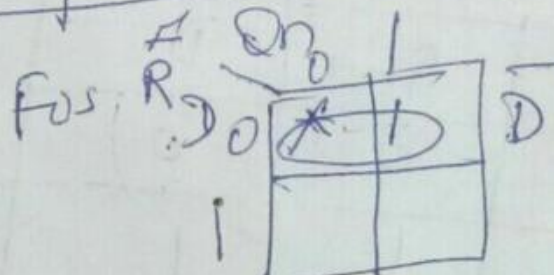
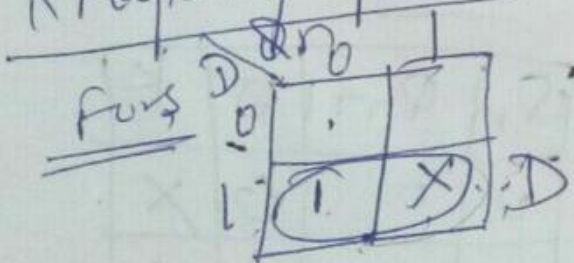
Realization of one Flipflop using other Flipflop

SR Flipflop to D Flipflop.

Input D	PS Q_n	NS Q_{n+1}	Flipflop Inputs	
			S	R
0	0	0	0	X
1	0	1	1	0
0	1	0	0	1
1	1	1	X	0

SR → D

K Map Simplification



SR to JK Flipflop

Input	ps	NS	S	R
J	K	Q_n	Q_{n+1}	
0	0	0	0	X
0	0	1	1	X
0	1	0	0	X
0	1	1	0	0
1	0	0	1	0
1	0	1	X	0
1	1	0	1	0
1	1	0	0	1

For S

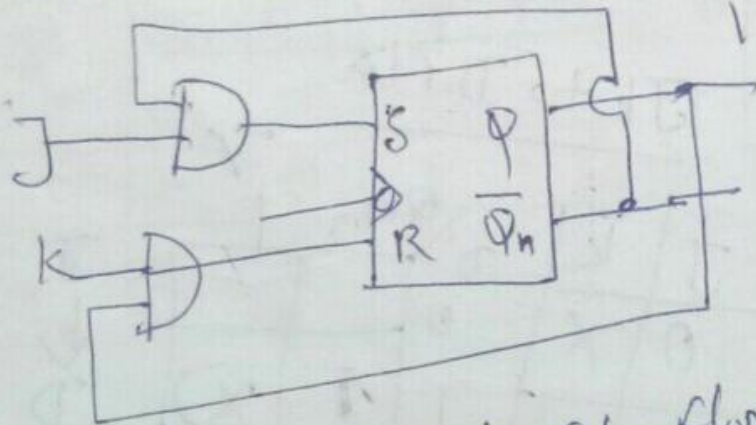
KQ_n	00	01	11	10
J	0	X	1	3
0	0	X	5	6
1	4	5	7	6

$$S = J \cdot \bar{Q}_n$$

For K

KQ_n	00	01	11	10
J	X	1	3	X
0	X	1	7	X
1	4	5	7	6

$$K = KQ_n$$



SR Flipflop to T flipflop

Input	ps	NS	S	R
T	Q_n	Q_{n+1}	S	R
0	0	0	0	X
0	1	1	X	0
1	0	1	1	0
1	1	0	0	1

S

Q_n	0	1
T	0	X
1	1	

$$S = T \bar{Q}_n$$

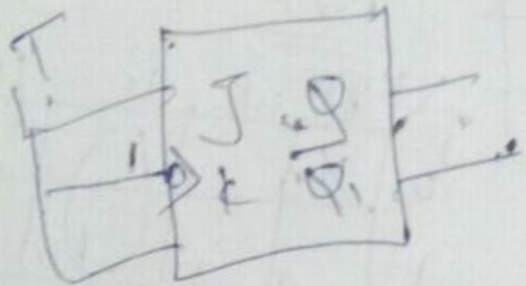
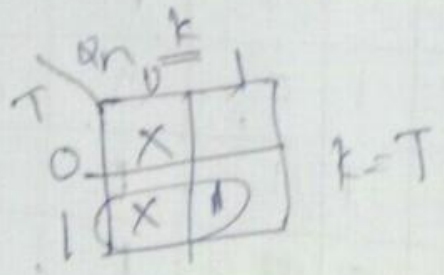
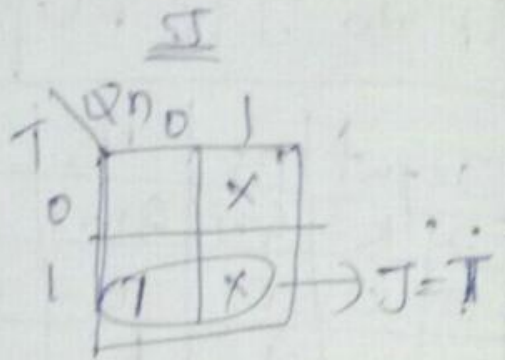
R

Q_n	0	1
T	0	X
1	X	1

$$R = TQ_n$$

JK flipflop to T flipflop

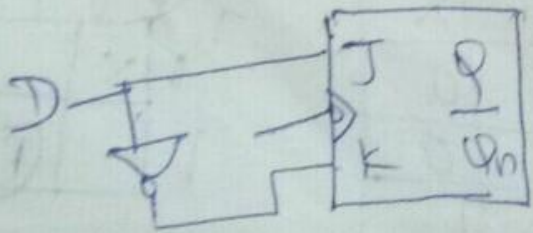
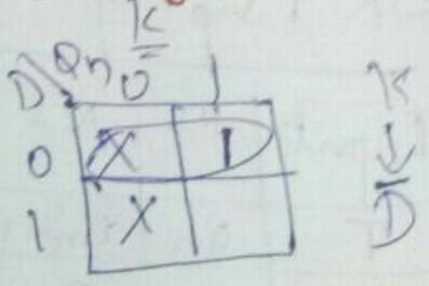
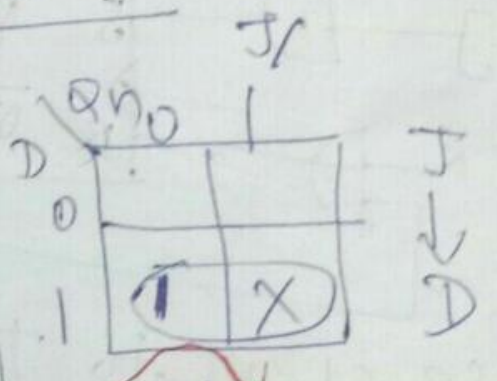
Input T	ps Q_n	NS Q_{n+1}	J	K
0	0	0	0	X
0	1	1	X	0
1	0	1	1	X
1	1	0	X	1



D FF to T FF

JK to D FF

D	ps Q_n	NS Q_{n+1}	J	K
0	0	0	0	X
0	1	0	X	1
1	0	1	1	X
1	1	1	X	0

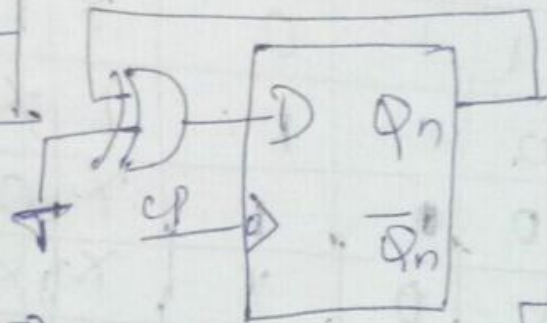


D Flip Flop T Flip Flop

T	Q _n	Q _{n+1}	D
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

T	Q _n	Q _{n+1}
0	0	1
1	1	0

$$D = T\bar{Q}_n + \bar{T}Q_n$$

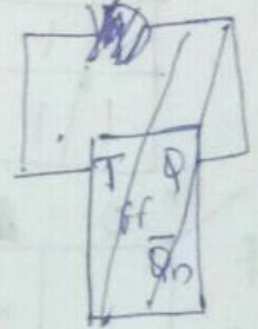
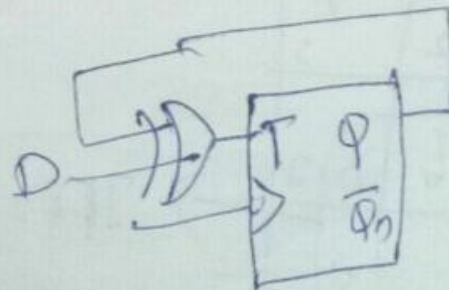


T flip flop to D flip flop

Input D	PS Q _n	NS Q _{n+1}	FF Input T
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

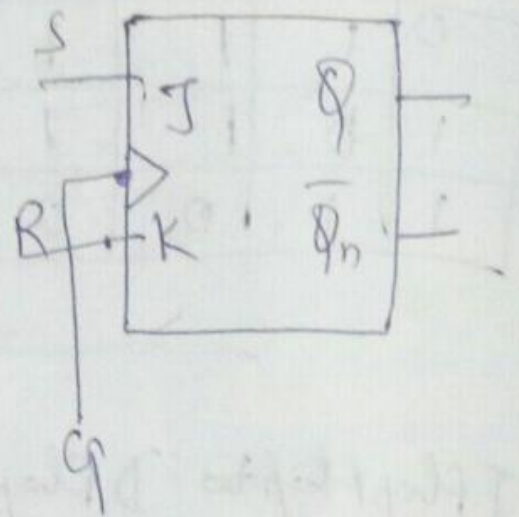
D	Q _n	Q _{n+1}
0	0	1
1	1	0

$$T = D\bar{Q}_n + \bar{D}Q_n$$



JF flip flop to SR flip flop

S	R	Q_n	Q_{n+1}	J	K
0	0	0	0	0	X
0	0	1	1	X	0
0	1	0	0	0	X
0	1	1	0	X	1
1	0	0	1	1	X
1	0	1	1	X	0
1	1	0	X	X	X
1	1	1	X	X	X



S	R	Q_n	Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	X
1	1	1	X

JFF = S

S	R	Q_n	Q_{n+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	X
1	1	1	X

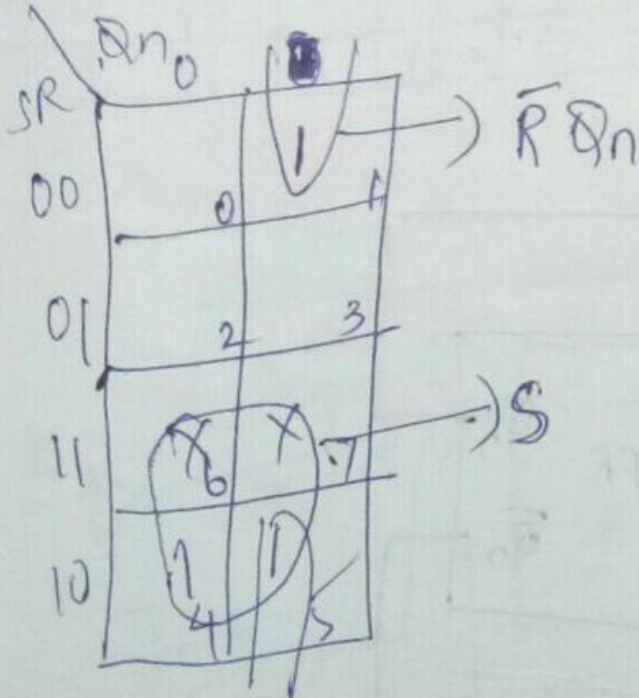
R

JFF

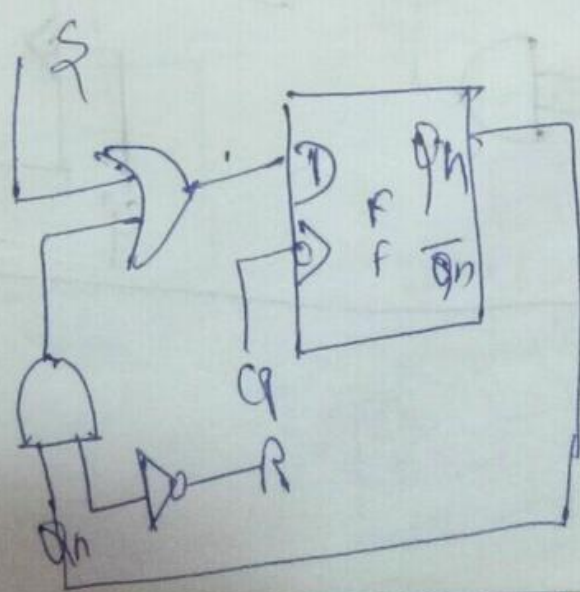
R flip flop

D Flipflop to SR Flipflop

	S	R	Q_n	Q_{n+1}	D
0	0	0	0	0	0
1	0	0	1	1	1
2	0	1	0	0	0
3	0	1	1	0	0
4	1	0	0	1	1
5	1	0	1	1	1
6	1	1	0	X	X
7	1	1	1	X	X

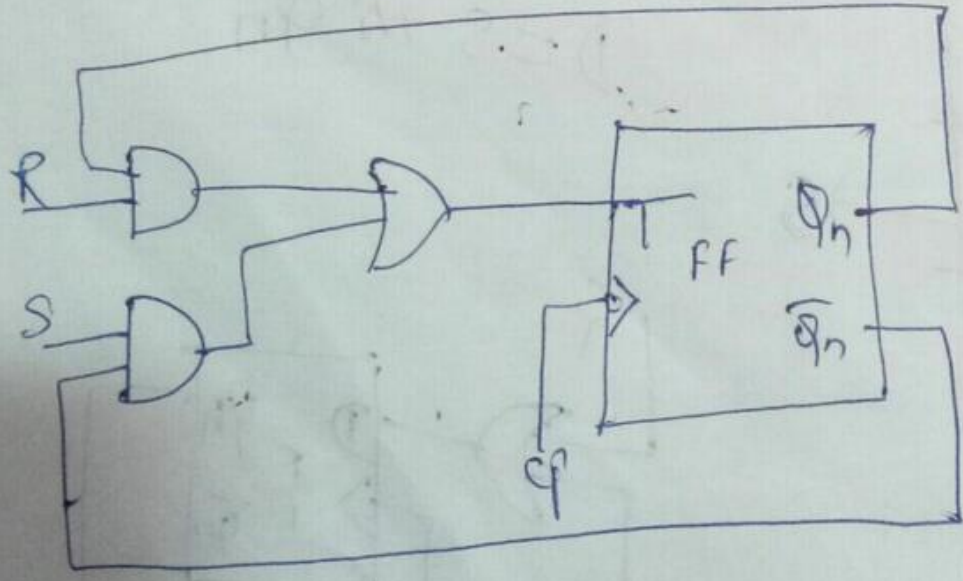
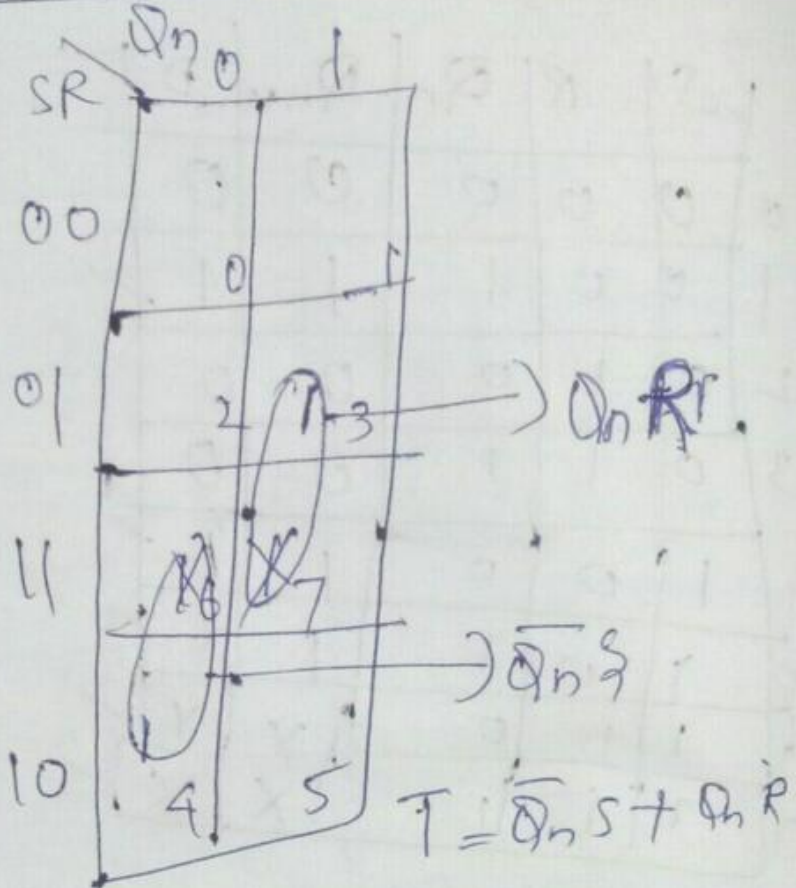


$$D = S + \bar{R}Q_n$$



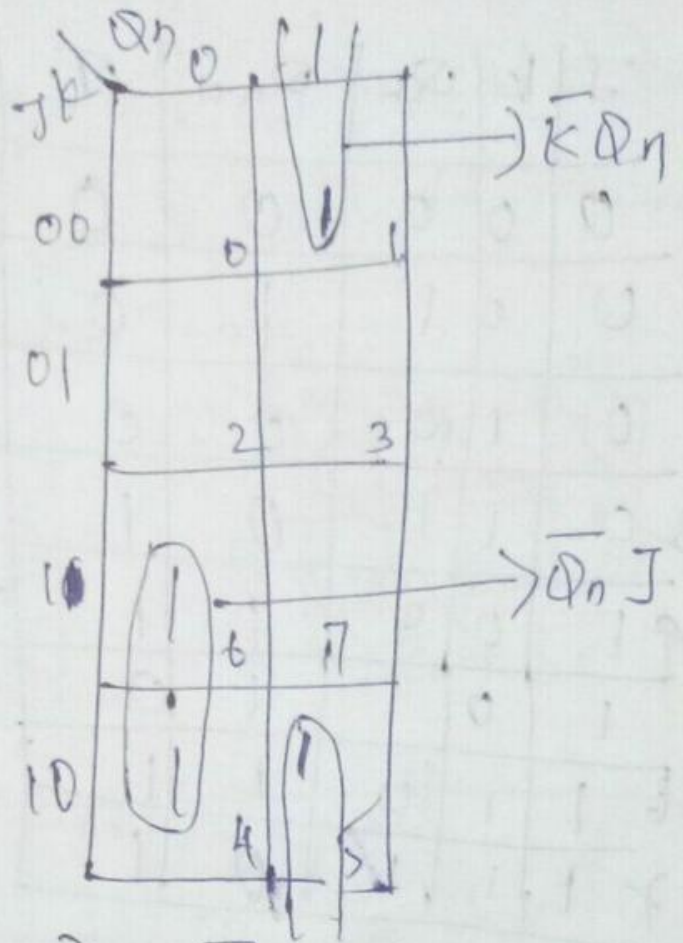
T Flipflop to SR flipflop

S	R	Q_n	Q_{n+1}	T
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	X	X
1	1	1	X	X

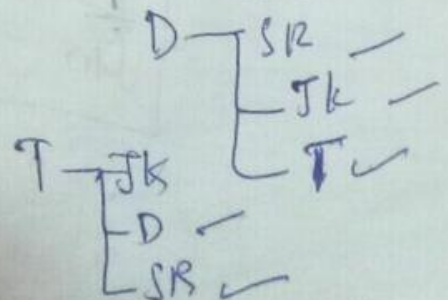
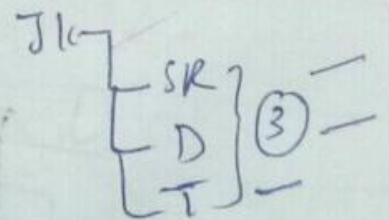
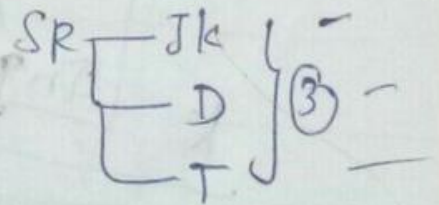
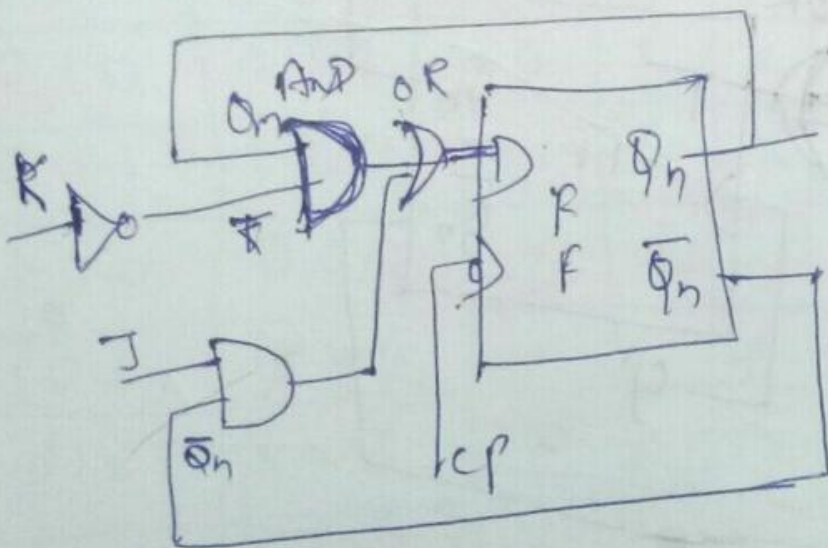


D Flip Flop to JK Flip Flop

J	K	Q_n	Q_{n+1}	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0



$$D = \overline{K}Q_n + Q_n J$$



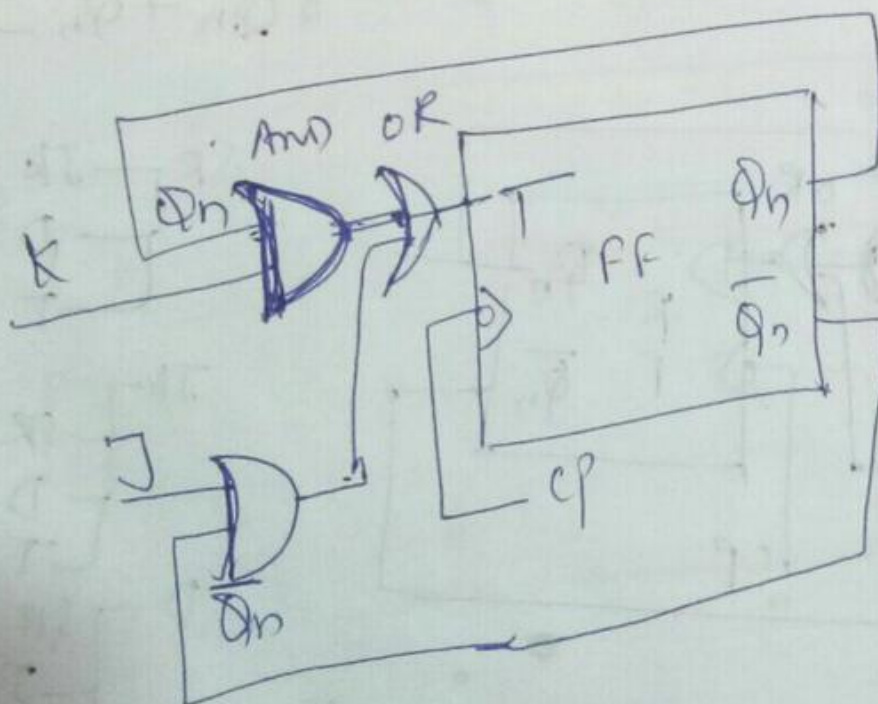
T flip flop to JK flip flop

J	k	Q_n	Q_{n+1}	ΣT
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1

JK	$Q_n = 0$	$Q_n = 1$
00	0	1
01	2	3
11	6	7
10	4	5

$\rightarrow kQ_n$ (circled cells 2, 3, 6, 7)
 $\rightarrow \bar{Q}_n J$ (circled cells 4, 5)

$$T = kQ_n + \bar{Q}_n J$$



Determine a Mod 5 Synchronous Counter using JK flipflop and implement it.

Step 1: Flipflop required $2^n = \text{no. of states}$
 $2^3 = 8$ but in the sum 5. 3 flipflop required

Step 2: JK Excitation Table

Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

	bc	00	01	11	10
a	0	0	1	1	0
1	1	1	0	0	1

$\Sigma = \bar{a} + \bar{b}c$

P_s	N_s	Flipflop Inputs					
		J_c	K_c	J_b	K_b	J_a	K_a
000	000	0	X	0	X	1	X
001	010	0	X	1	X	X	1
010	011	0	X	X	0	1	X
011	100	1	X	X	1	X	1
100	000	X	1	0	X	0	X
101	XXX	X	X	X	X	X	X
110	XXX	X	X	X	X	X	X
111	XXX	X	X	X	X	X	X

By using Kmap

	Q_B	Q_A		
	00	01	11	10
Q_C	0	1	1	2
1	X ₄	X ₅	X ₇	X ₆

	Q_B	Q_A		
	00	01	11	10
Q_C	0	X ₀	X ₁	X ₃
1	X ₄	X ₅	X ₇	X ₆

For $J_C = Q_B Q_A$

	Q_B	Q_A		
	00	01	11	10
Q_C	0	1	X ₃	X ₂
0	X ₄	X ₅	X ₇	X ₆

For $K_C = 1$

	Q_B	Q_A		
	00	01	11	10
Q_C	0	X ₁	X ₃	2
1	X ₄	X ₅	X ₇	X ₆

For $J_B = Q_A$

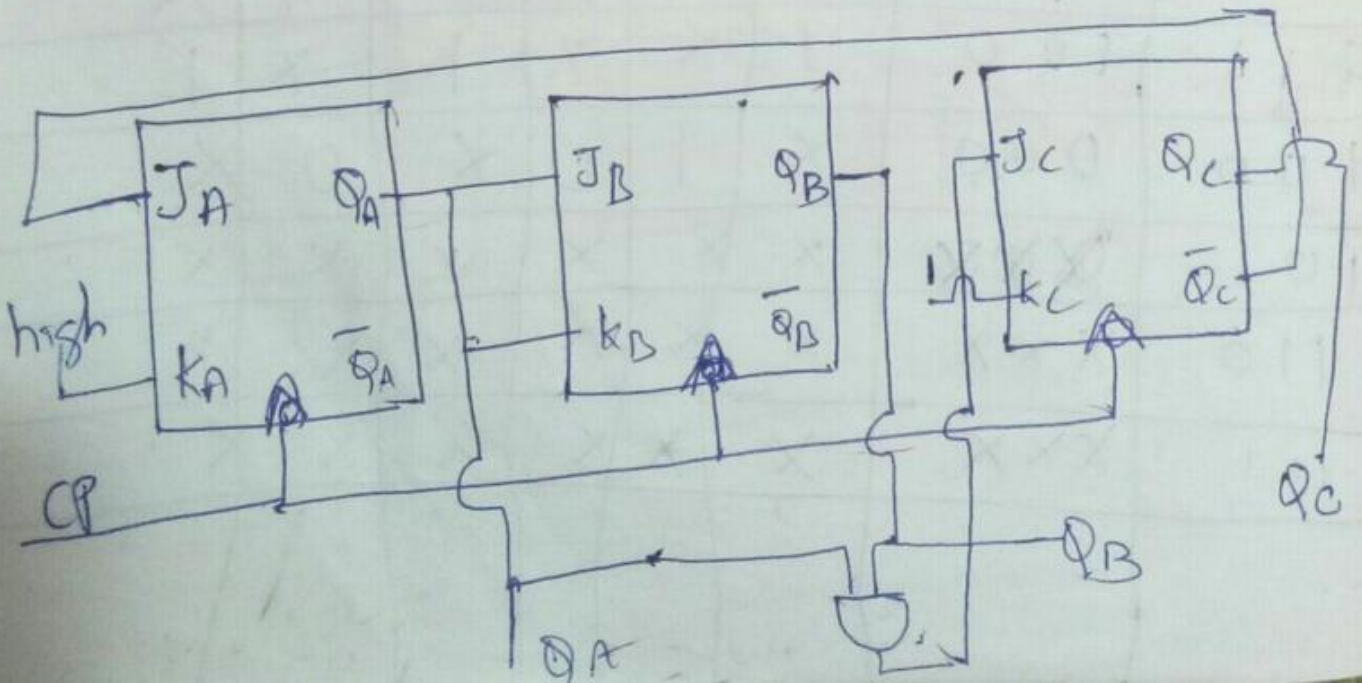
	Q_B	Q_A		
	00	01	11	10
Q_C	0	X ₁	X ₃	X ₂
1	X ₄	X ₅	X ₇	X ₆

For $K_B = Q_A$

	Q_B	Q_A		
	00	01	11	10
Q_C	0	X ₁	X ₃	X ₂
1	X ₄	X ₅	X ₇	X ₆

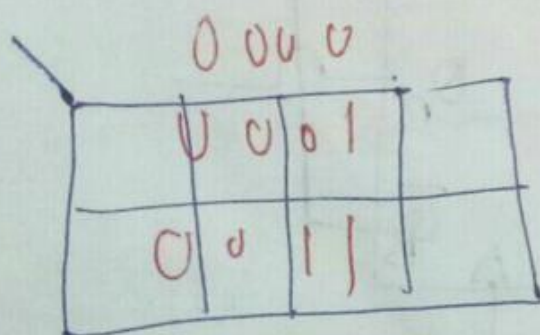
For $J_A = \bar{Q}_C$

For $K_A = 1$



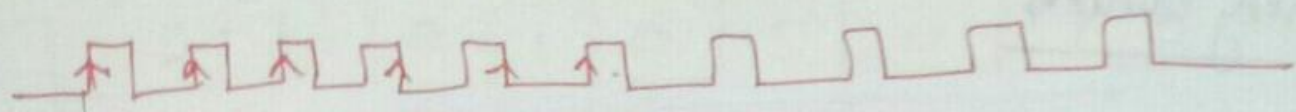
Design a Synchronous counter using JK FF to count the following sequence
 7, 4, 3, 1, 6, 0, 7

P _s		N _s			Flip flop Inputs						
A	B	A ⁺	B ⁺	A ⁺	J _c	K _c	J _B	K _B	J _A	K _A	
0	0	1	1	1	1	X	1	X	1	X	
0	0	1	1	0	1	X	1	X	X	1	
0	1	X	X	X	X	X	X	X	X	X	
0	1	0	0	1	0	X	X	1	X	0	
1	0	0	1	1	X	1	1	X	1	X	
1	0	X	X	X	X	X	X	X	X	X	
1	0	0	0	0	X	1	X	1	0	X	
1	1	1	0	0	X	0	X	1	X	1	



0 1 1 1
 1 1 1 1
 1 1 0 0
 1 0 0 0
 0 0 0 0

elsif (clock = '1'b1')
 begin
 count_temp =
 count_temp[2:0] ~
 count_temp[3];
 end
 assign count_out = count_temp;
endmodule



```

module four-bit-ring-counter(
input clock, reset;
output [3:0] q); reg [3:0] a;
always@ (posedge clock)
  if (reset)
    a = 4'b0001;
  else
    begin

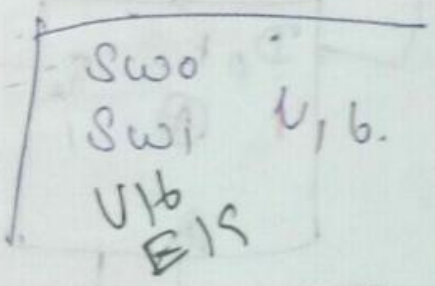
```

- 0001
- 0010
- 0100
- 1000
- 0001

```

      a <= a << 1 blocking
      a[0] <= a[3] assignment
    end
    assign q = a
end module

```



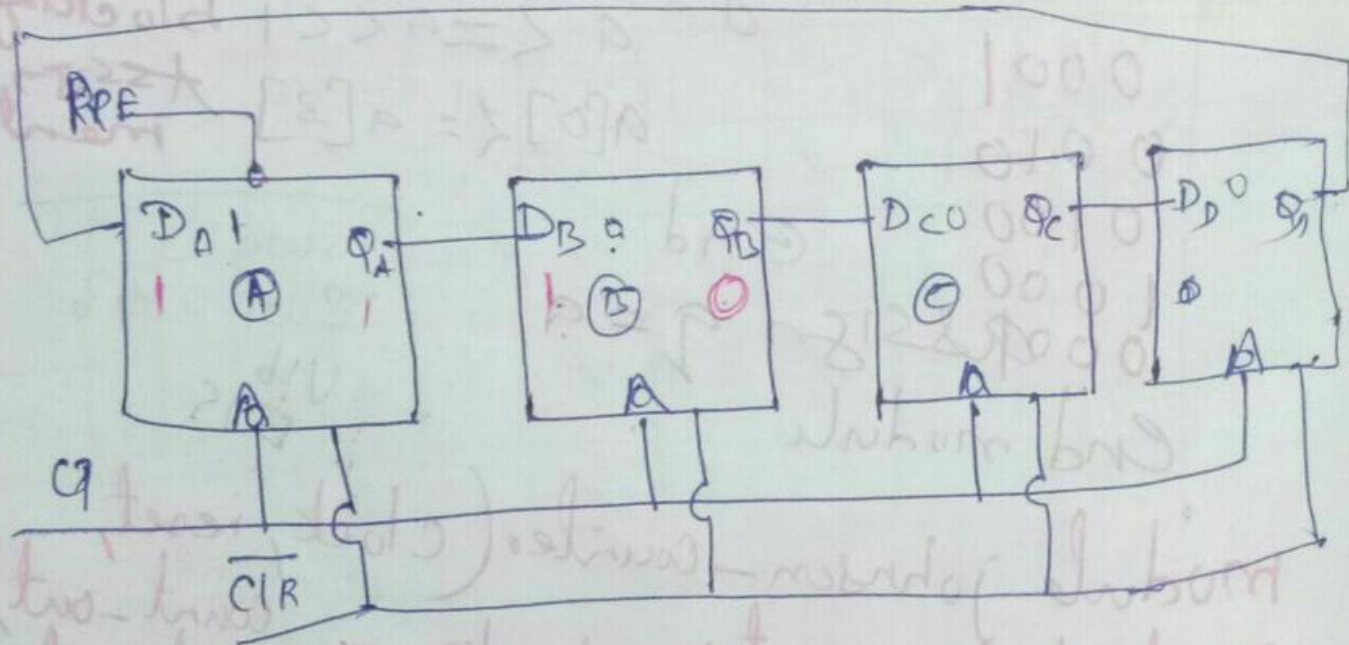
```

module johnson-counter (clock, reset,
                        count_out);
input clock, reset; output [3:0] count_out;
reg [3:0] count_temp;
always@ (posedge clock or reset)
  begin
    if (reset == 1'b1)
      count_temp = 4'b0000;
    end

```

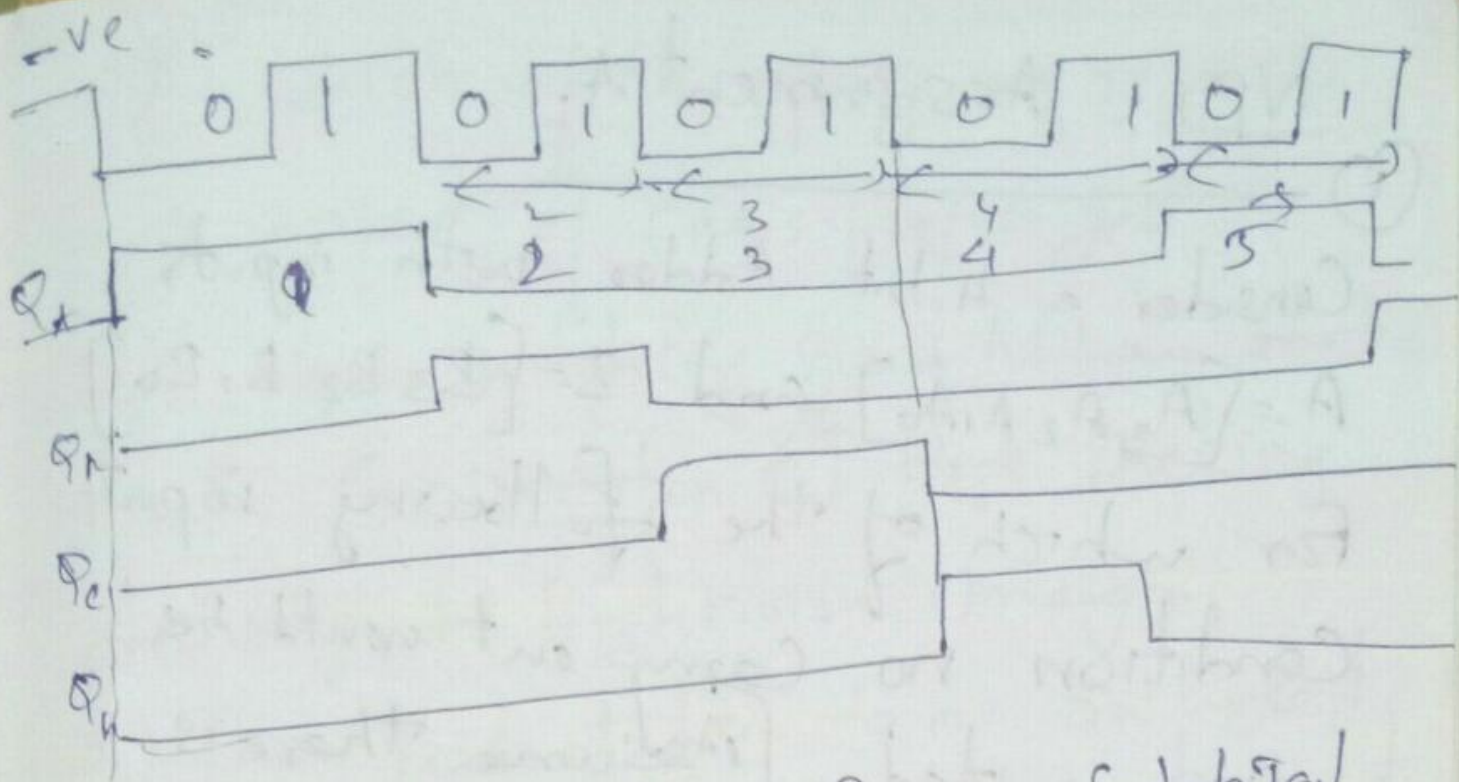
Ring counters

cp	Q _A	Q _B	Q _C	Q _D
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	1	0	0	0



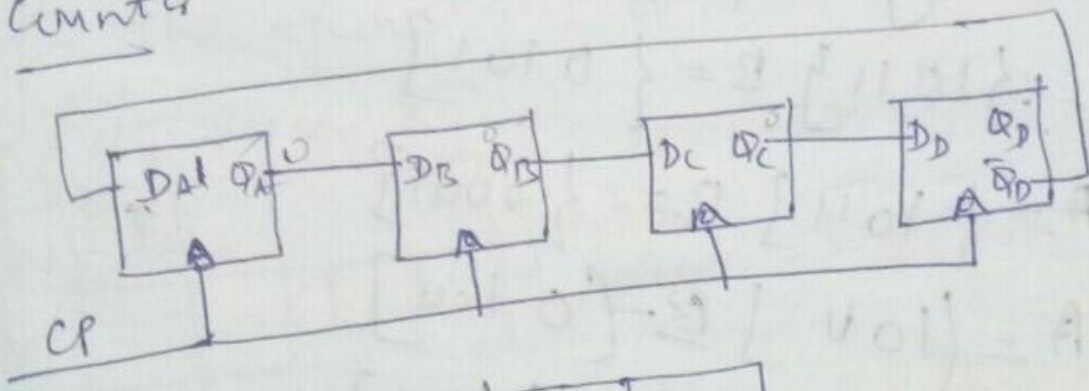
Even we can give $PRE = CIR$

PRE is used for take prev value of FF $Q_D \rightarrow D$



Johnson (or) Twisting Ring (or) Switch Tail

Counter



CP	Q _A	Q _B	Q _C	Q _D
↓	1	0	0	0
↓	1	1	0	0
↓	1	1	1	0
↓	1	1	1	1
↓	0	1	1	1
↓	0	0	1	1
↓	0	0	0	1

NPTTC Assignment 4.

①

Consider a 4 bit Adder with inputs $A = [A_3 A_2 A_1 A_0]$ and $B = [B_3 B_2 B_1 B_0]$ for which of the following input condition no carry out would be generated. [Assume there is no carry input]

a) $A = \{1011\}$ $B = \{0101\}$

b) $A = \{1011\}$ $B = \{0001\}$

c) $A = \{1011\}$ $B = \{0100\}$

d) $A = \{1001\}$ $B = \{0101\}$

a)
$$\begin{array}{r} 1011 \\ 0101 \\ \hline 10000 \end{array}$$

b)
$$\begin{array}{r} 1011 \\ 0001 \\ \hline 1100 \end{array}$$

①
$$\begin{array}{r} 1011 \\ 0100 \\ \hline 1111 \end{array}$$

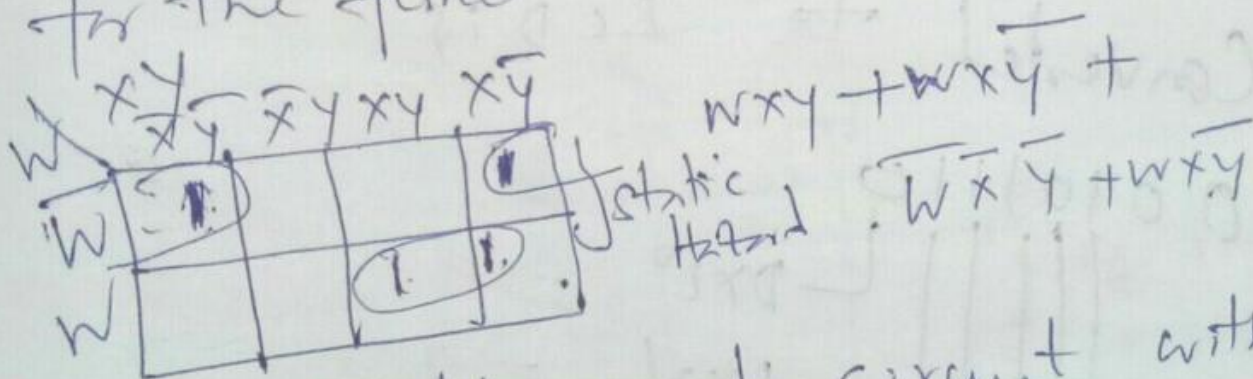
d)
$$\begin{array}{r} 1001 \\ 0101 \\ \hline 1110 \end{array}$$

2) Which of the following is not a property of CMOS logic gates?

- a) High switching speed b) Low static Power Consumption c) High packing density d) High noise margin

3) The Product term to be included to remove possible static hazard

for the function $WX + \overline{W}Y$ is



4) An odd parity circuit with

2^n inputs can be built with _____

XOR gates.

- a) 2^n b) 2^{n+1} c) 2^{n-1} d) $\lfloor \frac{n}{2} \rfloor$