BOARD QUESTION PAPER : JULY 2024

MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

*The question paper is divided into **FOUR** sections.*

(1) ***Section A:*** *Q. 1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.*
*Q. 2 contains **Four** very short answer type questions, each carrying **One** mark.*

(2) ***Section B:*** *Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)*

(3) ***Section C:*** *Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)*

(4) ***Section D:*** *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)*

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

(8) For each multiple choice type of question, only the first attempt will be considered for evaluation.

(9) Start answer to each section on a new page.

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

i. $\cos\left[\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]=$ ______.

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\pi}{4}$ (2)

ii. If θ is the angle between two vectors $\bar{a}$ and $\bar{b}$ and $|\bar{a}\cdot\bar{b}|=|\bar{a}\times\bar{b}|$ then θ is equal to ______.

(A) 0 (B) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π or $\frac{\pi}{6}$ (2)

iii. The angle between the lines $\bar{r}=(\hat{i}+2\hat{j}-3\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\bar{r}=(5\hat{i}-2\hat{j}+7\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$ is ______.

(A) $\cos^{-1}\left(\frac{17}{21}\right)$ (B) $\cos^{-1}\left(\frac{20}{21}\right)$ (C) $\cos^{-1}\left(\frac{18}{21}\right)$ (D) $\cos^{-1}\left(\frac{19}{21}\right)$ (2)

iv. The perpendicular distance of the plane $\bar{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})=5$, from the origin is ______.

(A) $\frac{5}{\sqrt{14}}$ units (B) $\frac{5}{14}$ units (C) 5 units (D) $\frac{\sqrt{14}}{5}$ units (2)

v. If $x=e^{\frac{x}{y}}$ then $\frac{dy}{dx}=$ ______.

(A) $1-\frac{y}{x}$ (B) $1+\frac{y}{x}$ (C) $\frac{x-y}{x\log x}$ (D) $\frac{x+y}{x\log x}$ (2)

vi. $y=c^2+\frac{c}{x}$ is solution of ______.

(A) $x^4\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}-y=0$ (B) $x^2\left(\frac{dy}{dx}\right)^2+y=0$

(C) $x^3\left(\frac{d^2y}{dx^2}\right)-x\frac{dy}{dx}+y=0$ (D) $x\frac{d^2y}{dx^2}=4y$ (2)

vii. Given that $X \sim B(n, p)$. If $n = 10$ and $p = 0.4$ then $E(X)$ and $Var(X)$ respectively are ________.
(A) 4, 0.24 (B) 0.4, 0.24 (C) 4, 2.4 (D) 3, 0.24 (2)

viii. The approximate value of $\tan(44°30')$, given that $1° = 0.0175^c$, is ________.
(A) 0.8952 (B) 0.9528 (C) 0.9285 (D) 0.9825 (2)

Q.2. Answer the following questions: **[4]**

i. Find the combined equation of the pair of lines $2x + y = 0$ and $3x - y = 0$ (1)

ii. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ (1)

iii. Evaluate: $\int \frac{5^x}{3^x}dx$ (1)

iv. Write the integrating factor (I.F.) of the differential equation $\frac{dy}{dx} + y = e^{-x}$. (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. If the statements p, q are true statements and r, s are false statements, then determine the truth value of the statement pattern:
$(q \wedge r) \vee (\sim p \wedge s)$ (2)

Q.4. Find the inverse of matrix A by elementary row transformations, where $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ (2)

Q.5. Find the polar co-ordinates of the point whose Cartesian co-ordinates are $(1, -\sqrt{3})$. (2)

Q.6. Find the acute angle between the lines represented by $xy + y^2 = 0$ (2)

Q.7. Using the truth table, show that the statement pattern $p \rightarrow (q \rightarrow p)$ is a tautology. (2)

Q.8. If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ then find the value of x, where $0 < 3x < 1$. (2)

Q.9. Find the points on the curve given by $y = x^3 - 6x^2 + x + 3$ where the tangents are parallel to the line $y = x + 5$. (2)

Q.10. Evaluate: $\int \frac{e^x(1+x)dx}{\sin^2(xe^x)}$. (2)

Q.11. The displacement of a particle at a time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. Find the time when acceleration is 14 ft/sec^2. (2)

Q.12. Evaluate: $\int_0^{\pi/4} \sqrt{1 + \sin 2x}\, dx$ (2)

Q.13. The probability distribution of X is as follows:

$X = x$	0	1	2	3	4
$P(X = x)$	0.1	k	2k	2k	k

Find (a) k
(b) $P(X < 2)$ (2)

Q.14. Find the particular solution of:
$r\frac{dr}{d\theta} + \cos\theta = 5$ at $r = \sqrt{2}$ and $\theta = 0$ (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. In ΔABC, if $a \cos A = b \cos B$ then prove that the triangle is either a right angled or an isosceles triangle. (3)

Q.16. Are the four points A(1, –1, 1), B(–1, 1, 1), C(1, 1, 1) and D(2, –3, 4) co-planar? Justify your answer. (3)

Q.17. Find the difference between the slopes of the lines given by
$(\tan^2\theta + \cos^2\theta)x^2 - 2xy\tan\theta + (\sin^2\theta)y^2 = 0$ (3)

Q.18. Find the vector equation of the line passing through the point $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and perpendicular to the vectors $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}-\hat{j}+\hat{k}$. (3)

Q.19. Let $\bar{a}$ and $\bar{b}$ be non-collinear vectors. If vector $\bar{r}$ is co-planar with $\bar{a}$ and $\bar{b}$ then prove that there exists unique scalars t_1 and t_2 such that $r = t_1 a + t_2 b$. Hence find t_1 and t_2 for $\bar{r} = \hat{i}+\hat{j}, \bar{a} = 2\hat{i}-\hat{j}, \bar{b} = \hat{i}-2\hat{j}$. (3)

Q.20. Find the equation of the plane passing through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the origin. (3)

Q.21. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\sqrt{\frac{3-x}{3+x}}\right)$ (3)

Q.22. Find the approximate value of $f(x) = x^3 + 5x^2 - 2x + 3$ at $x = 1.98$. (3)

Q.23. Evaluate: $\int \frac{\sin(x+a)}{\cos(x-b)}\,dx$ (3)

Q.24. Solve the differential equation $dr + (2r\cot\theta + \sin 2\theta)\,d\theta = 0$ (3)

Q.25. Let $X \sim B(10, 0.2)$. Find
(a) $P(X = 1)$
(b) $P(X \geq 1)$ (3)

Q.26. Find the expected value, variance and standard deviation of r.v. X whose p.m.f. is given as:

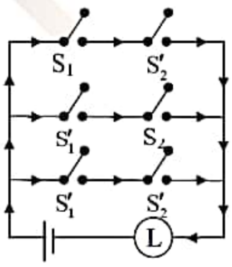
$X = x$	1	2	3
P(X)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

(3)

SECTION – D

Attempt any FIVE of the following questions: **[20]**

Q.27. Give an alternative arrangement for the following circuit, so that the new circuit has minimum switches:



(4)

Q.28. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then find A^{-1} by Adjoint method. (4)

Q.29. In ΔABC, D and E are points on BC and AC respectively such that $BD = 2DC$ and $AE = 3EC$. Let P be the point of intersection of AD and BE. Find ratio $\frac{BP}{PE}$ using the vector method. (4)

Q.30. A firm manufactures two products A and B on which profit earned per unit are ₹3 and ₹4 respectively. Each product is processed on two machines M_1 and M_2. The product A requires one minute of processing time on M_1 and two minutes on M_2, while product B requires one minute on M_1 and one minute on M_2.
Machine M_1 is available for use not more than 450 minutes, while M_2 is available for 600 minutes during any working day.
Find the number of units of products A and B to be manufactured to get maximum profit. (4)

Q.31. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is differentiable function of x such that the composite function $y = f[g(x)]$ is a differentiable function of x then prove that: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

Hence find $\frac{d}{dx}\left(\frac{1}{\sqrt{\sin x}}\right)$ (4)

Q.32. Evaluate: $\int x^2 \sin 3x\,dx$ (4)

Q.33. Prove that:

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx, \quad \text{if f is an even function}$$
$$= 0, \quad \text{if f is an odd function}$$

Hence find the value of $\int_{-1}^{1} \tan^{-1}x\,dx$ (4)

Q.34. Find the area of the ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Hence write area of $\frac{x^2}{25} + \frac{y^2}{16} = 1$ (4)