12/02/2022 Ch 18 : Trigonometric Identities

Exercise 18

(1) $\sin A = \frac{3}{5}$

$\therefore \cos^2 A = 1 - \sin^2 A$

$= 1 - \left(\frac{3}{5}\right)^2$

$\cos^2 A \Rightarrow 1 - \frac{9}{25} = \frac{25 - 9}{25}$

$\cos^2 A = \frac{16}{25}$

$\therefore \boxed{\cos A = \frac{4}{5}}$ ans.

$\tan A = \frac{\sin A}{\cos A}$

$\tan A = \frac{3}{5} \times \frac{5}{4}$

$\therefore \boxed{\tan A = \frac{3}{4}}$ ans

$\cot A = \frac{\cos A}{\sin A}$

$\cot A = \frac{4}{5} \times \frac{5}{3}$

$\therefore \boxed{\cot A = \frac{4}{3}}$ ans

$\sec A = \frac{1}{\cos \theta}$

$\sec A = \frac{1}{4} \times 5$

$\therefore \boxed{\sec A = \frac{5}{4}}$ ans

$\operatorname{cosec} A = \frac{1}{\sin}$

$\operatorname{cosec} A = \frac{1}{3} \times 5$

$\therefore \boxed{\operatorname{cosec} A = \frac{5}{3}}$ ans

(2) $\sec A = \frac{17}{8}$

$\cos A = \frac{1}{\sec A} \Rightarrow \cos A = \frac{1}{17} \times 8$

$\therefore \boxed{\cos A = \frac{8}{17}}$ ans

$\sin^2 A = 1 - \cos^2 A \Rightarrow \sin^2 A = 1 - \left(\frac{8}{17}\right)^2$

$\sin^2 A = \frac{289 - 64}{289} \Rightarrow \sin A = \sqrt{\frac{225}{289}}$

$\therefore \boxed{\sin A = \frac{15}{17}}$ ans

$\operatorname{cosec} A = \frac{1}{\sin A} \Rightarrow \boxed{\operatorname{cosec} A = \frac{17}{15}}$ ans

$\tan A = \frac{\sin A}{\cos A} \Rightarrow \tan A = \frac{15}{17} \times \frac{17}{8}$

$\therefore \boxed{\tan A = \frac{15}{8}}$ ans

$$\cot A = \frac{1}{\tan A} \Rightarrow \boxed{\cot A = \frac{8}{15}} \text{ ans.}$$

(3) $\sin^2 A = 1 - \cos^2 A$

$\sin A = \sqrt{1 - \cos^2 A}$ ans.

$\cos^2 A = 1 - \sin^2 A$

$\therefore \cos A = \sqrt{1 - \sin^2 A}$ ans.

$$\tan A = \frac{\sin A}{\cos A}$$

$$\therefore \tan A = \frac{\sin A}{\sqrt{1 - \sin^2 A}} \text{ ans.}$$

$$\sec A = \frac{1}{\cos A}$$

$$\therefore \sec A = \frac{1}{\sqrt{1 - \sin^2 A}} \text{ ans.}$$

(4) $$\tan A = \frac{1}{\sqrt{3}}$$

$$\therefore \boxed{\cot A = \sqrt{3}} \text{ ans.}$$

$\sec^2 A = 1 + \tan^2 A$

$\sec^2 A = 1 + \left(\frac{1}{\sqrt{3}}\right)^2$

$$\sec^2 A = \frac{3 + 1}{3}$$

$$\therefore \sec A \Rightarrow \sqrt{\frac{4}{3}} = \boxed{\frac{2}{\sqrt{3}}} \text{ ans.}$$

$$\cos A = \frac{1}{\sec A}$$

$$\therefore \boxed{\cos A = \frac{\sqrt{3}}{2}} \text{ ans.}$$

$$\sin A = \tan A \times \cos A$$

$$\sin A = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$$\therefore \boxed{\sin A = \frac{1}{2}} \text{ ans.}$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$\therefore \boxed{\operatorname{cosec} A = 2} \text{ ans.}$$

(5) $12 \operatorname{cosec}\theta = 13$

$$\operatorname{cosec}\theta = \frac{13}{12}$$

$$\sin\theta = \frac{12}{13}$$

$$\cos^2\theta = 1 - \sin^2\theta \Rightarrow \cos^2\theta = 1 - \left(\frac{12}{13}\right)^2$$

$$\cos^2\theta = \frac{169 - 144}{169} \Rightarrow \cos\theta = \boxed{\frac{25}{169}}$$

$$\cos\theta = \frac{5}{13}$$

$$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{\boxed{2 \times \frac{12}{13}} - \boxed{3 \times \frac{5}{13}}}{\boxed{4 \times \frac{12}{13}} - \boxed{9 \times \frac{5}{13}}}$$

$$\Rightarrow \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}} = \frac{\frac{24-15}{13}}{\frac{48-45}{13}}$$

$$\Rightarrow \frac{9}{13} \Rightarrow \frac{\cancel{9}^{3}}{\cancel{13}} \times \frac{\cancel{13}}{\cancel{3}} = \boxed{3} \text{ ans.}$$

$$\frac{3}{13}$$