Chapter-1 : Units and Dimension

Measurement :-

Measurement is essentially a comparison process. To determine the value of a physical quantity, we compare it with a standard reference. The result of this comparison gives us a number (magnitude), and the reference quantity is known as the unit.

Unit :-

A unit is a defined and accepted standard used to measure physical quantities of the same kind. It's universally accepted and helps ensure consistency in measurements.

Standard

A standard refers to the physical realization or representation of a unit.

Formula

Measurement = Numerical value (n) × Unit (u)

Eg:- Length of rod = 8m → 8 is numerical value, m(meter) is the unit.

Fundamental Physical Quantities and Units

These are basic quantities that do not depend on other physical quantities for their definition. The units used to measure them are called fundamental units.

Eg: In the MKS system :

Mass → kilogram

Length → meter

Time → second

Derived Physical Quantities and Units

Quantities that are formed by combining fundamental quantities are known as derived physical quantities. The units used to express them are called derived units.

Eg:- Velocity → m/s

Density → kg/m^3

Force → $kg.m/s^2$

Systems of Units

Historically, different systems of units were used. Today, the SI (International system of Units) is globally accepted. It includes seven base quantities.

1. CGS system : centimeter, gram, second

2. FPS system : foot, pound, second

3. MKS system : meter, kilogram, second

4. SI system : meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), candela (cd).

Supplementary units :-

Apart from the seven base units, there are two supplementary units :

Radian (rad) — plane angle

Steradian (sr) — solid angle

Advantages of the SI system

- Universally accepted
- Rational and coherent
- Metric based and user friendly
- Compatible with CGS and MKS systems
- Uses decimal notation for ease of conversion

Other common units of length :

Used to measure large or small distances

→ 1 Astronomical Unit (AU) = 1.496×10^{11} m

→ 1 Light year = 9.46×10^{15} m

→ 1 Parsec = 3.08×10^{16} m

→ 1 Fermi (f) = 10^{-15} m

→ 1 Angstrom (Å) = 10^{-10} m

Parallax Method

Used to measure distances to celestial objects.

Parallax is the apparent shift in the position of an object when viewed from two different positions.

Formula : $\text{Distance to star} = \dfrac{b}{\theta}$

where b = baseline , θ = parallax angle

Measuring small sizes

- Electron microscopes and tunneling microscopes help in measuring atomic sizes.
- Rutherford's α-particle experiment estimated nuclear sizes.

Time measurement devices

- pendulum clocks, mechanical watches, quartz watches
- Cesium atomic clocks – accuracy of 1 part in 10^{13}

Mass measurement at Atomic scale

SI unit of mass = kilogram

Atomic scale : unified atomic mass unit

$= 1.66 \times 10^{-27}$ kg

Base Quantity	Name	Symbol	SI Units Definition
Length	metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second. *(1983)*
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. *(1889)*
Time	second	s	The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. *(1967)*
Electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. *(1948)*
Thermodynamic Temperature	kelvin	K	The kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. *(1967)*
Amount of substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. *(1971)*
Luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. *(1979)*

Significant figures

The significant figures are a measure of accuracy of a particular measurement of a physical quantity. Significant figures in a measurement are those digits in a physical quantity that are known reliably plus the first digit which is uncertain.

→ The rules for determining the number of significant figures :-

(i) All non-zero digits are significant.

(ii) All zeros between non-zero digits are significant.

(iii) All zeros to the right of the last non-zero digit are not significant in numbers without decimal point.

(iv) All zeros to the right of a decimal point and to the left of a non-zero digit are not significant.

(v) All zeros to the right of a decimal point and to the right of a non-zero digit are significant.

(vi) In addition and subtraction, we should retain the least decimal place among the values operated, in the result.

(vii) In multiplication and divison, we should express the result with the least number of significant figures as associated with the least precise number in operation.

(vii) If scientific notation is not used :

(a) for a number greater than 1, without any decimal, the trailing zeros are not significant.

(b) for a number with a decimal, the trailing zeros are significant.

Error :-

The value we measure for a physical quantity is usually not exactly the true value. This is because all measuring instruments give results that are only close to the real value, not exact. This small difference is called uncertainity, and it is known as error. Any quantity we calculate using these measured values will also have some error.

Types of errors :-

→ Least count error
- Due to limited resolution of instrument
- Max error = least count

→ Instrumental error
- Due to faulty calibration or condition change
- May include zero error
- Needs correction

→ Random error

- Varies in repeated measurements
- follows normal distribution

→ Accidental error

- Unexpected large deviations
- too high or too low values
- discarded from calculations

→ Systematic error

- consistent bias (positive or negative)
- Eg : air buoyancy, heat loss
- can be corrected
- Sources : (i) instrumental error
  (ii) imperfect technique
  (iii) personal error

Absolute error, relative error and percentage error

* Absolute error $= |v_A - v_E|$

* Percentage error $= \left| \dfrac{v_A - v_E}{v_E} \right| \times 100\%$

* Relative error $= \left| \dfrac{v_A - v_E}{v_E} \right|$

v_A = approximate (measured) value

v_E = exact value

→ Combination of errors

$$Z = A \times B$$

$$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$$

$$= AB \pm B\Delta A \pm A\Delta B \pm \Delta A \Delta B$$

$$1 \pm \frac{\Delta Z}{Z} = 1 \pm \frac{\Delta A}{A} + \frac{\Delta B}{B} + \left(\frac{\Delta A}{A}\right)\left(\frac{\Delta B}{B}\right)$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

→ Dimensions :

The dimensions of a physical quantity are the powers to which the fundamental units of mass, length and time must be raised to represent a given physical quantity.

⇒ Dimensional formula :-

The dimensional formula of a physical quantity is an expression telling us how and which of the fundamental quantities enter into the unit of that quantity.

→ Applications of Dimensions

The concept of dimensions and dimensional formulae are put to the following uses :

(i) Checking the results obtained.

(ii) Conversion from one system of units to another.

(iii) Deriving relationships between physical quantities.

(iv) Scaling and studying of models.

The underlying principle for these uses, is the principle of homogeneity of dimensions. According to this principle, the net dimensions of the various physical quantities on both sides of a permissible physical relation must be the same; also only dimensionally similar quantities can be added to or subtracted from each other.

- If a given physical quantity has a dimensional formula $M^a L^b T^c$, then

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

where M_1, L_1, T_1 and M_2, L_2, T_2 are the units of mass, length and time in two systems and n_1 and n_2 the numerical values of the physical quantity in these unit systems.