SETS

A well defined collection of objects, is called a set. Sets are usually denoted by the capital letters A, B, C, X, Y, Z etc. The elements of a set are represented by small letters a, b, c, x, y, z etc.

If 'a' is an element of a set A, then we say that 'a' belongs to A. The word 'belongs to' denoted by Greek symbol $\in$ (epsilon)

Thus, in notation form, 'a' belongs to set A is written as $a \in A$ and 'b' does not belong to set A is written as $b \notin A$.

eg: (i) If $A = \{1, 2, 3, 4, 5\}$ then $3 \in A$ and $6 \notin A$

(ii) If P being the set of perfect square numbers, then $36 \in P$ but $5 \notin P$.

NOTE: Objects, elements and members of a set are synonymous terms.

DIFFERENCE BETWEEN NOT WELL DEFINED AND WELL DEFINED COLLECTIONS.

NOT WELL DEFINED COLLECTION	WELL DEFINED COLLECTION
1) A group of intelligent students	A group of students scoring more than 95% marks of your class.
2) A group of most talented writers of India	A group of odd natural numbers less than 25
3) Group of pretty girls	Group of girls of class XI of your school.

SOME OF THE SETS USED PARTICULARLY IN MATHEMATICS

Important

$N \rightarrow$ The set of Natural Numbers

$Z \rightarrow$ The set of all integers

$Q \rightarrow$ The set of all rational numbers

$R \rightarrow$ The set of real numbers (rational and irrational numbers)

$Z^+ \rightarrow$ The set of positive integers

$Q^+ \rightarrow$ The set of positive rational numbers

$R^+ \rightarrow$ The set of positive real numbers (positive rational and irrational numbers)

REPRESENTATIONS OF SETS

Sets are generally represented by following two ways:

1. Roaster Form or Tabular Form or Listing Method
2. Set-Builder Form or Rule Method

ROASTER FORM / TABULAR / LISTING METHOD

- In this form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within curly braces. { }

Example:

(i) The set of all natural numbers less than 10 is represented in roster form as $\{1,2,3,4,5,6,7,8,9\}$

(ii) The set of prime numbers is $\{2,3,5,7\ldots\}$
Here, three dots tells us that the list of prime numbers continue indefinetly

SET BUILDER FORM / RULE METHOD

In set builder form, all the elements of a set possess a single common property which is not possessed by any element outside the set.

Example:

In the set $\{a,e,i,o,u\}$, all the elements possess a common property, namely, each of them is a vowel in the English alphabet, and no other letter possess this property.
Denoting this set by V, we write:

$V = \{x : x$ is a vowel in English alphabet$\}$

SET NOTATION

SET LISTING METHOD

$\{1,3,5,7,9\}$

SET BUILDER NOTATION

$\{x \in N \mid 0 < x < 11,\ x \text{ is odd}\}$

- x: REPRESENTS ALL THE NUMBERS
- $\in$: THAT BELONG TO
- N: NATURAL NUMBERS
- $\mid$: SUCH THAT
- $0 < x < 11,\ x$ is odd: GENERAL PROPERTY OF ALL THE NUMBERS IN THE SET

INTERVALS:

Intervals are basically subsets of R. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows:

OPEN INTERVAL *(Important)*

$(a,b) = \{x : a < x < b\}$

i.e. end points are not included

CLOSED INTERVAL

$[a,b] = \{x : a \leq x \leq b\}$

i.e. end points are also included. This is possible only when both a and b are finite.

SEMI OPEN OR SEMI CLOSED INTERVAL

$(a,b] = \{x : a < x \leq b\}$;

$[a,b) = \{x : a \leq x < b\}$

INFINITE INTERVALS

The infinite intervals are defined as follows:

(i) $(a,\infty) = \{x : x > a\}$

(ii) $[a,\infty) = \{x : x \geq a\}$

(iii) $(-\infty,\infty) = R$

TYPES OF SETS

- **NULL SET OR EMPTY SET**: A set having no element in it is called an empty set/null set/void set, it is denoted by ϕ or $\{\}$ ϕ
- **SINGLETON SET**: A set consisting of a single element.
- **FINITE SET**: A set which has only finite numbers of elements.
- **ORDER OF A FINITE SET**: The number of elements in a finite set A is called the order of this set and denoted by O(A) or n(A). It is also called the cardinal number of the set.
  Eg: $A = \{a,b,c,d\} \Rightarrow \boxed{n(A) = 4}$
- **INFINITE SET**: A set which has an infinite number of elements.
- **EQUAL SETS**: Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A. If sets A and B are equal, we write $\boxed{A = B}$ and if A and B are not equal then $\boxed{A \neq B}$.

EQUIVALENT SETS: Two finite sets A and B are equivalent if the number of elements are same. i.e. $n(A) = n(B)$

eg: $A = \{1,3,5,7\}$, $B = \{a,b,c,d\}$ $\Rightarrow n(A) = 4$ and $n(B) = 4$

$\Rightarrow$ A and B are equivalent sets.

Equal sets are always equivalent but equivalent sets are/may not be equal.

SUBSET AND SUPERSET:

Let A and B be two sets. If every element A is an element B then A is called a subset of B and B is called superset of A. We write it as $A \subseteq B$

eg: $A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6,7\}$

$\Rightarrow$ $A \subseteq B$ If A is not a subset of B then we write $A \not\subseteq B$

PROPER SUBSET:

If A is a subset of B but $A \neq B$ then A is a proper subset of B and we write $A \subset B$. Set A is not proper subset of A so this is improper subset of A.

NOTE: (i) Every set is a subset of itself

(ii) Empty set ϕ is a subset of every set

(iii) $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$

(iv) The total number of subsets of a finite set containing n elements is 2^n

(v) Number of proper subsets of a set having n elements is $2^n - 1$

(vi) Empty set ϕ is proper subset of every set except itself.

POWER SET:

Let A be any set. The set of all subsets of A is called powerset of A and is denoted by P(A).

For eg: power set of $A = \{1,2\}$ is $P(A) = \{\{\},\{1\},\{2\},\{1,2\}\}$

UNIVERSAL SET:

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by U.

eg: If $A = \{1,2,3\}$, $B = \{2,4,5,6\}$, $C = \{1,3,5,7\}$ then $U = \{1,2,3,4,5,6,7\}$ can be taken as the universal set.

SOME OPERATION ON SETS

(i) UNION OF TWO SETS : $A \cup B = \{x : x \in A \text{ or } x \in B\}$

eg: $A = \{1,2,3\}$, $B = \{2,3,4\}$ then $A \cup B = \{1,2,3,4\}$

(ii) INTERSECTION OF TWO SETS : $A \cap B = \{x : x \in A \text{ and } A \in B\}$

eg: $A = \{1,2,3\}$, $B = \{2,3,4\}$ then $A \cap B = \{2,3\}$

(iii) DIFFERENCE OF TWO SETS : $A - B = \{x : x \in A \text{ and } x \notin B\}$. It is also written as $A \cap B'$.

Similarly: $B - A = B \cap A'$

eg: $A = \{1,2,3\}$, $B = \{2,3,4\}$; $A - B = \{1\}$

(iv) SYMMETRIC DIFFERENCE OF SETS : It is denoted by $A \Delta B$ and $A \Delta B = (A-B) \cup (B-A)$

(v) COMPLEMENT OF A SET : $A' = \{x : x \notin A \text{ but } x \in U\} = U - A$

eg: $U = \{1, 2 \ldots\ldots, 10\}$, $A = \{1,2,3,4,5\}$ then $A' = \{6,7,8,9,10\}$

(vi) DISJOINT SETS : If $A \cap B = \phi$, then A, B are disjoint

eg: If $A = \{1,2,3\}$, $B = \{7,8,9\}$ then $A \cap B = \phi$

SOME PROPERTIES OF COMPLEMENT SETS

COMPLEMENT LAWS

(i) $A \cup A' = U$

(ii) $A \cap A' = \phi$

DE-MORGAN'S LAW

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

LAW OF DOUBLE COMPLEMENTATION

$(A')' = A$

LAW OF EMPTY SET & UNIVERSAL SET

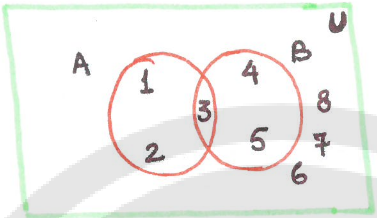
$\phi' = U$ and $U' = \phi$

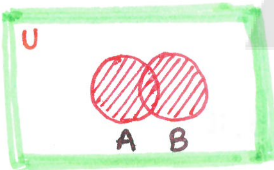
* These laws can be verified by using Venn diagrams.

VENN DIAGRAM:

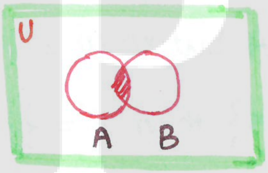
Most of the relationships between sets can be represented by means of diagrams which are known as venn diagrams. These diagrams consist of a rectangle for universal set and circles in the rectangle for subsets of universal set. The elements of the sets are written in respective circles.

For eg: If $A = \{1,2,3\}$, $B = \{3,4,5\}$, $U = \{1,2,3,4,5,6,7,8\}$, then their venn diagram is:

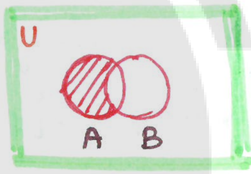




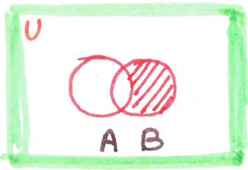
$A \cup B$



$A \cap B$

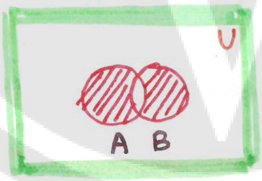


$A - B$

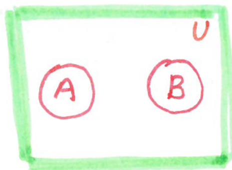


$B - A$

A'



$(A \Delta B) = (A-B) \cup (B-A)$



DISJOINT

- LAWS OF ALGEBRA OF SETS (PROPERTIES OF SETS)

COMMUTATIVE LAW $(A \cup B) = B \cup A$; $A \cap B = B \cap A$

ASSOCIATIVE LAW $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

DISTRIBUTIVE LAW $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

DE-MORGAN LAW $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

IDENTITY LAW $A \cap U = A$; $A \cup \phi = A$

COMPLEMENT LAW $A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$

IDEMPOTENT LAW $A \cap A = A$, $A \cup A = A$

NOTE: (i) $A - (B \cup C) = (A - B) \cap (A - C)$, $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cap \phi = \phi$, $A \cup U = U$

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS:

If A, B, C are finite sets and U be the finite Universal set then;

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A - B) = n(A) - n(A \cap B)$

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(iv) Number of elements in exactly two of the sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$$

(v) Number of elements in exactly one of the sets A, B, C

$$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$$