TGT/PGT
PHYSICS
Revision Book

Importent Facts, Formulas & Oneliners
Chapter, Topic & Subtopic Wise

Useful for : TGT/PGT/LT-GRADE/NVS/KVS/DSSSB/GIC/GDC/Assistant Professor EMRS/AWES/DIET/AEES and Other Competitive Exam

Chief Editor
A. K. Mahajan

Written & Compiled By
YCT Expert Team
Computer Graphics by

INDEX

- Fundamental Physical Constants ---------- 4
- Basic Mathematics used in Physics ---------- 5-10
- **General Physics (Mechanics) ---------- 11-125**
  - Units, Dimensions and Measurements ---------- 11
  - Scalar and Vectors ---------- 20
  - Curl, Divergence, Gauss & Stokes' Theorem & their Application ---------- 27
  - Motion in a Straight Line ---------- 32
  - Motion in a Plane ---------- 39
  - Newton's Law of Motion ---------- 50
  - Friction ---------- 56
  - Work, Energy and Power ---------- 60
  - Rotational Motion ---------- 67
  - Gravitation ---------- 76
  - Elasticity, Elasticity Constants & their Interrelations ---------- 84
  - Beam, Bending of Beams & Torsion of Cylindrical Bodies ---------- 89
  - Hydrostatics ---------- 91
  - Hydrodynamics ---------- 95
  - Surface Tension ---------- 100
  - Special Theory of Relativity ---------- 103
  - Exam Oriented Objective Problems ---------- 108-125
- **Heat & Thermodynamics ---------- 126-160**
  - Thermal Properties of Matter ---------- 126
  - Modes of Heat transfer (Conduction, Convection & Radiation) ---------- 132
  - Kinetic Theory of Gases ---------- 136
  - Thermodynamics ---------- 141
  - Exam Oriented Objective Problems ---------- 150-160
- **Ray optics ---------- 161-187**
  - Light ---------- 161
  - Reflection of Light ---------- 161
  - Mirror ---------- 162
  - Refraction of Light ---------- 167
  - Lens ---------- 170
  - Prism and Scattering of Light ---------- 175
  - Human Eye ---------- 176
  - Optical Instruments ---------- 178
  - Exam Oriented Objective Problems ---------- 181-187

- **Wave optics ---------- 188-207**
  - Introduction ---------- 188
  - Superposition of waves ---------- 190
  - Coherence ---------- 191
  - Interference of light ---------- 192
  - Diffraction of light ---------- 196
  - Polarisation & Polaroids ---------- 197
  - Exam Oriented Objective Problems ---------- 201-207
- **Wave & Oscillations ---------- 208-249**
  - Simple Harmonic Motions ---------- 208
  - Free, Forced, Damped Oscillations & Resonance ---------- 214
  - Travelling Waves ---------- 223
  - Standing Waves ---------- 227
  - Sound Waves and Doppler's Effect ---------- 232
  - Exam Oriented Objective Problems ---------- 241-249
- **Electricity & Magnetism ---------- 250-335**
  - Electrostatics ---------- 250
  - Capacitance & Capacitors ---------- 261
  - Current electricity & Heating effect of current ---------- 268
  - Magnetism and Matter ---------- 279
  - Alternating Current ---------- 285
  - Magnetic effect of current & Magnetism ---------- 298
  - Electromagnetic Induction ---------- 311
  - Electromagnetic waves ---------- 318
  - Exam Oriented Objective Problems ---------- 321-335
- **Modern Physics ---------- 336-367**
  - Atomic Physics ---------- 336
  - Nuclear Physics ---------- 342
  - Electrons, Photons and X-Rays ---------- 348
  - Exam Oriented Objective Problems ---------- 358-367
- **Electronics ---------- 368-390**
  - Introduction of Solids ---------- 368
  - Energy Bands of solids ---------- 372
  - Semiconductors ---------- 373
  - PN Junction and Diodes ---------- 375
  - Transistors ---------- 380
  - Logic Gates & Truth Table ---------- 382
  - Exam Oriented Objective Problems ---------- 390
- **Communications & Communications System ---------- 391-400**
  - Exam Oriented Objective Problems ---------- 399-400

Fundamental Physical Constants

S.No.	Physical Quantity	Symbol	Value	Units
1.	Speed of light	c	3×10^{8}	m/s
		h	6.63×10^{-34}	J s
2.	Planck's constant	$\hbar = h/2\pi$	1.055×10^{-34}	J s
		hc	1242	eV-nm
3.	Gravitation constant	G	6.67×10^{-11}	$m^{3}\ kg^{-1}\ s^{-2}$
4.	Acceleration due to gravity (at sea level)	g	9.81	ms^{-2}
5.	Boltzmann constant	k	1.38×10^{-23}	J/K
6.	Volume of Ideal gas (0°C Temperature and 1 atm Pressure)	V	22.4	$L\ mol^{-1}$
7.	Universal gas constant	R	8.314	J/(mol K)
8.	Mechanical Equivalent of Heat	J	4.184	$J\ cal^{-1}$
9.	Avogadro's number	NA	6.023×10^{23}	mol^{-1}
10.	Charge of electron	e	1.602×10^{-19}	C
11.	Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^{2}
12.	Permeability of vacuum	ϵ_0	8.85×10^{-12}	F/m
13.	Coulomb constant	$\frac{1}{4\pi\epsilon_0}$	9×10^{9}	$N\ m^{2}/C^{2}$
14.	Faraday constant	F	6485	C/mol
15.	Mass of electron	m_e	9.1×10^{-31}	kg
16.	Charge of electron	e	1.6×10^{-19}	C
17.	Unified atomic mass unit	1 u	1.661×10^{-27}	kg
18.	Electron volt	eV	1.6×10^{-19}	J
19.	Energy Equivalent of 1 u	uc^{2}	931.5	MeV
20.	Mass of proton	m_p	1.6726×10^{-27}	kg
21.	Mass of neutron	m_n	1.6749×10^{-27}	kg
22.	Atomic mass unit	u	1.66×10^{-27}	kg
			931.49	MeV/c^{2}
23.	Stefan-Boltzmann constant	σ	5.67×10^{-8}	$W/(m^{2}\ K^{4})$
24.	Rydberg constant	R_∞	1.097×10^{7}	m^{-1}
25.	Bohr magneton	μ_B	9.27×10^{-24}	J/T
26.	Bohr radius	a_0	0.529×10^{-10}	m
27.	Standard atmosphere	1 atm	1.01325×10^{5}	Pa
28.	Wien displacement constant	b	2.9×10^{-3}	mK

Basic Mathematics Used in Physics

Quadratic Equation

- Roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Sum of roots : $x_1 + x_2 = -\dfrac{b}{a}$
- Produce of roots $x_1 x_2 = \dfrac{c}{a}$

Binomial theorem

- $(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3 + ...$
- $(1-x)^n = 1 - nx + \dfrac{n(n-1)}{2!}x^2 - \dfrac{n(n-1)(n-2)}{3!}x^3 + ...$

If $x <<$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

Logarithm

- $\log mn = \log m + \log n$
- $\log \dfrac{m}{n} = \log m - \log n$
- $\log m^n = n \log m$
- $\ln m = \log_e m = 2.303 \log_{10} m$
- $\log 2 = 0.3010$ & $\ln 2 = 0.693$
- $\log 3 = 0.4771$ & $\ln 3 = 1.098$

Componendo and dividendo theorem

- If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

Algebraic Expressions:

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a - b)^3 = a^3 - b^3 - 3ab(a-b)$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
- $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
- $(a + b)(a - b) = a^2 - b^2$

Arithmetic progression - AP

$a, a + d, a + 2d, a + 3d, ..., a + (n-1)d$ here d = common difference

Sum of n terms $= S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Note: (i) $1+2+3+4+5.....+n=\frac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+.....+n^2=\frac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^3+3^3+.....+n^3=\left[\frac{n(n+1)}{2}\right]^2$

■ Geometrical progression - GP

a, ar, ar^2, ar^3, ... here, r = common ratio

Sum of n terms $S_n=\frac{a(1-r^n)}{1-r}$ Sum of ∞ terms $S_\infty=\frac{a}{1-r}$

■ Trigonometry

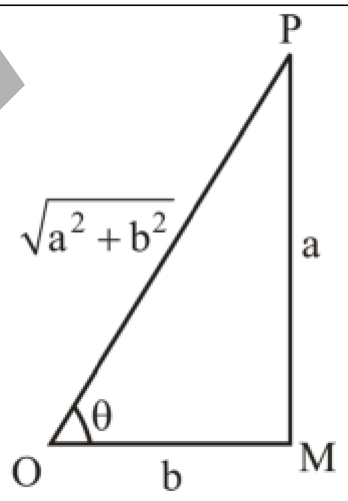
$\tan\theta=\frac{\text{Perpendicular}}{\text{Base}}=\frac{MP}{OM}$ $\cot\theta=\frac{\text{Base}}{\text{Perpendicular}}=\frac{OM}{MP}$

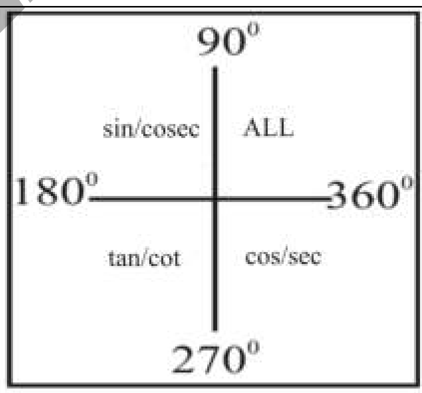
$\sec\theta=\frac{\text{Hypotenuse}}{\text{Base}}=\frac{OP}{OM}$ $\cos\theta=\frac{\text{Base}}{\text{Hypotenuse}}=\frac{OM}{OP}$

$\sin\theta=\frac{a}{\sqrt{a^2+b^2}}$ $\cos\theta=\frac{b}{\sqrt{a^2+b^2}}$

$\tan\theta=\frac{a}{b}$ $\text{cosec}\,\theta=\frac{1}{\sin\theta}$

$\sec\theta=\frac{1}{\cos\theta}$ $\cot\theta=\frac{1}{\tan\theta}$





$\sin(90°+\theta)=\cos\theta$	$\sin(180°-\theta)=\sin\theta$	$\sin(-\theta)=-\sin\theta$	$\sin(90°-\theta)=\cos\theta$
$\cos(90°+\theta)=-\sin\theta$	$\cos(180°-\theta)=-\cos\theta$	$\cos(-\theta)=\cos\theta$	$\cos(90°-\theta)=\sin\theta$
$\tan(90°+\theta)=-\cot\theta$	$\tan(180°-\theta)=-\tan\theta$	$\tan(-\theta)=-\tan\theta$	$\tan(90°-\theta)=\cot\theta$
$\sin(180°+\theta)=-\sin\theta$	$\sin(270°-\theta)=-\cos\theta$	$\sin(270°+\theta)=-\cos\theta$	$\sin(360°-\theta)=-\sin\theta$

cos (180º + θ) = – cos θ   cos (270º – θ) = – sin θ   cos (270º + θ) = sin θ   cos (360º – θ) = cos θ

tan (180º + θ) = tan θ   tan (270º – θ) = cot θ   tan (270º + θ) = – cot θ   tan (360º – θ) = – tan θ

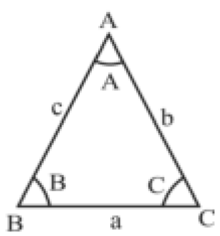
$\theta \rightarrow$	0° (0)	30° $\left(\frac{\pi}{6}\right)$	45° $\left(\frac{\pi}{4}\right)$	60° $\left(\frac{\pi}{3}\right)$	90° $\left(\frac{\pi}{2}\right)$	120° $\left(\frac{2\pi}{3}\right)$	135° $\left(\frac{3\pi}{4}\right)$	150° $\left(\frac{5\pi}{6}\right)$	180° (π)	270° $\left(\frac{3\pi}{2}\right)$	360° (2π)
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0	∞	0

Trigonometric Identities-

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \mathrm{cosec}^2\theta$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin 2A = 2\sin A\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- **For small θ**

$\sin\theta \approx \theta, \cos\theta \approx 1$ $\tan\theta \approx \theta, \sin\theta \approx \tan\theta$

- **Some Integration and Differentiation Formulae**

Differentiation	Integration
$\frac{d}{dx}[\text{Constant}(k)] = 0$	$\int dx = x + C$
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
$\frac{d}{dx}\ln\lvert x\rvert = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}e^{\alpha x+\beta} = \alpha e^{\alpha x+\beta}$	$\int e^{\alpha x+\beta} dx = \frac{1}{\alpha}e^{\alpha x+\beta} + C$
$\frac{d}{dx}a^x = a^x \ln a$	$\int a^x dx = \frac{1}{\ln a}a^x + C$
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$	$\int \frac{dx}{1+x^2} = -\cot^{-1}x + C$
$\frac{d}{dx}\sec^{-1}x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} = \sec^{-1}x + C$
$\frac{d}{dx}\mathrm{co}\sec^{-1}x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} = -\mathrm{co}\sec^{-1}x + C$
$\frac{d}{dx}[k\bullet f(x)] = k\bullet f'(x)$	$\int \ln x\,dx = x\ln x - x + C$
$\frac{d}{dx}[f(x)\pm g(x)] = f'(x)\pm g'(x)$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$
$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$
$\frac{d}{dx}f(g(x)) = f'(g(x))\bullet g'(x)$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$	$\int \mathrm{cosec}\,x\,dx = -\ln\lvert\mathrm{cosec}\,x + \cot x\rvert + C$

■ Maxima & Minima of a function y = f(x)

- For maximum value $\frac{dy}{dx} = 0 \;\&\; \frac{d^2y}{dx^2} = -ve$
- For minimum value $\frac{dy}{dx} = 0 \;\&\; \frac{d^2y}{dx^2} = +ve$

■ Average of a varying quantity

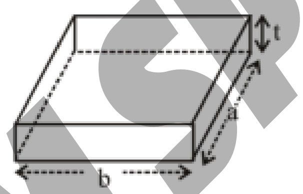
- If $y = f(x)$ than $\langle y \rangle = \bar{y} = \frac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

■ Formulae for determination of area

- Area of a square = $(\text{side})^2$
- Area of rectangle = length × breadth
- Area of a triangle $= \frac{1}{2}\times \text{base}\times\text{height}$

- Area of a trapezoid $= \frac{1}{2} \times (\text{distance between parallel sides}) \times (\text{sum of parallel sides})$
- Area enclosed by a circle = πr^2 (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base × height
- Area of curved surface of cylinder $= 2\pi r\ell$ $(r = \text{radius and } \ell = \text{length})$
- Area of whole surface of cylinder $= 2\pi r(r + \ell)$ $(\ell = \text{length})$
- Area of ellipse = πab (a & b are semi major and semi minor axis respectively)
- Surface area of a cube = 6 $(\text{side})^2$
- Total surface area of a cone $= \pi r^2 + \pi\ell$ where $= \pi r(r^2 + h^2)$ = lateral area

■ Formulae for determination of volume:



- Volume of a rectangular slab = length × breadth × height = abt
- Volume of a cube = $(\text{side})^3$
- Volume of a sphere $= \frac{4}{3}\pi r^3$ (r = radius)
- Volume of a cylinder = $= \pi r^2 \ell$ (r = radius and ℓ is length)
- Volume of a cone $= \frac{1}{3}\pi r^3 h$ (r = radius and h is height)

■ KEY POINTS:

- To convert an angle from degree to radian, we have to multiply if by $\frac{\pi}{180^\circ}$ and to convert an angle from radian to degree, we have to multiple it by $\frac{\pi}{180^\circ}$.
- By help of differentiation, if y is given, we can find $\frac{dy}{dx}$ and by help of integration, if $\frac{dy}{dx}$ is given, we can find y.
- The maximum and minimum values of function [A cosθ + B sinθ] are $\sqrt{A^2 + B^2}$ and $\sqrt{A^2 - B^2}$ respectively.

01.

General Physics (Mechanics)

(1) Unit, Dimensions and Measurement

■ Physical Quantity-

- A quantity which can be measured directly or indirectly or can be explained and expressed in the form of laws of physics are called physical quantity.
- A physical quantity is completely represented by its magnitude and unit.
- The magnitude of physical quantity and unit are inversely proportional to each other. Larger the unit smaller will be the magnitude.

Types of physical Quantity -

- **Ratio (Numerical value only)-** When a physical quantity is a ratio of two similar quantities. It has no unit. For **example-**

$$\text{Relative density} = \frac{\text{Density of Object}}{\text{Density of Water}}$$

- **Scalar-** A physical quantity which has magnitude only and do not have any direction.
  Example- Work, Energy, Length, Time.
- **Vector-** A physical quantity which has magnitude and direction both.
  Example- Displacement, Velocity, Acceleration etc.
- **Units-** Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called unit.

■ System of units-

- A system of unit is a complete set of unit. It is used to measure all kinds of fundamental and derived quantities.

Some system of units are as follows-

Physical quantity	CGS	MKS	FPS
Length	cm	meter	foot
Mass	gram	kg	pound
Time	second	second	second

Fundamental and Derived Unit-

- **Fundamental unit-** The units of those physical quantities which can neither be derived from one another, nor they can be further resolved into more simpler units. Example:- Units of Mass, Length etc.
- **Derived Unit-** Those units of physical quantities which are derived from units of fundamental quantities are Called Derived units. Example:- Units of Velocity, Acceleration, Force, Work etc.

S.I. Unit-

- The S.I. unit is the international system of units. This system contains seven fundamental units and two supplementary fundamental units.

Fundamental quantities in S.I. System and their units-

Sr. No.	Physical Quantity	Name of unit	Symbol of unit
1.	Mass	Kilogram	kg
2.	Length	Meter	m
3.	Time	Second	s
4.	Temperature	Kelvin	K
5.	Luminous intensity	Candela	Cd
6.	Electric Current	Ampere	A
7.	Amount of Substance	Mole	Mol

Supplementary S.I Unit- (Dimensionless Unit)

Sr. No.	Physical Quantity	Name of unit	Symbol of unit
1.	Plane angle	Radian	rad
2.	Solid angle	Steradian	Sr

■ Dimension of Physical Quantity-

- The powers to which fundamental quantities must be in order to express the given physical quantity is called its dimension.
- It is used to express derived quantity in terms of fundamental quantities.

For example- Force = Mass× Acceleration

$$= \frac{\text{Mass} \times \text{Velocity}}{\text{Time}}$$

$$= \text{Mass} \times \text{Length} \times \text{Time}^{-2}$$

$$= [MLT^{-2}]$$

S.I. Prefixes-

- The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	mili	m
10^{9}	giga	G	10^{-6}	mirco	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	K	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Units of Important Physical Quantities-

Physical Quantity	Unit	Physical Quantity	Unit
Angular Acceleration	Rad-s^{-2}	Frequency	Hertz
Moment of inertia	Kg-m^2	Resistance	Ohm
Self inductance	Henry	Surface tension	Newton/m
Magnetic Flux	Weber	Universal Gas Constant	Joule K^{-1} Mol^{-1}
Pole Strength	A-m	Dipole-moment	Coulomb-meter
Dynamic Viscosity	Pascal sec or kg/ms	Stefan Constant	Watt m^{-2} K^{-4}
Kinematic Viscosity	m^2/s	Permittivity of free space (ε_0)	$Coulomb^2/N\text{-}m^2$
Reactance	Ohm	Permeability of free space (μ_0)	Weber/A-m
Specific heat	J/Kg°C	Planck's constant	Joule-sec
Strength of magnetic field	Tesla	Entropy	J/K
Astronomical distance	Parsec	Angular Speed	Rad/sec

Dimensions of important Physical Quantities -

Physical Quantity	Dimensions	Physical Quantity	Dimensions
Momentum	$M^1L^1T^{-1}$	Capacitance	$M^{-1}L^{-2}T^4A^2$
Calorie	$M^1L^2T^{-2}$	Modulus of rigidity	$M^1L^{-1}T^{-2}$
Latent heat capacity	$M^0L^2T^{-2}$	Magnetic permeability	$M^1L^1T^{-2}A^{-2}$
Self inductance	$M^1L^2T^{-2}A^{-2}$	Solar constant	$M^1L^0T^{-3}$
Coefficient of thermal conductivity	$M^1L^1T^{-3}\theta^{-1}$	Magnetic flux	$M^1L^2T^{-2}A^{-1}$
Power	$M^1L^2T^{-3}$	Current density	$M^0L^{-2}T^0A^1$
Impulse	$M^1L^1T^{-1}$	Young's Modulus	$M^1L^{-1}T^{-2}$
Hole mobility of a semiconductor	$M^{-1}L^0A^1T^2$	Magnetic field intensity	$MT^{-2}A^{-1}$
Bulk modulus of elasticity	$M^1L^{-1}T^{-2}$	Magnetic induction	$M^1T^{-2}A^{-1}$
Light year	$M^0L^1T^0$	Permittivity	$M^{-1}L^{-3}T^4A^2$
Thermal resistance	$M^{-1}L^{-2}T^3\theta$	Electric field	$M^1L^1T^{-3}A^{-1}$
Coefficient of Viscosity	$M^1L^{-1}T^{-1}$	Resistance	$ML^2T^{-3}A^{-2}$

Physical quantities which are dimensionless-

Sr. No.	Physical Quantity	Dimensional Formula
1.	Specific gravity	$[M^0L^0T^0]$
2.	Strain	
3.	Angle (θ)	
4.	Avogadro's number (N)	
5.	Reynold's number (N_R)	
6.	Refractive Index (μ)	
7.	Mechanical equivalent of heat (J)	
8.	Dielectric Constant (K) or relative permittivity	
9.	Relative density	
10.	Trigonometric-ratios	
11.	Distance gradient	
12.	Relative permeability	

Physical quantities which have same dimensional formula -

S.No.	Physical Quantity	Dimensional Formula
1.	Speed or velocity	$M^0L^1T^{-1}$
2.	Velocity of light in Vacuum (c)	
3.	Distance travelled in n^{th} second ($S_n{}^{th}$)	
4.	Relative Velocity	
5.	Frequency (v)	$M^0L^0T^{-1}$
6.	Angular frequency	
7.	Angular velocity (ω)	
8.	Velocity Gradient	
9.	Work	$M^1L^2T^{-2}$
10.	Moment of force	
11.	Torque	
12.	Internal energy	
13.	Potential energy	
14.	Kinetic energy	
15.	Heat energy	
16.	Light energy	
17.	Coefficient of elasticity	$M^1L^{-1}T^{-2}$
18.	Pressure	
19.	Stress	
20.	Young's Modulus	
21.	Bulk Modulus	
22.	Modulus of rigidity	
23.	Energy density	
24.	Force	$M^1L^1T^{-2}$
25.	Weight	
26	Thrust	
27.	Energy gradient	
28.	Tension	
29.	Acceleration	$[M^0L^1T^{-2}]$
30.	Acceleration due to gravity	
31.	Gravitational field intensity	
32.	Plank's Constant (h)	$[M^1L^2T^{-1}]$
33.	Angular momentum	
34.	Mass	$[M^1L^0T^0]$
35.	Momentum	$[M^1L^1T^{-1}]$

36.	Impulse	
37.	Length	
38.	Radius of gyration (K)	$[M^0L^1T^0]$
39.	Wavelength (λ)	
40.	Force constant	
41.	Surface tension	$M^1L^0T^{-2}$
42.	Surface energy	
43.	Area	$M^0L^2T^0$
44.	Volume	$M^0L^3T^0$
45.	Density	$M^1L^{-3}T^0$
46.	Universal gravitational constant (G)	$M^{-1}L^3T^{-2}$
47.	Moment of Inertia	$M^1L^2T^0$
48.	Angular acceleration	$M^0L^0T^{-2}$
49.	Rate of flow	$M^0L^3T^{-1}$
50.	Mass per unit length	$M^1L^{-1}T^0$
51.	Rydberg constant (R)	$M^0L^{-1}T^0$
52.	Coefficient of viscosity (η)	$M^1L^{-1}T^{-1}$
53.	kinematic viscosity	$M^0L^2T^{-1}$
54.	Surface potential	$M^0L^2T^{-2}$
55.	Specific Volume	$M^{-1}L^3T^0$
56	Power	$M^1L^2T^{-3}$

■ Dimensional Analysis and Its Applications:-

- Dimensional analysis helps up in deducing certain relations among different physical quantities checking the derivation accuracy and dimensional consistency or homogeneity of various mathematical expressions.
- **Checking dimensional consistency of equations:** According to principle of homogeneity, dimensions of each term on both side of an equation must be same.

Example -

(i) Work done = force × displacement ;

$[ML^2T^{-2}] = [MLT^{-2}] \times [M^0LT^0]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

(ii) $S = ut + \frac{1}{2}at^2$; Dimensionally,

$[S] = [ut] = [at^2]$

$[M^0LT^0] = [M^0LT^{-1}]\,[M^0L^0T] = [M^0LT^{-2}]\,[M^0L^0T^2]$

$[M^0LT^0] = [M^0LT^0] = [M^0LT^0]$

- **To convert a physical quantity from one to another system of units**: $Q_1n_1 = Q_2n_2$; Where Q_1 = unit in 1st system, Q_2 = units in 2nd system n_1 and n_2 be constant value in 1st and 2nd system.

$$\therefore\ n_2 = \frac{Q_1n_1}{Q_2} \Rightarrow n_2 = n_1\left[\frac{Q_1}{Q_2}\right]$$

Example -

Conversion of SI unit of force from Newton (MKS) into dyne (CGS),

Let $n_2 = x$, Q_2 = dyne (g cm s^{-2}), $n_1 = 1$, Q_1 = N(kg m/s^{-2})

Applying $n_2 = \left[\frac{Q_1}{Q_2}\right]n_1$

$$\therefore\quad x = 1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c \text{ or } x = 1\times\left[\frac{kg}{g}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{s}{s}\right]^{-2}$$

$$\text{or } x = 1\times\left[\frac{1000\text{ g}}{1\text{ g}}\right]\times\left[\frac{100\text{ cm}}{\text{cm}}\right]\times\left[\frac{1\text{ s}}{\text{s}}\right] \Rightarrow \qquad x = 10^5$$

$\therefore$ 1 N = 10^5 dynes.

- **Deducing relation among the physical quantities:** If we know the dependency of a physical quantity on the other quantities then using dimensional analysis relation between them can be derived.

Example- Time period of simple pendulum depends on mass of bob (m), length (l) of string and acceleration due to gravity (g).

$\therefore\ T \propto m^a l^b g^c$ **;** $T = k\, m^a l^b g^c$

Here k is a dimensionless constant.

$[M^0L^0T] = [ML^0T^0]^a\,[M^0LT^0]^b\,[M^0LT^{-2}]^c$

$[M^0L^0T] = [M^aL^{b+c}T^{-2c}]$

Comparing the powers, we get a = 0, b + c = 0 and –2c = 1

$\therefore\ c = \frac{-1}{2}$ and $b = \frac{1}{2}$

Substituting values of a, b and c in equation (i), $T\ k\ m^0\ l^{1/2}\ g^{-1/2}$

$\therefore\ T = k\sqrt{\frac{l}{g}}$ (k = a constant cannot be determined using dimensions)

Limitations of dimensional analysis:

- Dimensional method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometric and logarithmic function can't be derived using this method. Formulae containing more than one term which are added or subtracted like S= $ut + \frac{1}{2}at^2$ also can't be derived.
- We cannot determine the value of constants in a relation.
- It gives no information whether a physical quantity is a scalar or a vector.
- In mechanics, the physical quantities depends on more than three quantities cannot be derived by dimensional method as there will be less number (= 3) of equations than the unknowns (> 3). However still we can check the correctness of equation dimensionally.
- Physical quantities having identical dimensions may be of entirely different in nature.

Application of Dimensional Analysis -

1. To convert physical quantity from one system of units to another.
2. To check correctness of a given physical relation.
3. To derive a relationship between different physical quantities.

■ Accuracy and Precision -

- **Accuracy-** Accuracy is the term used to indicate the closeness of a measured value to its accurate value.

$$\text{Accuracy} \propto \frac{1}{\text{Fractional or Relative error}}$$

- **Precision -** Precision is the closeness of a measurement of two or more measurement to each other.

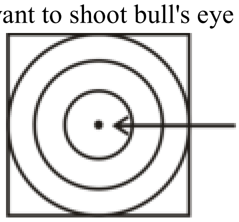
limit of precision = ± ½(least count of instrument)

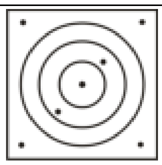
$$\text{Precision} \propto \frac{1}{\text{least count}}$$

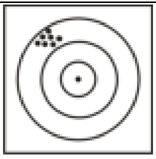
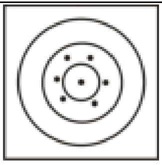
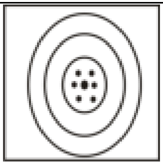
Precision $\propto$ Fractional error/Relative error

Example-

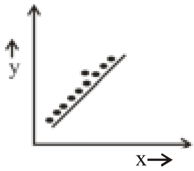
- Suppose, A cadate of soldier want to shoot bull's eye and they have six bullets.


Bull's Eye (target)

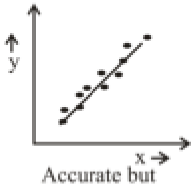
	Conditions	Conclusions
	(i) Maximum bullets are far from target. (ii) The separation between bullets are also large.	Low Accuracy and Low Precision

	(i) Bullets are for from target. (ii) Bullets are very close to each other.	Low Accuracy and High precision
	(i) Bullets are close to target. (ii) Separation between bullets are large.	High Accuracy and Low Precision
	(i) Bullets are close to target. (ii) Separation between bullets are low.	High Accuracy and High Precision

Thus,



Precise but inaccurate data



Accurate but imprecise data

■ Significant figure -

- The figure which express the required degree of accuracy, is called significant figure.
- In significant figure digits carry a meaningful representation.
- Accuracy $\propto$ No. of Significant figure.

Rules for counting the no. of Significant figure in a measured quantity-

(i) All non-zero digits are significant.
Ex- 13.75 S.f = 4

(ii) All zeros between two non- zero digits are significant.
Ex- 100.05 km S.f = 5

(iii) All zeros to the rights of a non-zero digits but to the left of an understood decimal point are not significant.
Ex- 86400 S.f = 3

But such zero are significant if they come from a measurement 86400 sec S.f = 5

(iv) All zero to the rights of a non-zero digits but to the left of the decimal point are significant.
Ex- 648700 S.f = 6

(v) All zeros to the rights of a decimal point are significant.

Ex-	S.f
161 cm	3
161.0 cm	4
161.00 cm	5

(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant.
0.161 cm $\rightarrow$ S.f = 3
0.0161 cm $\rightarrow$ S.f = 3

(vii) The no. of significant figure does not depend on the system of units.
16.4 cm $\rightarrow$ 3
0.164 m $\rightarrow$ 3
0.000164 km $\rightarrow$ 3

(viii) The power of 10 are not counted as significant digit.
$1.4\times10^{-7} \rightarrow 2$
$1.65\times10^{14} \rightarrow 3$

Rounding off a digit-

(i) If the no. lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1.

$x = 6.24$ rounding in 2 significant $\quad x = 6.2$

$x = 5.328$ rounding in 3 significant $\quad x = 5.33$

(ii) If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is increased by 1.

$x = 14.252 \quad x = 14.3$ (upto 3)

(iii) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is unchanged if it is even.

$x = 6.250$ or $x = 6.25$

$x = 6.2$ (upto 2)

(iv) If the digit to be dropped is 5 or 5 followed by zeros then the preceding digits is raised by one if it is odd.

$x = 6.350$ or $x = 6.35$

$x = 6.4$ (rounding off two significant)

■ Error

- The difference between the measured value and the true value of a quantity is known as the error in the measurement.

Error

Combination of error	Propagation of error
• When we know actual error in observation.	• When we do not know the actual error in observation.
• Systematic error	• Random error
• Upto 10%	

Combination of Error-

(i) Absolute error - The difference between the true value and the measured value of a quantity is called an absolute error.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \text{measured/mean value}$$

Absolute error -

$$\Delta x_1 = \bar{x} - x_1$$
$$\Delta x_2 = \bar{x} - x_2$$
$$\Delta x_3 = \bar{x} - x_3$$
$$\vdots$$
$$\Delta x_n = \bar{x} - x_n$$

Absolute error may be + ve or – ve.

(ii) Mean absolute error-

$$\Delta x_{mean} = \frac{|\Delta x_1| + |\Delta x_2| + ... + |\Delta x_n|}{n}$$

Final result can be written as–

$$x = \bar{x} \pm \Delta x_{mean}$$

(iii) Relative or Fractional error-

$$= \frac{\Delta x_{mean}}{x_m} = \frac{\Delta x_{mean}}{\bar{x}}$$

$$\% = \frac{\Delta x_{mean}}{\bar{x}} \times 100$$

Operation	**Formula (Z)**	**Absolute error (ΔZ)**	**Relative error** $\left(\frac{\Delta Z}{Z}\right)$	**Percentage error** $\frac{\Delta Z}{Z}\times 100$
Sum	$A+B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A+B}$	$\frac{\Delta A+\Delta B}{A+B}\times 100$
Difference	$A-B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A-B}$	$\frac{\Delta A+\Delta B}{A-B}\times 100$
Multiplication	$A\times B$	$A\Delta B+B\Delta A$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A+A\Delta B}{B^2}$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A}\times 100$
Root	$A^{1/n}$	$\frac{1}{n}A^{1/n-1}\Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A}\times 100$

Propagation of Error-

Case	**Propagation of Error**
$Z = ax \pm b$	$\delta z = a\cdot\delta x$
$Z = x \pm y$	$\delta z = [(\delta x)^2 + (\delta y)^2]^{1/2}$
$Z = cxy$	$\frac{\delta z}{z} = \left[\left(\frac{\delta x}{x}\right)^2+\left(\frac{\delta y}{y}\right)^2\right]^{1/2}$
$Z = c\frac{x}{y}$	$\frac{\delta z}{z} = \left[\left(\frac{\delta x}{x}\right)^2+\left(\frac{\delta y}{y}\right)^2\right]^{1/2}$
$Z = cx^a$	$\frac{\delta z}{z} = a\frac{\delta x}{x}$
$Z = cx^ay^b$	$\frac{\delta z}{z} = \left[\left(a\frac{\delta x}{x}\right)^2+\left(b\frac{\delta y}{y}\right)^2\right]^{1/2}$
$Z = \sin x$	$\frac{\delta z}{z} = \delta x \cot x$
$Z = \cos x$	$\frac{\delta z}{z} = \delta x \tan x$
$Z = \tan x$	$\frac{\delta z}{z} = \frac{\delta x}{\sin x \cos x}$

■ Measurement -

Pitch -

- The smallest value of length or any other units which can be read directly from a main scale accurately is called pitch.

$$\text{Pitch} = \frac{1\,\text{Unit}}{\text{No. of division in unit}}$$

Least Count-

- The minimum measurement which can be taken accurately by the measuring instrument.

$$\text{L.C} = \frac{\text{Value Measured}}{\text{No. of Division in that Measurement}}$$

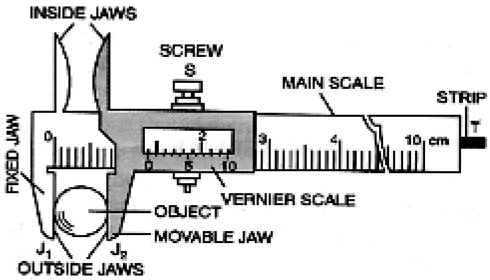
$$\text{or L.C} = \frac{\text{Pitch}}{\text{No. of V.S.D}} \quad \text{(for vernier)}$$

There are basically two types of precision instrument used for measurement:-

(i) Vernier Calipers

(ii) Screw Gauge

Vernier Calipers -

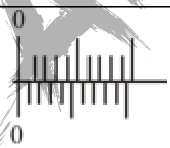
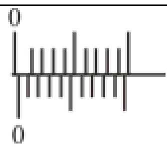
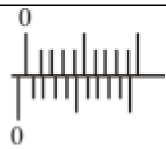


- A vernier scale is an auxiliary scale that slides along the main scale.
- The vernier scale is that in which a certain number, no. of division n on the vernier scale is equal in length to a different number (usually one less) of main-scale divisions.
  nV.S.D = (n – 1) M.S.D
  Where n = number of divisions on the vernier scale
  V.S.D = The length of one division on the vernier scale
  and M.S.D = Length of the smallest main-scale division
- Least count = M.S.D. – V.S.D = $\frac{1}{n}$M.S.D
- In the ordinary Vernier calipers, 1 M.S.D is 1 mm and 10 VSD coincide with 9 MSD

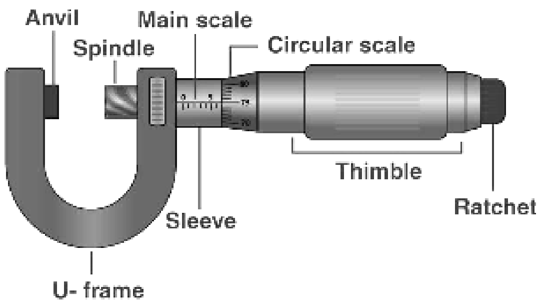
  1 VSD = $\frac{9}{10}$ MSD = 0.9 mm

  Least Count of Vernier 1 M.S.D – 1 V.S.D
  = 1 mm –0.9 mm
  = 0.1 mm

Types of zero error–

	No zero Error	Positive Zero Error	Negative Zero Error
Correction	None	Negative	Positive
Correction formula	None	– Coinciding division × L.C. of V.S.	+ Coinciding division × L.C. of V. S.
			
Corrected reading	Observed reading + correction		
Actual reading	Main scale reading + V.S. reading ± zero correction		

Screw Gauge-



- A Screw Gauge allows a measurement of the size of a body. It is one of the most accurate mechanical devices in common use.
- It consists of a main scale and a thimble
- Least Count = $\frac{\text{Pitch}}{\text{No. of Divisions on Circular Scale}}$
- L.C of Screw Gauge = 0.001 mm

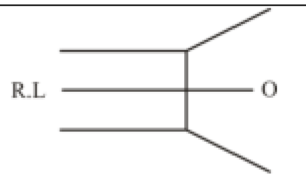
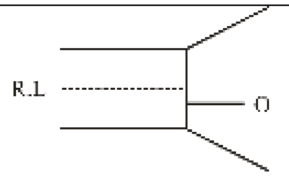
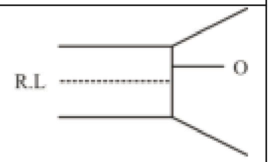
Method of Measurement

Step- I : Find the whole number of mm in the barrel

Step - II: Find the reading of barrel and multiply by 0.01

Step III: Add the value in Step- I and Step -II

Types of zero Error :–

	No zero Error	**Positive zero error**	**Negative zero error**
correction	None	Negative	Positive
Correction Formula	None	+zero error ×L.C	-zero error ×L.C
			
Actual reading	Observed reading -excess reading ×zero error		

Least Count of various measuring Instruments :-

Instrument	**Its least Count**
mm Scale	1 mm
Vernier Calipers	0.1 mm
Screw Gauge	0.001 mm

■ Key points -

- Trigonometric functions sin θ, cos θ, tan θ etc and their arrangements θ are dimensionless.
- Dimension of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$.
- Dimension of integrals $[\int y dx] = [yx]$ we can not add or subtract two physical quantities of different dimensions.
- Independent quantity may be taken as fundamental quantities in a new system of units.
- Measure of a physical quantity = Numerical value of the physical quantity × Size of the unit

  i.e $Q = n \times u$

  Thus, the numerical value (n) is inversely proportional to the size (u) of the unit.

  $n \propto \frac{1}{u}$ or nu = constant.

(2) Scalar and Vectors

■ **Scalars:-** Those physical quantity which require only magnitude but no direction for their complete representation are called Scalars. Ex. Distance, Speed, Work, Mass, Energy, Power, Temperature etc.

■ **Vectors:-** A physical quantity which requires magnitude and direction both, when it is expressed. Ex. Force, Displacements, Momentum, Acceleration, Velocity, Impulse, Pressure, Gravity etc.

Types of Vectors:-

(i) Equal vectors:-

- Two vectors of equal magnitude and having same direction are called equal vector. fig (i)

(ii) Negative Vectors:-

- Two vectors of equal magnitude but having opposite direction are called negative vectors. Fig(ii)

fig. (i)

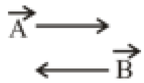


fig. (ii)

(iii) Zero Vector or null Vector:-

- A vector whose magnitude is zero known as a zero or null vector. Its direction is not defined. It is denoted by $\vec{0}$. Velocity of stationary object and resultant of two equal and opposite vectors are the example of null vector.

(iv) Unit vector:-

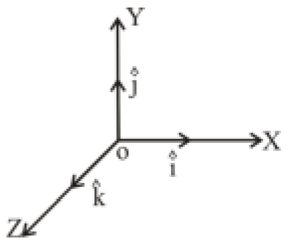
- A vector having unit magnitude is called a unit vector. A unit vector in the direction of vector A is given by

  $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

- A unit vector is unit less and dimensionless vector and represents direction only.

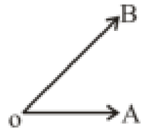
(v) Orthogonal Unit Vector:-

- The unit vectors along the direction of orthogonal axis i.e. x-axis, y-axis and z-axis are called orthogonal unit vectors. They are represented by $\hat{i}, \hat{j}$ and $\hat{k}$



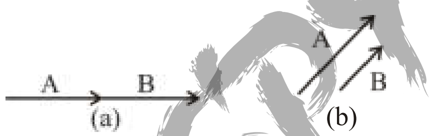
(vi) Co-initial vector :-

- Vectors having a common initial points are called co-initial vectors.



(vii) Colinear Vector :-

- Vector having equal or unequal magnitudes but acting along the same or parallel lines are called colinear vectors.



(viii) Co-planar vector:-

- Vectors acting in the same plane are called coplanar vectors.

(ix) Localised Vectors-

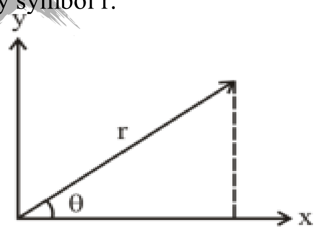
- A vector whose initial point is fixed is called localised vectors.

(x) Non-Localized or free vector-

- A vector whose initial point is not fixed is called a non-localized or free vectors.

(xi) Position Vectors-

- A vector which gives position of an object with reference to the origin of a co-ordinate system is called position vector. It is represented by symbol r.



$\vec{r} = x\hat{i} + y\hat{j}$

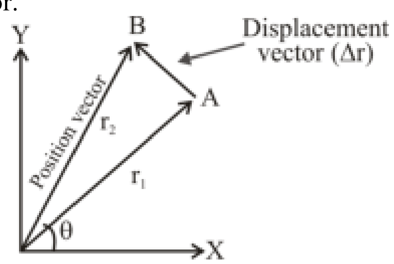
If $\vec{r}$ makes an angle θ with x-axis, then

x= rcos θ and y = rsin θ

$\vec{r} = r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

(xii) Displacement Vector-

- The vector which tells how much and in which direction an object has changed its position in a given interval of time is called a displacement vector.



r_1 & r_2 are position vector.
The displacement vector for AB is-

$\Delta r = r_2 - r_1$

■ Tensors-

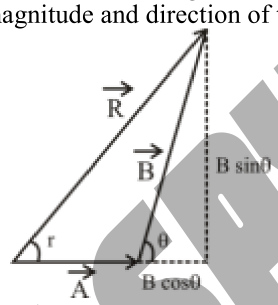
A quantity that has different values in different direction is called Tensor.
Tensors can be classified according to following order-

- **Zero-Order Tensors (Scalars):** Among some of the quantities that have magnitude but not direction are zero-order tensors e.g.: mass density, temperature, and pressure.
- **First-Order Tensors (Vectors):** Quantities that have both magnitude and direction e.g.: velocity, force. The first-order tensor is symbolized with a boldface letter and by and arrow at the top part of the vector, i.e.: $\vec{0}$.
- **Second-Order Tensors:** Quantities that have magnitude and two directions, e.g. stress and strain. The second-order and higher-order tensors are symbolized with a boldface letter.

■ Vector addition-

(i) Triangle law of vector addition -

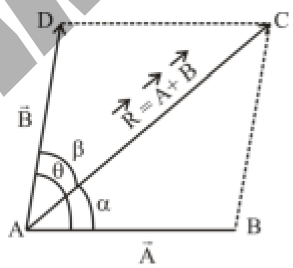
When two vectors are represented as two sides of the triangle with the order of magnitude and direction then the third side of the triangle represents the magnitude and direction of the resultant vector.



- $\vec{R} = \vec{A} + \vec{B}$

(ii) Parallelogram law of vector addition-

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point.



$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC} = \vec{R}$ or $\vec{A} + \vec{B} = \vec{R}$

(a) Resultant of vectors A and B is given by -

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(b) If the resultant vector R subtend an angle β with vector B and α with vector A, then

$$\tan\alpha = \frac{\beta\sin\theta}{A + B\cos\theta} \ \& \ \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

Case1- If A=B then R= $2A\cos\frac{\theta}{2}$ & $\alpha = \frac{\theta}{2}$

Case2- If $\theta = 0°$ then

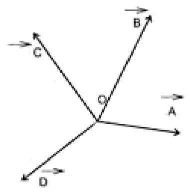
$R_{max} = A+B$

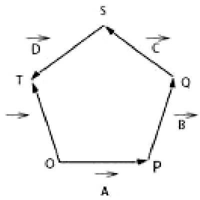
Case3- If $\theta = 180°$ then

$R_{min} = A-B$

(iii) Law of polygon (Addition of more than two vectors)-

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.





$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

OT = OP + PQ + QS + ST

Properties of vector Addition-

1. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
2. $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$ (associative property)
3. $\vec{A} + \vec{0} = \vec{A}$ (additive identity)
4. $\vec{A} + (-\vec{A}) = \vec{O}$ (additive inverse)
5. $|\vec{A} + \vec{B}| \le |\vec{A}| + |\vec{B}|$
6. $m(\vec{A} + \vec{B}) = m\vec{A} + m\vec{B}$ (distributive property)

■ Vector subtraction-

Subtraction of vector B from a vector A is defined as the addition of vector B (negative of vector B) to vector A.

Thus, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

Multiplication of a vector-

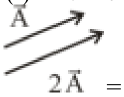
1. By a real no.-

When a vector A is multiplied by a + ve real no. λ gives a vector whose magnitude is changed by the factor λ but the direction is the same as that of $\vec{A}$

$|\lambda\vec{A}| = \lambda|\vec{A}|$

Example (i) if $\lambda > 0$

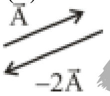
$\vec{A}$



$2\vec{A}$ = direction is same but magnitude is different.

Example (ii) If $\lambda < 0$ then

$\vec{A}$



$-2\vec{A}$ = Magnitude and direction both are different.

2. By a scalar-

Let $\vec{A}$ be a vector and λ be a scalar then the product of it is called a multiplication of a vector by the scalar λ i.e.

$|\lambda\vec{a}| = |\lambda||\vec{a}|$

When a vector is multiplied by a scalar quantity then the magnitude of the vector changes in accordance with the magnitude of the scalar but the direction of the vector remains unchanged.

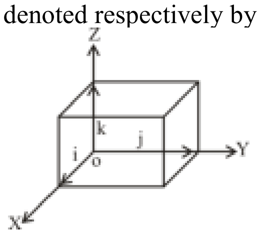
■ Rotation of a vector -

(i) If a vector is rotated through an angle θ, which is not an integral multiple of 2π the vector changes.

(ii) If the frame of reference is rotated or translated the given vector does not change, the components of a vector may change.

■ The rectangular unit vector-

It is an important set of unit vectors and are those vectors having the direction of the positive x,y and z axis of a three dimensional co-ordinate system and denoted respectively by i,j and k



Rectangular component-

When a vector is resolved along two mutually perpendicular directions the components so obtained are called rectangular components of a given vector.

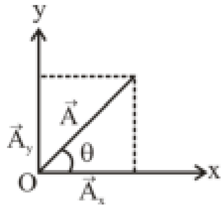
Rectangular components of a vector in a plane-

$\vec{A} = \vec{A}_x + \vec{A}_y$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

If $\vec{A}$ makes an angle θ with x-axis then-

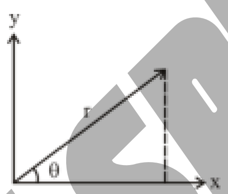
$A_x = A\cos\theta$ $A_y = A\sin\theta$



Magnitude of vector -

$A = \sqrt{A_x^2 + A_y^2}$ $\tan\theta = \frac{A_y}{A_x} \Rightarrow \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$

General vector in x-y plane-



$\vec{r} = x\hat{i} + y\hat{j}$

If $\vec{r}$ makes an angle θ with x-axis, then

$x = r\cos\theta$ and $y = r\sin\theta$

$\vec{r} = r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

Example-

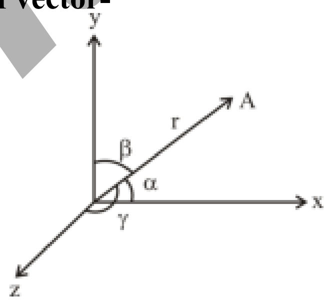
1. construct a vector of magnitude 6 units making an angle of 60° with x-axis-

solution- $\vec{r} = 6\left(\cos 60°\hat{i} + \sin 60°\hat{j}\right) = 3\hat{i} + 3\sqrt{3}\hat{j}$

2. construct an unit vector making an angle of 135° with x-axis-

solution- $\vec{r} = 1\left(\cos 135°\hat{i} + \sin 135°\hat{j}\right) = \frac{1}{\sqrt{2}}\left(-\hat{i} + \hat{j}\right)$

■ Direction cosine of vector-



- Angle made with x-axis $\cos\alpha = \frac{A_x}{A} = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \ell$
- Angle made with y-axis $\cos\beta = \frac{A_y}{A} = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$
- Angle made with z-axis $\cos\gamma = \frac{A_z}{A} = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$

Where, ℓ, m & n are called direction cosines.

- $\ell^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \dfrac{A_x^2 + A_y^2 + A_z^2}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = 1$.
- $\sin^2\alpha + \sin^2\beta + \sin^2\gamma\,\theta = 2$

■ The Dot or Scalar product -

The dot or scalar product of two vector is defined as the product of the magnitudes of A and B and the cosine of the angle θ between them.

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta \qquad (0 \le \theta \le \pi)$

Note:-

- $\vec{A}\cdot\vec{B}$ is a scalar not a vector.
- $\vec{A}\cdot\vec{B}$ is +ve if θ is acute.
- $\vec{A}\cdot\vec{B}$ is –ve if θ is obtuse.
- $\vec{A}\cdot\vec{B}$ is zero if θ is right angle.

Properties of Dot product-

1. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ (Commutative law)
2. $\vec{A}.\left(\vec{B}+\vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (Distributive law)
3. $m\left(\vec{A}\cdot\vec{B}\right) = \left(m\vec{A}\right)\vec{B} = \vec{A}.\left(m\vec{B}\right) = \left(\vec{A}.\vec{B}\right)m$, where m is scalar
4. $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
5. $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
6. If A = $A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and B = $B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$, then

   $\vec{A}.\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

   $\vec{A}.\vec{A} = A^2 = A_1^2 + A_2^2 + A_3^2$

   $\vec{B}\cdot\vec{B} = B^2 = B_1^2 + B_2^2 + B_3^2$
7. $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

Example of dot product-

- Work W = $\vec{F}.\vec{d} = F\,d\cos\theta$ where, F → force, d → displacement
- Power P = $\vec{F}.\vec{v} = F.v\cos\theta$ Where, F → force, v → velocity
- Electric Flux $\phi_E = \vec{E}.\vec{A} = E\,A\cos\theta$ Where, E → Electric field A → Area
- Magnetic flux $\phi_B = \vec{B}.\vec{A} = B\,A\cos\theta$ Where, B → magnetic field A → area
- Potential energy of dipole in uniform field $\vec{U} = -\vec{p}.\vec{E}$ Where, $\vec{p}$ → dipole moment, E → electric field.

■ Cross Product (or vector product)-

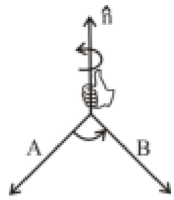
The magnitude of cross or vector product of A and B i.e. A×B is defined as the product of the magnitude of A and B and the sine of the angle θ between then,

$\vec{A}\times\vec{B} = |A||B|\sin\theta\,\hat{n}$

where, $\hat{n}$ is a vector perpendicular to $\vec{A}$ & $\vec{B}$ or their plane and its direction given by right hand thumb rule.

Right hand thumb rule-

Curl the fingers of your right hand from A to B through the smaller angle between them. Then, the direction of thumb represents A×B or $\hat{n}$.



Properties of cross product-

1. $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ (Anti Commutative law)
2. $\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$ (Distributive law)
3. $m\left(\vec{A}\times\vec{B}\right)=\left(m\vec{A}\right)\times\vec{B}=\left(\vec{A}\times\vec{B}\right)m$, where m is scalar
4. $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k};=0$
5. $\hat{i}\times\hat{j}=\hat{k},\hat{j}\times\hat{k}=\hat{i},\hat{k}\times\hat{i}=\hat{j}$
6. If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, then

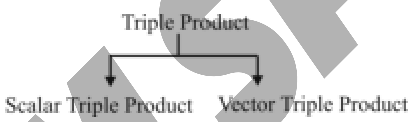
$$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

7. The magnitude of $\vec{A}\times\vec{B}$ is the same as the area of a parallelogram with sides A and B.
8. If $\vec{A}\times\vec{B}=0$ Then $\vec{A}$ and $\vec{B}$ are parallel and $\theta = 0°$

Examples of Cross product-

- Torque $\vec{\tau}=\vec{r}\times\vec{F}$ Where, $\vec{r}\rightarrow$ position vector, F $\rightarrow$ force
- Angular momentum $\vec{J}=\vec{r}\times\vec{p}$ Where, $\vec{r}\rightarrow$ position vector, p $\rightarrow$ linear momentum
- Linear velocity $\vec{V}=\vec{\omega}\times\vec{r}$ Where, $\vec{r}\rightarrow$ position vector, $\omega\rightarrow$ angular velocity
- Torque on dipole placed in electric field $\vec{\tau}=\vec{p}\times\vec{E}$ Where, p $\rightarrow$ dipole moment, E $\rightarrow$ Electric field

■ Triple product-



Scalar Triple Product - It is the dot product of a vector with the cross product of two other vectors. If a, b, c are three vectors, then, their scalar product is $\vec{a}.\left(\vec{b}\times\vec{c}\right)$.

Symbolically it is also written as [a b c] = $\vec{a}.\left(\vec{b}\times\vec{c}\right)$

Properties of Scalar Triple Product-

- The scalar triple product of three vectors is zero if any two of them are parallel, i.e., [a a b] = 0
- [(a + b) c d] = [a c d] + [b c d]
  LHS = [(a + b) c d]
  = (a + b) · (c × d)
  = a· (c × d) + b· (c × d)
  = [a c d] + [b c d]
  = RHS
- [λa b c] = λ [a b c], where λ is a real number.
- The scalar triple product of three non-zero vectors is zero if and only if they are coplanar,
- Since the scalar product is commutative, therefore we have
  - a · (b × c) = (b × c) · a
  - b· (c × a) = (c ×a) · b
  - c· (a × b) = (a × b) · c

Vector Triple Product - The vector triple product is the cross product of a vector with the cross-product of the other two vectors.

Mathematically, it can be represented by $\left(\vec{a}\times\vec{b}\times\vec{c}\right)$

Properties of Vector Triple Product–

- Vector triple product is a vector quantity.
- Unit vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{c}$ is $\pm\dfrac{\left(\vec{a}\times\vec{b}\right)\times\vec{c}}{\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|}$
- $\vec{a}\times\left(\vec{b}\times\vec{c}\right)\neq\left(\vec{a}\times\vec{b}\right)\times\vec{c}$
- $\left[\vec{a}\times\vec{b}\quad\vec{b}\times\vec{c}\quad\vec{c}\times\vec{a}\right]=\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2$

■ Reciprocal vectors-

The set of vectors a*, b*, c* are the reciprocal vectors of a, b, c respectively.

Properties of reciprocal vectors-

(i) $\vec{a}*\vec{a}=1$ (ii) $\vec{a}*\vec{b}=0$ (iii) $\vec{a}*\vec{c}=0$

$\vec{b}*\vec{b}=1$ $\quad \vec{b}*\vec{a}=0$ $\quad \vec{b}*\vec{c}=0$

$\vec{c}*\vec{c}=1$ $\quad \vec{c}*\vec{a}=0$ $\quad \vec{c}*\vec{b}=0$

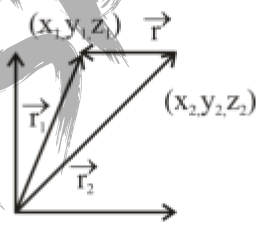
Where,
$$\vec{a}^* = 2\pi\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$$
$$\vec{b}^* = 2\pi\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$$
$$\vec{c}^* = 2\pi\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$$

■ Linearly Independent and dependent vector-

Let $\vec{A},\vec{B}$ and $\vec{C}$ are set of vectors,

- If $\left[\vec{A}\vec{B}\vec{C}\right]=0$ then these set of vectors are linearly dependent and coplanar
- If $\left[\vec{A}\vec{B}\vec{C}\right]\neq 0$ then these set of vectors are linearly independent and non-coplanar
- When a particle moved from (x_1, y_1, z_1) to (x_2, y_2, z_2) then its displacement vector-

$$\vec{r} = \vec{r}_2 - \vec{r}_1 = \left(x_2\hat{i}+y_2\hat{j}+z_2\hat{k}\right)-\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)$$
$$= \left(x_2-x_1\right)\hat{i}+\left(y_2-y_1\right)\hat{j}+\left(z_2-z_1\right)\hat{k}$$



Magnitude -

$$r = \left|\vec{r}\right| = \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

■ Key points-

- A Scalar is a zero rank tensor
- A vector is a first rank tensor.
- Electric current is not a vector as it does not obey the law of vector addition.
- A unit vector has no unit.
- A scalar or a vector can never be divided by a vector
- To a vector only a vector of same type can be added and resultant is a vector of same type.

(3) Curl Divergence, Gauss, Stokes Theorem and their application

■ The Vector differential Operator Del-

$$\nabla \equiv \frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k} \equiv \hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$$

The operator ∇ is also known as nabla.

■ The Gradient-

- Let (x, y, z) be defined as differential at each point in a certain region of space (i.e ϕ defines a differential scalar field). Then the $\nabla\phi$ can be written as-

$$\nabla\phi = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\phi = \left(\frac{\partial\phi}{\partial x}\hat{i}+\frac{\partial\phi}{\partial y}\hat{j}+\frac{\partial\phi}{\partial z}\hat{k}\right)$$

- $\nabla\phi$ defines a vector field.

Note :- The component of $\nabla\phi$ in the direction of a unit vector a is given by $\nabla\phi$.a and is called the directional derivative of ϕ in the direction a. Physically, it is the rate of change of ϕ (x, y, z) in the direction a.

■ The Divergence –

- Let V(x, y, z) = $V_1\hat{i}+V_2\hat{j}+V_3\hat{k}$ can be defined as differential at each point (x, y, z) in a certain region of space (i.e V defines a differential vector field). Then the divergence of V i.e ∇.V can be written as -

$$\nabla\cdot V=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial x}\hat{j}+\frac{\partial}{\partial x}\hat{k}\right)\cdot\left(V_1\hat{i}+V_2\hat{j}+V_3\hat{k}\right)$$

$$\nabla\cdot V=\left(\frac{\partial V_1}{\partial x}+\frac{\partial V_2}{\partial y}+\frac{\partial V_3}{\partial z}\right)$$

Note- $\nabla\cdot V \neq V\cdot\nabla$

■ The Curl -

- If V(x, y, z) is a differential vector field then curl of V can be written as -

$$\nabla\times V=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\times\left(V_1\hat{i}+V_2\hat{j}+V_3\hat{k}\right)$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ V_1 & V_2 & V_3\end{vmatrix}$$

Note :-

- $\nabla\phi$ is vector so we can take divergence and curl of it.

Formula involving ∇ –

- If A and B are differentiable vector functions and ϕ and ψ are differentiable scalar function of positions (x, y, z) then-

1. $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$
2. $\nabla\cdot(A + B) = \nabla\cdot A + \nabla\cdot B$
3. $\nabla\times(A + B) = \nabla\times A + \nabla\times B$
4. $\nabla\cdot(\phi A) = (\nabla\phi)\cdot A + \phi(\nabla\cdot A)$
5. $\nabla\times(\phi A) = (\nabla\phi)\times A + \phi(\nabla\times A)$
6. $\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$
7. $\nabla\times(A\times B) = (B\cdot\nabla)A - B(\nabla\cdot A) - (A\cdot\nabla)B + A(\nabla\cdot B)$
8. $\nabla (A\cdot B) = (B\cdot\nabla)A + (A\cdot\nabla) B + B\times(\nabla\times A) + A\times(\nabla\times B)$
9. $\nabla\cdot(\nabla\phi)=\nabla^2\phi=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}$

   Where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is called the Laplacian operator

10. $\nabla\times(\nabla\phi) = 0$ The curl of the gradient of ϕ is zero.
11. $\nabla\cdot(\nabla\times A) = 0$ The divergence of curl of A is.
12. $\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$

■ Integral Calculus-

(i) Line integral-

- The line integral expression is -

$$I=\int_a^b \vec{A}\cdot\vec{d\ell}\ =0$$

where, $\vec{A}$ = vector field

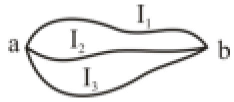
$\vec{d\ell}$ = infinitesimal displacement vectors

$\vec{d\ell}=\ dx\hat{i}+dy\hat{j}+dz\hat{k}$

- The close line integral -

$$\oint_{Path}\vec{A}.\vec{d\ell}$$

- In general the line integral is path dependent.



- There are some special vector function for which the line integral is independent of path

$I = I_1 = I_2 = I_3$

For example -

(a) $I = \int_a^b \vec{A}\cdot\vec{d\ell}$ = path independent (will depends on end points a and b,)

where, $\vec{A}$ = special vector function

(b) $\int \nabla T\cdot\vec{d\ell} = T(\vec{b}) - T(\vec{a})$

= independent of path

$$\oint \nabla T\cdot\vec{d\ell} = 0$$

Where, T is a function and ∇T is grad T

(ii) Surface (Double) integral -

The expression is of the form

$$I = \int_{\text{surface}} \vec{A}\cdot\vec{da}$$

where, $\vec{A}$ = vector function

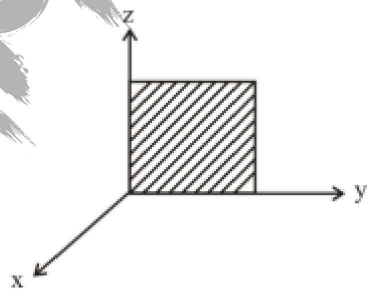
$\vec{da}$ = elementary area is called surface integral.

Case-1 When surface is parallel to fundamental plane (xy, yz, zx)

$\vec{da} = dydz\hat{i}$

$\vec{da} = dzdx\hat{j}$

$\vec{da} = dxdy\hat{k}$



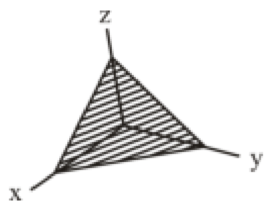
Case-2 When surface is not parallel to fundamental plane

$\vec{da} = da\,\hat{n}$

$\phi = c$ (equation of surface)

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|}$$

$\vec{da} = \dfrac{dxdy}{|\hat{k}\cdot\hat{n}|}\hat{n} \rightarrow$ In xy -plane



$\vec{da} = \dfrac{dydz}{|\hat{i}\cdot\hat{n}|}\hat{n} \rightarrow$ In yz -plane

$\vec{da} = \dfrac{dzdx}{|\hat{j}\cdot\hat{n}|}\hat{n} \rightarrow$ In xz -plane

(iii) Volume Integral -

- The expression of the volume integral is -

$$I = \int T dV$$

Where, T = Scalar function
dV = infinitesimal volume element
dV = dxdydz $\rightarrow$ In Cartesian co-ordinate system

■ Fundamental theorem of Calculus-

- The integral of a derivative is equal to the value of the function at end point or boundary

$$\frac{d}{dx}\int_a^x f(t)dt = f(x) \Rightarrow \int_a^b f(x)dx = F(b) - F(a)$$

Fundamental theorem of Gradient -

- The line integral of gradient is given by the value of the function at the boundaries (a and b)

$$\int_a^b \nabla T \cdot \vec{d\ell} = T(b) - T(a)$$

Where, $\vec{dl}$ = infinitesimal displacement vector in Cartesian co-ordinate.

The Fundamental theorem for Divergence (Green's Theorem, Gauss Divergence Theorem)

This theorem is applicable only for closed surfaces and this theorem is used to convert surface integral into volume integral and vice verse.

$$\oiint_s \vec{A} \cdot \vec{ds} = \oiint_s \vec{A} \cdot \hat{n} ds = \iiint_v (\nabla \cdot \vec{A}) dv$$

Where $\hat{n}$ is the outward normal to s indicating the +ve direction of s.

Fundamental theorem of curl-

- This theorem is applicable only for open surfaces and this theorem is used to convert surface integral into line integral and vice versa.
- If S is an open, two sided surface bounded by a closed, non-intersecting curve C and $\vec{A}$ is vector function of position with continuous derivatives then-

$$\oint_c \vec{A} \cdot \vec{dr} = \iint_s (\nabla \times \vec{A})\hat{n}\, ds = \iint_s (\nabla \times \vec{A}) \cdot \vec{ds}$$

Where C is traversed in the position (counter clockwise direction)

■ Co-ordinate System–

Co-ordinate system	$\vec{dl}$	h_1 h_2 h_3	u_1 u_2 u_3
Cartesian	$dx\hat{i} + dy\hat{j} + dz\hat{k}$	1 1 1	x y z
Spherical	$dr\hat{r} + r\, d\theta\hat{\theta} + r \sin\theta\, d\phi\hat{\phi}$	1 r r sin θ	r θ ϕ
Cylindrical	$ds\,\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z}$	1 S 1	s ϕ z

■ General Expression for Gradient, Divergence and Curl-

(i) Gradient - $\nabla\phi = \left(\frac{1}{h_1}\frac{\partial}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial}{\partial u_3}\hat{u}_3\right)\phi(u_1 u_2 u_3)$

(ii) Divergence - $\nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 A_1) + \frac{\partial}{\partial u_2}(h_1 h_3 A_2) + \frac{\partial}{\partial u_3}(h_1 h_2 A_3)\right]$

(iii) Curl - $\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3}\begin{bmatrix} h_1\hat{u}_1 & h_2\hat{u}_2 & h_3\hat{u}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{bmatrix}$

Formula for Gradient in co-ordinate system -

Cartesian - $\nabla\phi = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\phi(x, y, z)$

Spherical - $\nabla\phi = \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}\right)\phi(r, \theta, \phi)$

$$\text{Cylindrical - }\nabla\phi = \left(\frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}\right)\phi(s, \phi, z)$$

$$\text{Polar - }\nabla\phi = \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta}\right)\phi(r, \theta)$$

Formula for divergence in co-ordinate system-

$$\text{Cartesian - }\nabla\cdot\vec{A} = \left[\frac{\partial}{\partial x}A_1 + \frac{\partial}{\partial y}A_2 + \frac{\partial}{\partial z}A_3\right]$$

$$\text{Spherical - }\nabla\cdot\vec{A} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\, A_1\right) + \frac{\partial}{\partial\theta}\left(r\sin\theta\, A_2\right) + \frac{\partial}{\partial\phi}\left(r\, A_3\right)\right]$$

$$\text{Cylindrical - }\nabla\cdot\vec{A} = \frac{1}{s}\left[\frac{\partial}{\partial s}\left(s\, A_1\right) + \frac{\partial}{\partial\phi}A_2 + \frac{\partial}{\partial z}\left(s\, A_3\right)\right]$$

Formula for curl in co-ordinate system -

$$\text{Cartesian - }\nabla\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$\text{Spherical - }\nabla\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & (r\sin\theta)\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_1 & rA_2 & r\sin\theta A_3 \end{vmatrix}$$

$$\text{Cylindrical - }\nabla\times\vec{A} = \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_1 & sA_2 & A_3 \end{vmatrix}$$

■ Key Points-

(i) ∇ is not a vector, it mimics the behaviour of an ordinary vector.

(ii) ∇ is a vector operator that acts upon:

- A scalar function T : ∇T (the gradient)
- A vector function V : $\nabla\cdot V$ (the divergence)
- A vector function V : $\nabla\times V$ (the curl)

(iii) ∇ is not a vector that multiplies T.

(iv) Operator is meaningless without a function (scalar field, vector field). For eg. without a computer mobile pen drive (operator) is useless.

(v)

Function	Outcome
$\nabla\phi$	Vector
$\nabla\cdot\vec{A}$	Scalar
$\nabla\times\vec{A}$	Vector

(vi) $\nabla\cdot(\nabla\phi)$ = Divergence of gradient

$\nabla\times(\nabla\phi)$ = Curl of gradient

$\nabla(\nabla\cdot\vec{A})$ = Gradient of divergence

$\nabla\cdot(\nabla\times\vec{A}) = 0$ Divergence of curl

$\nabla\times(\nabla\times\vec{A})$ = Curl of curl

(vii) $\hat{i}$, $\hat{j}$ and $\hat{k}$ Constant unit vector and it is taken out from differential and integral sign.

(viii) Surface integral represent flow of vector through the surface or flux.

(ix) In flux through a closed surface become zero then it is Solenoid vector.

$$\oint\hat{A}\cdot\vec{ds} = 0 \text{ then } \vec{\nabla}\cdot\hat{A} = 0$$

(x) Volume integral represent amount of quantity in volume.

(xi) If line integral of a field along closed loop is zero then it is irrotational vector field.

$$L{\cdot}I = \oint \vec{F}\cdot\vec{dr} = 0$$

So, $\nabla \times \vec{F} = 0$

(xii) For conservative field

$$\nabla \times \vec{F} = 0$$

and $F = \nabla\phi$, then

$$\nabla\times(\nabla\,\phi) = 0 \text{ (always)}$$

(xiii) Physically gradient represent normal vector to the surface.

$$\vec{n} = \nabla T$$

$$\hat{n} = \frac{\nabla T}{|\nabla T|}$$

(xiv) The divergence measure how much the vector v spreads out (diverges) from point.

(xv) The curl is $\nabla\times V$ is measure of how much the vector V curl around the point .

(xvi) General form of Laplacian -

$$\nabla^2\varphi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\phi}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\phi}{\partial u_3}\right)\right]$$

(4) Motion in a Straight Line

■ **Mechanics-:** Mechanics is the branch of physics that deals with the condition of rest or motion of the material object around us.

■ **Rest:** When a body does not change its position with respect to time, the body is said to be in rest.

Example: A bed lying in a room is in the state of rest, because it does not change its position with respect to time.

■ **Motion :** When a body changes its position with respect to its surrounding, it is said to be in motion.

Example: A train moving on rails

Rest and motion as relative terms - Rest and motion are relative states. It means an object which is at rest in one frame of reference can be in motion in another frame of reference.

■ **Types of Motion –**

On the basis of direction:-

1. One dimensional Motion- if only one out of three co-ordinates specifying the position of the object with respect to time. Then it is called one dimensional motion or rectilinear motion.

For Example- (i) Motion of car on straight road.
(ii) Motion of a body under gravity.

2.Two dimensional Motion -

If only two out of three co-ordinates specifying the position of the object with respect to time, then the motion is called two dimensional motion.

For Example- (i) A gymnast on a balance beam.
(ii) Motion of planets around the sun.
(iii) A car moving along zig-zag path on a level road.

3. Three dimensional motion -

If all three coordinates specify the position of object with respect to time then it is called 3-D motion

For example- (i) Movement of gyroscope.
(ii) A like flying on a windy day.
(iii) Motion of an aeroplane in space.

On the basis of moving object in space:-

1. Uniform Motion: When moving objects cover equal distances in equal time intervals.

2. Non Uniform Motion: When moving objects cover different distances in unequal time intervals.

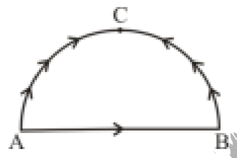
■ Distance and Displacement -

Distance (x)

- Total path ACB traveled by the body between initial and final position in definite interval is called Distance
- It is a scalar quantity.
- It have no direction
- Distance will be always positive.
- Distance have infinite function.

Displacement ($\vec{x}$)

- Displacement is the minimum possible path (AB) between initial and final position.
- It is a vector quantity.
- Its direction will be always from initial to final position.
- It may be +ve, –ve or zero.
- it have only one unique function.



■ Speed and Velocity-

(i) Speed-: The rate of change of position of an object with respect to time in any direction is called its speed.

$$\text{Speed}(V) = \frac{\text{distance travelled}(s)}{\text{time taken}(t)}$$

- It is a scalar quantity
- It is always +ve
- It's S.I unit is m/sec.

Uniform Speed- If a body covers equal distance in equal intervals of time it is said to be moving with uniform speed.

Example- (i) A rotating fan
(ii) A rocket moving in a space.

Variable speed or Non-Uniform speed:-

If a body covers unequal distances in equal intervals of time. It is said to be moving with a variable speed.

Example- (i) A train starting from a station.
(ii) a dog chasing a cat.

Average speed : The ratio of total distance travelled by the object to the total time taken is called average speed.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Total time taken}}$$

Instantaneous speed: If the speed of a body is continuously changing with time. Then the speed at some particular instant during the motion is called instantaneous speed.

For example- Speedometer of a moving automobile measures instantaneous speed.

(ii) Velocity : The rate of change of displacement with respect to time of body in specified direction is called velocity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

- It is a vector quantity.
- It may be +ve, -ve or zero.
- It's S.I. unit is m/sec.

Uniform velocity -

When a body covers equal distances in equal intervals of time in a particular direction the body is said to be moving with uniform velocity.

Non-uniform velocity- when a body covers unequal distances in equal intervals of time in a particular direction the body is said to be non-uniform velocity .

Average Velocity- The ratio of the total displacement to the total time taken by the body is called average velocity.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

Instantaneous Velocity - The velocity of a particle at any instant of time is known as instantaneous velocity

$$\text{Instantaneous velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

■ Acceleration -

The rate of change of velocity with respect to time is known as acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity } (\Delta V)}{\text{Time interval } (\Delta t)}$$

- Its S.I unit is m/sec^2
- It is a vector quantity
- It may be +ve, -ve or zero
- If velocity increases then acceleration is +ve
- If velocity decreases then it is known as retardation and 'it' is -ve.
- If velocity is constant then a = 0 (i.e uniform motion)

Uniform Acceleration - When a body describes equal changes in velocity in equal intervals of time , it is said to be moving with uniform acceleration.

Non- Uniform Acceleration-

If an object is moving with non-uniform acceleration , it means that change in velocity is unequal for equal interval of time.

Average Acceleration-

It is defined as the ratio of total change in velocity in given interval to the total time taken. Unlike acceleration the average acceleration is calculated for a given interval.

Instantaneous Acceleration-

It is defined as the acceleration of body at any instant of time.

$$\text{Instantaneous Acceleration} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

Formula and concept for uniformly accelerated motion in a straight line

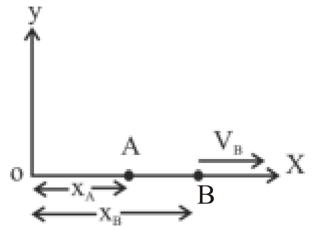
Scalar form	**Vector form**
$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$v^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

- Displacement of a particle in n^{th} second of its motion in uniformly accelerated motion-

$D_n = u + \frac{a}{2}(2n-1)$

■ Relative motion in one Dimension :-

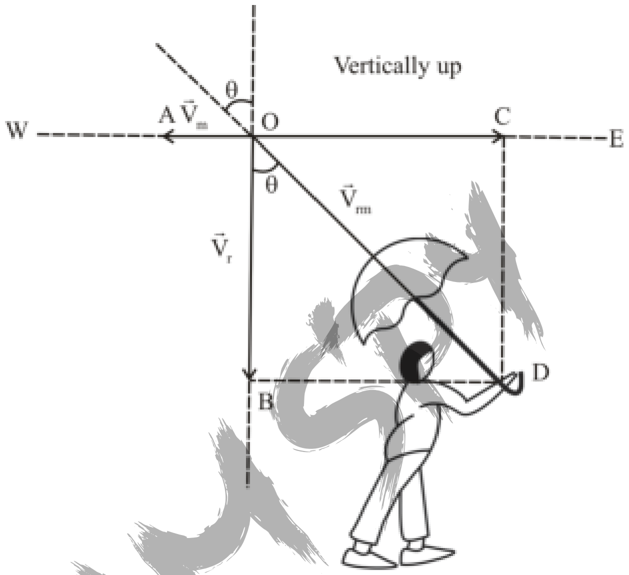
Let A and B are two objects and if $\vec{x}_A$ and $\vec{x}_B$ are their respective displacements with respect to the fixed origin. Then



- The relative displacement of B with respect to A is defined as
  $\vec{x}_{BA} = \vec{x}_B - \vec{x}_A$
- The relative velocity of B with respect to A is defined as-
  $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$
- The relative acceleration of B with respect to A is defined as -
  $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$

Relative velocity of Rain with respect to the Moving man -

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$ represented by $\overrightarrow{OB}$ as shown in



The relative velocity of rain with respect to man $\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$\therefore V_{rm} = \sqrt{V_r^2 + V_m^2 + 2V_rV_m \cos 90^\circ} = \sqrt{V_r^2 + V_m^2}$

If θ is the angle which $\vec{V}_{rm}$ makes with the vertical direction then

$\tan\theta = \dfrac{BD}{OB} = \dfrac{V_m}{V_r} \Rightarrow \theta = \tan^{-1}\dfrac{V_m}{V_r}$

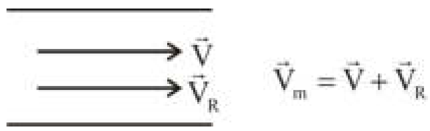
- **Swimming into the River-**

  A man can swim with velocity $\vec{V}$ i.e it is the velocity of man with respect to still water. If water is also flowing with velocity $\vec{V}_R$, then velocity of man relative to ground.

  $\vec{V}_m = \vec{V} + \vec{V}_R$

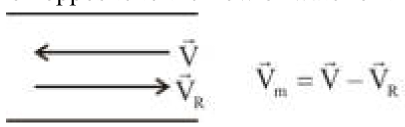
Case I -

- If the swimming is in the direction of flow of water or downstream then-



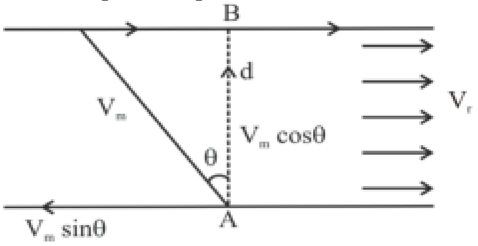
Case II -

- If the swimming is in the direction opposite to the flow of water or then-



Case-III To cross the river from one bank to another bank.

(i) To cross the river in minimum possible path.



d = width of river to reach from A to B,

$$V_m \sin\theta = V_r$$

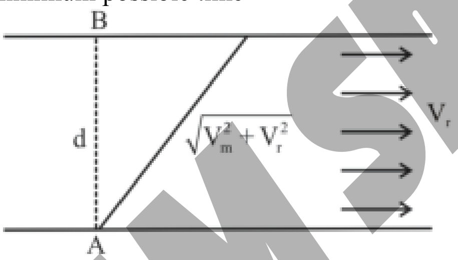
$$\sin\theta = \frac{V_r}{V_m}$$

$$\theta = \sin^{-1}\frac{V_r}{V_m}$$

(ii) Time taken to cross the river -

$$t = \frac{d}{V_m \cos\theta} = \frac{d}{\sqrt{V_m^2 - V_r^2}}$$

(iii) To cross the river in minimum possible time-



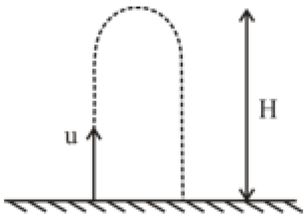
$$t = \frac{d}{V_m \cos\theta}$$

For minimum, $\theta = 0^\circ$

$$\left[t_{min} = \frac{d}{V_m}\right]$$

■ Motion Under Gravity –

- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance), then-



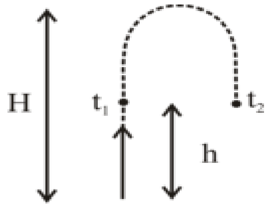
(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$

(iv) Velocity of body at the point of projection = u (downwards)

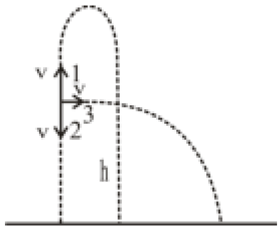
(v) **Galileo's law of odd numbers :** For a freely falling body ratio of successive distance covered in equal time interval 't' $S_1 : S_2 : S_3$ .... = 1 : 3 : 5 : .... : 2n – 1.



(vi) At any point on its path the body will have same speed for upward journey and downward journey.

(vii) If a body throws upward crosses point in time t_1 & t_2 respectively, then height of point $h = \frac{1}{2}gt_1t_2$ and maximum height $H = \frac{1}{2}g\left(t_1 + t_2\right)^2$.

(viii) A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then $t_3 = \sqrt{t_1t_2}$ & height from where the particle was thrown is- $H = \frac{1}{2}g\,t_1t_2$.



Important points about graphical analysis of motion -

- Instantaneous velocity is the slope of position time curve $\left(V = \frac{dx}{dt}\right)$.
- Slope of velocity time curve = instantaneous acceleration $\left(a = \frac{dv}{dt}\right)$.
- V-t curve area gives displacement, $\left[\Delta x = \int vdt\right]$.
- a-t curve area gives change in velocity $\left[\Delta v = \int adt\right]$

■ Key points -

- $\boxed{\text{Displacement}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\text{Velocity}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\text{Acceleration}}$
- Displacement $\leq$ Distance.
- $\frac{\text{Velocity}}{\text{Speed}} \leq 1$
- $\frac{\text{Average velocity}}{\text{Average speed}} \leq 1$
- $\frac{\text{Instantaneous velocity}}{\text{Instantaneous speed}} = 1$
- If distance > |displacement| this implies -
  (a) Atleast at one point in path, velocity is zero.
  (b) The body must have retarded during the motion.
- If particle travels distances S_1, S_2, S_3, ... with speeds V_1, V_2, V_3, ... then,

$$\text{Average speed} = \frac{S_1 + S_2 + S_3}{\left(\frac{S_1}{V_1} + \frac{S_2}{V_2} + \frac{S_3}{V_3} ......\right)}$$

- If particle travels equal distances ($S_1 = S_2 = S$) with velocities V_1, V_2, V_3, ... during time intervals t_1, t_2, t_3 then, $\text{Average speed} = \frac{V_1t_1 + V_2t_2 + V_3t_3}{t_1 + t_2 + t_3 + ...}$

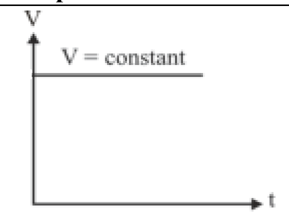
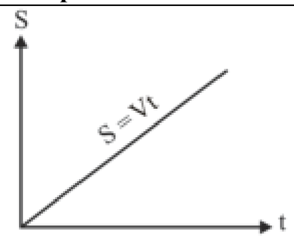
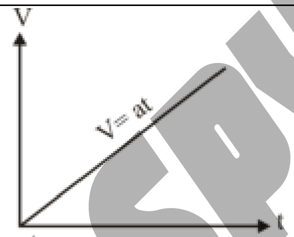
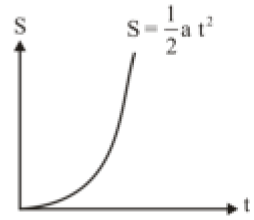
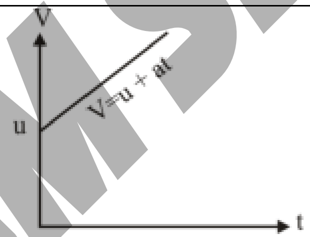
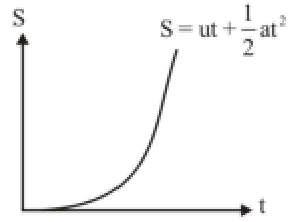
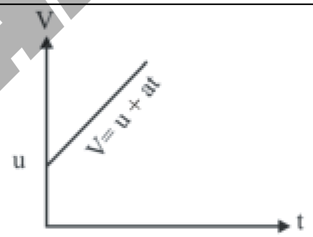
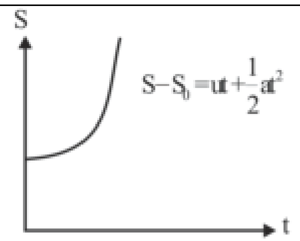
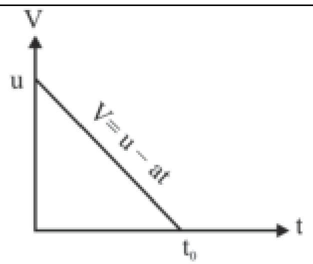
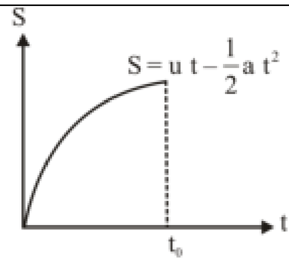
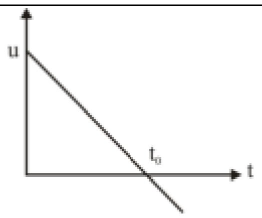
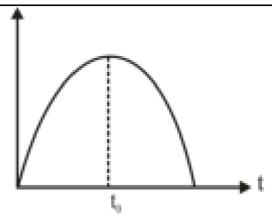
- If particle travels with speed V_1 and V_2 for equal time intervals i.e $t_1 = t_2 = t$, then

  Average speed = $\dfrac{V_1 + V_2}{2}$.

- When a body travels equal distances with speed V_1 and V_2, the average speed (V) is the harmonic mean of two speeds i.e

$$\frac{2}{V} = \frac{1}{V_1} + \frac{1}{V_2}$$

Different Motions and their Graphs :

Different Cases	**V-t Graph**	**S-t Graph**
1. Uniform motion		
2. Uniform accelerated motion with u = 0 at t = 0		
3. Uniformly accelerated with u ≠ 0 at t = 0		
4. Uniformly accelerated motion with u ≠ 0 and S= S_0 at t = 0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

(5) Motion in a Plane

■ Motion in a Plane

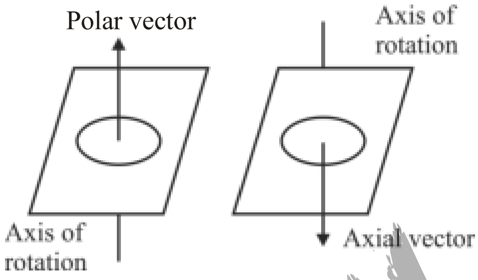
- Motion in a plane is also called as a motion in two dimension.
  For example- circular motion, projectile motion etc.

Polar Vectors - The polar vectors which have a starting point or point of application are called polar vectors.

Example- Displacement, velocity, force etc are polar vectors.

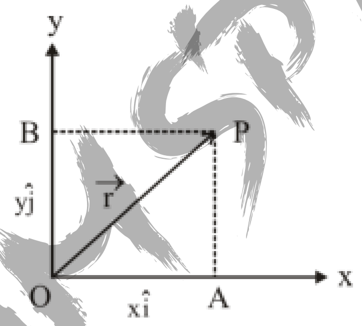
Axial Vectors- The vector which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vector.

Example:- Angular velocity, Torque, Angular momentum etc.



■ Terms related to motion in a plane -

Position vector

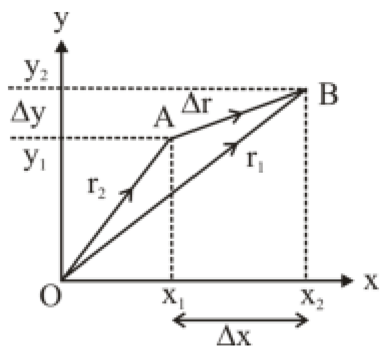


$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$$

$$\vec{r} = x\hat{i} + y\hat{j}$$

- This equation express position vector $\vec{r}$ in terms of its rectangular component x and y.

Displacement Vector -



- In plane, displacement can be represented as -

$$\Delta r = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

- Magnitude of displacement vector

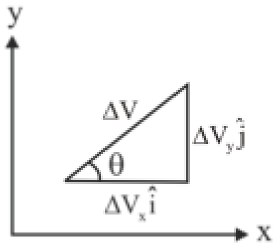
$$|\Delta r| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Direction of the displacement vector Δr is given by -

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

Velocity Vector-

(i) Average Velocity -



$$V_{av} = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{t_2 - t_1}$$

Average velocity in component form-

$$V_{av} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

$$= \Delta V_x \hat{i} + \Delta V_y \hat{j}$$

Direction of the velocity ΔV is given by-

$$\tan\theta = \frac{\Delta V_y}{\Delta V_x}$$

(ii) Instantaneous Velocity-

$$V = \lim_{x \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

$$V = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$V = V_x \hat{i} + V_y \hat{j}$$

Magnitude of Instantaneous Velocity-

$$|V| = \sqrt{V_x^2 + V_y^2}$$

Direction of V is given by-

$$\tan\theta = \frac{V_y}{V_x}$$

Acceleration Vector -

(i) Average Acceleration-

- The average acceleration vector is defined as the rate at which the velocity changes. It is in the direction of the change in velocity $\overrightarrow{\Delta V}$

$$\vec{a}_{av} = \frac{\overrightarrow{\Delta V}}{\Delta t}$$

$$\vec{a}_{av} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

(ii) Instantaneous Acceleration -

- It is defined as the limit of the average acceleration as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}$$

$$\vec{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta V_x}{\Delta t}\hat{x} + \frac{\Delta V_y}{\Delta t}\hat{y} + \frac{\Delta V_z}{\Delta t}\hat{z} \right)$$

■ Motion in a plane with uniform acceleration-

$V_x = V_{ox} + a_x t$

$V_y = V_{oy} + a_y t$

Path of particle under constant Acceleration-

$$x = x_0 + V_{ox}t + \frac{1}{2}a_x t^2 \text{ .... along x-axis}$$

$$y = y_0 + V_{oy}t + \frac{1}{2}a_y t^2 \text{ .... along y-axis}$$

■ Circular Motion -

- When object is moving on a circular path on the circumference of the circle, then the motion is called circular motion.

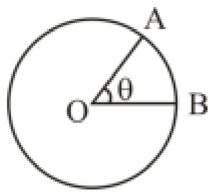
Uniform circular Motion-

- When object is moving on a circular path on the circumference of the circle, covers equal distances in equal intervals of time then the motion is called uniform circular motion.

Angular displacement (θ)-

- It is the angle traced out by the radius vector at the circular path.

Angle $(\theta) = \dfrac{\text{arc}}{\text{radius}}$



It is a vector quantity.

Angular Velocity $(\vec{\omega})$ -

- It is the time rate of change of angular displacement.

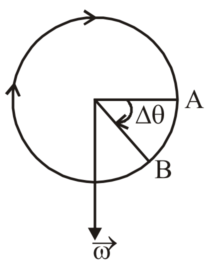
  SI unit is rad/sec .

  $\vec{\omega} = \dfrac{\text{Angular displacement}}{\text{Time taken}}$

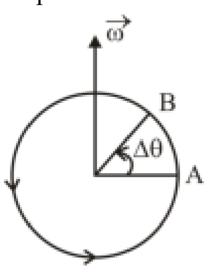
- Instantaneous angular velocity $\omega = \dfrac{d\theta}{dt}$

- Average angular velocity $\vec{\omega}_{av} = \dfrac{\text{Total angular displacement}}{\text{Total time taken}} = \dfrac{\Delta\theta}{\Delta t}$

- For clockwise rotation $\vec{\omega}$ is directed downwards



- For anti-clockwise rotation $\vec{\omega}$ is directed upwards.



■ **Time Period (T):-** The taken by object to complete one revolution on its circular path.
■ **Frequency (ν):-**The number of revolution per unit time on the circular path.
■ **Angular acceleration (α) -**

- It is the time rate of change of angular velocity

$$\alpha = \frac{d\vec{\omega}}{dt}$$

- SI unit radian/second^2
- When a body moves with constant angular velocity, its angular acceleration is zero.

Centripetal Acceleration (a_c) -

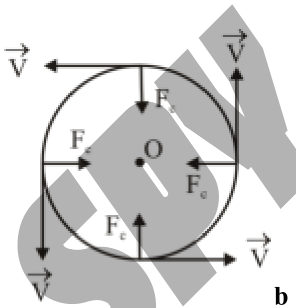
- Acceleration of an object moving uniformly on the circular path and is along the radius towards the centre of the circular path.

$$a_c = \omega^2 r = \frac{V^2}{r} = \omega V \qquad \left\{\because \quad \omega = \frac{V}{r}\right\}$$

$$\vec{a}_c = \vec{\omega} \times \vec{V}$$

Centripetal Force (F_c) -

$$F_c = \frac{mv^2}{r}$$
$$F_c = ma_c$$
$$F_c = m\omega^2 r$$
$$F_c = m\omega v$$
$$\vec{F}_c = m(\vec{v} \times \vec{\omega})$$



b

- The work done by centripetal force is zero.
- Centripetal force is essential for circular motion, without it the body cannot move in circular path.
- The K.E. and angular momentum cannot be increased by centripetal force.

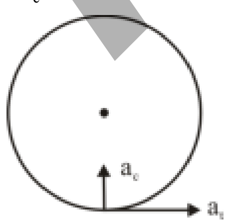
Tangential Acceleration (a_t) -

- The acceleration which acts along the tangent to the circular path.

$$a_t = \alpha r$$
$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

Total acceleration ($\vec{a}$) -

$$\vec{a} = \alpha \times \vec{r} + \vec{\omega} \times \vec{v}$$
$$a = \sqrt{a_t^2 + a_c^2}$$



Where, a_t = Tangential acceleration -
a_c = Centripetal acceleration

Some relations -

(i) Relation between time period and frequency $\nu = \dfrac{1}{T}$

(ii) Relation between frequency angular velocity and time. $\omega = \dfrac{\theta}{t} = \dfrac{2\pi}{T} = 2\pi\nu$

(iii) Relation between linear acceleration and angular acceleration.

$$a = \alpha r$$
$$\vec{a} = \alpha \times \vec{r}$$

■ Motion in Vertical circle -

- Motion in a vertical circle is non-uniform circular motion.

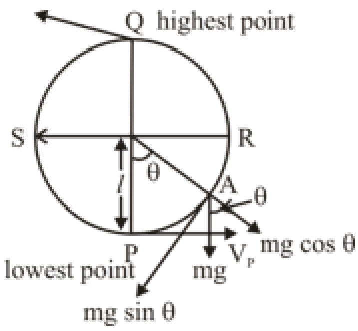
- Tension at the lowest point P

$$T_P = \frac{mV_P^2}{l} + mg$$

- Tension at the highest point Q.

$$T_Q = \frac{mV_Q^2}{l} - mg$$

$$T_Q = \frac{mV_P^2}{l} - 5\,mg$$



- Tension at point R-

$$T_R = \frac{mV_R^2}{l}$$

$$T_R = \frac{mV_P^2}{l} - 2\,mg$$

- $T_P > T_R > T_Q$
- $T_P - T_Q = 6\,mg$
- $T_P - T_R = 3\,mg$
- Tension at any point A -

$$T = \frac{mV^2}{r} + mg\cos\theta$$

- **Minimum velocity for vertical circular motion -**

(a) V_P at $P \geq \sqrt{5gl}$

(b) V_Q at $Q \geq \sqrt{gl}$

(c) V_R at $R \geq \sqrt{3gl}$

- **In case of minimum velocity-**

(a) $T_P \geq 6\,mg$

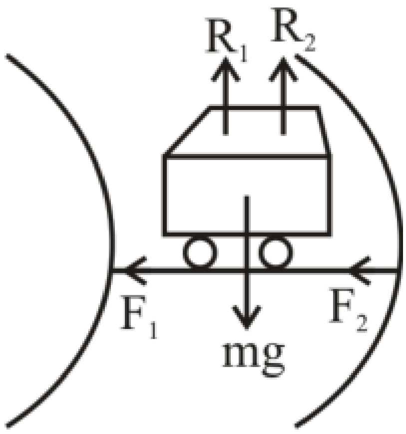
(b) $T_Q = 0$

(c) $T_R \geq 3\,mg$

- If $T_{min} < 0$, the string will slack and the body will fall down from the highest point. Hence, for "looping the loop" or completing the vertical circle $T_{min} \geq 0$.
- If $V_P = \sqrt{2gl}$, velocity and tension becomes zero at R and S and particle will oscillate along semi-circular path.
- If $V_P < \sqrt{2gl}$, velocity becomes zero between P and R and particle oscillate about with the lower point P.
- If $V_P > \sqrt{5gl}$ tension never becomes zero and particle will just complete the circle.
- For leaving the vertical circle somewhere between 90º < θ < 180º. Tension becomes zero (T = 0) at the point of leaving but the velocity will not be zero.

$\sqrt{2gl} < V_P < \sqrt{5gl}$

■ Rounding a level curved Road -



- $\dfrac{mv^2}{r} \le (F_1 + F_2)$

  Where, $F_1 = \mu R_1$ and $F_2 = \mu R_2$ μ = Coefficient of friction between tyres and road.

- $V \le \sqrt{\mu rg}$, $V_{max} = \sqrt{\mu rg}$

  This is the maximum speed without skidding.

- If centripetal force is obtained only by the banking of roads, then the speed (v) of the vehicle for a safe turn.

  $v = \sqrt{r\,g\tan\theta}$

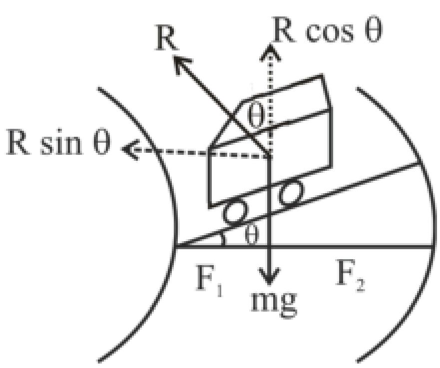
- If speed of vehicle is less than $\sqrt{r\,g\tan\theta}$. Then it will move inward (down) and r will decrease and if speed is more than $\sqrt{r\,g\tan\theta}$ then it will move toward (up) and r will increases.

- In normal life, the centripetal force is obtained by the friction force between the road and tyres as well as by the banking of the roads.

- Therefore the maximum permissible speed for the vehicle is much greater than the optimum value of the speed on a banked road.

- When centripetal force is obtained from friction force as well as banking of roads then maximum safe value of speed of vehicle.

  $V_{max} = \sqrt{\dfrac{rg(\tan\theta + \mu_s)}{(1-\mu_s \tan\theta)}}$ Where μ_s = coefficient of static friction



Bending of cyclist-

- When a cyclist takes turn at road, he inclines himself from the vertical slows down his speed and moves on a circular path of larger radius.

  If a cyclist is inclined at an angle θ, then

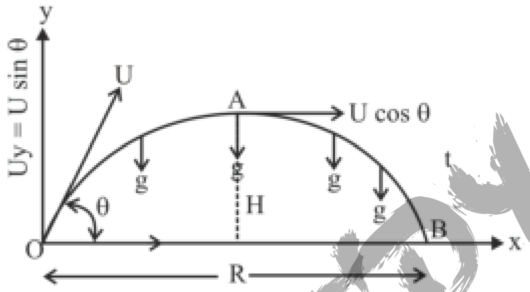
  $$\tan\theta = \frac{V^2}{rg}$$

  Where, V = Speed of the cyclist
  r = Radius of path
  g = Acceleration due to gravity

■ Projectile Motion –

- When any object is thrown from horizontal at an angle θ except 90º then it moves on a parabolic known as trajectory. The object is called projectile and its motion is called projectile motion.

Projectile motion in two dimensional motion :-



$U_x = U\cos\theta$
= Horizontal motion
= Responsible for range produced
= Constant acceleration (∵ a = 0)

But,
$U_y = U\sin\theta$
= Vertical component
= Variable ($a_y = -g$)
= Responsible for height produced.

Concept -

x-axis	y-axis
$U_x = U\cos\theta$	$U_y = U\sin\theta$
$a_x = 0$	$a_y = -g$

- **Time of assending (t) -**
  along y axis-
  $V_y = U_y + a_y t$
  $$t = \frac{U\sin\theta}{g}$$
- **Time of Flight (T) -**
  T = 2t
  $$T = \frac{2U\sin\theta}{g}$$
- **Height attained by the body in projectile motion -**
  $$H = \frac{U^2\sin^2\theta}{2g}$$
- **Condition for maximum height attained (H_{max}) –**
  For maximum height,
  θ = 90º, Sin θ = 1
  $$H_{max} = \frac{U^2}{2g}$$

- **Range produced by the body in projectile motion (R) -**

$$R = \frac{U^2 \sin 2\theta}{g}$$

- **Condition for maximum Range : (R_{max}) –**

Sin 2θ = 1 = max = sin90º

θ = 45º

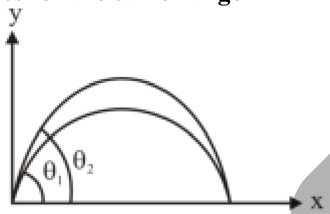
$$R_{max} = \frac{U^2}{g}$$

- **Ratio -**

$$\frac{R_{max}}{H_{max}} = \frac{\frac{U^2}{g}}{\frac{U^2}{2g}} = 2$$

$R_{max} = 2\times H_{max}$

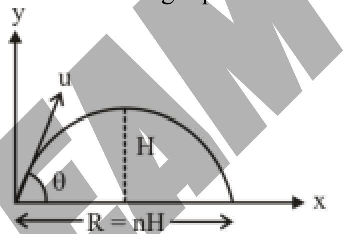
- **Two projective angles for the same range -**



$$\theta_1 + \theta_2 = 90^\circ = \frac{\pi}{2}$$

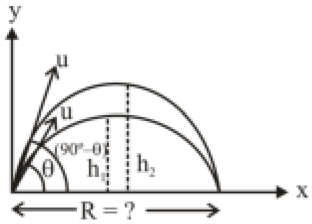
Special Cases –

- If Horizontal range is n-times of height produced then to determine projection angle.



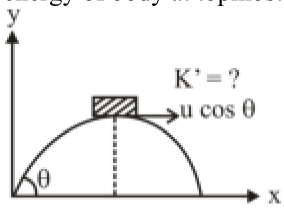
$$\theta = \tan^{-1}\left(\frac{u}{H}\right)$$

- If two bodies are projected with equal speed u such that their range produced are same but height produced are different.

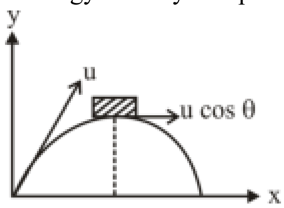


$$R = 4\sqrt{h_1 h_2}$$

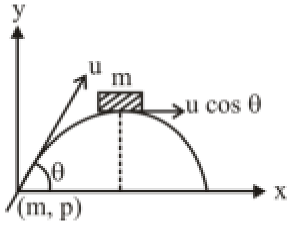
- To determine kinetic energy of body at topmost point in projectile motion ⇒ $K' = K\cos^2\theta$ where K=initial K.E



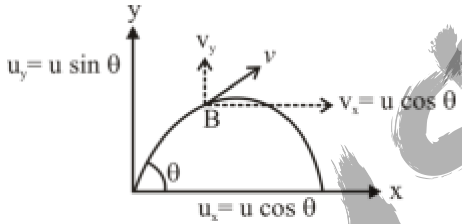
- To determine potential energy of body at topmost point in projectile $\Rightarrow U = K \sin^2\theta$



- Ratio of potential energy and kinetic energy at topmost point in projectile motion $\Rightarrow \frac{U}{K'} = \tan^2\theta$
- To determine linear momentum of body at topmost point in projectile of initial linear momentum p is given by-
  $p' = p \cos\theta$



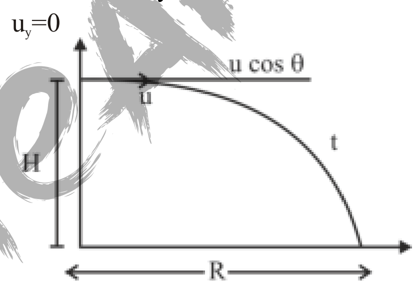
- To determine the change in linear momentum of body after time t in projectile motion ($\Delta P = ?$)



$|\Delta p| = |\Delta p_y|$

$|\Delta p| = mg \times t$

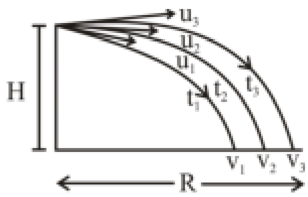
- When projectile projected horizontally-



Time of flight $= T = \sqrt{\frac{2H}{g}}$

Range $= u_x \times \sqrt{\frac{2H}{g}}$

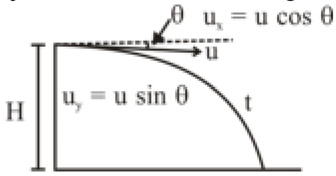
Concept -



$$T = T_1 = T_2 = T_3 = \sqrt{\frac{2H}{g}}$$

$$V_3 > V_2 > V_1 > V$$

- When projectile is projected downward at an angle θ with Horizontal -

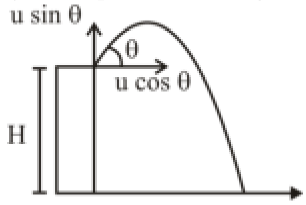


$R = u \cos\theta \times t$

$H = (u \sin\theta)t + \frac{1}{2}gt^2$

Time of flight, $T = \frac{-2u \sin\theta}{2g} \pm \frac{\sqrt{4u^2\sin^2\theta + 8gh}}{2g}$

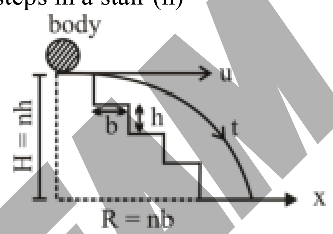
- When projectile is projected upward at an angle θ with Horizontal.



$H = -(u \sin\theta)t + \frac{1}{2}gt^2$

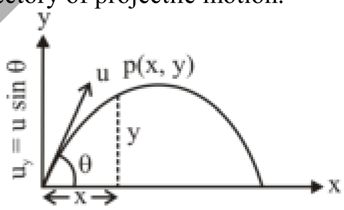
$T = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}}$

- To determine no of steps in a stair (n)



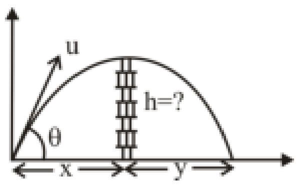
$n = \frac{2hu^2}{gb^2}$

- To determine equation of trajectory of projectile motion.



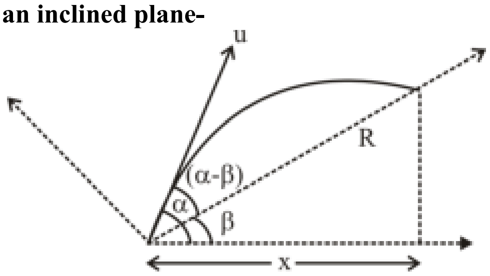
$t = \frac{x}{u\cos\theta}$

$y = x\tan\theta - \frac{gx}{2u^2\cos^2\theta}$

- 

$h = \left(\frac{xy}{x+y}\right)\tan\theta$

- **Projectile motion on an inclined plane-**



Initial velocity along the inclined plane = u cos (α – β).
Initial velocity perpendicular to the inclined plane = u sin (α – β).
Acceleration along the inclined plane = g sin β.
Acceleration perpendicular to the inclined plane = g cos β.

Time of flight, $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$

Maximum height, $H = \dfrac{u^2\sin^2(\alpha-\beta)}{2g\cos\beta}$

Horizontal range, $x = \dfrac{2u^2\sin^2(\alpha-\beta)\cos\alpha}{g\cos\beta}$

Range on inclined plane, $R = \dfrac{x}{\cos\beta} = \dfrac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$

Range on inclined plane will be maximum,

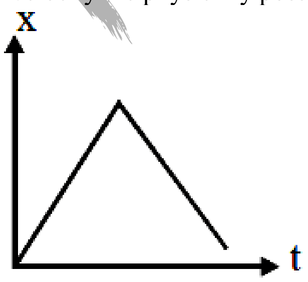
when, $\alpha = 45^\circ + \dfrac{\beta}{2}$

$R_{max} = \dfrac{u^2}{g(1+\sin\beta)}$

For angle of projectile α and (90° – α + β), The range an inclined plane are same.
If the projectile is thrown downwards, then maximum range is -

$R_{max} = \dfrac{u^2}{g(1-\sin\beta)}$

- A positive acceleration can be associated with a "slowing down" of the body because the origin and the positive direction of motion are a matter of choice.
- The x-t graph for a particle undergoing rectilinear motion cannot be as shown in figure because infinitesimal changes in velocity are physically possible only in infinitesimal time.



- In oblique projection of a projectile the speed gradually decreases up to the highest point and then increases because the tangential acceleration opposes the motion till the particle reaches the highest point, and then it favours the motion of the particle.
- In free fall the initial velocity of a body may not be zero.
- A body can have acceleration even if its velocity is zero at an instant.
- Average velocity of a body may be equal to its instantaneous velocity.
- The trajectory of an object moving under constant acceleration can be straight line or parabola.
- The path of one projectile as seen from another projectile is a straight line as relative acceleration of one projectile with respect to another projectile is zero.

(6) Newtons's Law of Motion

■ Force -

- A push or pull that one object exerts on another.

Force in Nature -

- There are four fundamental forces in nature-

1. Gravitational force   2. Electromagnetic force
3. Strong nuclear force   4. Weak force

Interaction	Particle affected	Range	Relative Strength	Characterstics time	Particle exchange	Role in universe
Strong Nuclear force	Quarks Hadrons	$\sim 10^{-15}$ m	1	10^{-23} Sec	Gluons Mesons	Holds quark together to form nucleon. Hold nucleons together to form atomic nuclei.
Electromagnetic	Charged particles	∞	10^{-2}	10^{-20} sec	Photons	Determine structure of atoms, solids and liquid is important factor in astronomical universe.
Weak nuclear force	Quark & Leptons	$\sim 10^{-16}$ m	10^{-13}	10^{-10} sec	Intermediate boson	Mediate transformations of quarks & leptons helps determine composition of atomic nuclei
Gravitational	All	∞	10^{-39}	10^{-16}	Gravitons Not experimentally detected	Assemble matter into planet, stars and galaxies

■ Types of forces on macroscopic objects –

(a) Field Force or Range Forces -

- These are the forces in which contact between two objects is not necessary.

  Ex. (i) Gravitational force between two bodies.

  (ii) Electrostatic force between two charges.

(b) Contact force -

- Contact forces exist only as long as the objects are touching each other.

  (i) Normal force   (ii) Frictional force.

(c) Attachment to another body -

- Tension (T) in a string and spring force (F = kx) comes in this group.

■ Newton's Law of Motion –

(i) First law (Galileo's law of inertia)
(ii) Second law (Law of force)
(iii) Third law (Law of action and reaction)

(i) Newton's First law (Galileo's law of inertia)

- If $\vec{F}_{external} = 0$ and $\vec{V} = 0$ i.e body is in rest then it will always remain in rest.

Ex. A person who is standing freely in bus is thrown backward when the bus starts suddenly.

(ii) Newton's Second law (Law of force) -

- The rate of change of linear momentum w.r.t. time is equal to applied force and change in momentum takes place in the direction of applied force.

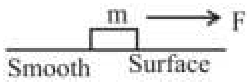
$$F = \frac{\Delta P}{\Delta t}$$

If $P = f(t)$ then,

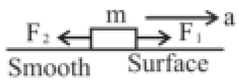
$$F = \frac{dP}{dt}$$

- If $\frac{\Delta P}{\Delta t} = \frac{P_f - P_i}{\Delta t} = \frac{mv - mu}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma$
- $\vec{F} = m\vec{a}$
- Unit - $1\ Kg\text{-}m/s^2 = 1$ Newton (In M.K.S.)
  1 Newton = 10^5 Dyne (In C.G.S.)
- For Horizontal motion -
  F = ma
  Where, m = Inertial mass
- For vertical motion -
  W = mg
  Where, m = Gravitational mass.

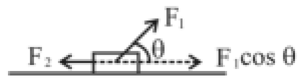
Concept -

(a) 

F = ma

(b) 

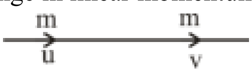
$F_1 - F_2 = ma$

(c) 

$F_1 \cos\theta - F_2 = ma$

Momentum (P) -

- It is the product of the mass and velocity of a body i.e
  $\vec{P} = m\vec{v}$
- Change in linear momentum (ΔP) -



$\Delta P = P_f - P_i$ Where, P_f = final momentum, P_i = Initial momentum.
$\Delta P = mv - mu$
$\Delta P = m(v - u)]$
$\Delta P = m\,(v - u)$

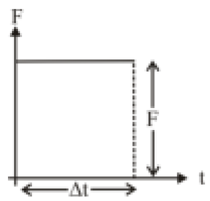
- S.I. unit is $Kg\text{-}m\text{-}s^{-1}$ or N-sec
- Dimension of linear momentum = $[MLT^{-1}]$.

Impulse -

- If a force is applied on the body for very small time interval then-
  $I = \vec{F}\,\Delta t$
  $I = \frac{\Delta P}{\Delta t} \times \Delta t$
  $\boxed{\vec{I} = \Delta\vec{P}}$ Impulse = chang in momentum.

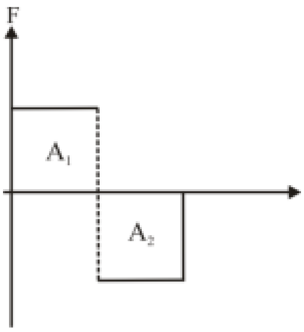
Area bounded by F - T Graph -

(a) If constant force acting on the body-
- Area under the force time graph give impulse

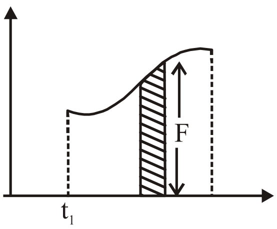


Area = length × breadth
= F × Δt

(b) If direction of force on the body is reversed. $I = \Delta P = |A_1| - |A_2|$



(c) If variable force is acting on the body i.e $F = f(t)$

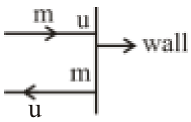


$I = \int f(t)\,dt$

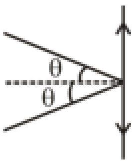
Concept -

(a) 

$$F = \frac{\Delta p}{\Delta t} = \frac{mv - mu}{\Delta t}$$

(b) 

$$F = \frac{2mu}{\Delta t}$$

(c) 

$$F = \frac{\Delta p}{\Delta t} = \frac{2mu\cos\theta}{\Delta t}$$

(iii) Newton's Third Law (Law of action and reaction) -

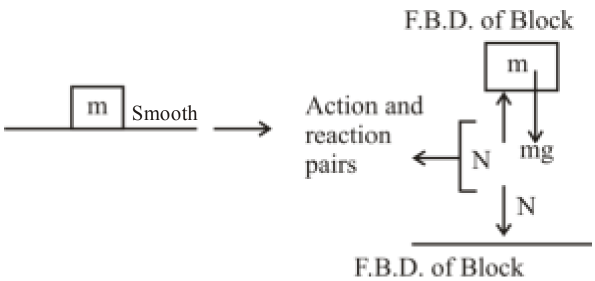
- According to this law, for every action there is always equal and opposite reaction i.e the forces of action and reaction are always equal and opposite.

Example -

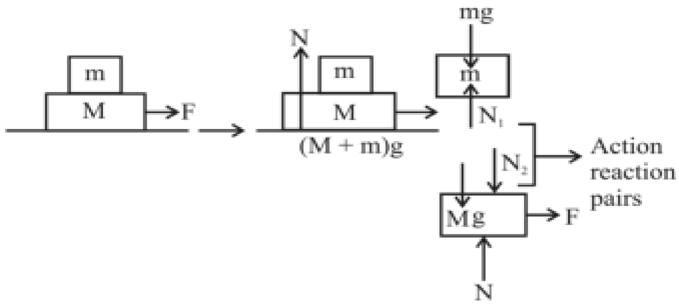
- When you walk you interact with the floor, you push against the floor and the floor pushes against you. The pair of forces occurs at the same time.
- Likewise, the tires of car push against the road while the road pushes back on the tires the tires and the road simultaneously push against each car.

NOTE :-Normal force will be perpendicular to the surface of contact.

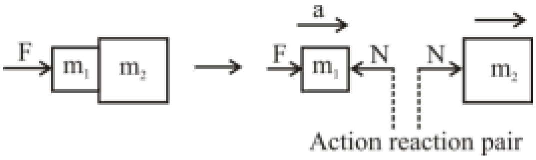
Representing normal reactions and weight in free body diagram -

(i) 

(ii)



(iii)



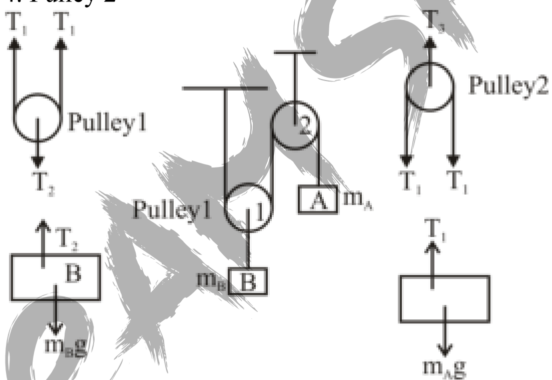
■ Tension -

- Tension force always pulls a body.
- Tension can never push a body.
- Tension across massless pulley or frictionless pulley remains constant.
- Ropes become slack when tension force becomes zero.

Representation of Tension force in different situations -

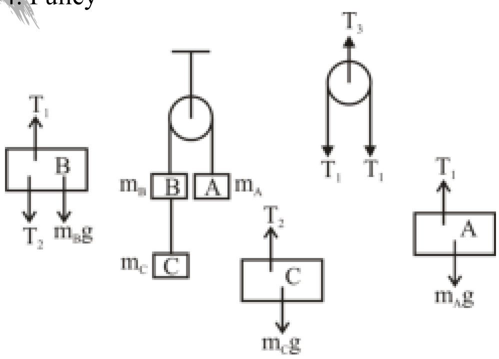
Case-1 Two blocks of masses m_A and m_B are arranged in the diagram as shown. The free body diagram of -

1. Block m_A 2. Block m_B
3. Pulley 1 4. Pulley 2



Case -2 Three blocks of masses m_A, m_B and m_C are arranged in the diagram as shown. The free body diagram of-

1. Block m_A 2. Block m_B
3. Block m_C 4. Pulley



■ Frame of Reference -

A frame in which an observer is situated and makes his observations is known as "Frame of reference."

- **Inertial frame of reference -** A frame of reference which is at rest or which is moving with a uniform velocity along a straight line is called an inertial frame of reference.
  Ex. Car moving with constant velocity on a straight road.
- **Non-Inertial frame of reference -** Accelerated frame of references are called non-inertial frame of reference.
  Ex. Car moving in uniform circular motion.

Inertial observer –

- Rest
- Moving with constant velocity
- $a = 0$
- Non-acceleration
- Newton's law is valid for it.

Non-inertial observer –

- Accelerated observer or retarded observer
- $a \neq 0$
- Newton's law is not valid.
- Newton's law is valid when Pseudo force is applying.
- F_{Pseudo} = Mass of observer body× Acceleration of observer

$F_{Pseudo} = ma$

■ Apparent Weight and Weight -

- Apparent Weight of an object refers to weight of an object whenever gravity force acting on object is not balanced by an equal and opposite normal force
- If whole system is in rest, then-

  $R = mg$
- If acceleration increases or decreases then-

  $R > mg$ or $R < mg$

  $R = m(g \pm a)$

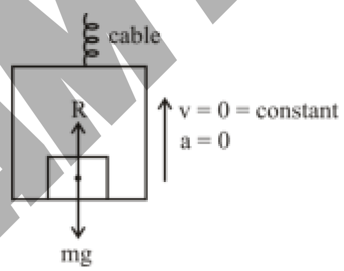
  Where, R = apparent weight

  = observed weight

  = reading of spring balance

Different cases for lift Motion –

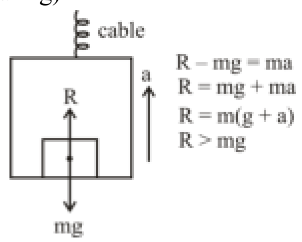
(i) When lift is in rest, then-



$R = mg$

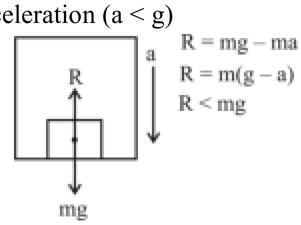
The weighing machine will read the actual weight.

(ii) If lift upward with some acceleration ($a > g$)



The weighing machine will read the apparent weight which is more than the actual weight.

(iii) If lift moves downward with some acceleration ($a < g$)



The weighing machine will read the apparent weight, which is less than actual weight.

(iv) When lift is falling freely under gravity (g = a), then

$R = m(g - g)$

$R = 0$

The apparent weight of body becomes zero.

(v) If lift is accelerating downward with acceleration greater than g, then body will be lifted from floor to ceiling of the lift.

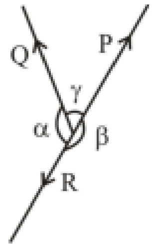
■ Key Points-

(i) **Inertia of direction -** It is the inability of body to change by itself the direction of motion.

Ex. The rotating wheel of any vehicle throw out mud.

(ii) The equilibrium of a particle in mechanics refers to the situation when the net external force acting on the particle is zero.

(iii) **Lami's Theorem-** When three coplanar & concurrent forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

(iv) The force on a body due to acceleration of non-inertial frame is called fictitious or apparent or Pseudo force.

(v) Aeroplane always fly at low altitudes because according to Newton's IIIrd law of motion as aeroplane displaces air & at low altitudes density of air is high.

(vi) Rocket move by pushing the exhaust gases out so they can fly at low or high altitude.

(vii) A man in a closed cabin (lift) falling freely does not experiences gravity as inertial and gravitational mass have equivalence.

(viii) Inertia is proportional to mass of the body.

(ix) If a number of force $\vec{F}_1, \vec{F}_2, \vec{F}_3$ .... act on the body then it is in equilibrium when $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 .... = 0$

(x) A body in equilibrium cannot change the direction of motion.

(xi) If a body moves along a curved path, then it is certainly acted upon by a force

(xii) A single isolated force cannot exist.

(xiii) Newton's second law of motion give the measure of force i.e., (f = ma), force is a vector quantity.

(xiv) Absolute units of force are dyne in C.G.S. system and Newton (N) in SI.

(xv) A body starting from rest moves along a smooth inclined plane of length ℓ, height h and having angle of inclination θ.

(1) Its acceleration down the plane is g sin θ

(2) Velocity at the bottom of the inclined plane will be $\sqrt{2gh} = \sqrt{2gl\sin\theta}$

(3) Time taken to reach the bottom will be –

$$t = \sqrt{\frac{2\ell}{g\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$$

(4) If the angle of inclination is changed keeping the height constant then–

$$\frac{t_1}{t_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

(xvi) Action and reaction forces never act on the same body they act on different bodies. If they act on the same body, the resultant force on the body will be zero i.e., the body will be in equilibrium.

- Action and reaction forces act along the line joining the centres of two bodies.
- Newton's third law is applicable whether the bodies are at rest or in motion.

(7) Friction

- Friction is the force between two surface in contact, or a force of a medium acting on a moving object (i.e air on aircraft).
- Frictional forces arise due to molecular interactions. In some cases frictions acts as a supporting force and in some cases it acts as a opposing force.

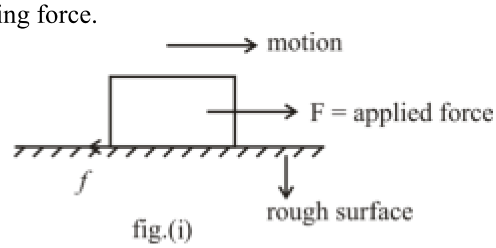


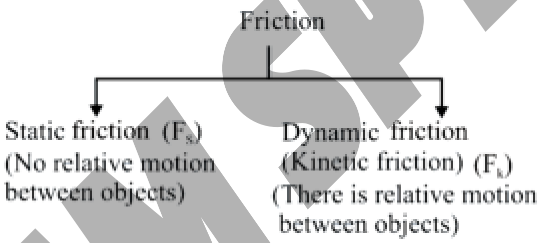
fig.(i)

■ **Cause of friction -** Friction arises on account of strong atomic or molecular force of attraction between the two surfaces at the points of actual contact.

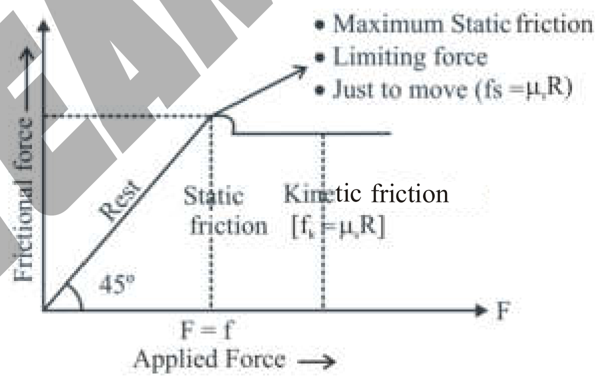
Concept –

- If motion is in equilibrium (fig (i)) then equation of motion
  $(F - f) = ma$
- For just to move $(a = 0)$
  $F - f = m \times 0$
  $F = f$

■ Types of Friction -



■ Graph between applied force and force of friction -

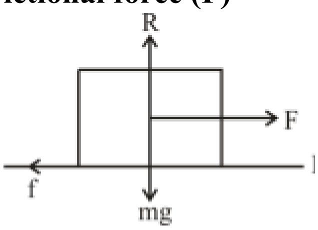


Where, μ_s = coefficient of static friction
μ_k = coefficient of kinetic friction

Concept :-

- If $\mu_k < \mu_s$, $F_k < F_s$
  then block is in rest.

■ Coefficient of friction (μ) and Frictional force (F) -



$F \propto R$

$F = \mu R$

but here $R = mg$

$\Rightarrow \quad F = \mu R = \mu\, mg$

$\Rightarrow \quad \mu = \dfrac{F}{R}$

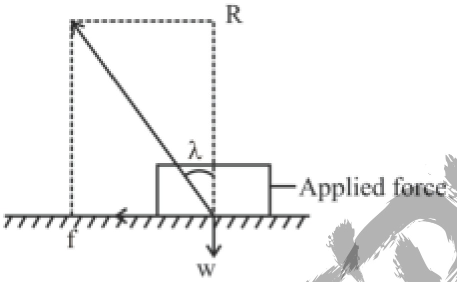
$\Rightarrow$ It has no unit. i.e. Dimensionless $[M^0L^0T^0]$

$\Rightarrow$ Coefficient of friction depends upon nature of surface

- For perfectly smooth surface $(\mu) = 0$
- For perfectly rough surface $(\mu) = 1$
- Value of μ, $0 \le$ to ≤ 1

■ Angle of Friction (λ) -

The angle between the resultant of friction force and the normal reaction is called the angle of friction.



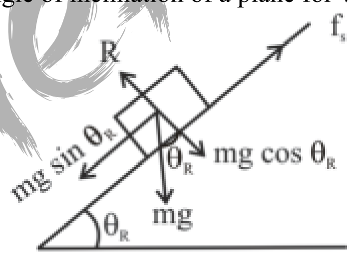
- $\tan \lambda = \dfrac{f}{R} = \dfrac{\mu R}{R} = \mu$
- $\lambda = \tan^{-1}(\mu)$
- For smooth surface $\lambda = 0$
- Resultant -

$S = \sqrt{f^2 + R^2} = \sqrt{(\mu R)^2 + R^2}$

$S = R\sqrt{1+\mu^2}$

■ Angle of repose or angle of sliding -

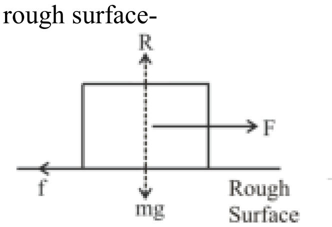
- The maximum angle of inclination of a plane for which a block remains stationary on this plane.



- $\tan \theta_R = \mu_S$
- For smooth surface $\theta_R = 0$

■ Special Cases for friction -

Case-1 For horizontal rough surface-



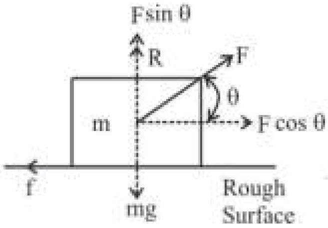
- Equation of motion -

$F - f = ma$

- Just to move $a = 0$

$F - f = 0$

Case-2 If force is applied on block at an angle θ -

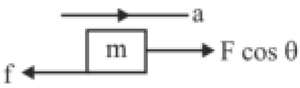


$R = mg - F \sin\theta$

$f = \mu R = \mu(mg - F \sin\theta)$

Case (2.1)

- If block is move in horizontal direction then-



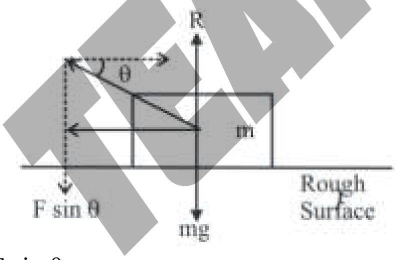
- Equation of motion is -
  $F \cos\theta - f = ma$
- For just to move a = 0
  $F \cos\theta = f$
  $F \cos\theta = \mu(mg - F \sin\theta)$
- Applied force at an angle θ upward for just to move -

$$F = \frac{\mu\, mg}{\cos\theta + \mu \sin\theta}$$

$$F = \frac{mg \sin\lambda}{\cos(\theta - \lambda)}$$

- If F_{min} then -
  $\cos(\theta - \lambda) = 1$
  $\theta - \lambda = 0$
  $\theta = \lambda$

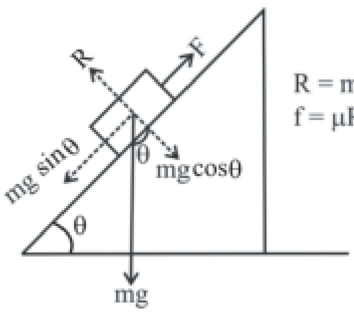
Case-2.2-



- $R = mg + F \sin\theta$
- $f = \mu R = \mu(mg + F \sin\theta)$
- Force applied at angle θ in downward for just to move $F = \dfrac{\mu\, mg}{\cos\theta - \mu \sin\theta}$

Case -3 Inclined Rough surfaces -

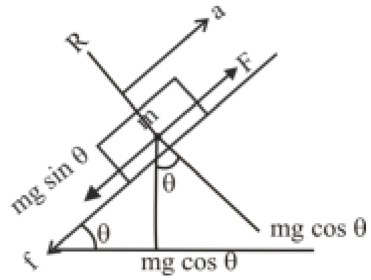
(a) Upward motion of body on the inclined plane -



$R = mg \cos\theta$

$f = \mu R = \mu mg \cos\theta$

(b) If body moves upward with some acceleration-



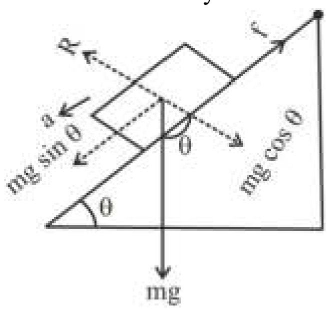
$F = mg \sin\theta + \mu\, mg \cos\theta = ma$

(c) If a body projected in upward direction on the rough inclined plane-

$a = g \sin\theta + \mu \cos\theta$ = retardation

Case- 4

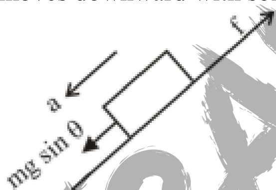
(a) Downward motion of a body on the rough inclined plane-



$R = mg \cos\theta$

$f = \mu R = \mu\, mg \cos\theta$

(b) If block moves downward with some acceleration -



$a = g \sin\theta - \mu g \cos\theta$ = Downward acceleration

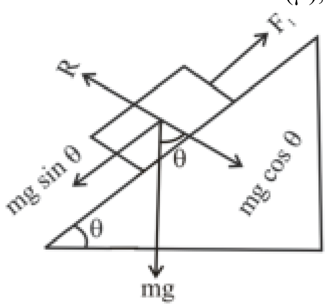
(c) For just to move -

$a = 0$

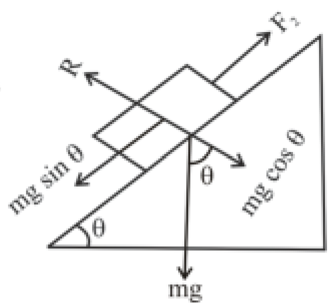
$\theta = \tan^{-1}(\mu) = \lambda$ = angle of repose

Concept -

- If $\theta = \lambda$ i. e $\theta = \tan^{-1}(\mu)$, then body will be in condition of just to move.
- If $\theta > \lambda$ i. e $\theta > \tan^{-1}(\mu)$, then body moves downward.
- If $\theta < \lambda$ i. e $\theta < \tan^{-1}(\mu)$, then body remains in rest on the inclined plane.



m = light mass



m = heavy mass

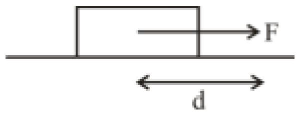
- Force applied for just to move the block $F_1 = mg \sin\theta + f$
- Force applied for just to prevent to sliding down $F_2 = mg \sin\theta - f$

■ Key Points -

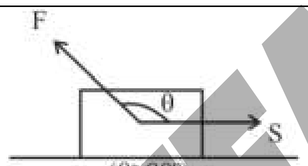
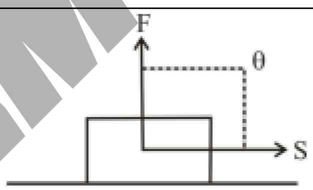
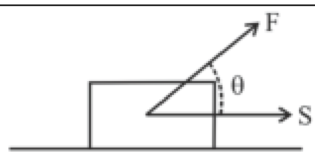
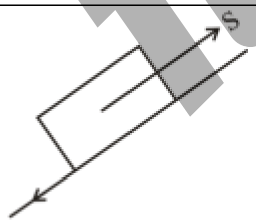
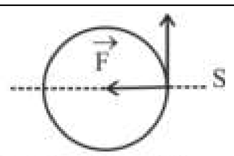
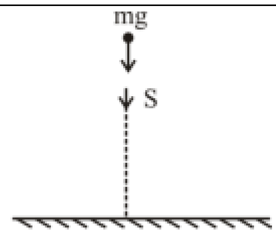
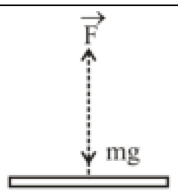
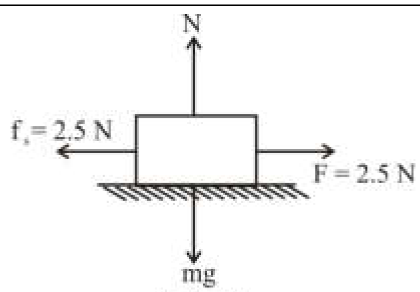
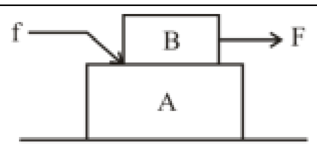
- While walking on ice, one should take small steps to avoid slipping. This is because smaller step increases the normal reaction and that ensure larger friction.
- Pulling is easier than pushing on a rough horizontal surface because normal reaction is less in pulling than in pushing.
- Force of friction is non-conservative force.
- Force of Friction acts in a direction opposite to that of the tendency of relative motion between the surfaces.
- Rolling friction is much less than the sliding friction at slow speed. This knowledge was used by man to invent the wheels.
- The atomic and molecular forces of attraction between the two surface at the point of contact give rise to friction between the surface.

(8) Work, Energy and Power

■ Workdone- $W = \int dw = \int \vec{F} \cdot \vec{dr} = \int F\, dr \cos\theta$

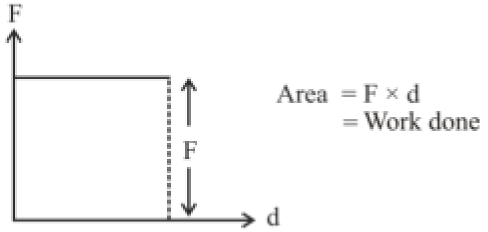


- $W = \vec{F}.\vec{d}$ For constant force.
- Unit = Newton – Meter.
- $W = \int dw = \int F\, dx$ For unidirectional force.
- **Nature of work done -**Although work done is scalar quantity, yet its value may be +ve, –ve or zero.

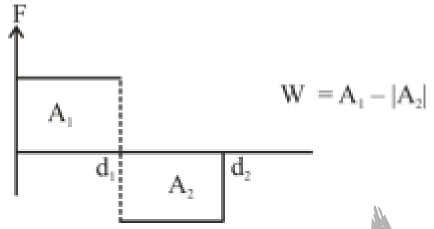
Negative Work	Zero Work	Positive Work
 ($\theta > 90^o$)	 ($\theta = 90^o$)	 ($\theta < 90^o$)
 Workdone by friction force $\theta = 180^o$	 Motion of particle on circular path $\theta = 90^o$	 Motion under gravity ($\theta = 0^o$)
 Workdone by gravity ($\theta = 180^o$)	 As f = F, S = 0	 Workdone by friction force on block A ($\theta = 0^o$)

■ Graph between force and displacement -

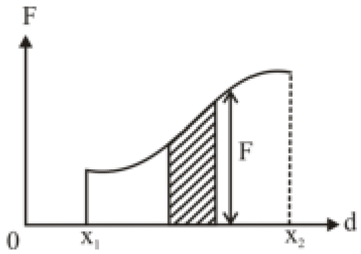
(i) If constant force is acting on body



(ii) If direction of force on the body is reversed -



(iii) If variable force is applied on the body



$dw = Fdx$

$\int dw = \int_{x_1}^{x_2} Fdx$

$w = \int_{x_1}^{x_2} Fdx$

Concept-

(i) If $F = f(x)$

$$W = \int_{x_1}^{x_2} Fdx$$

(ii) If $F = f(y)]$

$$W = \int_{y_1}^{y_2} Fdy$$

(iii) If $F = f(z)$

$$W = \int_{z_1}^{z_2} Fdz$$

But for more than one variable -

$$W = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} \vec{F}\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

■ Conservative Forces -

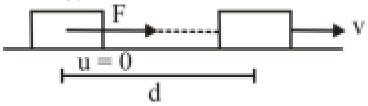
- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc are conservative forces.
- When only a conservation forces acts within a system, the kinetic energy and potential energy can change into each other. However, their sum the mechanical energy of system doesn't change.
- Work done is completely recoverable.
- If $\vec{F}$ is a conservative force then- $\nabla \times \vec{F} = \vec{0}$ (i.e curl of $\vec{F}$ is zero)

■ Non-Conservative Forces -

- Work done depends upon path.
- Work done in a round trip is not zero.
- Force are velocity dependent & regarding in nature eg. friction, viscous force etc.
- Work done against a non-conservative force may be dissipated as heat energy.
- Work done is not recoverable.

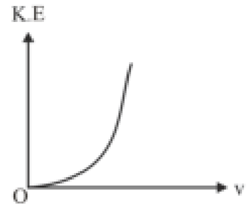
■ Energy -

- The capacity of doing work is called energy.



■ Kinetic Energy-

- $K.E. = \frac{1}{2}mv^2$
- $K.E \propto v^2$



- Percentage error in K.E -

  $\frac{\Delta K}{K}\times 100 = 2\frac{\Delta v}{v}\times 100$

- Relation between Kinetic energy (K.E) and Linear Momentum –

  $K.E = \frac{P^2}{2m}$

  $P = \sqrt{2mK}$

- For same K.E of two particles -

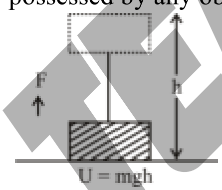
  $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$

Concept -

- Work done due to all types of forces acting on the body will be equal to only change in K.E of that body.

■ Potential Energy -

- The energy possessed by any object by virtue of its position or configuration is called potential energy.



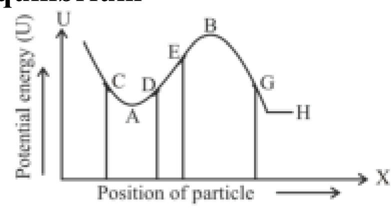
- Potential energy is relative quantity.
- Potential energy is defined only for conservative force field.
- Relationship between conservative force field and potential energy.

  $\vec{F} = -\vec{\nabla}U = -\text{grad}(U) = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$

- If force varies only with one dimension (along x-axis) then

  $F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F\,dx$

■ Potential energy curve and equilibrium -



- It is a curve which shows change in potential energy with position of a particle.

Stable Equilibrium -

- When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium then it is said to be in stable equilibrium.

  At Point C : Slope $\frac{dU}{dx}$ is negative so F is positive.

  At Point D : Slope $\frac{dU}{dx}$ is positive so F is negative.

  At Point A : It is the point of stable equilibrium.

  At Point A : $U = U_{min}$, $\frac{dU}{dx}$ and $\frac{d^2U}{dx^2}$ = positive.

Unstable Equilibrium –

- When a particle is slightly displaced from equilibrium and it tends to move away from equilibrium position then it is said to be is unstable equilibrium.

  At Point E : Slope $\frac{dU}{dx}$ **is positive** So, F is negative.

  At Point G : Slope $\frac{dU}{dx}$ is negative so F is positive.

  At Point B : It is the point of unstable equilibrium

  At Point B : $U = U_{max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = negative.

Neutral Equilibrium –

- When a particle is slightly displaced from equilibrium position and no force acts on it then equilibrium is said to be neutral equilibrium point.

U = constant : $\frac{dU}{dx} = 0, \frac{d^2U}{dx^2} = 0$

■ **Total Mechanical Energy of body-**

- Mechanical energy = K.E + P.E
- $E = K + U$
- $E = K_{max} + U_{min}$
- $E = K_{min} + U_{max}$

■ **Work energy theorem for constant force-**

- It states that the work done by the net force acting on a body is equal to the change in the K.E of the body.
  $W = K_f - K_i$
  Change in K.E of body = work done in the body by net force.

Work Energy theorem for variable force -

$W = \Delta$ K.E
Change in Kinetic energy = Work done by all forces.

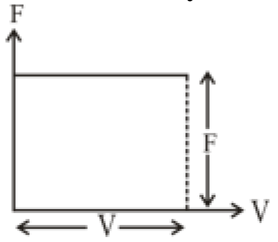
■ **Power -**

- The rate of doing work with respect to time is power.

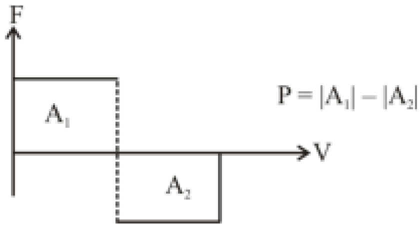
  $P_{avg} = \frac{W}{t}$ & $P_{instantaneous} = \frac{dW}{dt}$

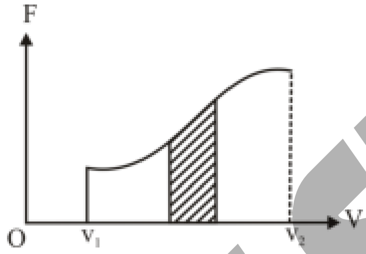
- Power is scalar quantity.
- S.I. unit of power is J/S or Watt.
- 1 mW = 10^{-3} W 1 MW = 10^{6} W
  1 KW = 10^{3} W 1 Horse Power = 746 Watt
- $P = \vec{F} \cdot \vec{V}$
  $= F V \cos\theta$
- If $\theta = 0°$: $P = FV = +ve$
  If $\theta = 180°$: $P = -FV = -ve$
  If $\theta = 90°$: $P = 0$

■ Graph between force and velocity –

(i) 

Area = FV = P = Power

(ii) 

(iii) 

$P = \int_{V_1}^{V_2} F\,dv$

■ Principle of Conservation of Energy –

- This law states that energy can neither be created nor destroyed but it can changed from one form to another.

Example – When a body falls freely, under gravity its potential energy gradually changes into Kinetic energy, But total mechanical energy (Kinetic energy + Potential energy) remains constant at any point of its motion.

■ Law of Conservation of Linear Momentum –

- This law states if no external force acts on a system, then its linear momentum remains constant.

Example - A rifle givens backward kick on firing a bullet before firing, both the bullet and the rifle are at rest and initial momentum of the system is zero. As soon as bullet is fired, it moves forward with a large velocity. In order to conserve momentum the rifle moves backward with such a velocity that final momentum of the system is zero.

■ Collision of Bodies –The event of the process, on which two bodies in contact with each other or due to mutual interaction at distance apart, affect each other motion (velocity, momentum, energy or direction of motion) is defined as a collision.

In Collision –

- The particle comes closer before collision and after collision they either stick together or move away from each other.
- The particles need not come on contact with each other for a collision.
- The law of conservation of linear momentum is necessary applicable in a collision whereas the law of conservation of mechanical energy is not.

Type of Collision –

- **On the basis of direction**

(i) One dimensional collision or Head on collision- The collision in which the particles move along the same straight line before and after the collision is defined as one dimensional collision.

(ii) Two dimensional collision or oblique collision- The collision in which the particles move in the same plane at different angles before and after collision is defined as oblique collision.

- **On the Basis of Kinetic Energy-**

(i) Elastic collision-A collision is said to be elastic, if the total kinetic energy before and after the collision remains same.

(ii) Inelastic collision - A collision is said to be inelastic, If the total K.E does not remains constant.

(iii) Perfectly Inelastic collision- The collision in which the particles gets stacked together after the collision it is called perfectly inelastic collision, in this type of inelastic collision loss of energy is maximum.

Coefficient of restitution --

- $$e = -\frac{\text{Velocity of Seperation along line of impact}}{\text{Velocity of approach along line of impact}}$$

⇒ **Value of e-**

For elastic collision, $e = 1$

For perfectly inelastic collision, $e = 0$

For inelastic collision, $0 < e < 1$

Head on collision-

Head on inelastic collision of two bodies-

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ .... (i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law- $e = \dfrac{V_2 - V_1}{u_1 - u_2}$ .... (ii)

By solving (i) and (ii) we get-

$$v_1 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

$$v_2 = \frac{m_1u_1 + m_2u_2 - m_1e(u_2 - u_1)}{m_1 + m_2}$$

Elastic collision (e = 1)

- **If the two bodies of equal masses : $m_1 = m_2 = m$, $v_1 = u_2$, $v_2 = u_1$.**
  Thus, if two bodies of equal masses undergoes elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.
- **If the mass of body is negligible as compared to other -**
  If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$. When a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will moves with velocity double that of A.
  If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$.

Loss in Kinetic Energy in inelastic collision–

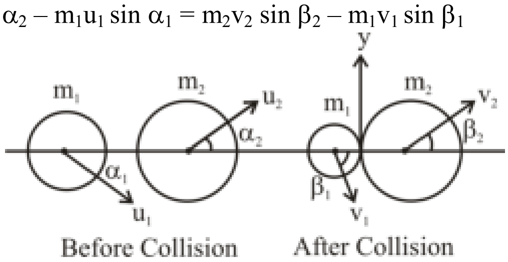
$$\Delta K = \frac{m_1m_2}{2(m_1 + m_2)}(1 - e^2)(u_1 - u_2)^2$$

Oblique Collision –

- Conserving the momentum of system in direction along normal (x-axis in our case) and tangential (y-axis in our case).

$$m_1u_1 \cos\alpha_1 + m_2u_2 \cos\alpha_2 = m_1v_1 \cos\beta_1 + m_2v_2 \cos\beta_2$$

$$m_2u_2 \sin\alpha_2 - m_1u_1 \sin\alpha_1 = m_2v_2 \sin\beta_2 - m_1v_1 \sin\beta_1$$



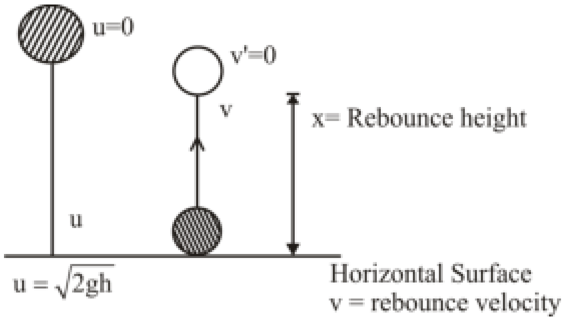
Since number of force is acting on m_1 and m_2, along the tangent (i.e y-axis) the individual momentum of m_1 and m_2 remains conserved.

$m_1u_1 \sin\alpha_1 = m_1v_1 \sin\beta_1$ & $m_2u_2 \sin\alpha_2 = m_2v_2 \sin\beta_2$

By using Newton's experimental law along the line of impact-

$$e = \frac{v_2 \cos\beta_2 - v_1 \cos\beta_1}{u_1 \cos\alpha_1 - u_2 \cos\alpha_2}$$

- **To Determine Rebounce Velocity and Rebounce Height of Body when Collision Takes Place Between A Body And Horizontal Rigid Surface-**



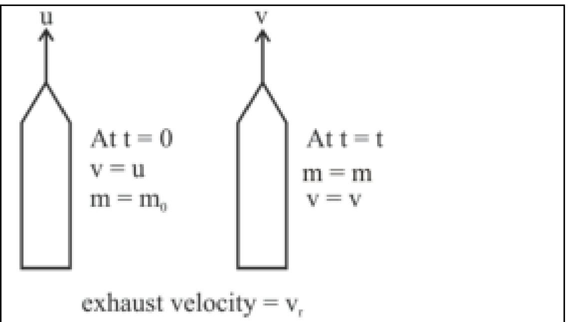
- **Newton's Experimental Formula-**
  $V_1 - V_2 = -e\,(U_1 - U_2)$
  $V - 0 = -e\,(U - 0)$
  $V = -eU$
- **Rebounce Velocity -**
  $|V| = eU = E\sqrt{2gh}$
- **n^{th} Rebounce Velocity**
  $V_n = e^n u = e^n \times \sqrt{2gh}$
  Here, n = 1,2,3,4,5........
- **Rebounce Height-**
  $h = \frac{e^2}{x}$
- **n^{th} Rebounce Height-**
  $x_n^{th} = e^{2n} \times h$
  $h = \frac{x_n^{th}}{e^{2n}}$
  here, n = 1,2,3.......

■ Rocket propulsion –

Thrust force on the rocket = $v_r\left(-\frac{dm}{dt}\right)$

Velocity of rocket at any instant

$V = u - gt + v_r \, ln\left(\frac{m_0}{m}\right)$



■ Key points-

- A quick collision between two bodies is more violent then slow collision, even when initial and final velocity are equal because the rate of change of momentum determines that the impulsive force small or large.
- Impulse momentum theorem is equivalent to Newton's 2^{nd} law of motion.
- For a system conservation of linear momentum is equivalent to Newton's third law of motion.
- A body may gain P.E. simultaneously because principle of conservation of mechanical energy may not be valid every time.
- Comets move around the sun in elliptical orbits. The Gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force is zero in complete round trip because gravitational force is conservative force.
- Rocket move by pushing the exhaust gases out so they can fly at low or high altitude.
- A man in a closed cabin (lift) falling freely does not experiences gravity as inertial and gravitational mass have equivalence

(9) Rotational Motion

■ **Rigid body -** In physics a rigid body (also known as a rigid object) is a solid body in which deformation is zero or so small that it can be neglected.

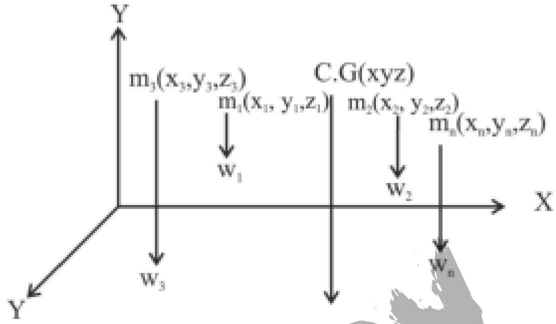
■ **Center of Mass-**

A point where total mass of body is concentrated.

■ **Centre of Gravity (C.G) -**

A point where total weight of body is concentrated.

To Determine Position (Co-ordinate) of Centre of Gravity of a Body-



Concept -

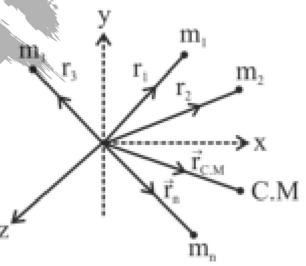
$$x_{C.G} = x_{C.M}$$

$$x_{C.M} = \frac{m_1x_1 + m_2x_2 + ...+ m_nx_n}{m_1 + m_2 + m_3 + ...m_n}$$

$$y_{CM} = \frac{m_1y_1 + m_2y_2 + ...+ m_ny_n}{m_1 + m_2 + m_3 + ...m_n}$$

$$z_{CM} = \frac{m_1z_1 + m_2z_2 + ...+ m_nz_n}{m_1 + m_2 + m_3 + ...m_n}$$

Position of center of mass in vector from-

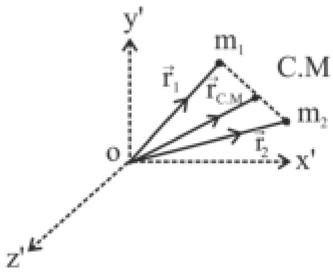


(i) For many particle System-

$$r_{C.M} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_3\vec{r_3}.......+ m_n\vec{r_n}}{m_1 + m_2 + m_3 + ...+ m_n}$$

(ii) For two particle system-

$$r_{C.M} = \frac{m_1r_1 + m_2r_2}{m_1 + m_2}$$



Velocity of centre of mass -

(i) For many particle system-

$$\vec{V}_{C.M} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + ... + m_n\vec{V}_n}{m_1 + m_2 + m_3 + ... + m_n}$$

(ii) For two Particle system-

$$\vec{V}_{C.M} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2}$$

- **Acceleration of centre of Mass-**

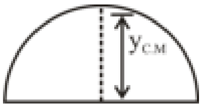
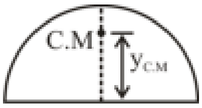
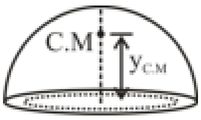
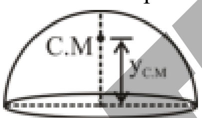
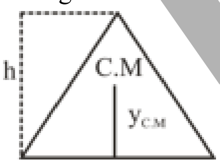
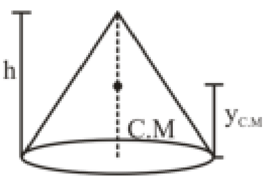
(i) For many particle system-

$$\vec{a}_{C.M} = \frac{d\vec{V}_{C.M}}{dt}$$

(ii) For two particle system-

$$\vec{a}_{C.M} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

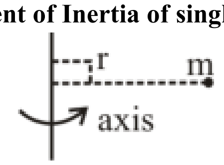
■ **Position of centre of Mass of some Regular shape-**

Regular Shape	Centre of Mass
(i) Semi circular ring 	$y_{C.M} = \frac{2R}{\pi}$
(ii) Semi circular disc 	$y_{C.M} = \frac{4R}{3\pi}$
(iii) Hollow Hemi Sphere 	$y_{C.M} = \frac{R}{2}$
(iv) Solid Hemi Sphere 	$y_{C.M} = \frac{3R}{8}$
(v) Triangular Lamina 	$y_{C.M} = \frac{h}{3}$
(vi) For hollow cone 	$y_{C.M} = \frac{h}{3}$ (From Base) & $\frac{2h}{3}$ (From Apex)

■ **Moment of Inertia (I)**

It always opposes the rotational motion of body about any of rotation.

Moment of Inertia of single particle-



→ $I = mr^2$

→ Unit $kg - m^2$

→ $[ML^2T^0]$

→ It is the tensor quantity and for a fixed axis it is Scalar.

→ If r = 0 then I = 0

• **Radius of gyration** $K = \sqrt{\dfrac{I}{M}}$

• **Radius of gyration of particle system**

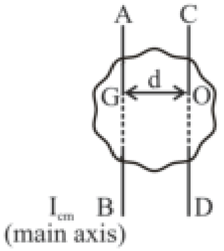
$$K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

Parallel - Axis Theorem:

The parallel axis theorem states that, the moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its center of mass and the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.

$I = I_{cm} + Md^2$

Where, I_{cm} - moment of inertia about an axis through the centre of mass, M - total mass of the body and d - Perpendicular distance between the two parallel axis.

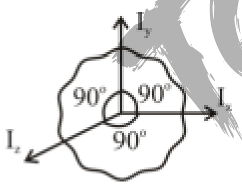


G = centre of gravity

O = origin

Perpendicular Axis Theorem (Applicable for only planar bodies)

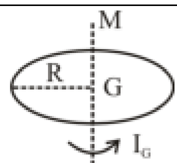
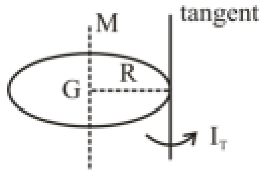
The perpendicular axis theorem states that the moment of inertia, for any axis which is perpendicular to the plane, is equal to the sum of any two perpendicular axes of the body which intersects with the first axis.

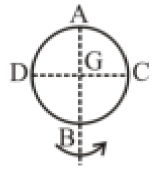
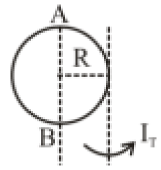


$\mathbf{I_z = I_x + I_y}$

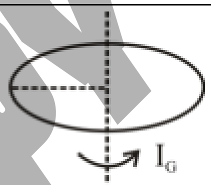
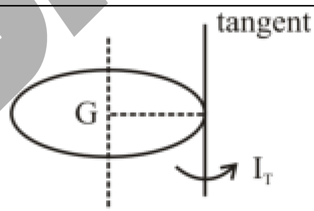
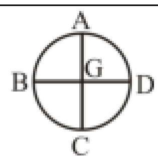
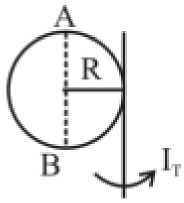
Moment of Inertia of several Uniform object-

1. For a Thin circular Ring

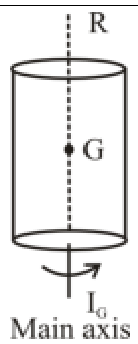
Sr. No.	**Axis of Rotation**	**Moment of Inertia**
(i)	About its main axis $I_G = I_{mainaxis} = MR^2$	
(ii)	About its tangent perpendicular to its plane $I_T = I_G + MR^2$ $= 2MR^2$	

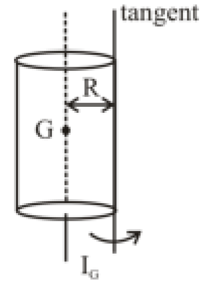
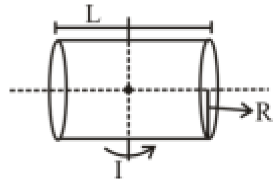
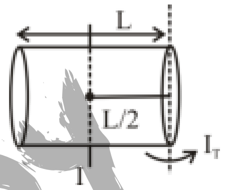
(iii)	About its diameter $I_X + I_Y = I_Z$ $I_{CD} + I_{AB} = I_Z$ $I_{dia} + I_{dia} = MR^2$ $I_{dia} = \frac{MR^2}{2}$	
(iv)	About its tangent parallel to diameter $I_T = I_{AB} + MR^2$ $= \frac{1}{2}MR^2 + MR^2$ $\frac{3}{2}MR^2$	

2. For a disc (M.R) -

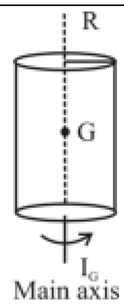
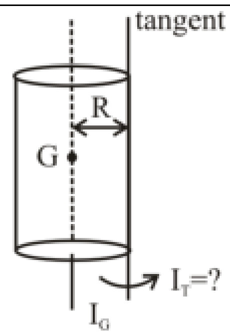
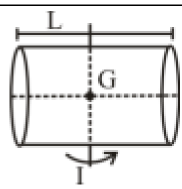
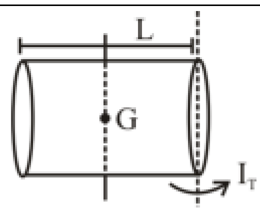
Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = I_{mainaxis} = \frac{1}{2}MR^2$	
(ii)	About its tangent perpendicular to its plane $I_T = I_G + MR^2$ $= \frac{3}{2}MR^2$	
(iii)	About its diameter $I_{dia} = \frac{1}{4}MR^2$	
(iv)	About its tangent parallel to diameter $I_T = \frac{5}{4}MR^2$	

3. For a hollow cylinder (M,R,L)-

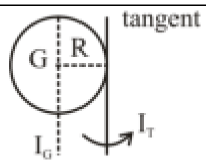
Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = I_{mainaxis} = MR^2$	

(ii)	About its tangent $I_T = I_G + MR^2$ $= MR^2 + MR^2$ $= 2MR^2$	
(iii)	About an axis passing through its diameter of circular surface $I = M\left[\frac{L^2}{12} + \frac{R^2}{2}\right]$	
(iv)	About its tangent parallel to diameter $I_T = M\left[\frac{L^2}{3} + \frac{R^2}{2}\right]$	

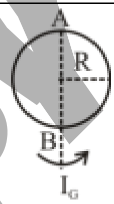
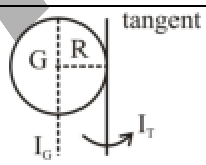
4. For Solid Cylinder [M.L.R]-

Sr. No.	**Axis of Rotation**	**Moment of Inertia**
(i)	About its main axis $I_G = I_{mainaxis} = MR^2/2$	
(ii)	About its tangent $I_T = \frac{3}{2}MR^2$	
(iii)	About an axis passing through its diameter of circular surface $I = M\left[\frac{L^2}{12} + \frac{R^2}{4}\right]$	
(iv)	About its tangent parallel to diameter $I_T = M\left[\frac{L^2}{3} + \frac{R^2}{4}\right]$	

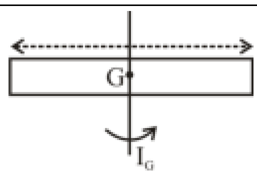
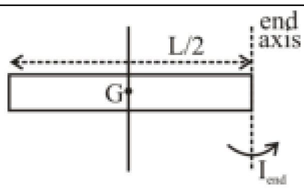
5. Moment of inertia of Hollow Sphere-

Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = I_{mainaxis} = \frac{2}{3}MR^2$	
(ii)	About its tangent $I_T = \frac{5}{3}MR^2$	

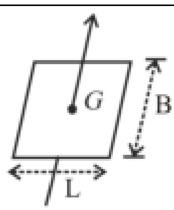
6. For solid Sphere-

Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = I_{mainaxis} = I_{dia} = \frac{2}{5}MR^2$	
(ii)	About its tangent $I_T = \frac{7}{5}MR^2$	

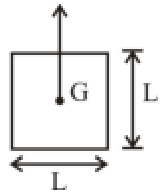
7. For rod (M,L)-

Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = I_{mainaxis} = \frac{1}{12}ML^2$	
(ii)	About its end points $I_{end} = \frac{1}{3}ML^2$	

8. For Rectangular Lamina (Plate)-

Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = \frac{1}{12}M\left(L^2 + B^2\right)$	

9. For Square Lamina -

Sr. No.	Axis of Rotation	Moment of Inertia
(i)	About its main axis $I_G = \frac{1}{12}M\left(L^2 + L^2\right)$ $I_G = \frac{1}{6}ML^2$	

10. Moment of inertia of di-atomic molecule about its centre of gravity-

$I = \mu r^2$

Where , $\mu = \frac{m_1 m_2}{m_1 + m_2}$

= Reduced Mass

= Average Mass of two Mass system.

Moment of force (τ)-

- Torque or moment of force about the axis of rotation

  $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta\hat{n}$
- It is a vector quantity
- It is also known as moment of force or couple .
- Its S.I. Unit is N-m.

Sign convention for torque-

- If the object is rotating in a clockwise direction then torque is considered negative.

  $\boxed{\tau = -Fd}$
- If the object is rotating in a anti-clockwise direction then torque is considered positive.

  $\boxed{\tau = +Fd}$

Relation between τ and I is

$\vec{\tau} = I\vec{\alpha}$

Where, α = angular acceleration, I = moment of inertia

■ Angular Momentum

The moment of linear momentum is called angular momentum.

It is denoted by L.

$L = I\omega = mvr = \vec{r} \times m\vec{v}$

Its SI unit is J-s and its dimensional formula is $[ML^2T^{-1}]$.

Relation between torque and angular momentum-

$\tau = T_{ext}$ = Rate of change in angular momentum

$\tau = \frac{\Delta L}{\Delta t} = \frac{L_2 - L_1}{\Delta t} = \frac{dL}{dt}$ = external torque on the system.

Law of conservation of angular momentum-

The Law of conservation of angular momentum states that if no external force acts on a system, then its angular momentum remains constant

Angular momentum (L) = Moment Inertia (I) ×angular speed (ω)

$L = I\omega$ = constant

$L_1 = L_2$

$I_1\omega_1 = I_2\omega_2$

Example - While revolving in its elliptical orbit, when a planet approaches the sun Its moment of Inertia about the sun decreases. To conserve the angular momentum, its angular speed increases.

■ **Kinetic Energy of rotational motion-**

$$K.E = \frac{1}{2}I\omega^2$$

Where, I = moment of inertia, ω = angular speed

■ **Power delivered by the Torque-**

$$P = \int_{\omega_1}^{\omega_2} \tau d\omega = \tau.\omega$$

■ **Work done by the Torque-**

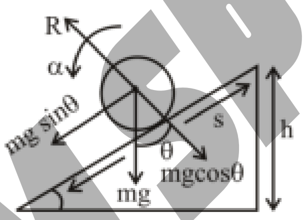
$$\tau = f(\theta)$$

$$W = \int_{\theta_1}^{\theta_2} \tau(\theta)\, d\theta = \tau\theta$$

■ **Motion of a body Rolling down without slipping on an inclined Plane-**

- **Acceleration of the body-**

$$\alpha = \frac{mg\sin\theta}{m+\frac{I}{r^2}} = \frac{g\sin\theta}{1+\frac{K^2}{r^2}}$$



Where, K = radius of gyration

m = mass of the body.

θ = inclination of the Plane.

- **Velocity attained at the bottom-**

$$V = \sqrt{\frac{2gh}{1+\frac{K^2}{r^2}}} = \sqrt{\frac{2gS\sin\theta}{1+\frac{K^2}{r^2}}} \quad \left(\because S = \frac{h}{\sin\theta}\right)$$

- Time taken by the rolling body to reach at the bottom of inclined plane-

$$\text{Time} = \sqrt{\frac{2S\left(1+\frac{K^2}{r^2}\right)}{g\sin\theta}}$$

Where, S = length of slope, r = radius of rolling body and h = height of slope.

Concept-

- If a cylinder, ring, disc and sphere rolls on inclined plane, then the sphere will reach the bottom first with greater velocity while the ring will reach the bottom with least velocity.
- If a solid and hollow body of same shape are allowed to roll down an inclined plane, then solid body will reach the bottom first with greater velocity.
- For rolling without slipping - the minimum value of coefficient friction-

$$\mu = \frac{F}{R} = \frac{\left(I\alpha/r^2\right)}{mg\cos\theta}$$

■ **Pure rolling (or rolling without slipping) on stationary surface-**

→ Condition : $V_{cm} = R\omega$

→ In accelerated motion $a_{cm} = R\alpha$

→ If $V_{cm} > R\omega$ then rolling with forward slipping.

→ If $V_{cm} < R\omega$ then rolling with backward slipping.

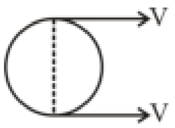
→ Total kinetic energy of rolling object-

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

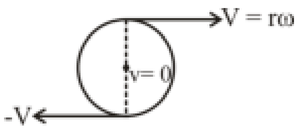
Concept-

In rolling all points of the body have same angular speed but different linear speed.

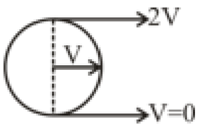
- In pure Translational motion-



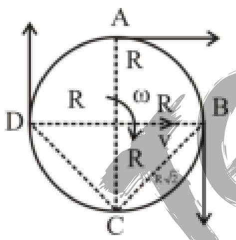
- In pure rotational motion-



- In combined motion i.e translational as well as rotational motion-



- To determine linear velocity of any point on the rolling bodies if velocity of its center of mass is V-



- $V_A = r\omega = 2R \times \frac{V}{R} = 2V$
- $V_B = r\omega = R\sqrt{2} \times \frac{V}{R} = V\sqrt{2}$
- $V_D = r\omega = R\sqrt{2} \times \frac{V}{R} = \sqrt{2}V$

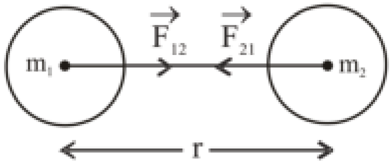
■ **Key Points-**

- A ladder is more opt to slip, when you are high up rather than when you just began to climb because at high up on a ladder the torque is large and on climbing up the torque is small.
- The hardboiled egg and raw egg can be distinguished on the basis of spinning of both.
- Instantaneous axis of rotation is stationary with respect to ground.
- All particles of a rigid body, which do not lie on an axis of rotation move on circular paths with centers at an axis of rotation.
- The relative angular velocity between any two points of a rigid body is zero at any instant.

(10) Gravitation

Gravitation - It is the force of attraction between any two bodies.

■ Newton's Law of Gravitation-



- Force of attraction between two masses m_1 and m_2 separated by a distance r -

$$F = \frac{Gm_1m_2}{r^2}$$

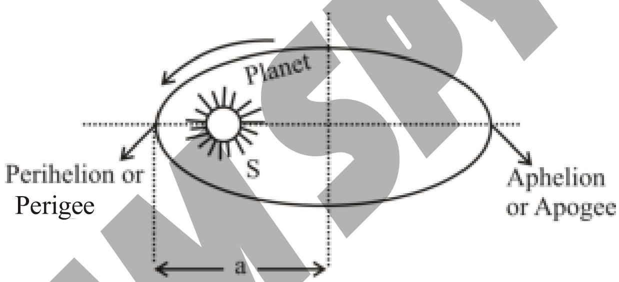
Where, G = Universal gravitational constant

$G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

$\vec{F}_{12} = -\vec{F}_{21}$

■ Kepler's Law's of Planetary Motion –

(i) Laws of orbit -

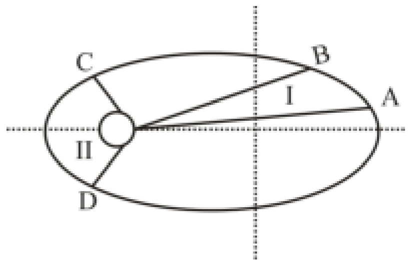


- Every planet revolves around the earth in an elliptical orbit and sun is at its focus.

(ii) Law of area -

- The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time i.e that areal velocity of the planet around the sun is constant.

$$\text{Areal velocity of planet } \frac{\text{Area swept}(\Delta A)}{\text{Time}(\Delta t)} = \frac{L}{2m} = \text{constant}$$



Where, L = angular momentum

m = mass of the planet

(iii) Laws of period -

- The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of its elliptical orbit.

$$T^2 \propto a^3 \text{ or } \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

■ Acceleration due to gravity –

The uniform acceleration produced in a freely falling object due to the gravitational pull of the earth.

$$g = \frac{GM}{R^2} = \frac{4}{3}(\pi\rho GR)$$

- for same material on two planet -

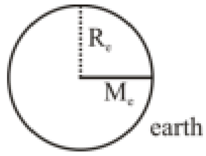
$$g \propto R$$

$$\frac{g_1}{g_2} = \frac{R_1}{R_2}$$

Where, ρ = density of earth, M = mass of earth and R = radius of earth

Acceleration due to gravity at earth's surface –

$$g_{earth} = \frac{GM_e}{R_e^2} = 9.8 \text{ m/sec}^2$$



Where, $M_e = 6\times10^{24}$ Kg

$R_e = 6.4\times10^6$ m or 6400 km

Acceleration due to gravity at moon surface –

$$g_{moon} = \frac{GM_m}{R_m^2}$$

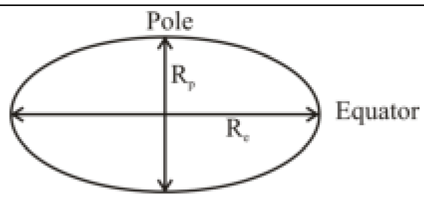
$$g_{moon} = \frac{g_{earth}}{6}$$

Where, $M_m = 7.34\times10^{22}$ Kg

$R_m = 1.74\times10^6$ m or 1737.4 km

■ Variation of gravity -

(i) Due top shape of earth-

$g = \frac{GM_e}{R_e^2}$ $g \propto \frac{1}{R_e^2}$ $R_{equator} - R_{polar} = 21$ Km $R_{equator} > R_{Polar}$ $g_{equator} < g_{polar}$	

(ii) Due to height from earth surface -

$$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} = g\left(1-\frac{2h}{R}\right)$$

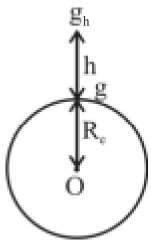
- The value of 'g' decrease with height. It varies inversely as the square of the distance from the center of the earth.

If $h = R_e$, $g_h = \frac{g}{4}$

If $h \to \infty$, $g_h = 0$

Decrease in the value of g with height is $g_h - g = \frac{2hg}{R_e}$

Fractional decrease in the value of $g = \frac{g_h - g}{g} = \frac{2h}{R}$



(iii) Due to depth from earth surface -

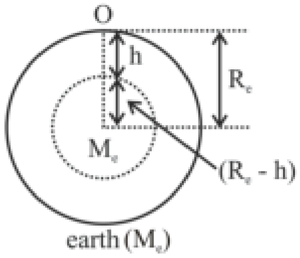
$$g' = g\left(1 - \frac{h}{R_e}\right)$$

If $h = R_e$, $g'_{centre} = 0$

Decrease in the value of 'g' with depth is $\Delta g = g - g' = \frac{gh}{R_e}$

Fractional decrease in the value of 'g' with depth is $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{h}{R_e}$

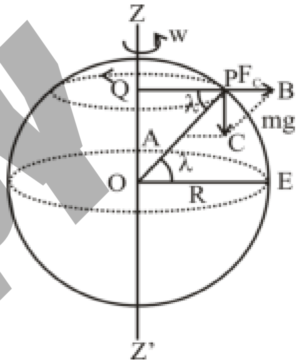
Percentage error in g is $= \frac{\Delta g}{g} \times 100 = \frac{h}{R_e} \times 100$



(iv) Rotation of earth about its own axis-

If ω is the angular velocity of rotation of earth about its own axis then acceleration due to gravity at a place having latitude λ is given by - $g' = g - R\omega^2\cos^2\lambda$

- ♦ At poles $\lambda = 90°$ and $g' = g$. Therefore, there is no effect of rotation of earth about its own axis at poles.
- ♦ At equator $\lambda = 0°$ and $g' = g - R\omega^2$. Therefore value of g is minimum at equator.
- ♦ If earth stops its rotation about its own axis, then g will remain unchanged at poles but increase by $R\omega^2$ at equator.



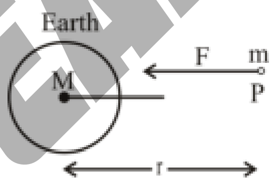
■ Gravitational field –

- The space around a body in which its gravitational pull works, is called gravitational field.
- The gravitational force acting per unit mass at any point in gravitational field is called intensity of gravitational field at that point.
- It is denoted by $\vec{E}$.

$$\vec{E} = \frac{\vec{F}}{m}$$

$$\vec{F} = \frac{GMm}{r^2}$$

$$\vec{E} = -\frac{GM}{r^2}\vec{e}_r$$



Where, $\vec{e}_r$ is the unit vector along $\vec{r}$

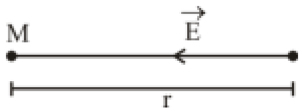
– ve sign shows force of attraction

- Intensity is zero at infinite distance from the body.

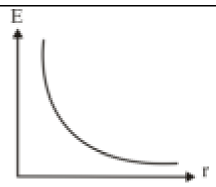
Gravitational field intensity for different bodies -

(1) Intensity due to point mass -

$$\vec{E} = -\frac{GM}{r^2}\vec{e}_r$$

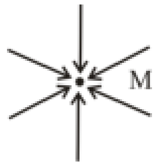


Where, $\vec{e}_r$ is the unit vector along $\vec{r}$

If r = 0, E = ∞ If r = ∞, E = 0	

Concept

- Gravitational field intensity always direct towards the mass (M) of the body.



(2) Intensity due to uniform solid sphere –

- Outside the Surface , r > R

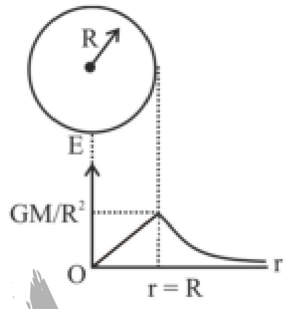
$$\vec{E} = -\frac{GM}{r^2}$$

- On the Surface, r = R

$$\vec{E} = -\frac{GM}{R^2}$$

- Inside the Surface, r < R

$$E = -\frac{GMr}{R^3}$$



(3) Intensity due to Spherical shell -

- Outside the Surface , r > R

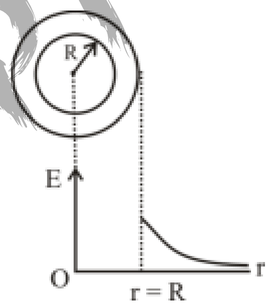
$$E = -\frac{GM}{r^2}$$

- On the Surface, r = R

$$E = -\frac{GM}{R^2}$$

- Inside the Surface, r < R

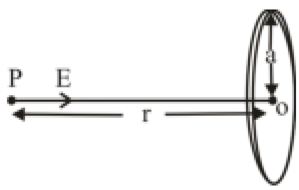
E = 0



(4) Intensity due to uniform circular ring-

- At a point on its axis

$$E = -\frac{GMr}{(a^2+r^2)^{3/2}}$$



- At the centre of ring, E = 0

■ Gravitational Potential (V) -

- It is amount of work done in bringing a unit mass m_0 from infinity to a point in the gravitational field without acceleration.

$$V = \int_{+\infty}^{r} \frac{GM}{r^2}\,dr$$

$$V = -\frac{GM}{r} = \frac{W}{r}$$

- If r = R (Surface of earth)

$$V = \frac{-GM}{R}$$

- If r = ∞, V = 0 means 'V' is maximum at infinity.
- Its S.I. units is J/Kg.
- It is a scalar quantity.
- Since, work W is obtained i.e. it is negative, the gravitational potential is always negative.

■ Gravitational Potential Energy (U) -

- It is amount of work done in bringing a massive body from infinity to a point in the gravitational field without acceleration.

$$U = -\frac{GMm}{r}$$

- Negative sign shows attractive force.
- When $r = \infty$, $U = 0$ (Maximum gravitational P.E.)
- Change in P,E. of a body, when it moves from distance r_1 to distance r_2

$$\Delta U = \int_{r_1}^{r_2} \frac{GMm}{r^2}\, dr$$

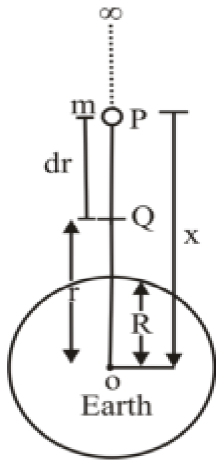
$$\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

- ♦ Gravitational potential energy at height 'h' from surface of earth

$$U_h = -\frac{GMm}{R+h} = \frac{mgR}{1+\frac{h}{R}}$$

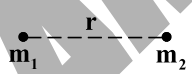
- ♦ P.E. of the body decreases when it brought close to the earth.
- ♦ Change in P.E. of a body, which is at a height 'h' above the surface of the earth.

  $\Delta U = mgh$



Gravitational Potential Energy of a two particle system-

$$U = -\frac{Gm_1m_2}{r}$$



Gravitational Potential Energy for a System of more than two particles- (say m_1, m_2, m_3 & m_4)

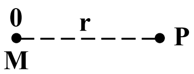
$$U = -G\left[\frac{m_4m_3}{r_{43}} + \frac{m_4m_2}{r_{42}} + \frac{m_4m_1}{r_{41}} + \frac{m_3m_2}{r_{32}} + \frac{m_3m_1}{r_{31}} + \frac{m_2m_1}{r_{21}}\right]$$

$\therefore$ for n particle system there are $\frac{n(n-1)}{2}$ pairs

■ Gravitational Potential for different bodies -

(i) Potential due to a point mass -

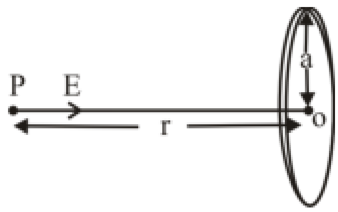
$$V = -\frac{GM}{r}$$



(ii) Potential due to uniform ring –

- ♦ At a point on its axis

$$V = -\frac{GM}{\sqrt{a^2 + r^2}}$$



- ♦ At the centre of ring, $V = -\frac{GM}{a}$

(iii) Potential due to spherical shell -

- Outside the Surface , r > R

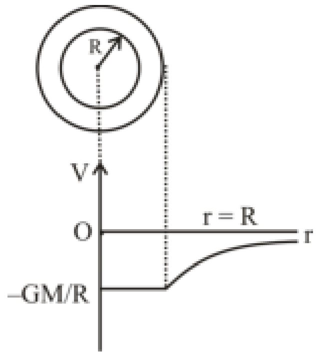
$$V = -\frac{GM}{r}$$

- On the Surface, r = R

$$V = -\frac{GM}{R}$$

- Inside the Surface, r < R

$$V = -\frac{GM}{r}$$



(iv) Potential due to uniform solid sphere -

- Outside the Surface , r > R

$$V = -\frac{GM}{r}$$

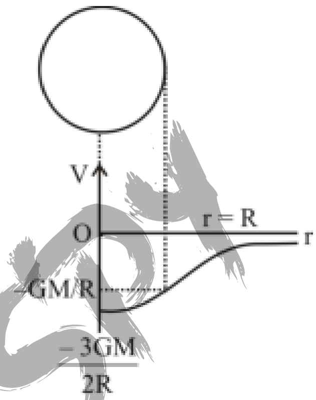
- On the Surface, r = R

$$V_{surface} = -\frac{GM}{R}$$

- Inside the Surface,

$$V = -\frac{GM}{2R}\left[3-\left(\frac{r}{R}\right)^2\right]$$

$$V_{centre} = \frac{-3}{2}\frac{GM}{R}$$



■ Relation between Gravitational Field and Potential –

$$dV = -E.dr$$

Where, $E = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$$dr = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$dV = -E_x dx - E_y dy - E_z dz$$

$$E_x = \frac{-\partial V}{\partial x},\; E_y = \frac{-\partial V}{\partial y},\; E_z = \frac{-\partial V}{\partial z},$$

■ Orbital Velocity -

- Orbital velocity of satellite is the minimum velocity required to put the satellite into a given orbit around earth.

$$V_o = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{R+h}}$$

Where, M = mass of the planet
R = radius of the planet
h = height of the satellite from planets surface.

- If satellite revolving near the earth's surface then orbital velocity $V_o \approx 7.92$ Km/s.
- If V is the speed of satellite in its orbit and V_o is the required orbital velocity.

(i) If $V < V_o$, then satellite will move on a parabolic path and satellite will fall back to earth.
(ii) If $V = V_o$, the satellite will revolve in circular path/orbit around earth.
(iii) If $V_o < V < V_e$, then satellite will revolve around earth in elliptical orbit.

■ Escape Velocity -

- Escape velocity of earth is the minimum velocity with which a body has to be projected vertically upwards from the earth surface, so that it just crosses the earth's gravitational field and never returns.

Case-1

- Escape velocity does not depend upon the mass, shape and size of the body as well as direction of projection of body.

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$V_e \propto m^o$

Case-2 Escape velocity in terms of density of that planet -

$$\rho = \frac{M}{V}$$

$$V_e = R\sqrt{\frac{8}{3}\pi G\rho}$$

Case- 3 – Condition for black hole of any planet –

Escape velocity of the planet $= \sqrt{\frac{2\,GM}{R}} \geq$ light velocity (3×10^8)

Case-4

- If Projection Velocity (V_p) of a body from earth-surface is more than escape velocity (V_e) then to determine the velocity of that body in space -

$$V_s = \sqrt{V_p^2 - V_e^2}$$

Some important escape velocity –

Heavenly body	**Escape Velocity**
Moon	2.3 Km/sec
Mercury	4.28 Km/sec
Earth	11.2 Km/sec
Jupiter	60 Km/sec
Sun	618 Km/sec
Neutron star	2×10^5 Km/sec

Relation between escape velocity and orbital velocity–

$$V_e = \sqrt{2}V_o$$

■ Motion of Satellite –

- Orbital velocity $V_o = \sqrt{\frac{GM}{r}}$
- Time period $T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$
- Kinetic energy $K = \frac{GMm}{2r}$
- Potential energy $U = -\frac{GMm}{r}$
- Total mechanical energy $E = -\frac{GMm}{2r}$
- Near the surface of earth,

  $r \approx R$ and $V_o = \sqrt{\frac{GM}{R}} = \sqrt{Rg} = \frac{V_e}{\sqrt{2}} = 7.9\text{Kms}^{-1}$

  This is the maximum speed of any earth's satellite.
- Time period of such satellite would be

  $$T = \frac{2\pi}{\sqrt{GM}} R^{3/2} = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

  This is the minimum time period of any earth's satellite.
- Binding energy B.E $= +\frac{GMm}{2r}$
- Angular momentum of Satellite, $L = \sqrt{m^2GMr}$
- Effective weight in a satellite, W = 0 and satellite behaves like a free fall body.

■ Geostationary or Parking Satellites –

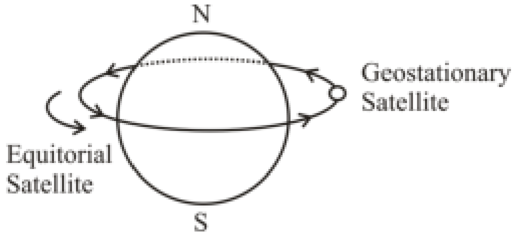
- A satellite which appears to be at a fixed position at a definite height to an observer on earth is called geostationary or parking satellite. They rotate from west to east.

  Height from earth's surface = 36000 Km

  Time period = 24 hour

  Orbital velocity = 3 Km/sec

  Angular velocity = $\frac{2\pi}{24} = \frac{\pi}{12}$ rad/min



- These satellite are used in communication purpose.
- INSAT 2B and INSAT 2C are geostationary satellite of India.

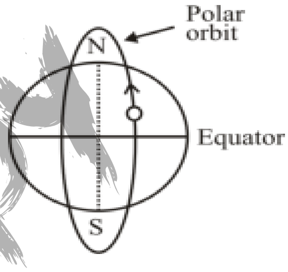
■ Polar Satellites –

- These are those satellites which revolve in polar orbits around earth.
- Height form earth's surface ≈ 880 km

  Time period ≈ 90 min

  Orbital velocity ≈ 8 km/sec

  Angular velocity ≈ $\frac{2\pi}{90} = \frac{\pi}{45}$ rad/min



- These satellite revolve around the earth in polar orbits.
- These satellites are used in forecasting weather studying the upper region of the atmosphere in mapping etc.
- PSLV series satellites are polar satellites of India.

■ Key Points -

(i) At the centre of earth, a body has centre of mass, but no centre of gravity.

(ii) The centre of mass and centre of gravity of body coincide if gravitational field is uniform.

(iii) We does not experience gravitational force in daily life due to objects of same size as value of G is very small.

(iv) Moon travellers tie heavy weight at there back before landing on moon due to smaller value of g at moon.

(v) Space rockets are usually launched in equatorial line from west to east because g is minimum at equator and earth rotates from west to east about its axis.

(vi) Angular momentum is gravitational field is conserved because gravitational force is central force.

(vii) Kepler's second law or constancy of areal velocity is a consequence of conservation of angular momentum.

(viii) The energy required by satellite to leave its orbit around the earth (planet) and escape to infinity is called Binding Energy of satellite.

(11) Elasticity

Deforming Force – A force which produces a change in configuration of the objects on applying it.

Elasticity – The property of the body to regain its original configuration after removing the deforming force .

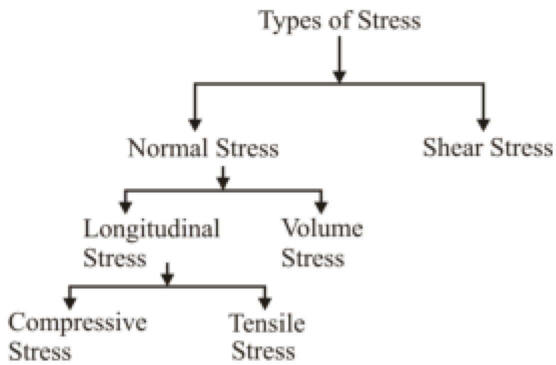
■ **Stress** – The stress is internal restoring force acting per unit area of a deformed body.

Stress = $\frac{F}{A}$ = restoring force per unit area.

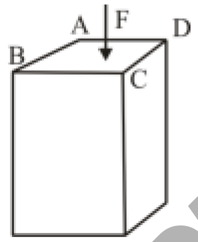
- If the configuration of the body is not permanently changed then stress in defined as –

Stress = $\frac{\text{External deforming force}}{\text{Area}}$.

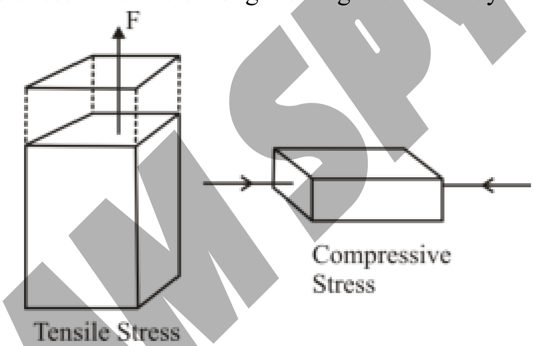
- Stress is tensor quantity.
- S.I. unit of stress is N/m^2, Pascal (P_a).



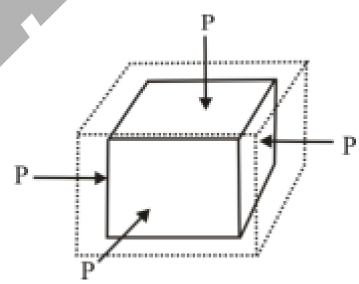
(1) Normal Stress – The stress which acts perpendicular to the cross sectional area of any object or body.



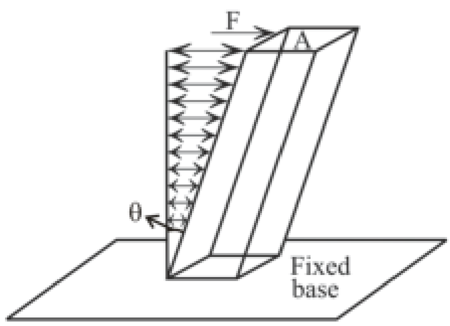
(a) Longitudinal Stress - The stress which acts along the length of the body.



(b) Volumetric Stress - The stress which acts from all dimension resulting in the change in the volume of the object.



(2) Shear stress - The stress which acts parallel to the cross-sectional area of any object or body is called shear stress.



■ **Strain -** The effect of stress in the body or object is called strain.

The ratio of change in configuration to the original configuration is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

- It is unit less.

Types of strain -

(i) Longitudinal Strain = $\frac{\text{Change in Length}}{\text{Original length}}$.

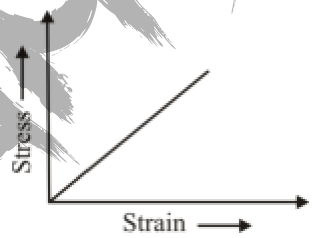
(ii) Volumetric Strain = $\frac{\text{Change in Volume}}{\text{Original Volume}}$.

(iii) Shear Strain = Angular displacement of the plane perpendicular to the fixed surface.

$= \tan\theta = \frac{\Delta x}{\ell}$

Hooke's law -

- Within the limit of elasticity, the stress is proportional to strain.



Stress $\propto$ Strain

Stress = E × Strain

E = Modulus of elasticity = $\frac{\text{Stress}}{\text{Strain}}$.

- A material is more elastic when its E is large.

■ **Elastic Modulus -** The ratio of stress and strain is called modulus of elasticity or elastic moduli.

Types of modulus of elasticity -

(1) Young's Modulus of elasticity -

- It is defined as the ratio of longitudinal stress to the longitudinal strain within the elastic limit

$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$Y = \frac{F}{A}\frac{l}{\Delta l}$$

- Young's modulus is defined only for Solids.
- When impurity in the solids increases, Y increases.
- When temperature increases, Y decreases.

- It's S.I. unit is N/m^2 or Pascal.
- Dimensional formula is $[ML^{-1}T^{-2}]$.

■ Force Constant of wire -

- Force required to produce unit elongation in a wire is called force constant of a material of a wire. It is denoted by k. $k = \frac{YA}{l}$.

  Where, Y = Young's modulus of elasticity

  A = Cross – section area of wire.

(2) Bulk Modulus of elasticity -

- It is defined as the ratio of volumetric stress to the volumetric strain within the elastic limit.

$$K = \frac{\text{Volumetric Stress}}{\text{Volumetric Strain}}$$

$$K = -\frac{FV}{A\,\Delta V} = -\frac{\Delta P.V}{\Delta V}$$

Where, $\Delta P = \frac{F}{A}$ = change in pressure.

- – ve sign indicates that when pressure decreases volume increases and vice -versa.
- Bulk modulus of elasticity is defined for all the three states of matter.
- Its S.I. unit is N/m^2 or Pascal.
- Its dimensional formula is $[ML^{-1}T^{-2}]$
- **Compressibility** of the material $C = \frac{1}{K}$
- S.I. unit of compressibility is $Pascal^{-1} = N^{-1}\,m^2$.
- CGS unit of compressibility is = $dyne^{-1}\,cm^2$.

(3) Modulus of rigidity (η) -

- It is defined as the ratio of tangential stress to the shearing strain.

$$\eta = \frac{\text{Tangential Stress}}{\text{Shearing Strain}}$$

$$\eta = \frac{F}{A\theta}$$

- It is define only for Solid.
- It's S.I. unit is N/m^2 or Pascal.
- It's dimensional formula is $[ML^{-1}T^{-2}]$

Factor affecting elastic limit–

(i) Modulus of elasticity of material decreases with the rise in temperature, except for invar steel.

(ii) By annealing elasticity of the material decreases.

(iii) By hammering or rolling elasticity of the material increases.

(iv) Addition of impurities affects elastic properties depending on whether impurities are themselves more or less elastic.

(v) For liquids, modulus of rigidity is zero.

■ Poisson's Ratio –

- When a deforming force is applied at the free and of a suspended wire of length and Radius R, then its length increases by dl but radius decreases by dR.

$$\text{Poisson's ratio} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = \frac{-\Delta R/R}{\Delta l/l}.$$

- Practical value lies between 0 and 0.5.;
- Theoretical limits of poisson 'σ' $= -1 < \sigma < \frac{1}{2}$

■ Relation between Y, K, η and σ -

(i) $Y = 3K(1 - 2\sigma)$

(ii) $\sigma = \dfrac{3K - 2\eta}{2\eta + 6K}$

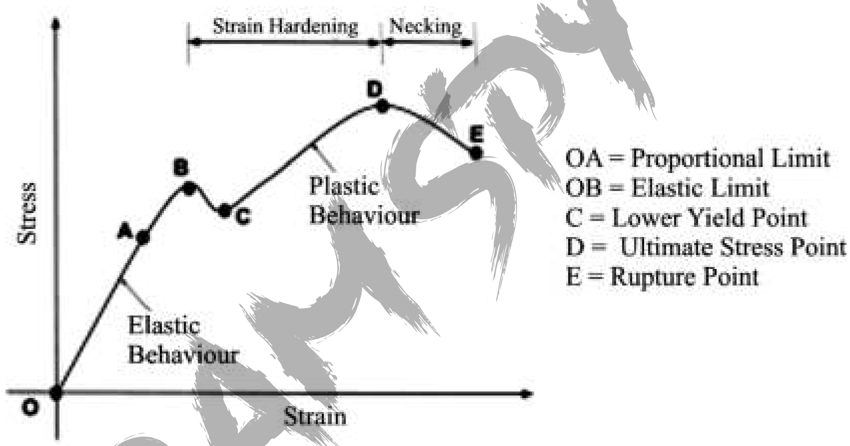
(iii) $Y = 2\eta[(1 + \sigma)]$

(iv) $\dfrac{9}{Y} = \dfrac{1}{K} + \dfrac{3}{\eta}$

(v) $Y = \dfrac{9K\eta}{\eta + 3K}$

■ Stress-Strain Curve -

- It is the plot of stress (y-axis) and strain (x-axis) of a material or metal in the graph.
- The stress and Strain diagram for different material is different.



(i) Proportional limit -

It is the region in the stress-strain curve which obey's Hooke's law.

(ii) Elastic limit -

It is the point in the graph up to which the material returns to its original position when the load acting on it is completely removed. Beyond this limit the material cannot retain its original position and plastic deformation starts to appear in it.

(iii) Yield point or Yield stress point –

it is defined as the point at which the material starts to deform plastically after the yield point is passed there is permanent deformation in the material and which is not reversible.

(iv) Ultimate stress point -

It is the point corresponding to the maximum stress that a material can handle the maximum load.

(v) Fracture or Breaking point –

It is the point in the curve at which the failure of the material takes place. The fracture or breaking of material takes place at this point.

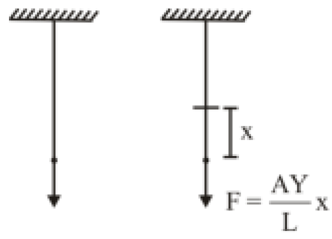
- If large deformation takes place between the elastic limit and the fracture point, the material is called ductile.
- If it breaks soon after the elastic limit is crossed it is called brittle.

■ Elastic potential Energy - in a stretched wire –

When a body is in its natural shape, its potential energy corresponding to the molecular forces is minimum and we may take the potential energy in this state to be zero.

When a body deformed, internal forces appears and work has to be done against these forces.

Thus, the potential energy of the body is increased. This is called the elastic potential energy.



$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2} \times \left(Y\frac{l}{L}\right) \times \left(\frac{l}{L}\right) \times (AL)$$

■ Work done per unit Volume (or Energy per unit Volume in a strain) -

(i) Longitudinal Strain –

Work done per unit volume of the wire or strain energy per unit volume of the wire –

$$\frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2}\left(\frac{F}{A}\right) \times \left(\frac{l}{L}\right)$$

(ii) Volume Strain -

Work done (or strain energy) per unit volume.

$$= \frac{1}{2} \times P \times \frac{\Delta V}{V}$$

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

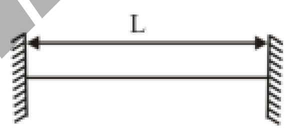
(iii) Shearing Strain -

Work done (or strain energy) per unit volume

$$= \frac{1}{2}\left(\frac{F}{A}\right)(\theta)$$

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

■ Thermal Stress -



- Thermal stress is induced in a body when the temperature of the body is raised or lowered and the body is not allowed to expand or contract freely. Thermal stress includes both heat and cold stress.

Thermal Stress = $\frac{F}{A} = Y\,\alpha\,\Delta\theta$

Where, α = coefficient of linear expansion of the material of the rod.
Y = Young's modulus of elasticity
$\Delta\theta$ = change in temperature.

(12) Beams, Bending of Beams & Torsion of Cylindrical Bodies

■ **Beam -** A rod or a bar of a circular or rectangular cross sections with its length very much greater than its thickness (so that there are no shearing stresses over any section of it) is called a beam.

■ Bending moment of beam -

- Bending moment $= \dfrac{\text{Flexural Rigidity}}{R} = \dfrac{YI_g}{R}$

  Where, Y = Young's modulus of elasticity.

- For a beam of rectangular cross section, $Ig = \dfrac{bd^3}{12}$.

  Where, b = breadth of the beam
  d = depth of the beam
  I_g = geometrical moment of inertia.

- For a beam of circular cross section, $I_g = \dfrac{\pi R^4}{4}$.

  Where, R = radius of its cross section.

■ Stiffness of beam -

- The stiffness of a beam is taken to be the ratio between the maximum deflection of its loaded end and its span and is usually denoted by the symbol $1/n$.
- For steel girders with a large span, n should lie between 1000 to 2000, and for those with a smaller span, between 500 to 700. For timber beam, it should be 360 or above.

■ The Cantilever (Depression of its loaded end) -

- If the beam is fixed only at one end and loaded at the other end, then it is called the cantilever.

Case (i) - Weight of the cantilever ineffective - $\delta = \dfrac{WL^3}{3YI_g}$.

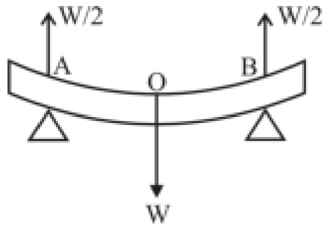
Where, δ = depression of the loaded end
L = length of Cantilever
Y = Young's modulus of rigidity
I_g = Geometrical moment of Inertia
W = Weight (loaded at its free end)

Case (ii) - Cantilever uniformly loaded - $\delta = \dfrac{WL^3}{8YI_g}$.

In case there if no load placed on the cantilever and it bends under its own weight i.e. $W = W_0$, $\delta = \dfrac{W_0L^3}{8YI_g}$.

■ Depression of Supported and fixed beam -

Case (i) - Supported beam, centrally loaded and when the weight of the beam is ineffective -



$\delta = \dfrac{WL^3}{48YI_g}$.

Where, L = Length of cantilever.

Case (ii) - Supported beam uniformly loaded - $\delta = \frac{5}{8}\left(\frac{WL^3}{48YI_g}\right)$.

If there is no load placed on the beam i.e. $W = W_0$ then - $\delta = \frac{5}{8}\left(\frac{W_0L^3}{48YI_g}\right)$.

Case (iii) - Fixed beam, centrally loaded - $\delta = \frac{1}{4}\left(\frac{WL^3}{48YI_g}\right)$.

i.e. the depression of the mid point of beam is one fourth of that when it is simply supported at the end.

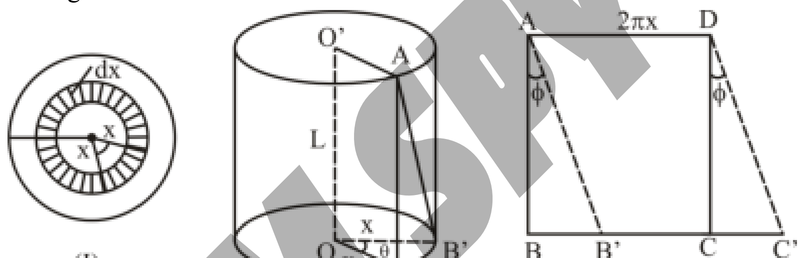
Concept -

- Equivalence of a shear to a tensile and a compressive strain at right angles to each other and each equal half the shear.
- Equivalence of a shearing stress to an equal tensile and an equal compressive stress at right angle to each other.

■ Twisting Couple of a Cylinder -

(i) Case of a solid cylinder or wire-

Let a solid cylinder (or wire) of length L and radius R be fixed at its upper end and let a couple be applied to its lower end in a plane perpendicular to its length (with its axis coinciding with that of cylinder) such that it is twisted through an angle 'θ'.



(I) (II) (III)

- The moment of this force about the axis OO' of the cylinder is $= \frac{2\pi\eta\theta}{L}x^3dx$.
- Twisting couple on the whole cylinder, $\int_0^R \frac{2\pi\eta\theta}{L}x^3dx = \frac{\pi\eta R^4}{2L}\theta$.
- Twisting couple per unit twist of the cylinder or wire, is called torsional rigidity of its material is given by - $C = \frac{\pi\eta R^4}{2L}$.

(ii) Case of Hollow Cylinder -

- If the cylinder be a hollow one of inner and outer radii R_1 and R_2 respectively we have-
- Twisting couple on the cylinder $= \int_{R_1}^{R_2} \frac{2\pi\eta\theta}{L}x^3dx = \frac{\pi\eta}{2L}\left(R_2^4 - R_1^4\right)\theta$.
- Twisting couple per unit twist - $C' = \frac{\pi\eta}{2L}\left(R_2^4 - R_1^4\right)$.

Concept -

- If we consider two cylinders of the same material, of density ρ, and of same mass M and length L but one solid of radius R and the other hollow of inner and outer radii R_1 and R_2 respectively -

  $\frac{C'}{C} = \frac{R_2^2 + R_1^2}{R^2}$ and $\frac{C'}{C} > 1$ or $C' > C$.

  The twisting couple per unit twist is greater for a hollow cylinder than for a solid one of the same material, mass and length.

Variation of Stress in a Twisted Cylinder -

(i) Along the radius -

- Shearing stress at distance x from the axis = ηϕ.

- Maximum shearing stress at distance R from the axis = $\eta\phi_m$,

$$\frac{\text{Shearing stress at distance x}}{\text{Maximum shearing stress}} = \frac{\eta\Phi}{\eta\Phi_m} = \frac{\Phi}{\Phi_m} = \frac{x}{R}$$

- Stress at distance x from the axis = $\frac{x}{R} \times$ maximum stress.

(ii) Along the length -

- Strain or shear on the surface of the cylinder - $\theta = \frac{L}{R}\phi_m$

So, shear on the surface of the cylinder $\frac{R}{L} \times \frac{L}{R}\phi_m = \phi_m$ is same as in the plane at distance L from the fixed end.

Thus, the shearing stress on the surface of the cylinder is quite independent of the distance from either end of it.

■ Strain Energy in a twisted cylinder -

- The work done or energy stored up in a cylinder twisted through an angle θ is - $\frac{1}{2}C\theta^2$

Where, C = couple per unit twist.

(13) Hydrostatics

Hydrostatics- The branch of physics that deals with the study of fluids at rest is called fluid static or Hydrostatics.

■ Fluids-

- Fluids are those substances which has no fixed shape and can flow when external force is applied on them.
- Liquids and gases can flow and therefore called fluids.
- An ideal fluid is incompressible, Non-viscous, Irrotational

■ Thrust-

- A force acting normally on a surface is called thrust
- Thrust is a vector quantity.
- Thrust is measured in dynes in C.G.S system and in Newton's (N) in S.I. system.
- The gravitational unit of thrust is kilogram force Kgf.
  Where, 1 kgf = 9.8 N

■ Pressure - The amount of force exerted (thrust) on a surface per unit area is called pressure.

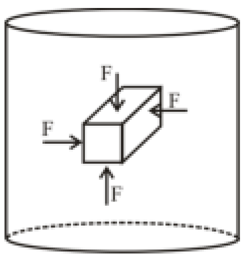
$$\text{Pressure} = \frac{\text{Thrust}}{\text{Area}} = \frac{F_\perp}{A}$$

- Pressure is a scalar quantity.
- Unit of pressure -
  - In S.I. unit = Nm^{-2} = Pascal = Pa
  - 1atm = 1.01×10^5 Pa ≈ 1 bar ≈ 760 mm of Hg ≈ 760 torr
  - 1 torr = 133.32 Pa
  - In gravitational unit = $Kgfm^{-2}$
  1 $Kgfm^{-2}$ = 10^5 Pa

■ Pressure in fluids-

When an object is submerged in a fluid at rest, the fluid exerts a force on its surface . This force is always normal to the object's surface.

$$\text{Pressure} = \frac{\text{Force}_\perp}{\text{Area}}$$



■ Density -

- The density of any material is defined as its mass per unit volume. If a body of Mass M occupies volume V, then its density is.-

$$\rho = \frac{M}{V}$$

- Density is positive scalar quantity.
- As liquids are incompressible, their density remains constant at all pressures.
- Density of a gas varies largely with pressure.

Example- water = 1.0×10^3 Kg-m^{-3} , Mercury = 13.6×10^3 kg-m^{-3} , Air = 1.29 kg-m^{-3}

■ Relative density/ Specific Gravity-

- Relative density of a substance is the ratio of its density to the density of water at 4 °C

$$R.D = \frac{\text{Density of sustance}}{\text{Density of water at 4°C}}$$

- Water has highest density at 4°C = 1g / cm^3 =10^3kg/m^3
- Relative density is a unit less quantity.

Example -

$$(R.D)_{Hg} = \frac{13.6\times10^3\, kg/m^3}{10^3\, kg/m^3} = 13.6$$

■ Density of a mixture of two or more liquids-

Case I - In the proportion of Mass -

The density of mixture is $\rho = \dfrac{M_1 + M_2 + M_3 + ...}{\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + ...}$

Case (II) - In the proportion of volume-

The density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + }{V_1 + V_2 + V_3 + }$..........

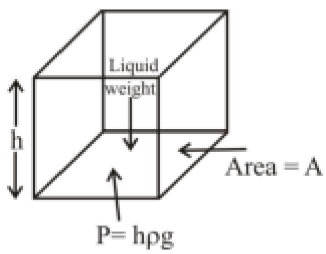
■ Pressure Exerted by Liquid Column-

The pressure exerted by a liquid column at rest is proportional to

(i) Height of liquid column (h)

(ii) Density of the liquid (ρ)

$P = h\rho g$



■ Absolute Gauge, Atmospheric and vacuum pressures-

(i) Absolute pressure -

- It is the zero reference to a perfect vacuum which exist in the air free space of the universe.
- It is the sum of gauge pressure and atmospheric pressure.

$P = P_0 + \rho gh$

Where, $P_{gauge} = \rho gh$

(ii) Gauge pressure -

- It is the pressure, zero referenced against ambient Pressure
- It is also called overpressure. Gauge pressure is + ve for pressure above the atmosphere and negative for a pressure below it.

$P_{gauge} = P - P_0$

(iii) Atmospheric Pressure-

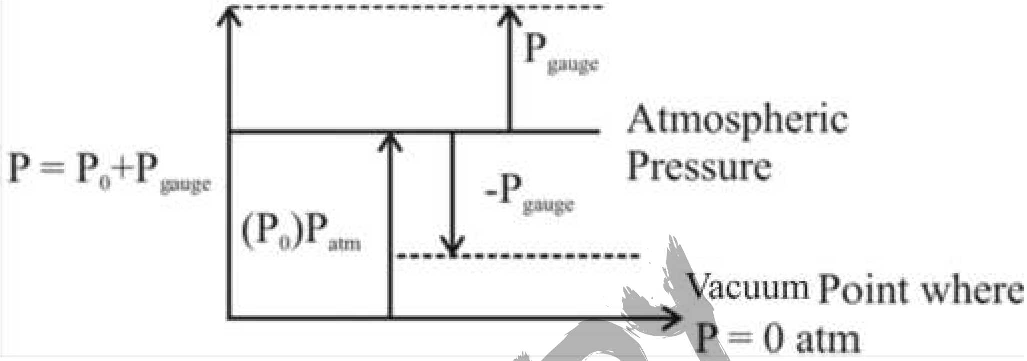
- It is the pressure exerted by the weight of the atmosphere.
- The standard value of air pressure at sea level is 14.696 psi at 59 deg F heat
- In imperial unit and metric units - it is 1013.25 milibar at 15 deg C

(iv) Vacuum Pressure -

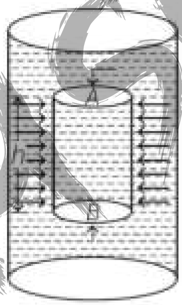
It is the pressure below atmospheric pressure measured by vacuum gauges. It indicates the difference between atmospheric pressure and absolute pressure.

$P_{vac} = P_0 - P$

$P_{vac} = - P_{gauge}$



■ Effect of gravity on Fluid Pressure-



$P_B = P_A + \rho gh$

Where, h = vertical height.

- The liquid pressure is the same at all points at the same horizontal level or same depth.
- Pressure at any point inside the fluid depends on the depth h.
- Pressure does not depend on the cross- section or base area or the shape of the vessel.

■ Pascal's law -

- This law tell us how pressure can be transmitted in a fluid.
- The pressure exerted at any point on an enclosed liquid is transmitted equally in all direction.
- The pressure in a fluid at rest is same at all points if we ignore gravity.

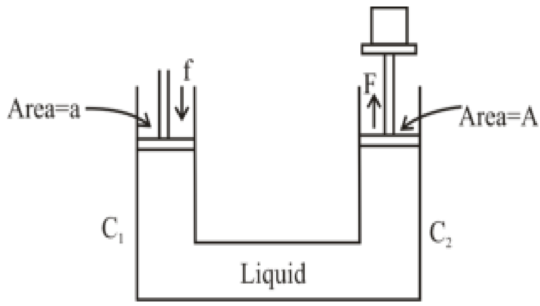
Application of Pascal's law-

(i) Hydraulic lift-

- Hydraulic lift is an application of Pascal's law.
- It is used to lift heavy objects.
- It is a force multiplier.
- According to Pascal's Law , the same pressure P is also transmitted to the larger piston of cross- sectional area A.
- Force on larger piston is-

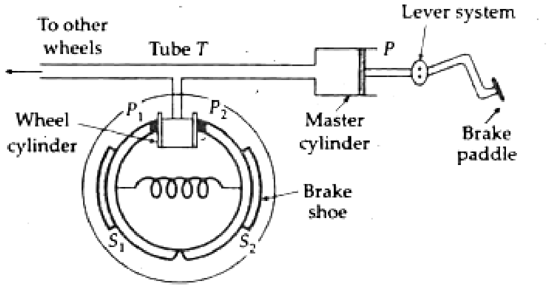
$$F = P\times A = \frac{f}{a}A = \frac{A}{a}\times f$$

As A > a , therefore , F > f



(ii) Hydraulic brakes-

The hydraulic brakes used in automobiles are based on Pascal's Law of transmission of pressure in a liquid.



- The Working of hydraulic lift and hydraulic brakes are based on Pascal's law.

■ Hydrostatic Paradox-

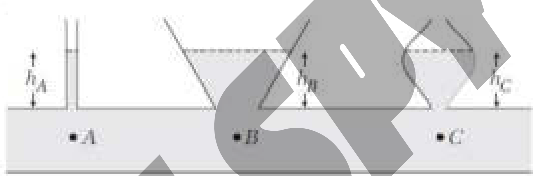
The liquid pressure at a point is independent of the quantity of liquid but depends on the depth of point below the liquid surface this is known as hydrostatic paradox.

Important Point related to fluid pressure-

(i) The pressure exerted at any point on an enclosed liquid is transmitted equally in all direction.

(ii) Pressure will be same at all point at the same level in a liquid.

(iii) The pressure exerted by a liquid depends only on the height of fluid column and is independent of the shape of the containing vessels.



If $h_A = h_B = h_C$, then $p_A = p_B = p_C$

(iv) Pressure at a point inside the liquid increases with the increase in the density of liquid.

■ Buoyancy-

The upthrust force experienced by a body when partly or wholly immersed in a fluid is called upthrust or buoyant force .The phenomena responsible for this force is called buoyancy

■ Archimedes Principle -

- Archimedes Principle states that when a body is partially or wholly immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced by it and its upthrust acts through the center of gravity of the displaced fluid.

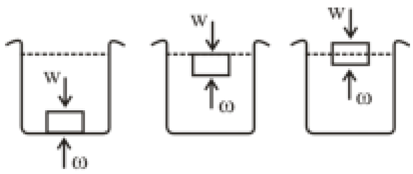
  Upthrust or buoyant force = weight of liquid displaced -
- Apparent weight of immersed body = Actual weight – Buoyant force

  $W_{app} = W\left(1-\frac{\rho}{\sigma}\right)$ Where, $W = V\sigma g$ = weight of the body, σ = density of the body, ρ = density of liquid

  V = volume of liquid displaced.

■ Law of floatation-

A body will float in a liquid, if the weight of the body is equal to or less than weight of the liquid displaced by the immersed part of the body.



(a) $w > \omega$ (b) $w = \omega$ (c) $w < \omega$

If W is the weight of the body and ω is the buoyant force, then -

(a) If $W > \omega \Rightarrow V\sigma g > V\rho g$ or $\sigma > \rho$, Then a body sinks in a liquid.

(b) If $W = \omega \Rightarrow V\sigma g = V\rho g$ or $\sigma = \rho$, then the body will be float fully immersed in the liquid.

(c) If $W < \omega$, then the body will float partially submerged in the liquid.

- The floating body will be in stable equilibrium, if meta-centre (center of buoyancy) lies vertically above the center of gravity of body.

- The floating body will be in unstable equilibrium, if meta center (centre of buoyancy) lies vertically below the centre of gravity of the body.
- The floating body will be in neutral equilibrium if meta centre coincides with the centre of gravity of the body.
- Fraction of volume of a floating body outside the liquid.

$$\frac{V_{out}}{V} = \left[1 - \frac{\rho}{\sigma}\right]$$

where, ρ = density of body

σ = density of liquid

If two different bodies A and B are floating in the same liquid , then

$$\frac{\rho_A}{\rho_B} = \frac{(V_{in})_A}{(V_{in})_B}$$

If the same body is made to float in different liquids of densities σ_A and σ_B respectively then-

$$\frac{\sigma_A}{\sigma_B} = \frac{(V_{in})_A}{(V_{in})_B}$$

(14) Hydrodynamics

Fluid dynamics:- The branch of physics that deals with the study of fluids in motion is called fluid dynamics or hydrodynamics.

■ Types of fluid flow -

- **Steady and unsteady flow -**

Steady flow	**Unsteady flow**
Fluid property do not change with time.	Change with time
$\frac{\partial V}{\partial t} = 0, \frac{\partial P}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0$	$\frac{\partial V}{\partial t} \neq 0, \frac{\partial P}{\partial t} \neq 0, \frac{\partial \rho}{\partial t} \neq 0$

This is applicable for all properties.

- **Uniform and non-uniform flow :**

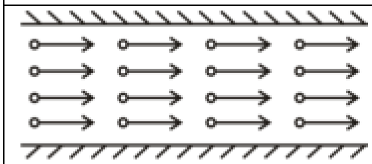
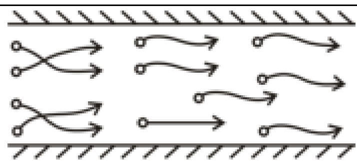
Uniform	**Non-uniform**
Velocity at any given, time does not change with respect to space. $\left(\frac{\partial V}{\partial s}\right)_{t=c} = 0$	Velocity at any given time changes with respect to space. $\left(\frac{\partial V}{\partial s}\right)_{t=c} \neq 0$

It is applicable for velocity only.

∂s = length of flow in the direction s.

∂V = change of velocity.

- **Laminar and Turbulent flow -**

Laminar flow	**Turbulent flow**
Fluid particle move along well defined path. The adjacent layer do not cross each other.	Fluid particle move in random order of zig-zag way.
	

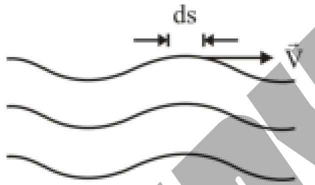
Reynolds number < 2000 (for pipe flow) Reynolds number < 5×10^5 (for flow over the flat plate)	Reynolds number > 4000 (for pipe flow) Reynolds number = 5×10^5 (For flow over flat plate)

- **Compressible and incompressible flow -**

Compressible flow	Incompressible flow
Density of fluid varies point to point ($\rho \neq c$). Density of fluid varries with respect to Pressure	Density remains constant ($\rho = c$). Density of fluid is constant with respect to Pressure

■ Type of flow lines -

(1) Stream line : Stream line is an imaginary line or curve in a flow field such that the tangent drawn at any point gives the direction of instantaneous velocity at that point.



- Flow never takes place perpendicular to stream line.
- Stream line equation in 3D flow.

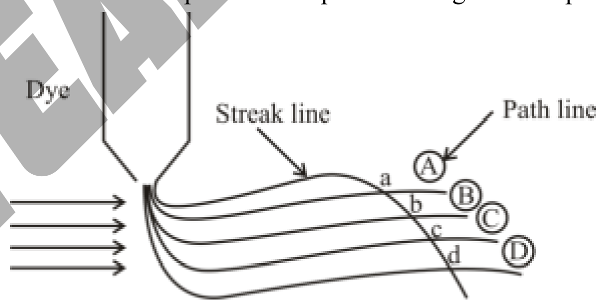
$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

- Stream line equation in 2D flow.

$$\frac{dx}{u} = \frac{dy}{v}$$

(2) Streak Line :-

- At any instant of time it is locus of all particles i.e. passes through a fixed point in a flow fluid.



(3) Path line -

- Path travelled by a single individual particle over same time interval is called path line.

Equation for path line, $u = \frac{dx}{dt}, v = \frac{dy}{dt}, w = \frac{dz}{dt}$

■ Reynold's Number :-

- It is dimensionless parameter whose value decides the nature of flow of a liquid through a pipe. It is given by -

$$R_e = \frac{\rho v D}{\eta}$$

Where, ρ = density of the liquid.
v = velocity of the liquid.
η = coefficient of viscosity of the liquid.
D = diameter of the pipe.

■ Equation of continuity -

- Conservation of mass (m) ; Mass flow rate is constant.
- For compressible flow, $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$.
- For incompressible flow, Density ρ will be constant $A_1V_1 = A_2V_2$. (Q = AV is discharge)
- General continuity equation -

$$\frac{\partial \rho}{dt} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Where, u, v & w are the components of velocity in x, y & z direction.

- Continuity equation for steady flow $\left(\frac{\partial \rho}{\partial t} = 0\right)$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

- Continuity equation for incompressible flow i.e. ρ = constant

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\nabla \vec{V} = 0$$

$$\text{div } \vec{V} = 0$$

■ Energy of a fluid in a steady state -

Energy	**Kinetic Energy**	**Pressure Energy**	**Potential Energy**
Per unit mass	$\frac{1}{2}V^2$	$\frac{P}{\rho}$	gh
Per unit volume	$\frac{1}{2}\rho V^2$	P	ρgh

■ Bernoulli's Equation -

Assumptions -

1. Fluid is ideal i.e. viscosity is zero.
2. Steady flow.
3. The flow is incompressible.
4. The flow is irrotational.
5. The flow is along the stream line.

Mathematically, Equation in form of energy per unit volume

$$P + \frac{1}{2}\rho V^2 + \rho g h = \text{constant}$$

Equation in form of head

$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \text{constant}$$

Where, $\frac{P}{\rho g}$ = pressure head.

$\frac{V^2}{2g}$ = velocity head or K.E. head

h = gravitational head.

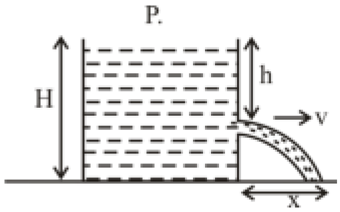
Application of Bernoulli's Equation -

1. Venturimeter (To measure the rate of discharge or flow rate).
2. Orificemeter (To measure the rate of discharge or flow rate).
3. Pitot tube (To measure the fluid velocity).

■ Torricelli's Theorem or law of efflux -

- Consider a vessel containing a liquid upto some height H. A small hole is punched in the wall at a depth h below the free surface. The speed of liquid coming out of the hole is called the velocity of efflux of the liquid.

$V = \sqrt{2gh}$



- Time taken by liquid to reach base levels.

$t = \sqrt{\frac{2(H-h)}{g}}$

- The distance at which liquid hits the ground.

$x = 2\sqrt{h(H-h)}$

■ Viscosity -

- It is a measure of resistance to flow due to internal; frictional force that develops between adjacent layers of fluid when they try to move over each other.
- These internal frictional forces are called viscous forces.
- Viscous forces are intermolecular forces acting between the molecules of different layers of liquid moving with different velocities.

Viscous force $(F) = -\eta A \frac{dv}{dx}$

$$\eta = -\frac{F}{A\left(\frac{dv}{dx}\right)}$$

Where, η = Coefficient of viscosity and it depends on the nature of fluid.

A = Area of cross section.

$\frac{dv}{dx}$ = Velocity gradient or rate or change of velocity with distance.

- Negative sign indicates opposition to relative motion η is measured in N-s/m^2 or dyne-s/cm^2 or poise

Variation of Viscosity -

- The Viscosity of liquids decreases with increase in temperature.
- The viscosity of liquids increases with increase in pressure but the viscosity of water decreases with increase in pressure.
- The viscosity of gases increases with increase of temperature because when temperature of gas increase then rate of diffusion increases.

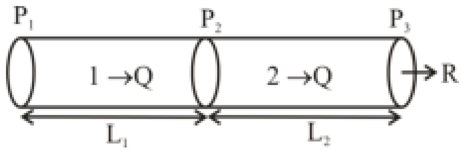
■ Flow of fluid through narrow tube, Poiseuille's Formula -

- The rate of flow of a fluid of coefficient of viscosity η through a tube of length *l* and radius of cross-section r when subjected to a pressure difference P is-

$V = \frac{\pi}{8}\frac{Pr^4}{\eta \ell}$

Where, $R = \frac{8\eta\ell}{\pi r^4}$ called liquid resistance.

(i) When two tubes are connected in series -

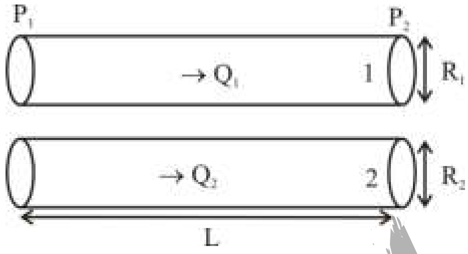


(a) Rate of flow of liquid is same through both tubes.

(b) Resultant pressure difference, $P = P_1 + P_2$

(c) Equivalent liquid resistance - $R = R_1 + R_2$

(ii) When two tubes are connected in parallel -



(a) Pressure difference (P) is same across both tubes.

(b) Rate of flow of liquid $V = V_1 + V_2$.

(c) Equivalent liquid resistance $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

■ Stoke's Law -

- When a spherical body of radius r falls through a fluid of viscosity η with a terminal velocity v a viscous drag acts on it is given by

  $F = 6\pi\eta r v$

  This is called Stoke's Law.

■ Terminal Velocity -

- When a small spherical body falls in a long liquid column, then after sometimes it falls with a constant velocity, called terminal velocity.

  $$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

  Where, ρ = density of body.

  σ = density of liquid.

  η = coefficient of viscosity of liquid.

  g = acceleration due to gravity.

(i) if $\rho > \sigma$, the body falls down wards.

(ii) if $\rho < \sigma$, the body moves upwards with the constant velocity.

(iii) if $\sigma << \rho$, $v = \frac{2r^2\rho g}{9\eta}$.

■ Critical Velocity -

- The critical velocity is the velocity of liquid flow, below which its flow is streamlined and above which its become turbulent.

  $$V_c = \frac{\eta R_e}{r\rho}$$

  Where, R_e = Reynolds number.

  η = Coefficient of viscosity of liquid.

  r = Radius of capillary tube.

  ρ = Density of the liquid.

■ Key Points -

(i) For water specific gravity is 1.0 at 4ºC and mercury specific gravity is 13.6.

(ii) Specific gravity varies with temperature therefore it should be determined at specified temperature (4ºC to 25ºC).

(iii) Density of water is 1000 kg/m^3.

(iv) In case of steady flow two stream lines can never intersect each other.

(v) A path line is an outcome of lagrangian approach.

(vi) A path line can intersect itself or other path line.

(vii) In case of steady flow, path line, stream line and streak line are identical.

(viii) Fluid flow is possible where mass is conserved and continuity equation is satisfied.

(ix) If the flow is steady but rotational then Bernoulli's can only be applied between those points which comes in same stream line because constant of integration will be different for different stream line.

(x) Terminal velocity depends on the radius of the sphere in such a way that, if radius becomes n times, then terminal velocity will becomes n^2 times.

(xi) Stokes' Law helps a man coming down with the help of parachute.

(xii) Stoke's Law accounts for the formation of clouds.

(xiii) Stoke's Law was used in the determination of electronic charge by Milikan in his oil drop experiment.

(15) Surface Tension

■ Surface Tension :-

- It is the elastic tendency of a fluid surface which makes it acquire the least surface area possible.
- Surface tension allows insects to float and stride on a water surface.
- It is a scalar quantity.
- It is denoted by σ .
- Surface Tension (σ) = $\dfrac{\text{Tension Force (F)}}{\text{Lenght } (\ell)}$.
- SI unit = N/m
- CGS unit = dyne/cm
- Dimension = $[MT^{-2}]$
- Small liquid drops are spherical due to property of surface tension.
- Surface tension of liquid depends only on the nature of liquid and is independent of the surface area of film or length of line considered.
- Surface tension of a liquid is numerically equal to its surface energy.

Factor Affecting Surface Tension -

(i) Surface tension of liquid decreases with increase in temperature and becomes zero at critical temperature.

(ii) At boiling point, surface tension of a liquid becomes zero and becomes maximum at freezing point.

(iii) Surface tension decreases when partially soluble, impurities such as soap, detergent, dettol, phenol etc are added in water.

(iv) Surface tension increases when highly soluble impurities such as salt is added in water.

(v) When dust particles or oil spreads over the surface of water. its surface tension decreases.

Adhesion - The force of attraction between the molecules of the different substances is called the force of Adhesion.

Example **(i)** Chalk particles to the black board.
(ii) Sticking of glue to the wood.
(iii) force of attraction acting between water molecules with capillary tube.

Cohesion -

- The force of attraction between molecules of same substance is called the force of Cohesion.
- Decreasing order of force of Cohesion
  Solid > Liquid > Gas.
- Due to maximum force of cohesion in solids, it have definite shape and resist all deforming forces,

Example - (i) Force of attraction acting between water molecules.

▪ Surface Energy -

- The potential energy per unit area of the surface film is called the surface energy.

$$\text{Surface Energy} = \frac{\text{Workdone in increasing the surfaec area}}{\text{Increase in surface area}}$$

Excess Pressure due to Surface Tension -	
Excess pressure	**Excess pressure (ΔP)**
Inside a liquid drop	$\Delta P = \frac{2T}{R}$
Inside a liquid or soap bubble	$\Delta P = \frac{4T}{R}$
Inside an air bubble	$\Delta P = \frac{2T}{R}$
For a spherical surface ($R_1 = R_2 = R$)	$\Delta P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, $\Delta P = \frac{2T}{R}$
For a cylindrical surface ($R_1 = R$ & $R_2 = \infty$)	$\Delta P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, $\Delta P = \frac{T}{R}$
On plane surface	$\Delta P = 0$
In liquid film of unequal radii	$\Delta P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Where, T = surface tension, R = radius of curvature

- Work done in spraying a liquid drop of radius R into n droplets of radius r -

  $= T \times$ increase in surface area $= 4\pi TR^3\left(\frac{1}{r} - \frac{1}{R}\right)$

  Fall in temperature, $\Delta\theta = \frac{3s}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

  Where, J = 4.2 J/cal.

- When n small drops are combined into a bigger drop, then work done is given by, $W = 4\pi R^2 T\left(n^{1/3} - 1\right)$

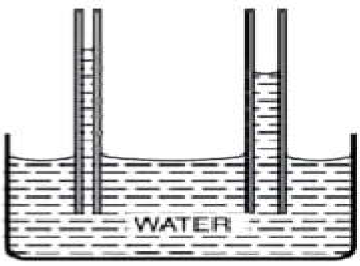
  Temperature increases, $\Delta\theta = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

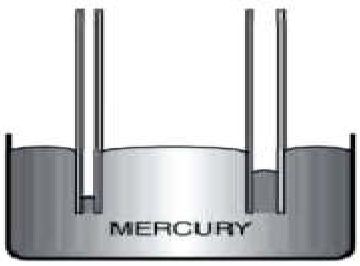
- When two soap bubbles of radii r_1 and r_2 ($r_2 > r_1$) are brought in contact with each other, then the interface acquires radius of curvature $r = \frac{r_2 r_1}{r_2 - r_1}$

- When two soap bubbles collapse to form a single bubble, the radius of curvature of the new bubble formed is r $= \sqrt{r_1^2 + r_2^2}$

▪ Capillarity or Capillary Action -

- Capillary is a phenomenon by which a liquid (depending upon its specific gravity) rises or fall into a thin glass tube above or below its general level. This phenomenon is due to the combined effect of Cohesion and Adhesion of liquid particles. The rise of liquid surface is known as capillary rise while the fall of liquid surface is known as capillary depression.





- Ascent of liquid column in capillary tube is given by -

$$h = \frac{2T\cos\theta}{r\,\rho g}$$

- If capillary is very narrow, then -

$$h = \frac{2T\cos\theta}{r\rho g}$$

Where, r = radius of capillary tube.
ρ = density of the liquid.
θ = angle of contact.
T = surface tension of liquids.

- If $\theta < 90^\circ$, h = positive (capillary rise) $\rightarrow$ Adhesive force is greater than cohesive force.
- If $\theta > 90^\circ$, h = negative (capillary fall) $\rightarrow$ Cohesive force is greater than Adhesive force.
- $\theta = 0^\circ$, for pure water and clean glass tube.
- $\theta = 130^\circ$, for mercury and clean glass tube.

Practical Applications of Capillarity –

(i) Blotting paper absorbs ink by capillary action.
(ii) Sap (moisture) rises in plant by capillary actions.
(iii) Coffee power is easily soluble in water because water immediately wets the fine granules of coffee by the action of capillarity.
(iv) We use a towel to dry our body after bath, which soaks water due to capillary action.

Angle of Contact -

- The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called the angle of contact or the capillary angle.
- The angle of contact depends on the nature of the solid and liquid in contact.
- Angle of contact $\theta < 90^\circ \Rightarrow$ Concave shape & Liquid rise up.
- Angle of contact $\theta > 90^\circ \Rightarrow$ Convex shape & Liquid falls.
- Angle of contact $\theta = 90^\circ \Rightarrow$ Plane shape liquid neither rise nor falls.
- Angle of contact for pure water and glass is zero.
- For ordinary water and glass it is 8° ($\theta < 90^\circ$).
- For mercury and glass it is 138° ($\theta > 90^\circ$).
- For pure water and silver it is 90°.
- For alcohol and clean glass $\theta = 0^\circ$.

Some Properties of Surface tension -

- It acts tangentially to liquid surface.
- Surface tension is always produced due to cohesive force.
- More is the cohesive force, more is the surface tension.
- When surface area of liquid increases, molecules from the interior of the liquid rise to the surface. For this work is done against the downward Cohesive force.

■ Key Points -

- The maximum distance upto which a molecule can attract some other molecule is known as Range of intermolecular forces.
- Surface tension of liquid is molecular phenomena.
- Energy required to split bigger drop into smaller drop -

$$\Delta U = \Delta S \times T$$

Where, ΔS = increase in surface area
T = surface tension of liquid.

- When a thin soap film is formed by dipping a U-shaped wire loop having a slider in a soap solution of surface tension T, then the maximum weight attached to the slider, the film can support before it will breaks, is given by- $Mg = 2(T \times l)$

Where, l= length of slider.

- If a bigger drop of radius R is splits into n small drops of radius r, then-

(i) The radius of small drop formed is given by -

$$r = n^{-1/3} R$$

(ii) The increase in surface area is given by -

$$\Delta S = \left(n^{1/3} - 1\right) \times 4\pi R^2$$

(iii) The energy required to split the bigger drop is given by -

$$\Delta U = \left(n^{1/3} - 1\right) \times 4\pi R^2 T$$

(iv) The decrease in temperature is given by -

$$\Delta\theta = \frac{3T}{R\rho s}\left(n^{1/3} - 1\right) = \frac{3T}{\rho s}\left(\frac{1}{r} - \frac{1}{R}\right)$$

Where, ρ density of the liquid

s = specific heat

- When a capillary tube of insufficient length is dipped in a liquid, the liquid will not overflow. It will rise upto the top end of the tube and then increase the radius of curvature of its meniscus to a suitable value R', so that R'h' = Rh (Zurin's Law).

Where, R = radius of curvature of meniscus, when the liquid rises in the capillary tube to a height h.

h' = insufficient length of the capillary tube.

(16) Special Theory of Relativity

■ Frame of Reference -

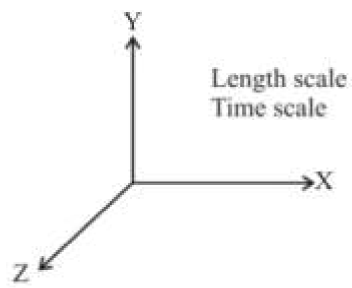
(i) Inertial frame of reference -

- In this case Newton's I[st] law of motion is valid i.e. $\vec{F} = 0$ then $\vec{a} = 0$,
  But in daily life we observe Newton's I[st] law of motion for example (If ball is in rest until the external force is applied).
- But in the case of bus and train without applying the external force we bend when the bus and train stops. (In this case Newton's I[st] law is not valid for this another frame is valid).

(ii) Non-Inertial frame of reference -

- Newton's I[st] law is not valid.
- Therefore, we impartly apply Newton's first law and this force is called Pseudo force.

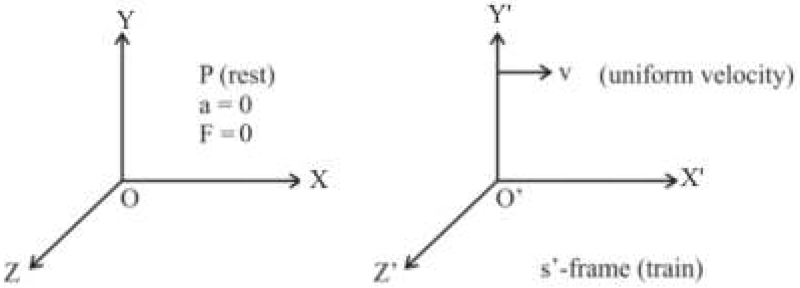
■ Axis System -



■ Galilean Transformation -

- When the particle velocity is less than velocity of light i.e. v <<< c then in this condition we use Galilean Transformation.

- For high velocity $v \approx c$ Galilean transformation is not adequate.

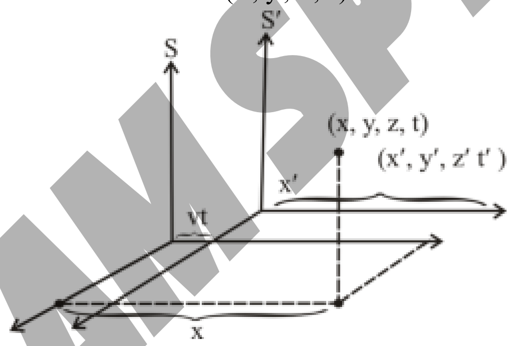


S-frame (Platform)

- If S' moves with uniform velocity w.r.t. S, then S' is also inertial.
- When O' crossing O at t = t' = 0
  - (i) X-X' axis are on same line.
  - (ii) Y-Y' axis are parallel.
  - (iii) Z-Z' axis are parallel.

■ Galilean Transformation equation -

- Measurement of event from s-frame is (x, y, z, t).
- Similarly, measurement of event form s'-frame is (x', y', z', t').



- $x' = x - vt$ $\quad\quad$ $x = x' + vt$
  $y' = y$ $\quad\quad$ $y = y'$
  $z' = z$ $\quad\quad$ $z = z'$
  $t' = t$ $\quad\quad$ $t = t'$

Invariance of Newton's Laws of motions under Galilean transformation (GT) -

- $$\left.\begin{aligned} V'_x &= V_x - V \\ V'_y &= V_y \\ V'_z &= V_z \end{aligned}\right\} \text{Velocity transformation equation}$$
- $$\begin{aligned} a'_x &= a_x \\ a'_y &= a_y \\ a'_z &= a_z \end{aligned}$$
- Acceleration of any particles are invariant in all inertial frame.
- Above result is true for only Galilean transformation.
- Newton's laws of motion are invariant in all inertial frames under G.T.

Postulates of Special theory of relativity -

(a) The laws of physics are the same in all inertial system. No preferred inertial system exist. (The Principle of relativity).

(b) The speed of light in free space has the same value c in all inertial system. (The principle of the constancy of the speed of light).

■ Lorentz Transformation -

- Lorentz transformation have to be such that -

(a) It is linear in x and x' so that a single event in frame S corresponds to a single event in frame S'.

(b) For lower velocity it reduces to Galilean transformation.

(c) The inverse transformation exist.

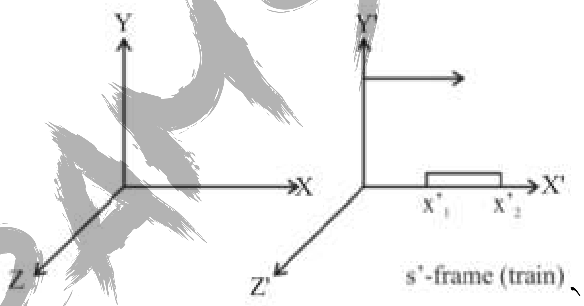
Lorentz Transformation	Inverse Lorentz Transformation
$x' = \dfrac{x - vt}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	$x = \dfrac{x' + v't}{\sqrt{1 - \dfrac{v^2}{c^2}}}$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \dfrac{t - \dfrac{vx}{c^2}}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	$t' = \dfrac{t' + \dfrac{vx'}{c^2}}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Consequences of Lorentz Transformation -

(i) Length Contraction -

- In order to measure the length of an object in motion relative to observer, the position of two end points recorded simultaneously, the length of object in direction of motion appeared smaller to observer.

$$\ell = \ell_0 \sqrt{1 - \frac{V^2}{C^2}}$$



- Thus $l < l_0$ this means that the length of rod as measured by an observer relative to which rod is in motion, is smaller than its proper length.
- Such a contraction of length in direction of motion relative to observer is called Lorentz Fitzgerald contradiction.

■ Time Dilation -

- A clock being at rest in the S' frame measures the time t'_2 and t'_1 of two events occurring at a fixed position x'. The time interval Δt measures from s-frame appear slow (Δt_0) from s-frame.i.e. to the observer the moving clock will appear to go slow.

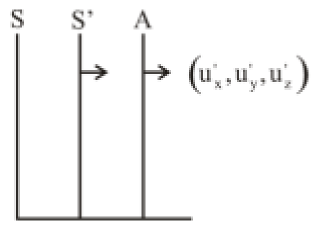
$$\Delta t = \Delta t' = t_2' - t_1' = \gamma\left(t_2' + \frac{vx'}{c^2}\right) - \gamma\left(t_1' + \frac{vx'}{c^2}\right)$$

$$\Delta t = \frac{t_2' - t_1'}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{\Delta t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$\Delta t_0 \rightarrow$ proper time, $\Delta t \rightarrow$ time interval measured from s-frame $\Delta t > \Delta t_0$.

■ Relative velocity -

- There is one inertial frame S, S' is another inertial frame moving with respect to S in x-direction and A is another inertial frames which is moving with respect to S' with velocity component (u'_x, u'_y, u'_z)



So,

$$u'_x = \frac{dx'}{dt'},\ u'_y = \frac{dy'}{dt'},\ u'_z = \frac{dz'}{dt'},$$

The velocity component of A from s-frame is given by -

$$u_x = \frac{dx}{dt},\ u_y = \frac{dy}{dt},\ u_z = \frac{dz}{dt},$$

■ Relative velocity are -

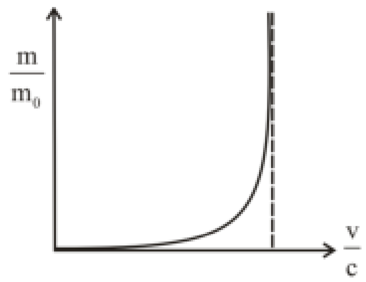
$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} \qquad u'_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x}$$

$$u_y = \frac{u'_y}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)} \qquad u'_y = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

$$u_z = \frac{u'_z}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)} \qquad u'_z = \frac{u_z}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

■ Relativistic Mass -

- The mass of a body moving at the speed v relative to an observer is larger than its mass then at rest relative to the observer by the factor $\dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$



Thus, $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where, m_0 is rest mass of body and m is observed mass.

■ Relativistic Momentum -

$$P = mv = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

■ Relativistic second law of motion -

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) = \frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

■ Relativistic Energy-

- Einstein suggested if m is relativistic mass of body then relativistic energy E is given by -

$$E = mc^2 \text{ and } E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Rest Energy -

- If rest mass of particles is m_0 then rest mass energy is given by m_0c^2.

■ Relativistic Kinetic Energy -

$$K = mc^2 - m_0c^2$$

$$K = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$$

■ Relationship between total energy and momentum -

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } \quad P = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$E^2 = m_0^2c^4 + P^2c^2$$

$$E = \sqrt{m_0^2c^4 + P^2c^2}$$

■ Four Vector and Relativistic Invariance -

- Four position vector ds = (dx, dy, dz, icdt)
- Four velocity vector $\frac{ds}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}, \frac{icdt}{dt},\right)$

$$u = \gamma\left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}, \frac{icdt}{dt},\right)$$

$$u = \gamma(u_x, u_y, u_z, ic)$$

$$u = \gamma(\vec{u}, ic)$$

- Four momentums -four energy vector-

$$P = \gamma(m_0u, i\, m_0c) = (m_0\vec{u}, imc) = \left(\vec{P}, \frac{imc^2}{c}\right) \Rightarrow \vec{P} = \left(\vec{P}, \frac{iE}{c}\right).$$

- Four force $\quad F = \gamma\left(\vec{F}, ic\frac{dm}{dt}\right)$

■ The invariant interval –

- Suppose event A occurs at $(x_A^\circ, x_A^1, x_A^2, x_A^3)$ and event B at $(x_B^\circ, x_B^1, x_B^2, x_B^3)$. The difference

$\Delta x^u \equiv x_A^u - x_B^u$ is the displacement 4-vector.

The scalar product of Δx^u with itself is a quantity of special importance.

The interval between two events -

$I \equiv (\Delta x)_u(\Delta x)^u$

$= -(\Delta x^0)^2 + (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2$

$= -c^2t^2 + d^2$

1. If I < 0 we call the interval time like, for this is the sign we get when the two occur at the same place (d = 0) and separated only temporally.
2. If I > 0 we call the interval space like, for this is the sign we get when the two occur at the same time (t = 0) and are separated only spatially.
3. If I = 0 we call the interval light like, for this is the relation that holds when the two events are connected by a signal traveling at the speed of light.

■ Key Points -

(i) Photons are charge less and mass less particles that travels at speed of light.

(ii) The rest mass of photon is zero. This can be proved from Einstein special theory of relativity.

Exam Oriented Objective Problems

General Physics (Mechanics)

Units, Dimensions and Measurements]

1. The SI unit of a physical quantity is Pascal-second. The dimensional formula of this quantity will be

(a) $[ML^{-1}T^{-1}]$ (b) $[ML^{-1}T^{-2}]$

(c) $[ML^{2}T^{-1}]$ (d) $[M^{-1}L^{3}T^{0}]$

JEE MAIN 2022

2. The dimension of mutual inductance is :

(a) $[ML^2\,T^{-2}\,A^{-1}]$ (b) $[ML^2\,T^{-3}\,A^{-1}]$

(c) $[ML^2\,T^{-2}\,A^{-2}]$ (d) $[ML^2\,T^{-3}\,A^{-2}]]$

JEE MAIN 2022

3. One main scale division of a Vernier calipers is 'a' cm and nth division of the Vernier scale coincide with (n – 1)th division of the main scale. The least count of the calipers in mm is:

(a) $\frac{10na}{(n-1)}$ (b) $\frac{10a}{(n-1)}$

(c) $\left(\frac{n-1}{10n}\right)a$ (d) $\frac{10a}{n}$

JEE MAIN 2021

4. If surface tension (*S*), moment of inertia (*I*) and Planck's constant (*h*), were to be taken as the fundamental units, the dimensional formula for linear momentum would be

(a) $S^{1/2}\,I^{1/2}\,h^{-1}$ (b) $S^{3/2}\,I^{1/2}\,h^{0}$

(c) $S^{1/2}\,I^{1/2}\,h^{-1}$ (d) $S^{1/2}\,I^{1/2}\,h^{-1}$

JEE MAIN 2019

5. Plane angle and solid angle have

(a) Both units and dimensions

(b) Units but no dimensions

(c) Dimensions but no units

(d) No units and no dimensions

NEET 2022

6. The area of a rectangular field (in m2) of length 55.3 m and breadth 25 m after rounding off the value for correct significant digits is

(a) 14×10^{2} (b) 138×10^{1}

(c) 1382 (d) 1382.5

NEET 2022

7. A screw gauge gives the following readings when used to measure the diameter of a wire

Main scale reading: 0 mm

Circular scale reading: 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions on the circular scale. The diameter of the wire from the above data is

(a) 0.052 cm (b) 0.52 cm

(c) 0.026 cm (d) 0.26 cm

NEET 2021

8. Taking into account of the significant figures, what is the value of 9.99 m – 0.0099 m?

(a) 9.98 m (b) 9.980 m

(c) 9.9 m (d) 9.9801 m

NEET 2020

9. A screw gauge has least count of 0.01 mm and there are 50 divisions in its circular scale.

The pitch of the screw gauge is:

(a) 0.25 mm (b) 0.5 mm

(c) 1.0 mm (d) 0.01 mm

NEET 2020

10. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 divisions above the reference level. If screw gauge has a zero error of –0.004 cm, the correct diameter of the ball is

(a) 0.521 cm (b) 0.525 cm

(c) 0.053 cm (d) 0.529 cm

NEET 2018

11. Given the equation → Angular Momentum $=\left(\alpha\beta-\frac{\gamma}{\text{time}}\right)\times$velocity which of the following are possible dimensions of α, β and γ?

(a) M, L, T (b) M, L, MLT

(c) ML, ML, MLT (d) M^2L^{-2}, ML, T

UPTGT Science - 2021

12. The unit of Poynting vector is -

(a) Watt (b) Watt/meter^2

(c) Watt - meter (d) Newton

UPPSC Poly. Lect. 2021 Paper - II

13. The dimensional formula for latent heat is -

(a) $\left[M^0L^2T^{-2}\right]$ (b) $\left[MLT^{-2}\right]$

(c) $\left[ML^2T^2\right]$ (d) $\left[ML^2T^{-1}\right]$

UPPSC Poly. Lect. 2021 Paper - I

(UPPGT 2005, 2010)

UPPCS Pre 2006

14. **If force (F), velocity (V) and Time (T) are taken as the fundamental dimensions, instead of mass, length and time, what will be dimensions of linear momentum (P)?**

(a) FVT^{-1} (b) FT
(c) FT^{-1} (d) VT^{-2}

UPTGT Science - 2021

15. **Unit of Gravitational constant in S.I. system is :**

(a) Nm^2kg^{-1} (b) Nm^2kg^{-2}
(c) Nm^{-1} (d) Nm^{-2}

TGT 2013

16. **The dimensions of a/b in the relation $P=\frac{a+x}{b}$ where P is pressure and x is distance, are–**

(a) $[M^1L^{-1}T^{-2}]$ (b) $[M^{-1}L^1T^2]$
(c) $[M^1L^2T^{-1}]$ (d) $[M^{-1}L^{-2}T^1]$

(UPPGT 2011)

Scalar and Vectors

17. **Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A}+\vec{B}$ is equal to two times the magnitude of $\vec{A}-\vec{B}$ then the angle between $\vec{A}$ and $\vec{B}$ will be:**

(a) $\sin^{-1}\left(\frac{3}{5}\right)$ (b) $\sin^{-1}\left(\frac{1}{3}\right)$
(c) $\cos^{-1}\left(\frac{3}{5}\right)$ (d) $\cos^{-1}\left(\frac{1}{3}\right)$

JEE MAIN 2022

18. **Find the torque about the origin when a force of $3\hat{j}$ N acts on a particle whose position vector is $2\hat{k}$ m.**

(a) $6\hat{j}$ Nm (b) $-6\hat{i}$ Nm
(c) $6\hat{k}$ Nm (d) $6\hat{i}$ Nm

NEET 2020

19. **Match List-I (Vector Combination) with List-II (Physical Quantities) and choose correct answer using codes given below from the list.**

	List-I		**List-II**
A.	Scalar	1.	Mass
B.	Vector	2.	Torque
C.	Scalar product	3.	Velocity
D.	Vector Product	4.	Work done

Codes:

	A	**B**	**C**	**D**
(a)	1	3	4	2
(b)	2	4	1	3
(c)	4	1	3	2
(d)	3	1	4	2

UPPSC GDC - 2021

20. **Rain is falling vertically on the ground at speed $5\sqrt{3}$ m/s. If a man walks towards the East with speed 5 m/s, he will feel the rain falling at what angle to the vertical?**

(a) 0° (b) 30°
(c) 45° (d) 60°

UPTGT Science - 2021

21. **Two forces $\vec{F}$ and $-3\vec{F}$ acts at a perpendicular distance'd' from each other. This is equivalent to which one of the following?**

(a) Net linear force $-2\vec{F}$ only
(b) Net linear force zero only
(c) A net couple of moment 2Fd only
(d) A net linear force of $-2\vec{F}$ and a net couple of moment Fd

UPTGT Science - 2021

22. **If A and B are non–zero vectors which obey the relation $|A+B|=|A-B|$ then the angle between them is :**

(a) 0^0 (b) 60^0
(c) 90^0 (d) 120^0

UP TGT Physics 2016

23. **If $|\vec{A}\times\vec{B}|=AB$, then angle between $\vec{A}$ and $\vec{B}$ is :**

(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

UP PGT Physics 2016 (Exam : 02.02.2019)

24. **Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ will be coplanar only if :**

(a) $\vec{A}.\vec{B}.=\vec{B}.\vec{C}=\vec{A}.\vec{C}=0$ (b) $\vec{A}.(\vec{B}\times\vec{C})=0$
(c) $\vec{A}\times(\vec{B}\times\vec{C})=0$ (d) $(\vec{A}\times\vec{B})\times\vec{C}=0$

UP PGT Physics 2016 (Exam : 02.02.2019)

25. **Which of the following physical quantity is a scalar?**

(a) Current (b) Area
(c) Momentum (d) Angular velocity

UPPCS Pre 2010

26. **Surface area is -**

(a) Scalar
(b) Vector
(c) Neither scalar nor vector
(d) Both scalar and vector

TGT 2011

27. **The magnitude of the sum of two vectors $\vec{A}$ and $\vec{B}$ comes out**

(a) Less than $|\vec{A}|-|\vec{B}|$
(b) More than $|\vec{A}|+|\vec{B}|$

(c) Either less than $|\vec{A}|-|\vec{B}|$ or greater than $|\vec{A}|+|\vec{B}|$

(d) $\geq|\vec{A}|-|\vec{B}|$ and $\leq|\vec{A}|+|\vec{B}|$

UPPSC Ashram Paddhati PGT - 2021

28. If the resultant $\vec{R}$ of two vectors $\vec{P}$ and $\vec{Q}$ is such that $R^2 = P^2 + Q^2$, the angle between vector $\vec{P}$ and $\vec{Q}$ is

(a) Zero (b) 45°

(c) 60° (d) 90°

UP PGT - 2021

29. The value of λ for which vectors $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-\lambda\hat{j}+4\hat{k}$ are perpendicular is -

(a) 14 (b) 10

(c) 7 (d) 3

UPPSC Poly. Lect. 2021 Paper - I

30. For two vectors $\vec{A}$ and $\vec{B}$, $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$, is always true when

(a) $|\vec{A}|=|\vec{B}|\neq 0$

(b) $\vec{A}\perp\vec{B}$

(c) $|\vec{A}|=|\vec{B}|\neq 0$ and $\vec{A}$ and $\vec{B}$ are parallel

(d) $|\vec{A}|=|\vec{B}|\neq 0$ and $\vec{A}$ and $\vec{B}$ are anti-parallel

RPSC School Lect. 08.01.2020

31. If magnitude of two vectors are 7 and 3 respectively, which of the following could not be magnitude of resultant of these two vectors?

(a) 3 (b) 4 (c) 5 (d)7

RPSC School Lect. 08.01.2020

Curl, Divergence, Gauss & Stokes' Theorem & their Application

32. According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0°C

B. The mean free path of gas molecules decreases if the density of molecules is increased.

C. The mean free path of gas molecules increases if temperature is increased keeping pressure constant.

D. Average kinetic energy per molecule per degree of freedom is $\frac{3}{2}k_BT$ (for monoatomic gases)

Choose the most appropriate answer from the options given below:

(a) A and C only (b) B and C only

(c) A and B only (d) C and D only

JEE MAIN 2022

33. In coplanar vectors $\vec{A}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{B}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{C}=3\hat{i}+\alpha\hat{j}+5\hat{k}$ the value of unknown α in $\vec{C}$ will be

(a) 3 (b) −3.6

(c) 4 (d) None of these

UPPSC Ashram Paddhati PGT - 2021

34. $\vec{\nabla}$ represents

(a) A differential operator

(b) A differential vector operator

(c) A scalar operator

(d) A Laplacian operator

UPPSC Ashram Paddhati PGT - 2021

35. Which of the following is not correct?

(a) $\vec{\nabla}\times\vec{B}=\mu_0\left(\vec{j}+\frac{\partial\vec{D}}{\partial t}\right)$ (b) $\vec{\nabla}\times\vec{B}=\mu_0\vec{j}$

(c) $\vec{\nabla}.\vec{B}=0$ (d) $\vec{\nabla}.\vec{B}=\frac{\rho}{\varepsilon_0}$

UPPSC Poly. Lect. 2021 Paper - II

36. The value of grad $\left(\frac{1}{r}\right)$ is

grad

(a) $-\frac{\vec{r}}{r}$ (b) zero

(c) $-\frac{\vec{r}}{r^3}$ (d) $-\frac{\vec{r}}{r^2}$

UP PGT - 2021

37. The gauge transformation in which $\vec{\nabla}.\vec{A}=0$, is called

(a) Lorentz gauge

(b) Gauss gauge

(c) Coulomb gauge

(d) Longitudinal gauge

UPPSC GDC - 2021

38. The divergence of position vector $\vec{r}$ is :

(a) 1 (b) 0 (c) 3 (d) 2

(UPPGT 2011)

39. If $\oint\vec{A}.\vec{dl}=0$ for each closed curve, this implies

(a) $\vec{\nabla}\times\vec{A}=0$ (b) $\vec{\nabla}.\vec{A}=0$

(c) $\vec{\nabla}\times(\vec{\nabla}.\vec{A})=0$ (d) $\vec{\nabla}.(\vec{\nabla}.\vec{A})=0$

UPPSC Ashram Paddhati PGT - 2021

40. Single magnetic pole does not exist because :

(a) $\vec{\nabla}\times\vec{B}=0$ (b) $\vec{\nabla}.\vec{B}=0$

(c) $\vec{\nabla}\times\vec{B}=0$ (d) $\vec{\nabla}\times\vec{B}=\vec{\nabla}.\vec{B}$

(UPPGT 2004)

41. **A charge q is placed at one corner of a cube. The electric flux linked with cube will be :**

(a) $\frac{q}{\epsilon_0}$ (b) $\frac{q}{2\epsilon_0}$

(c) $\frac{q}{4\epsilon_0}$ (d) $\frac{q}{8\epsilon_0}$

(UPPGT 2004)

Motion in a Straight Line

42. **Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by $X_P(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both me buses have same velocity?**

(a) $\frac{\alpha - f}{1+\beta}$ (b) $\frac{\alpha + f}{2(\beta - 1)}$

(c) $\frac{\alpha + f}{2(\beta + 1)}$ (d) $\frac{\alpha + f}{2(\beta + 1)}$

JEE MAIN 2022

43. **Two blocks of masses 10 kg and 30 kg are placed on the same straight line with coordinates (0, 0) cm and (x, 0) cm respectively. The block of 10 kg is moved on the same line through a distance of 6 cm towards the other block. The distance through which the block of 30 kg must be moved to keep the position of centre of mass of the system unchanged is :**

(a) 4 cm towards the 10 kg block

(b) 2 cm away from the 10 kg block

(c) 2 cm towards the 10 kg block

(d) 4 cm away from the 10 kg block

JEE MAIN 2022

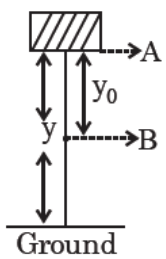
44. **When a ball is dropped into a lake from a height 4.9 m above the water level, it hits the water with a velocity v and then sinks to the bottom with the constant velocity v. It reaches the bottom of the lake 4.0 s after it is dropped. The approximate depth of the lake is :**

(a) 19.6 m (b) 29.4 m

(c) 39.2 m (d) 73.5 m

JEE MAIN 2022

45. **In the given figure, the block of mass m is dropped from the point 'A'. The expression for kinetic energy of block when it reaches point 'B' is :**



(a) $\frac{1}{2}mg y_0^2$ (b) $\frac{1}{2}mg y^2$

(c) $mg(y - y_0)$ (d) mgy_0

JEE MAIN 2022

46. **A block of mass M placed inside a box descends vertically with acceleration 'a'. The block exerts a force equal to one-fourth of its weight on the floor of the box. The value of 'a' will be :**

(a) $\frac{g}{4}$ (b) $\frac{g}{2}$

(c) $\frac{3g}{4}$ (d) g

JEE MAIN 2022

47. **A shell of mass m is at rest initially. It explodes into three fragments having mass in the ratio 2 : 2 : 1. If the fragments having equal mass fly off along mutually perpendicular directions with speed v, the speed of the third (lighter) fragment is**

(a) $3\sqrt{2}v$ (b) v

(c) $\sqrt{2}v$ (d) $2\sqrt{2}v$

NEET 2022

48. **A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80 m/s. The height of the tower is (g = 10 m/s2)**

(a) 360 m (b) 340 m

(c) 320 m (d) 300 m

NEET 2020

49. **Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = (at + bt^2)$ and $x_Q(t) = (ft - t^2)$. At what time do the cars have the same velocity?**

(a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(1-b)}$

(c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$

NEET-II 2016

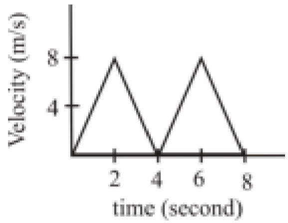
50. **Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time $t1$. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time $t2$. The time taken by her to walk up on the moving escalator will be**

(a) $\frac{t_1 t_2}{t_2 - t_1}$ (b) $\frac{t_1 t_2}{t_2 + t_1}$

(c) $t_1 - t_2$ (d) $\frac{t_1 + t_2}{2}$

NEET 2017

51. **In above velocity-time graph of a particle, the distance covered in first 4 seconds is**



(a) 12m (b) 16m
(c) 20m (d) Zero

UPPSC Ashram Paddhati PGT - 2021

Motion in a Plane

52. **For a particle in uniform circular motion, the acceleration $\vec{a}$ at any point P(R,θ) on the circular path of radius R is (when θ is measured from the positive x –axis and v is uniform speed) :**

(a) $-\frac{v^2}{R}\sin\theta\hat{i}+\frac{v^2}{R}\cos\theta\hat{j}$

(b) $-\frac{v^2}{R}\cos\theta\hat{i}+\frac{v^2}{R}\sin\theta\hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$

(d) $-\frac{v^2}{R}\hat{i}+\frac{v^2}{R}\hat{j}$

JEE MAIN 2022

53. **Given below are two statements. One is labeled as Assertion A and the other is labeled as Reason R. Assertion A :Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h1 and h_2 respectively, then R = $4\sqrt{h_1h_2}$**

Reason R: Product of said heights.

$$h_1h_2=\left(\frac{u^2\sin^2\theta}{2g}\right)\left(\frac{u^2\cos^2\theta}{2g}\right)$$

Choose the CORRECT answer :

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

JEE MAIN 2022

54. **A projectile is launched at an angle 'α' with the horizontal with a velocity 20 ms^{-1} . After 10 s, its inclination with horizontal is 'β'. The value of tanβ will be : (g = 10 ms^{-2})**

(a) tan α + 5 sec α (b) tan α – 5 sec α
(c) 2 tan α – 5 sec α (d) 2 tan α + 5 sec α

JEE MAIN 2022

55. **A girl standing on road holds her umbrella at 45° with the vertical to keep the rain away. If she starts running without umbrella with a speed of $15\sqrt{2}kmh^{-1}$, the rain drops hit her head vertically. The speed of rain drops with respect to the moving girl is :**

(a) 30 kmh^{-1} (b) $\frac{25}{\sqrt{2}}kmh^{-1}$

(c) $\frac{30}{\sqrt{2}}kmh^{-1}$ (d) 25 kmh^{-1}

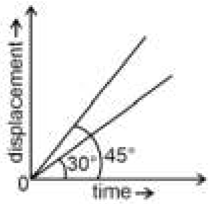
JEE MAIN 2022

56. **A stone tide to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as it reaches a position where the string is horizontal, is $\sqrt{x(u^2-gL)}$ .The value of x is**

(a) 3 (b) 2
(c) 1 (d) 5

JEE MAIN 2022

57. **The displacement-time graphs of two moving particles make angles of 30° and 45° with the *x*-axis as shown in the figure. The ratio of their respective velocity is**



(a) $1:\sqrt{3}$ (b) $\sqrt{3}:1$
(c) 1 : 1 (d) 1 : 2

NEET 2022

58. **A ball is projected with a velocity, 10 ms^{-1}, at an angle of 60° with the vertical direction. Its speed at the highest point of its trajectory will be**

(a) 10 ms^{-1} (b) Zero
(c) $5\sqrt{3}$ ms^{-1} (d) 5 ms^{-1}

NEET 2022

59. **A small block slides down on a smooth inclined plane, starting from rest at time t = 0. Let S_n be the distance travelled by the block in the interval t = n – 1 to t = n. Then, the ratio $\frac{S_n}{S_{n+1}}$ is**

(a) $\frac{2n}{2n-1}$ (b) $\frac{2n-1}{2n}$

(c) $\frac{2n-1}{2n+1}$ (d) $\frac{2n+1}{2n-1}$

NEET 2021

60. A car starts from rest and accelerates at 5 m/s^2. At t = 4 s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at t = 6 s?

(a) $20\sqrt{2}$ m/s, 10 m/s^2 (b) 20 m/s, 5 m/s^2

(c) 20 m/s, 0 (d) $20\sqrt{2}$ m/s, 0

NEET 2021

61. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be

(a) $1:2\sqrt{3}$ (b) $1:\sqrt{2}$

(c) $\sqrt{2}:1$ (d) $1:\sqrt{3}$

NEET 2019

62. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is, given by

(a) 45° west (b) 30° west

(c) 0° (d) 60° west

NEET 2019

63. A mass *m* is attached to a thin wire and whirled in a vertical circle. The wire is most likely to break when

(a) inclined at an angle of 60° from vertical

(b) the mass is at the highest point

(c) the wire is horizontal

(d) the mass is at the lowest point

NEET 2019

64. What is the minimum velocity with which a body of mass *m* must enter a vertical loop of radius *R* so that it can complete the loop?

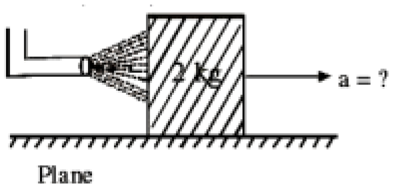
(a) $\sqrt{3gR}$ (b) $\sqrt{5gR}$

(c) $\sqrt{gR}$ (d) $\sqrt{2gR}$

NEET 2016

Newton's Law of Motion

65. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be :

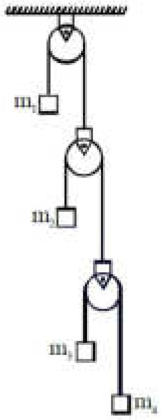


(a) 3 (b) 6

(c) 5 (d) 4

JEE MAIN 2022

66. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the accelerations of masses m_1, m_2, m_3 and m_4 respectively. Which of the following relation is true for this arrangement?



(a) $4a_1 + 2a_2 + a_3 + a_4 = 0$

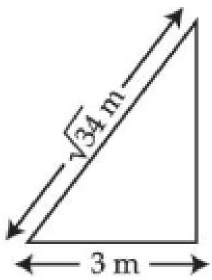
(b) $a_1 + 4a_2 + 3a_3 + a_4 = 0$

(c) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$

(d) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

JEE MAIN 2022

67. A $\sqrt{34}$m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor 3 m away from the wall as shown in the figure. If Ff and Fw are the reaction forces of the floor and the wall, then ratio of F_w/F_f will be : (Use g = 10 m/s^2)



(a) $\frac{6}{\sqrt{110}}$ (b) $\frac{3}{\sqrt{113}}$

(c) $\frac{3}{\sqrt{109}}$ (d) $\frac{2}{\sqrt{109}}$

JEE MAIN 2022

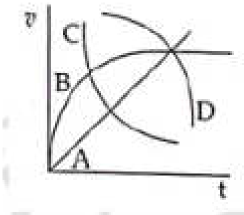
68. Amass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the mass, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (Take, g = 10 ms^{-2})

(a) 70 N (b) 200 N

(c) 100 N (d) 140 N

JEE MAIN 2019

69. A spherical ball is dropped in a long column of a highly viscous liquid. The curve in the graph shown, which represents the speed of the ball (v) as a function of time (t) is



(a) D (b) A
(c) B (d) C

NEET 2022

70. A particle is released from height S from the surface of the Earth. At a certain height its kinetic energy is three times its potential energy. The height from the surface of earth and the speed of the particle at that instant are respectively

(a) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$ (b) $\frac{S}{4}, \frac{3gS}{2}$
(c) $\frac{S}{4}, \frac{\sqrt{3gS}}{2}$ (d) $\frac{S}{2}, \frac{\sqrt{3gS}}{2}$

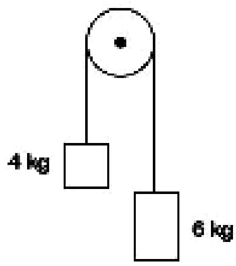
NEET 2021

71. A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80 m/s. The height of the tower is : (g = 10 m/s^2)

(a) 340 m (b) 320 m
(c) 300 m (d) 360 m

NEET 2020

72. Two bodies of mass 4 kg and 6 kg are tied to the ends of a mass less string. The string passes over a pulley which is frictionless (see figure). The acceleration of the system in terms of acceleration due to gravity (g) is :



(a) g/2 (b) g/5
(c) g/10 (d) g

NEET 2020

73. One end of string of length *l* is connected to a particle of mass *m* and the other end is connected to a small peg on a smooth horizontal table. If the particle moves in circle with speed *v*, the net force on the particle (directed towards centre) will be (*T* represents the tension in the string)

(a) $T + \frac{mv^2}{\ell}$ (b) $T - \frac{mv^2}{\ell}$
(c) zero (d) T

NEET 2017

74. A wire of length L has mass μ per unit length. It is used to pull up a body of mass M upwards with acceleration α. What is the tension at the midpoint of the wire?
(g = acceleration due to gravity)

(a) $M(g + \alpha)$ (b) $(M + \mu L)g$
(c) $\left(M + \frac{\mu L}{2}\right)(g + \alpha)$ (d) $\left(M + \frac{\mu L}{2}\right)g$

UPTGT Science - 2021

75. A force of 10 N acts on a particle in vertically upward direction and it is displaced downward direction by a distance 5.0 m. The work done is:

(a) +10 J (b) +50 J
(c) –50 J (d) –10 J

UKPSC Lecturer (Mains) 2020

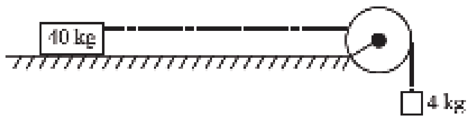
76. A 2.5 kg iron ball has the same diameter as a 1.25 kg aluminium ball. The balls are dropped at the same time from a cliff. Just before they reach the ground, they have same :

(a) Acceleration (b) Momentum
(c) Kinetic energy (d) Potential energy

UP TGT Physics 2016

Friction

77. A block of mass 40 kg slides over a surface, when a mass of 4 kg is suspended through an inextensible massless string passing over frictionless pulley as shown below. The coefficient of kinetic friction between the surface and block is 0.02. The acceleration of block is.
(Given g = 10 ms^{-2}.)



(a) 1 ms^{-2} (b) 1/5 ms^{-2}
(c) 4/5 ms^{-2} (d) 8/11 ms^{-2}

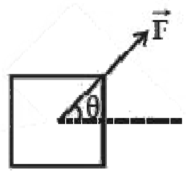
JEE MAIN 2022

78. **A block of mass 2 kg moving on a horizontal surface with speed of 4 ms^{-1} enters a rough surface ranging from x = 0.5 m to x = 1.5 m. The retarding force in this range of rough surface is related to distance by F = – kx where $k = 12 Nm^{-1}$. The speed of the block as it just crosses the rough surface will be :**

(a) zero (b) 1.5 ms^{-1}
(c) 2.0 ms^{-1} (d) 2.5 ms^{-1}

JEE MAIN 2022

79. **A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μK. Then, the block's acceleration 'a' is given by : (g is acceleration due to gravity)**



(a) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(b) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(c) $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$

(d) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

JEE MAIN 2021

80. **Aman (mass = 50kg) and his son (mass = 20kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms−1 with respect to the man. The speed of the man with respect to the surface is**

(a) 0.28 ms^{-1} (b) 0.20 ms^{-1}
(c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

JEE MAIN 2019

81. **An electric lift with a maximum load of 2000 kg (lift + passengers) is moving up with a constant speed of 1.5 ms^{-1}. The frictional force opposing the motion is 3000 N. The minimum power delivered by the motor to the lift in watts is : (*g* = 10 m s^{-2})**

(a) 23500 (b) 23000
(c) 20000 (d) 34500

NEET 2022

82. **Which one of the following statements is incorrect?**

(a) Rolling friction is smaller than sliding friction.
(b) Limiting value of static friction is directly proportional to normal reaction.
(c) Frictional force opposes the relative motion.
(d) Coefficient of sliding friction has dimensions of length.

NEET 2018

83. **A 40 N crate is resting on a rough horizontal floor. A 12 N horizontal force is then applies to it. If the coefficient of frictions are μ_s = 0.5 and μ_s = 0.4, the magnitude of the frictional force on the crate is**

(a) 8 N (b) 12 N
(c) 16 N (d) 20 N

RPSC School Lect. 26.07.2016

Work, Energy and Power

84. **A body of mass M at rest explodes into three pieces, in the ratio of masses 1:1:2. Two smaller pieces fly of perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third pieces will be:**

(a) 15 ms^{-1} (b) 25 ms^{-1}
(c) 35 ms^{-1} (d) 50 ms^{-1}

JEE MAIN 2022

85. **A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to :-**

(a) $t^{2/3}$ (b) $t^{3/2}$
(c) t (d) $t^{1/2}$

JEE MAIN 2021

86. **A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?**

(a) 850 J (b) 900 J
(c) 950 J (d) 875 J

JEE MAIN 2019

87. **A particle of mass *m* is moving with speed 2*v* and collides with a mass 2*m* moving with speed *v* in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass *m*, which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be**

(a) $\sqrt{2}v$ (b) $\frac{v}{\sqrt{2}}$
(c) $\frac{v}{(2\sqrt{2})}$ (d) $2\sqrt{2}v$

JEE MAIN 2019

88. **From a circular ring of mass 'M' and radius 'R' an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is 'K' times 'MR^2'. Then the value of 'K' is**

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$

(c) $\frac{7}{8}$ (d) $\frac{1}{4}$

NEET 2021

89. A ball of mass 0.15 kg is dropped from a height 10 m, strikes the ground and rebounds to the same height. The magnitude of impulse imparted to the ball is (g = 10 m/s^2) nearly

(a) 1.4 kg m/s (b) 0 kg m/s

(c) 4.2 kg m/s (d) 2.1 kg m/s

NEET 2021

90. A wheel of mass m and moment of inertia I is in motion with linear velocity v and angular velocity ω. Its kinetic energy is

(a) $\frac{1}{2}mv^2$ (b) $\frac{1}{2}I\omega^2$

(c) $\frac{1}{2}I\omega^2+\frac{1}{2}Imv^2$ (d) $\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

UPPSC Ashram Paddhati PGT - 2021

91. Two bodies of masses m_1 and m_2 are connected through a spring of force constant k. When a force F is applied in the direction of the length of spring, the distance between m_1 and m_2 changes by x. The stored potential energy in spring will be-

(a) $\frac{1}{2}k^2x$ (b) kx

(c) $\frac{1}{2}kx^2$ (d) $\frac{1}{2}k^2x^2$

UPPSC Ashram Paddhati PGT - 2021

92. A spring 40mm long is stretched by the application of a force. If 10N force is required to stretch the spring through 1mm, then the work done in stretching the spring through 40 mm is

(a) 84J (b) 64J

(c) 25J (d) 8J

UPPSC Ashram Paddhati PGT - 2021

93. Potential energy per unit volume u of a strained wire is

(a) u = stress × strain

(b) $u=\frac{1}{4}$ stress × strain

(c) $u=\frac{1}{2}$ stress × strain

(d) $u=\frac{3}{4}$ stress × strain

UP PGT - 2021

94. An object of mass M moving with velocity v explodes and breaks up into three equal pieces. Immediately after the explosion two pieces become stationary, but the third piece keeps moving. What is the kinetic energy of this piece?

(a) 2 Mv^2 (b) $\frac{3}{2}$Mv^2

(c) Mv^2 (d) $\frac{1}{2}$Mv^2

UP PGT - 2021

95. Consider a solid cylinder of mass M and radius R rolling down and inclined plane without slipping from a height h. The speed of its centre of mass when the cylinder reaches the bottom, is

(a) $\sqrt{\frac{4}{3}gh}$ (b) $\sqrt{2gh}$

(c) $\sqrt{\frac{3}{2}gh}$ (d) $\sqrt{\frac{3}{5}gh}$

UP PGT - 2021

96. What is conserved in an inelastic collision?

(a) Kinetic energy (b) Potential Energy

(c) Total energy (d) Momentum

UPPSC Poly. Lect. 2021 Paper - I

Rotational Motion

97. What percentage of kinetic energy of a moving particle is transferred to a stationary particle when it strikes the stationary particle of 5 times its mass? (Assume the collision to be head-on elastic collision)

(a) 50.0% (b) 66.6%

(c) 55.5% (d) 33.3%

JEE MAIN 2022

98. One end of a mass less spring of spring constant k and natural length l_0 is fixed while the other end is connected to a small object of mass m lying on a frictionless table. The spring remains horizontal on the table. If the object is made to rotate at an angular velocity ω about an axis passing through fixed end, then the elongation of the spring will be:

(a) $\frac{k-m\omega^2 l_0}{m\omega^2}$ (b) $\frac{m\omega^2 l_0}{k+m\omega^2}$

(c) $\frac{m\omega^2 l_0}{k-m\omega^2}$ (d) $\frac{k+m\omega^2 l_0}{m\omega^2}$

JEE MAIN 2022

99. Solid spherical ball is rolling on a frictionless horizontal plane surface about is axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is :-

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$

(c) $\frac{1}{5}$ (d) $\frac{7}{10}$

JEE MAIN 2022

100. **Match List-I with List-II**

	List-I		List-II
A	Moment of inertia of solid sphere of radius R about any tangent	I	$\frac{5}{3}MR^2$
B	Moment of inertia of hollow sphere of radius (R) about any tangent	II	$\frac{7}{5}MR^2$
C	Moment of inertia of circular ring of radius (R) about its diameter.	III	$\frac{1}{4}MR^2$
D	Moment of inertia of circular disc of radius (R) about any diameter.	IV	$\frac{1}{2}MR^2$

Question: Choose the correct answer from the options given below

(a) A-II, B-II, C-IV, D-III
(b) A-I, B-II, C-IV, D-III
(c) A-II, B-I, C-III, D-IV
(d) A-I, B-II, C-III, D-IV

JEE MAIN 2022

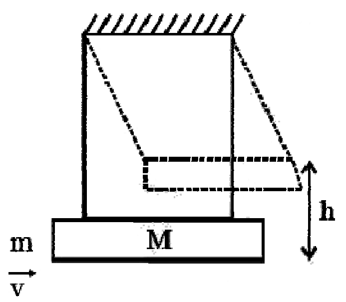
101. **A thin circular ring of mass M and radius r is rotating about its axis with an angular speed w. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become :**

(a) $\omega\frac{M}{M+m}$ (b) $\omega\frac{M+2m}{M}$

(c) $\omega\frac{M}{M+2m}$ (d) $\omega\frac{M-2m}{M+2m}$

JEE MAIN 2021

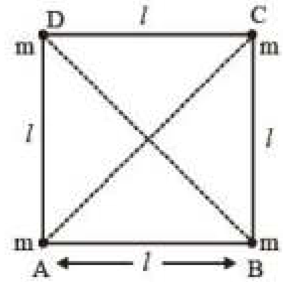
102. **A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is : (Take g = 9.8 ms^{-2})**



(a) 841.4 m/s (b) 811.4 m/s
(c) 831.4 m/s (d) 821.4 m/s

JEE MAIN 2021

103. **Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :**



(a) ml^2 (b) $2ml^2$
(c) $3\ ml^2$ (d) $\sqrt{3}ml^2$

JEE MAIN 2021

104. **The angular speed of a fly wheel moving with uniform angular acceleration changes from 1200 rpm to 3120 rpm in 16 seconds. The angular acceleration in rad/s^2 is**

(a) 104π (b) 2π
(c) 4π (d) 12π

NEET 2022

105. **A particle moving in a circle of radius R with a uniform speed takes a time T to complete one revolution. If this particle were projected with the same speed at an angle 'θ' to the horizontal, the maximum height attained by it equals 4R. The angle of projection, θ, is then given by :**

(a) $\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$ (b) $\theta = \cos^{-1}\left(\frac{gT^2}{\pi^2 R}\right)^{1/2}$

(c) $\theta = \cos^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$ (d) $\theta = \sin^{-1}\left(\frac{\pi^2 R}{gT^2}\right)^{1/2}$

NEET 2021

106. **Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle is nearly at a distance of**

(a) 33 cm (b) 50 cm
(c) 67 cm (d) 80 cm

NEET 2020

107. **The moment of inertia of a uniform circular disc of radius R and mass M about an axis passing from the edge of the disc and normal to the disc is–**

(a) $\frac{1}{2}MR^2$ (b) MR^2

(c) $\frac{7}{2}MR^2$ (d) $\frac{3}{2}MR^2$

(UPPGT 2011)

108. We have one solid sphere and one very thin spherical shell. They have identical masses and moment of inertia about their respective diameters. The ratio of their radii is given by:

(a) $\sqrt{3}:\sqrt{7}$ (b) $3:7$
(c) $5:3$ (d) $\sqrt{5}:\sqrt{3}$

UPPCS Pre 2001, (UPPGT 2013)

109. One quarter sector is cut from a uniform circular disc of radius R, the sector has mass M. It is made to rotate about a line perpendicular to It's plane and passing through the centre of the original disc. It's moment of inertia about the axis of rotation is:

(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$
(c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}MR^2$

RPSC School Lect. 08.01.2020 (UPPGT 2002)

Gravitation

110. The distance of the Sun from earth is 1.5×10^{11} m and its angular diameter is (2000) s when observed from the earth. The diameter of the Sun will be :

(a) 2.45×10^{10} m (b) 1.45×10^{10} m
(c) 1.45×10^{9} m (d) 0.14×10^{9} m

JEE MAIN 2022

111. The time period of a satellite revolving around earth in a given orbit is 7 hours. If the radius of orbit is increased to three times its previous value, then approximate new time period of the satellite will be :

(a) 40 hours (b) 36 hours
(c) 30 hours (d) 25 hours

JEE MAIN 2022

112. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be :

(a) $\sqrt{3}$ years (b) 3 years
(c) 9 years (d) $3\sqrt{3}$ years

JEE MAIN 2022

113. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is:

(a) 9T (b) 27T
(c) 12T (d) 3T

JEE MAIN 2021

114. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Take, mass of planet = 8×10^{22} kg, radius of planet = 2×10^{6} m, gravitational constant $G = 667 \times 10^{-11}$. N-m^2/kg^2]

(a) 11 (b) 17
(c) 13 (d) 9

JEE MAIN 2019

115. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and $2M$. The gravitational field at distance $3a$ from the centre will be

(a) $\frac{GM}{9a^2}$ (b) $\frac{2GM}{9a^2}$
(c) $\frac{GM}{3a^2}$ (d) $\frac{2GM}{3a^2}$

JEE MAIN 2019

116. A body of mass 60 g experiences a gravitational force of 3.0 N, when placed at a particular point. The magnitude of the gravitational field intensity at that point is

(a) 180 N/kg (b) 0.05 N/kgj
(c) 50 N/kg (d) 20 N/kg

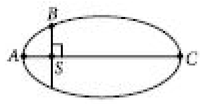
NEET 2022

117. The escape velocity from the Earth's surface is v. The escape velocity from the surface of another planet having a radius, four times that of Earth and same mass density is

(a) 4v (b) v
(c) 2v (d) 3v

NEET 2021

118. The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are KA, KB and KC, respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then



(a) $K_A < K_B < K_C$ (b) $K_A > K_B > K_C$
(c) $K_B < K_A < K_C$ (d) $K_B > K_A > K_C$

NEET 2018

119. A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\frac{v_1}{v_2}$ is

(a) $(r_1/r_2)^2$ (b) r_2/r_1
(c) $(r_2/r_1)_2$ (d) r_1/r^2

NEET 2011

120. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will
(a) move towards each other
(b) move away from each other
(c) will become stationary
(d) keep floating at the same distance between them.

NEET 2017

121. If the mass of the Sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which of the following is not correct?
(a) Raindrops will fall faster.
(b) Walking on the ground would become more difficult.
(c) Time period of a simple pendulum on the Earth would decrease.
(d) g on the Earth will not change.

NEET 2018

122. A solid sphere of constant density ρ has mass M and radius R. What is the gravitation potential difference between point P which is at distance $\frac{R}{2}$ from the centre and its surface?
(i.e. $V_p - V_{surface}$)
(a) Zero (b) $\frac{GM}{R}$
(c) $-\frac{3GM}{8R}$ (d) $-\frac{3GM}{2R}$

UPTGT Science - 2021

123. The earth has mass M and radius R. If 'g' is acceleration due to gravity on the surface of the earth, what is its value at height $\frac{R}{3}$ above the earth's surface?
(a) $\frac{9}{16}g$ (b) $\frac{4}{9}g$
(c) $\frac{g}{3}$ (d) zero

UPTGT Science - 2021

124. A gunman and an apple are both at height 98 m above the ground, at distance 200 m from each other. At t = 0, a bullet (m = 5 gm, velocity 200 m/s) is fired horizontally aimed at the apple. At the same time (t = 0) the apple starts to move downwards. Given g = 9.8 m/s^2, the bullet will hit the apple only if it moves at constant
(a) velocity 9.8 m/s
(b) velocity 4.9 m/s
(c) acceleration 9.8 m/s^2
(d) acceleration 4.9 m/s^2

UPTGT Science - 2021

125. The radius of the orbit of a satellite is 16 times the radius of the orbit of a geo-stationary satellite. The period of the satellite is
(a) 4 days (b) 16 days
(c) 64 days (d) 96 days

UP PGT - 2021
UPPCS Pre 2009

126. Steering of satellite is based on conservation of which of the following?
(a) Energy (b) Linear momentum
(c) Angular momentum (d) None of the above

UPPSC Poly. Lect. 2021 Paper - I

127. Two satellites A and B are revolving around the earth in the same orbit. Mass of B is thrice to that of A. Then the correct statement is
(a) speeds of A and B are equal
(b) speeds of A and B are greater
(c) P.E.s of A and B are equal
(d) None of the above

(UPPGT 2010)

Elasticity, Elasticity Constants & their Interrelations

128. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is :
(a) 3×10^8 Nm^{-2} (b) 9×10^8 Nm^{-2}
(c) 6×10^8 Nm^{-2} (d) 12×10^8 Nm^{-2}

JEE MAIN 2022

129. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment ?
(a) 0.14% (b) 0.9%
(c) 9% (d) 1.4%

JEE MAIN 2021

130. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to
(a) 1.3 mm (b) 1.5 mm
(c) 1.9 mm (d) 1.7 mm

JEE MAIN 2019

131. The bulk modulus of a spherical object is 'B'. If it is subjected to uniform pressure 'p', the fractional decrease in radius is
(a) $\frac{B}{3p}$ (b) $\frac{3p}{B}$
(c) $\frac{p}{3B}$ (d) $\frac{p}{B}$

NEET 2017

132. **Copper of fixed volume V is drawn into wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Dl. Which of the following graphs is a straight line?**

(a) Δl versus $1/l$ (b) Δl versus l^2

(c) Δl versus $1/l^2$ (d) Δl versus l

NEET 2014

133. **When a linear load W is applied to a metal wire of length l, its length increases by Δl. The fractional change in its volume $\frac{\Delta V}{V}$ will be proportional to**

(a) $\sqrt{\frac{\Delta l}{l}}$ (b) $\left(\frac{\Delta l}{l}\right)^2$

(c) $\frac{\Delta l}{l}$ (d) $\left(\frac{\Delta l}{l}\right)^3$

UP PGT - 2021

134. **Which of the following correctly represents theoretical limits of Poisson ratio 'σ' ?**

(a) $1 < \sigma < 2$ (b) $-1 < \sigma < \frac{1}{2}$

(c) $-2 < \sigma < 1$ (d) $\frac{1}{2} < \sigma < 1$

UPPSC Poly. Lect. 2021 Paper - I

UPPCS Pre 2007

135. **Increase in length of a wire on stretching is 0.025%. If its Poisson ratio is 0.4. then what will be the percentage decrease in the diameter of the wire?**

(a) 0.01 (b) 0.02

(c) 0.03 (d) 0.05

UPPSC Poly. Lect. 2021 Paper - I

136. **Steels is -**

(a) Same elastic as rubber

(b) More elastic than rubber

(c) Less elastic than rubber

(d) Neither elastic nor plastic

UPPSC Poly. Lect. 2021 Paper - I

137. **Choose the correct relation among elastic constants Y, K and η from the following -**

(a) $\frac{1}{\eta} = \frac{1}{3Y} + \frac{1}{9K}$ (b) $\frac{1}{3\eta} = \frac{1}{Y} + \frac{1}{9K}$

(c) $\frac{1}{3Y} = \frac{1}{\eta} + \frac{1}{9K}$ (d) $\frac{1}{Y} = \frac{1}{3\eta} + \frac{1}{9K}$

UPPSC Poly. Lect. 2021 Paper - I

138. **The volume of a solid does not vary with pressure. What will be the Poisson ratio?**

(a) 0.2 (b) 0.3

(c) 0.4 (d) 0.5

UPPSC Poly. Lect. 2021 Paper - I

Beam, Bending of Beams & Torsion of Cylindrical Bodies

139. **The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit?**

(a) 0.90 mm (b) 1.00 mm

(c) 1.16 mm (d) 1.36 mm

JEE MAIN 2019

140. **A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1p ms^{-2}, what will be the tensile stress that would be developed in the wire?**

(a) 6.2×10^6 Nm^{-2} (b) 5.2×10^6 Nm^{-2}

(c) 3.1×10^6 Nm^{-2} (d) 4.8×10^6 Nm^{-2}

JEE MAIN 2019

141. **One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is**

(a) 0.25 (b) 0.50

(c) 2.00 (d) 4.00

IIT 2013

142. **Two objects of mass 10 kg and 20 kg respectively are connected to the two ends of a rigid rod of length 10 m with negligible mass. The distance of the center of mass of the system from the 10 kg mass is**

(a) 5 m (b) $\frac{10}{3}$ m

(c) $\frac{20}{3}$ m (d) 10 m

NEET 2022

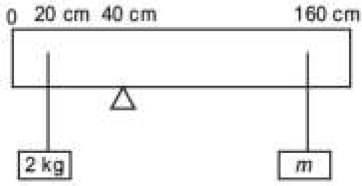
143. **The ratio of the radius of gyration of a thin uniform disc about an axis passing through its centre and normal to its plane to the radius of gyration of the disc about its diameter is**

(a) $1:\sqrt{2}$ (b) 2 : 1

(c) $\sqrt{2}:1$ (d) 4 : 1

NEET 2022

144. **A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass 'm' is suspended from the rod at 160 cm mark as shown in the figure. Find the value of 'm' such that the rod is in equilibrium. (g = 10 m/s^2)**



(a) $\frac{1}{12}$ kg (b) $\frac{1}{2}$ kg
(c) $\frac{1}{3}$ kg (d) $\frac{1}{6}$ kg

NEET 2021

145. A wire of length L, area of cross section A is hanging from a fixed support. The length of the wire changes to L_1 when mass M is suspended from its free end. The expression for Young's modulus is :

(a) $\frac{Mg(L_1-L)}{AL}$ (b) $\frac{MgL}{AL_1}$
(c) $\frac{MgL}{A(L_1-L)}$ (d) $\frac{MgL_1}{AL}$

NEET 2020

146. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass.
The centre of mass of the system from the 5 kg particle is nearly at a distance of :

(a) 50 cm (b) 67 cm
(c) 80 cm (d) 33 cm

NEET 2020

147. The stress-strain curves are drawn for two different materials *X* and *Y*. It is observed that the ultimate strength point and the fracture point are close to each other for material *X* but are far apart for material *Y*. We can say that materials *X* and *Y* are likely to be (respectively)

(a) ductile and brittle (b) brittle and ductile
(c) brittle and plastic (d) plastic and ductile

Odisha NEET 2019

148. A wire of length *L*, area of cross section *A* is hanging from a fixed support. The length of the wire changes to *L*1 when mass *M* is suspended from its free end. The expression for Young's modulus is

(a) $\frac{MgL_1}{AL}$ (b) $\frac{Mg(L_1-L)}{AL}$
(c) $\frac{MgL}{AL}$ (d) $\frac{MgL}{A(L_1-L)}$

NEET 2020

149. Two wires are made of the same material and have the same volume. The first wire has cross-sectional area *A* and the second wire has cross-sectional area 3*A*. If the length of the first wire is increased by D*l* on applying a force *F*, how much force is needed to stretch the second wire by the same amount?

(a) 9F (b) 6F
(c) 4F (d) F

NEET 2018

150. Torsional couple required to twist a cylinder of length *l* / 2, radius r and modulus of rigidity η by one complete turn is :

(a) $\frac{2\pi^2\eta r^4}{l}$ (b) $\frac{\pi\eta r^4}{l}$
(c) $\frac{\pi^2\eta r^4}{l}$ (d) $\frac{2\pi^2\eta r^4}{2l}$

UP LT Grade Science 2018 (Exam : 29.07.2018)

151. Twisting couple per unit angular twist is :

(a) $\frac{\pi\eta a^4}{1}$ (b) $\frac{2\pi\eta a^4}{1}$
(c) $\frac{\pi\eta a^4}{21}$ (d) $\frac{\pi\eta a^4}{31}$

TGT 2005

152. Two cylindrical rods have same length, mass and material, but one is hollow having cross-sectional internal and external radius r_1, and r_2 respectively and the other is solid having radius r. These are twisted by the same angle (by fixing at their respective ends), then the restoring couples C_h and C_s required will have ratio.

(a) $\frac{C_h}{Cs}=\frac{r_2^2-r_1^2}{r^2}$ (b) $\frac{C_h}{Cs}=\frac{r_2^2+r_1^2}{r^2}$
(c) $\frac{C_h}{Cs}=\frac{r^2}{r_2^2-r_1^2}$ (d) $\frac{C_h}{Cs}=\frac{r^2}{r_2^2+r_1^2}$

UPPCS Pre 2007, 2009

153. A cantilever of length 50 cm is depressed by 15mm at the loaded end. The depresssion at distance 30 cm from the fixed and will be :

(a) 3.24×10^{-3} (b) 12.96×10^{-3}
(c) 6.48×10^{-3} (d) 3.24×10^{-3}

(UPPGT 2009)

Hydrostatics

154. The velocity of a small ball of mass 'm' and density d_1, when dropped in a container filled with glycerin, becomes constant after some time. If the density of glycerin is d_2, then the viscous force acting on the ball, will be :

(a) $mg\left(1-\frac{d_1}{d_2}\right)$ (b) $mg\left(1-\frac{d_2}{d_1}\right)$
(c) $mg\left(\frac{d_1}{d_2}-1\right)$ (d) $mg\left(\frac{d_2}{d_1}-1\right)$

JEE MAIN 2022

155. **The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be : (Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})**

(a) $\frac{200}{3}\%$ (b) $\frac{200}{5}\%$

(c) $\frac{5}{200}\%$ (d) $\frac{3}{200}\%$

JEE MAIN 2021

156. **The velocity of a small ball of mass M and density d, when dropped in a container filled with glycerin becomes constant after some time. If the density of glycerin is $\frac{d}{2}$, then the viscous force acting on the ball will be**

(a) 2Mg (b) $\frac{Mg}{2}$

(c) Mg (d) $\frac{3}{2}$Mg

NEET 2021

157. **A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of the water in the capillary is 5 g. Another capillary tube of radius 2r is immersed in water. The mass of water that will rise in this tube is :**

(a) 5.0 g (b) 10.0 g

(c) 20.0 g (d) 2.5 g

NEET 2020

158. **A small hole of area of cross-section 2 mm^2 is present near the bottom of a fully filled open tank of height 2 m. Taking *g* = 10 m/s2, the rate of flow of water through the open hole would be nearly**

(a) 6.4×10^{-6} m^3/s (b) 12.6×10^{-6} m^3/s

(c) 8.9×10^{-6} m^3/s (d) 2.23×10^{-6} m^3/s

NEET 2019

159. **A fluid is in streamline flow across a horizontal pipe of variable area of cross section. For this which of the following statements is correct?**

(a) The velocity is maximum at the narrowest part of the pipe and pressure is maximum at the widest part of the pipe.

(b) Velocity and pressure both are maximum at the narrowest part of the pipe.

(c) Velocity and pressure both are maximum at the widest part of the pipe.

(d) The velocity is minimum at the narrowest part of the pipe and the pressure is minimum at the widest part of the pipe.

(Karnataka NEET 2013)

160. **The streamline flow of an incompressible and non-viscous liquid is described by**

(a) Boyle's law

(b) Poiseuille's law

(c) Archmedes principle

(d) Bernoulli's principle

UP PGT - 2021

161. **Viscosity of a gas is due to transport of**

(a) momentum (b) energy

(c) mass (d) none of these

UP PGT - 2021

162. **The velocity of oil in a 5 cm internal diameter pipe is 0.63 m/s. The velocity in a 3 cm internal diameter pipe that connects the first one is:**

(a) 1.75 m/s (b) 1.50 m/s

(c) 3.0 m/s (d) 9.0 m/s

UPPCS Pre 2005

163. **How high would water rise in the pipes of a building if the water pressure gauge shows the pressure at the ground floor to be 300 KPa :**

(a) 95.5 m/95.5 m (b) 45.3 m/45.5 m

(c) 40.2 m/40.2 m (d) 30.6 m/30.6 m

UPPCS (Pre) 2003

Hydrodynamics

164. **If p is the density and η is coefficient of viscosity of fluid which flows with a speed v in the pipe of diameter d, the correct formula for Reynolds number R_e is :**

(a) $R_e = \frac{\eta d}{\rho v}$ (b) $R_e = \frac{\rho v}{\eta d}$

(c) $R_e = \frac{\rho v d}{\eta}$ (d) $R_e = \frac{\eta}{\rho v d}$

JEE MAIN 2022

165. **What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.**
(Density of water : 10^3 kg/m^3)

(a) Unsteady to steady flow

(b) Remains steady flow

(c) Remains turbulent flow

(d) Steady flow to unsteady flow

JEE MAIN 2021

166. **Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional force are 10% of the input energy. How much power is generated by the turbine? (g = 10 m/s^2)**

(a) 7.0 kW (b) 10.2 kW

(c) 8.1 kW (d) 12.3 kW

NEET 2021

167. **Two small spherical metal balls, having equal masses, are made from materials of densities ρ_1 and $\rho_2(\rho_1 = 8\rho_2)$ and have radii of 1mm and 2 mm, respectively. They are made to fall vertically (from rest) in a viscous medium whose coefficient of viscosity equals h and whose density is 0.1 ρ_2. The ratio of their terminal velocities would be**

(a) $\frac{79}{72}$ (b) $\frac{19}{36}$

(c) $\frac{39}{72}$ (d) $\frac{79}{36}$

Odisha NEET 2019

168. **Venturimeter is based on -**

(a) Stokes law
(b) Archimedes principle
(c) Bernoulli theorem
(d) Equation of continuity

UPPSC Poly. Lect. 2021 Paper - I

169. **The working of an atomizer is based on**

(a) Bernoulli's principle (b) Boyle's law
(c) Archimedes principle (d) Hooke's law

RPSC School Lect. 08.01.2020

170. **The fictitious force on a body of mass 5 kg in a frame of reference moving vertically upwards with an acceleration of 5 m/s^2 is**

(a) 25 N, vertically downwards
(b) 25 N, vertically upwards
(c) 74 N, vertically upwards
(d) 74 N, vertically downwards

RPSC School Lect. 26.07.2016

171. **Equation for Bernoullis Theorem is :**

(a) $P + \rho v^2 + \rho gh = \text{constant}$
(b) $P + mv^2 + mgh = \text{constant}$
(c) $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$
(d) $P + \frac{1}{2}mv^2 + mgh = \text{constant}$

UP PGT Physics 2016 (Exam : 02.02.2019)

172. **Water rises to a height of 10 cm in a capillary. Mercury falls to a depth of 3.42 cm in same capillary. If the relative density of mercury is 13.6 g/cc and its angle of contact is 135°, the ratio of surface tension of water and mercury is :**

(a) 0.05 (b) 0.10
(c) 0.15 (d) 0.20

UP PGT Physics 2016 (Exam : 02.02.2019)

173. **Three capillaries of equal lengths but internal radii r, 2r, and 3r respectively are connected in series, a liquid flows in streamline motion through them. If the pressure difference across the entire system is 77cm (of water), then the pressure difference across the first capillary is :**

(a) 71.64cm (b) 70.32cm
(c) 76.00cm (d) 77.00cm

UP PGT Physics 2016 (Exam : 02.02.2019)

174. **Bernoulli's principle is based on the law of conservation of :**

(a) mass (b) energy
(c) linear momentum (d) angular momentum

UPPCS (Pre) 1999, 2000

Surface Tension

175. **Sixty four conducting drops each of radius 0.02 m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface density of bigger drop to the smaller drop will be :**

(a) 1 : 4 (b) 4 : 1
(c) 1 : 8 (d) 8 : 1

JEE MAIN 2022

176. **A water drop of diameter cm is broken into 64 equal droplets. The surface tension of water is 0.075 N/m. In this process the gain in surface energy will be :**

(a) 2.8×10^{-4} J (b) 1.5×10^{-3} J
(c) 1.9×10^{-4} J (d) 9.4×10^{-5} J

JEE MAIN 2022

177. **If a soap bubble expands, the pressure inside the bubble**

(a) Is equal to the atmospheric pressure
(b) Decreases
(c) Increases
(d) Remains the same

NEET 2022

178. **A capillary tube of radius *r* is immersed in water and water rises in it to a height *h*. The mass of the water in the capillary is 5 g. Another capillary tube of radius 2*r* is immersed in water. The mass of ater that will rise in this tube is**

(a) 2.5 g (b) 5.0 g
(c) 10.0 g (d) 20.0 g

NEET 2020

179. **The surface tension of a liquid**

(a) Increases with surface area
(b) Decreases with surface area
(c) Increases with temperature
(d) Decreases with temperature

UPPSC Ashram Paddhati PGT - 2021

180. **Surface tension of soap solution is 1.9×10^{-2} N/m Pressure difference on both sides of a soap bubble of diameter 2 cm will be -**

(a) 7.2 N/m^2 (b) 3.8 N/m^2
(c) 3.5 N/m^2 (d) 7.6 N/m^2

UPPSC Poly. Lect. 2021 Paper - I

181. Two capillary tubes of same material having radii 1 mm and 2 mm respectively are immersed in a liquid. If the liquid rises to 30 cm in the first, then the height in second will be-

(a) 7.5 cm (b) 60 cm
(c) 15 cm (d) 120 cm

UPPSC Poly. Lect. 2021 Paper - I

182. Two water drops of same radii are falling in air with terminal velocity v. If both drops are collapsed into a big drop then terminal velocity will be shall be :

(a) $2^{1/3}v$ (b) $2^{2/3}v$
(c) $\frac{1}{2}v$ (d) 2v

UP LT Grade Science 2018 (Exam : 29.07.2018)

183. The excess pressure in a soap bubble of 2 mm radius, if surface tension is 0.03N/m, is :

(a) $30N/m^2$ (b) $40N/m^2$
(c) $60N/m^2$ (d) $120N/m^2$

UP LT Grade Science 2018 (Exam : 29.07.2018)

184. A water drop of radius 10^{-2} m is broken into 1000 equal droplets. If the surface tension of water is 0.075 N/m, the gain in surface energy will be

(a) 8.5×10^{-4} J. (b) 8.1×10^{-4} J
(c) 7.7×10^{-4} J (d) 7.5×10^{-4} J

UP PGT - 2021

185. Air is pushed into soap bubble of radius r to double its radius. If the surface tension of the soap solution is S, the work done in the process is :

(a) $8\pi r^2S$ (b) $12\pi r^2S$
(c) $16\pi r^2S$ (d) $24\pi r^2S$

UPPCS Pre 2006

Special Theory of Relativity

186. When light propagates through a material medium of relative permittivity ε_r and relative permeability μ_r, the velocity of light, v is given by (c-velocity of light in vacuum)

(a) $v=\frac{c}{\sqrt{\varepsilon_r\mu_r}}$ (b) $v = c$
(c) $v=\sqrt{\frac{\mu_r}{\varepsilon_r}}$ (d) $v=\sqrt{\frac{\varepsilon_r}{\mu_r}}$

NEET 2022

187. The source of solar energy is :

(a) Chemical reactions
(b) Nuclear fusion
(c) Nuclear fission
(d) Gravitational interaction with other solar systems

UP PGT Physics 2016 (Exam : 02.02.2019)

188. If rest mass of an electron is 9.11×10^{-31} kg its rest energy will be:

(a) 8.2×10^{-11} J (b) 4.1×10^{-14} J
(c) 5.1×10^{-12} J (d) 8.2×10^{-14} J

UPPCS Pre 2005

189. If the speed of light were $\frac{2}{3}$ of its present value the energy released in a given atomic explosion:

(a) decreased by a factor of $\frac{2}{3}$
(b) decreased by a factor of $\frac{4}{9}$
(c) decreased by a factor of $\frac{5}{9}$
(d) decreased by a factor of $\frac{1}{3}$

UPPCS (Pre) 2004

190. A car is moving with a speed 108 km/hr. Its length is l and height h to a stationary observer, these will appear as :

(a) $l\gamma, h\gamma$ where $\gamma=\frac{1}{\sqrt{1-10^{-18}}}$
(b) $\frac{l}{\gamma},\frac{h}{\gamma}$ where $\gamma=\frac{1}{\sqrt{1-10^{-11}}}$
(c) $l\gamma, h$ where $\gamma=\frac{1}{\sqrt{1-10^{-14}}}$
(d) $\frac{l}{\gamma}, h$ where $\gamma=\frac{1}{\sqrt{1-10^{-11}}}$

UPPCS PRE 1996

191. The total energy of a relativistic particle with rest mass m_0 and moving with velocity v is given by :

(a) $\frac{1}{2}m_0v^2$ (b) $\frac{1}{2}\frac{m_0v^2}{\sqrt{1-\frac{v^2}{c^2}}}$
(c) $\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ (d) $m_0c^2\sqrt{1-\frac{v^2}{c^2}}$

UPPCS Pre 2006

192. For a particle of zero rest mass, the energy can be expressed in terms of its linear momentum p and its velocity v by E = pv, only if :

(a) $v < c$ (b) $v = c$
(c) $v > c$ (d) $v = 0$

UPPCS Pre 2001

193. A particle of rest mass m_0 moves at speed 0.6 c relative to the observer. The observed mass will be:

(a) $1.25\ m_0$ (b) $1.33\ m_0$
(c) $1.67\ m_0$ (d) $1.76\ m_0$

UPPCS Pre 2001

194. A body of mass m_o is placed in a rocket. The rocket is moving with a speed of 0.6 c. Then the mass of the body as observed by a person sitting in the rocket is

(a) m_0 (b) $\frac{5}{4}m_0$

(c) $\frac{4}{5}m_0$ (d) $\frac{3}{5}m_0$

UPPCS Pre 2008

195. The velocity at which the relativistic momentum of the particle becomes twice its Newtonian momentum is :

(a) c (b) $c\sqrt{3}$

(c) $\frac{2c}{\sqrt{3}}$ (d) $\frac{c\sqrt{3}}{2}$

UPPCS Pre 2008

196. The increase in the relativistic mass of a particle of rest mass 1 gm when it is moving with 0.8 c velocity is :

(a) 1 gm (b) 0.667 gm

(c) 1.667 gm (d) zero

UPPCS Pre 2008

197. The equivalent relativistic mass of a photon of wavelength 662 nm shall be [$h = 6.62\times10^{-34}$ J.s]

(a) Zero (b) 3.3×10^{-36} kg

(c) 3.3×10^{-34} kg (d) Infinity

UPPCS Pre 2007

198. A cube is moving with a velocity v along one of its edges. If L_o be the proper length of each edge, the volume of moving cube would be :

(a) $L_0^3\left(1-\frac{v^2}{c^2}\right)^{3/2}$ (b) $L_0^3c\sqrt{1-\frac{v^2}{c^2}}$

(c) $\frac{L_0^3}{\sqrt{1-\frac{v^2}{c^2}}}$ (d) $L_0^3\sqrt{1-\frac{v^2}{c^2}}$

UPPCS Pre 2008

199. If L_0^3 is the volume of a cube at rest, them the volume viewed from a reference frame moving with uniform velocity v in a direction parallel to an edge of the cube shall be:

(a) L_0^3 (b) $\frac{L_0^3}{\left[1-\frac{v^2}{c^2}\right]^{1/2}}$

(c) $L_0^3\left[1-\frac{v^2}{c^2}\right]^{1/2}$ (d) $L_0^3\left[1-\frac{v^2}{c^2}\right]$

UPPCS Pre 2007

200. The mass a of relativistic particle :

(a) increases with increasing velocity
(b) decreases with increasing velocity
(c) first increases until a critical value then starts decreasing
(d) remains independent of its velocity

UPPCS Pre 2005

Answer Key

1. (a)	2. (c)	3. (d)	4. (c)	5. (b)	6. (a)	7. (a)	8. (a)	9. (b)	10. (d)
11. (b)	12. (b)	13. (a)	14. (b)	15. (b)	16. (a)	17. (c)	18. (b)	19. (a)	20. (b)
21. (d)	22. (c)	23. (c)	24. (b)	25. (a)	26. (a)	27. (d)	28. (d)	29. (a)	30. (b)
31. (a)	32. (b)	33. (b)	34. (b)	35. (d)	36. (c)	37. (c)	38. (c)	39. (a)	40. (b)
41. (d)	42. (d)	43. (c)	44. (b)	45. (d)	46. (c)	47. (d)	48. (d)	49. (d)	50. (b)
51. (b)	52. (c)	53. (a)	54. (b)	55. (c)	56. (b)	57. (a)	58. (c)	59. (c)	60. (a)
61. (d)	62. (b)	63. (d)	64. (b)	65. (c)	66. (a)	67. (c)	68. (c)	69. (c)	70. (a)
71. (c)	72. (b)	73. (d)	74. (c)	75. (c)	76. (a)	77. (d)	78. (c)	79. (b)	80. (b)
81. (d)	82. (d)	83. (b)	84. (b)	85. (b)	86. (b)	87. (d)	88. (b)	89. (c)	90. (d)
91. (c)	92. (d)	93. (c)	94. (b)	95. (a)	96. (d)	97. (c)	98. (c)	99. (b)	100. (a)
101. (c)	102. (c)	103. (c)	104. (c)	105. (a)	106. (c)	107. (d)	108. (d)	109. (a)	110. (c)
111. (b)	112. (d)	113. (b)	114. (a)	115. (c)	116. (c)	117. (a)	118. (b)	119. (b)	120. (a)
121. (d)	122. (c)	123. (a)	124. (c)	125. (c)	126. (c)	127. (a)	128. (c)	129. (d)	130. (d)
131. (c)	132. (b)	133. (c)	134. (b)	135. (a)	136. (b)	137. (d)	138. (d)	139. (c)	140. (c)
141. (c)	142. (c)	143. (c)	144. (a)	145. (c)	146. (b)	147. (b)	148. (d)	149. (a)	150. (a)
151. (c)	152. (b)	153. (a)	154. (b)	155. (a)	156. (b)	157. (b)	158. (b)	159. (a)	160. (d)
161. (a)	162. (a)	163. (d)	164. (c)	165. (d)	166. (c)	167. (d)	168. (c)	169. (a)	170. (a)
171. (c)	172. (c)	173. (a)	174. (b)	175. (b)	176. (a)	177. (b)	178. (c)	179. (d)	180. (d)
181. (c)	182. (b)	183. (c)	184. (a)	185. (d)	186. (a)	187. (b)	188. (d)	189. (c)	190. (c)
191. (b)	192. (b)	193. (a)	194. (a)	195. (d)	196. (c)	197. (b)	198. (d)	199. (c)	200. (a)

02.

Heat and Thermodynamics

(1) Thermal Properties of Matter

■ Temperature and Heat

Temperature:- Temperature is a relative measure or indication of hotness or coldness of the body.

- The SI unit of temperature is Kelvin (K)
- Commonly used unit of temperature is degree Celsius (ºC)

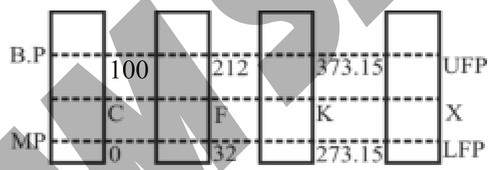
Heat:- Heat is a form of energy that flows between a body and its surrounding medium by virtue of temperature difference between them.

- The SI unit of heat energy transferred is expressed in Joule (J).

■ Types of Temperature scale

Name of the scale	Symbol for each scale	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of Divisions on the scale
Celsius	ºC	0ºC	100 ºC	100
Fahrenheit	ºF	32 ºF	212 ºF	180
Kelvin	K	273.15K	373.15K	100

■ Relationship among different Temperature scale.



$$\frac{C-0}{100-0}=\frac{F-32}{212-32}=\frac{K-273.15}{373.15-273.15}=\frac{X-LFP}{UFP-LFP}$$

- Two familiar temperature scales are the Fahrenheit temperature scale and the Celsius temperature scale.
- The Celsius Temperature (t_C) and the Fahrenheit temperature (t_F) are related by

$$\boxed{t_F=\frac{9}{5}t_C+32}$$

•Absolute Zero Temperature (–273.15K)

Absolute zero is the temperature at which a system is in the state of lowest possible energy. It is also known as thermodynamic temperature.

■ Measurement of Temperature

•Thermometer:- The instrument used to measure the temperature is called thermometer. The thermometers work on the thermometric property, i.e the property which changes with temperature like any physical quantity such as length, volume pressure and resistance etc. These vary varies linearly with a certain range of temperature. Common liquids used in glass thermometers are mercury, alcohol etc. whose volume varies linearly with temperature over a wide range.

■ Different types of thermometers

(1) Mercury thermometer:-

In these thermometers the length of mercury thread is taken as thermometric property if length of mercury column at 0º and 100º are ℓ_0 and ℓ_{100} respectively.

(2) Gas Thermometer:-

In such thermometers, the pressure of a given mass of an ideal gas (at constant volume) or volume of a given mass of an ideal gas (at constant pressure) is the thermometric property. The reference temperature is taken as 0ºC (ice point) and 100ºC (Boiling point). The unknown temperature can be calculated as

$$t(^{\circ}C) = \left(\frac{P_t - P_0}{P_{100} - P_0}\right) \times 100 \text{ (Constant volume gas thermometer)}$$

$$t(^{\circ}C) = \left(\frac{V_t - V_0}{V_{100} - V_0}\right) \times 100 \text{ (Constant pressure gas thermometer)}$$

(3) Platinum Resistance Thermometer

In this thermometer, the resistance of platinum wire is taken as thermometric property. The reference temperatures are ice point (0ºC) and steam point (100ºC). The unknown temperature in ºC is given by.

$$t(^{\circ}C) = \left(\frac{R_t - R_0}{R_{100} - R_0} \times 100\right)^{\circ}C = \left(\frac{R_t}{R_{tr}} \times 273.16\right)K$$

Here, temperature coefficient of Resistance (α) is given by

$$\alpha = \left(\frac{R_{100} - R_0}{R_0 \times 100}\right)$$

S. No.	Thermometer	Thermometric property	Principle
1.	Constant volume gas thermometer	Pressure	Ideal gas equation
2.	Constant pressure gas thermometer	Volume	Idea gas equation
3.	Electrical resistance thermometer	Resistance	Wheat- stone bridge
4.	Thermocouple	Thermal E.M.F.	Seebeck effect
5.	Liquid- in- glass thermometer	Length	Volumetric expansion
6.	Pyrometer	Infrared radiation flux	Radiation
7.	Magnetic thermometer	Paramagnetic	Curie's law

■ **Thermometry:-** The branch of physics which deals with the measurement of temperature is called thermometry.

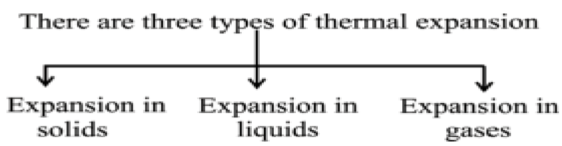
Few Terms used in Thermometry:-

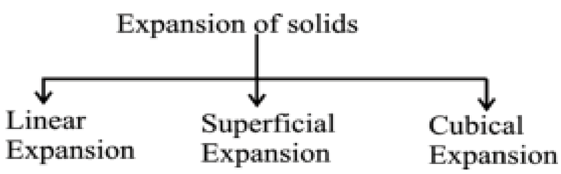
- **Melting: -** Conversion of solid into liquid state at constant temperature is called melting.
- **Fusion and Freezing point:-** The process of change of state from liquid to solid is called fusion. The temperature at which liquid starts to freeze is known as the freezing point of the liquid.
- **Evaporation: -** Conversion of liquid into vapour at all temperatures (even below its boiling point) is called evaporation
- **Sublimation: -** The conversion of a solid into vapour state is called sublimation.
- **Hoar - Frost:-** The conversion of vapour into solid state is called Hoar frost.

■ Thermal Expansion

Almost all substances (solids, liquids and gases) expand on heating and contract on cooling.
The expansion of a substance on heating is called thermal expansion of substance.

Note:- Water from 0ºC to 4 ºC and silver iodide from 80ºC to 141ºC. Which contract on heating and expand on cooling.





(i) Linear expansion:- By Experiments it is observed that on heating a rod, the increase in its length is directly proportional to its original length, as increase in its temperature.

$$\boxed{\Delta\ell = \ell\alpha\Delta T} \Rightarrow \boxed{\alpha = \frac{\Delta\ell}{\ell \times \Delta T}}$$

Where,
$\Delta\ell$ = Increase in length
ℓ = Original length
α = Coefficient of linear expansion
ΔT = Change in Temperature

S. No.	Materials	Coefficient of linear expansion (α) in K^{-1}
1.	Aluminium	2.5×10^{-5}
2.	Brass	1.8×10^{-5}
3.	Iron	1.2×10^{-5}
4.	Copper	1.7×10^{-5}
5.	Silver	1.9×10^{-5}
6.	Gold	1.4×10^{-5}
7.	Glass (pyrex)	0.32×10^{-5}
8.	Lead	0.29×10^{-5}

(ii) Superficial Expansion:- Superficial expansion of the material of a lamina is equal to the increase in unit area of the lamina when its temperature rises to 1ºC.

$$\boxed{\Delta A = A\beta\Delta T} \Rightarrow \boxed{\beta = \frac{\Delta A}{A \times \Delta T}}$$

Where,
ΔA = Change in Area
A = Original Area
ΔT = change in Temperature
β = Coefficient of superficial expansion

(iii) Cubical (volumetric) Expansion:- Expansion in volume on heating is called cubical expansion.

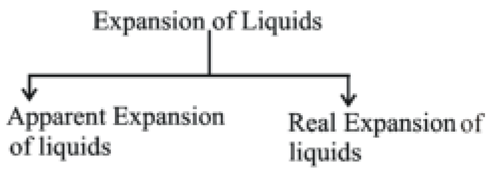
$$\boxed{\Delta V = V\gamma\Delta T} \Rightarrow \boxed{\gamma = \frac{\Delta V}{V\Delta T}}$$

Where,
ΔV = Increase in Volume
V = Original Volume
γ = Coefficient of cubical expansion
ΔT = Increase in temperature.

- **Relation between α , β and γ are**

$$\beta = 2\alpha \text{ and } \gamma = 3\alpha$$
$$\boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$$

- Dimension of α, β and γ are $[\theta^{-1}]$ and units are k^{-1} or $(^{\circ}C)^{-1}$



(i) Apparent Expansion of Liquids:-
When expansion of the container containing liquid, on heating is not taken into account, then observed expansion is called apparent expansion of liquids.
Coefficient of apparent expansion of a liquid.

$$\gamma_a = \frac{\text{apparent (or observed) increase in volume}}{\text{original volume} \times \text{change in temperature}}$$

$$\gamma_a = \left(\frac{\Delta V_a}{V \times \Delta T}\right) {}^{\circ}C^{-1}$$

(ii) Real Expansion of Liquids

When expansion of the container, containing liquid, on heating is also taken into account then observed expansion is called real expansion of liquids.

Real Expansion of Liquid = Apparent expansion of liquid + volume expansion of vessel.

Coefficient of real expansion of a liquid

$$\gamma_r = \frac{\text{real increase in liquid}}{\text{original volume} \times \text{change in temperature}}$$

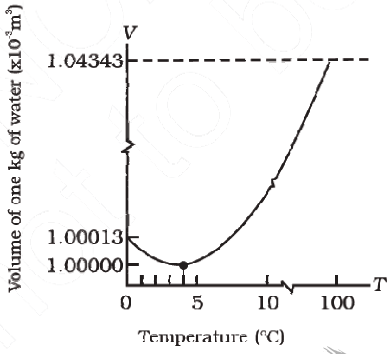
$$= \left(\frac{\Delta V_r}{V \times \Delta T}\right) {}^{o}C^{-1}$$

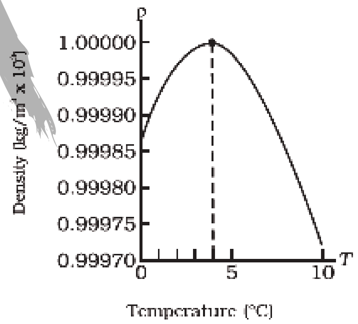
Relation between coefficient of real and apparent expansions-

$$\boxed{\gamma_r = \gamma_a + Y_g}$$

■ Anomalous Expansion of water

When temperature of water is increased from 0ºC, then its volume decreases upto 4ºC, becomes minimum at 4ºC and then increases. Hence density of water is maximum at 4ºC because at this point volume is minimum. This behaviour of water around 4ºC is called anomalous expansion of water.





■ Thermal Expansion of Gases

There are two types of coefficient of expansion in gases.

(i) Volume Coefficient (γ_v) : At constant pressure, the change in volume per unit volume per degree Celsius is called volume coefficient.

$$\gamma_V = \frac{\Delta V}{V_0 \times (\Delta T)}$$

Where,

ΔV = change in volumes of the gas

V_0 = original volume

ΔT = change in Temperature.

(ii) Pressure coefficient (γ_P)

At constant volume, the change in pressure per unit pressure per degree Celsius is called pressure coefficient.

$$\gamma_P = \frac{\Delta P}{P_0 \times (\Delta T)}$$

Where,

ΔP = change in pressure

P_0 = original pressure

ΔT = change in Temperature

■ Practical Applications of Thermal Expansion

(i) A glass stopper Jammed in the neck of a glass bottle can be taken out by heating the neck of the bottle.

(ii) When rails are laid down of the ground space is provided between the ends of two rails.

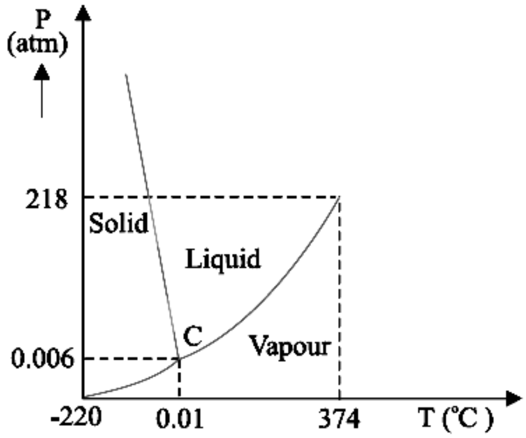
(iii) The transmission cable are not tightly fixed to the poles.

■ Triple point of water-

The values of pressure and temperature at which water coexists in equilibrium in all three states of matter, i.e. ice, water and vapour is called triple point of water.

Temperature at triple point = 0.01ºC = 273.16 K.

Pressure at triple point = 0.006 P_{atm} = 4.58 mm of Hg



■ **Specific Heat:-**

Specific heat of a substance is the amount of heat required to raise the temperature of the unit mass of the substance through a unit degree (1° C or 1K). It is denoted by C or S.

- Its SI unit is Joule/kilogram- °C or J kg^{-1} K^{-1} and its dimensional formula is $[L^2T^{-2}\theta^{-1}]$

The value of specific heat (S) will depend upon nature of the substance, and will obviously be different for different substances.

$$S = \frac{Q}{m\Delta T}$$

- The specific heat of water = 4180 J kg^{-1} K^{-1}
- Specific heat for Hydrogen is maximum (3.5 cal/(g°C)) and for water 1 cal g^{-1} °C^{-1}.
- For Radon and Actinium, specific heat is minimum (= 0.022 cal g^{-1} °C^{-1})
- Specific heat of a substance depends also on the state of the substance i.e solid, liquid or gas. For example

Specific heat of ice = 0.5 cal g^{-1} °C^{-1}

Specific heat of water = 1 cal g^{-1} °C^{-1}

Specific heat of steam = 0.47 cal g^{-1} °C^{-1}

Substance	Specific heat capacity ($J\ kg^{-1}K^{-1}$)	Substance	Specific heat capacity ($J\ kg^{-1}K^{-1}$)
Aluminium	900.0	Ice	2060
Carbon	506.5	Glass	840
Copper	386.4	Iron	450
Lead	127.7	Kerosene	2118
Silver	236.1	Edible oil	1965
Tungsten	134.4	Mercury	140
Water	4186.0		

- Gases have two types of specific heat such as specific heat capacity at constant volume C_v and at constant pressure C_p.

For molar specific heats $C_p - C_v = R$ where R = gas constant and this relation is called **Mayer's formula.**

- Specific heat ratio of adiabatic index is given as,

$$\gamma = \frac{C_p}{C_v}$$

The value of γ depends on atomicity of the gas.

■ Debye Model

- Debye model is a method developed by Peter Debye in 1912 for estimating the Phonon contribution to the specific heat (heat capacity) in a solid. This model correctly explains the low temperature dependence of the heat capacity.
- According to Debye model, the lattice heat capacity at low temperature varies as proportional to T^3.

The low temperature heat capacity,

$$\boxed{C = \frac{dU}{dT} = \frac{16\pi^5 K_B{}^4}{5h^3C_5^3}T^3}$$

■ **Heat capacity or Thermal Capacity**

The capacity of a body is defined as the amount of heat required to raise the temperature of the (whole) body through 1°C or 1K.

$$\Delta Q = ms\Delta T$$

When,
$\Delta T = 1$
$\Delta Q = ms$
ΔQ = Thermal capacity
Hence thermal capacity of a body is the product of mass and specific heat of the body.
Its unit is Jk^{-1} or Cal $^{o}C^{-1}$

■ **Water Equivalent:-**
It is the quantity of water whose thermal capacity is same as the heat capacity of the body. It is denoted by W
W = ms = heat capacity of the body.
Its expressed in the unit gram.

■ **Latent Heat:-**
Latent heat of a substance is the amount of heat energy required to change the state of unit mass of the substance from solid to liquid and from liquid to gas/vapour without any change in Temperature.

$$\text{Latent heat (L)} = \frac{\text{Quantity of heat (Q)}}{\text{Mass of substance (M)}}$$

- Its unit is measured in J kg^{-1} or Cal g^{-1}
- Latent heat of fusion of ice = 80 cal. $g^{-1} = 3.33 \times 10^5$ J/kg
- Latent heat of vaporisation of water is 540 cal g^{-1} or 22.6×10^5 J/kg.

■ **Few examples to understand Latent heat**
- It is more painful to get burnt by steam rather than by boiling water at 100ºC.
- Steam converted to water at 100ºC , then it gives out 536 cal of heat, so it is clear that steam at 100ºC has more heat than water at 100ºC (i.e boiling of water).
- After snow falls, the temperature of the atmosphere becomes very low. This is because the snow absorbs the heat from the atmosphere to melt down. So in the mountains, when snow falls, one does not feel too cold but when ice melts he feels too cold.
- There is more shivering effect of ice-cream on teeth as compared to that of water (obtained from ice). This is because when ice-cream melts down, it absorbs large amount of heat from teeth.

■ **Joule's Law:-**
According to Joule, when ever heat is converted into work or work is converted into heat, then the ratio between work and heat is constant.

$\frac{W}{Q} = J$, Where J is mechanical equivalent of heat and J = 4.2 J. Cal^{-1}

■ **Calorimetry**
This is the branch of heat transfer that deals with the measurement of heat. The heat is usually measured in calories or kilo calories.

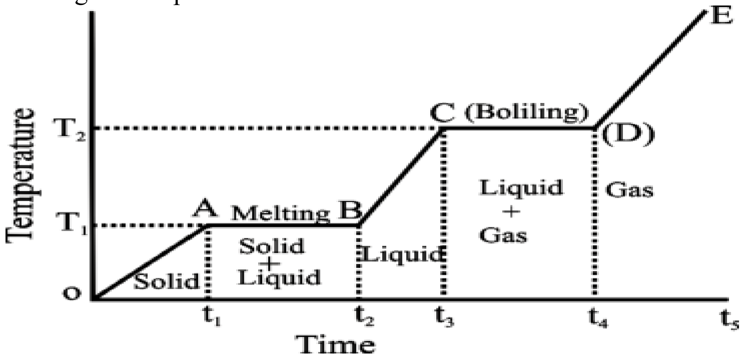
■ **Principle of calorimetry:-**
When a hot body is mixed with a cold body, then heat lost by hot body is equal to the heat gained by cold body.
Heat lost = Heat gained.
If two substances having masses m_1 and m_2, specific heats C_1 and C_2 kept at temperature T_1 and T_2 $(T_1 > T_2)$ are mixed, such that,
Temperature of mixture at equilibrium is T_{mix}, then

$$m_1C_1(T_1 - T_{mix}) = m_2C_2(T_{mix} - T_2) \text{ or}$$

$$\boxed{T_{mix} = \frac{m_1C_1T_1 + m_2C_2T_2}{m_1C_1 + m_2C_2}}$$

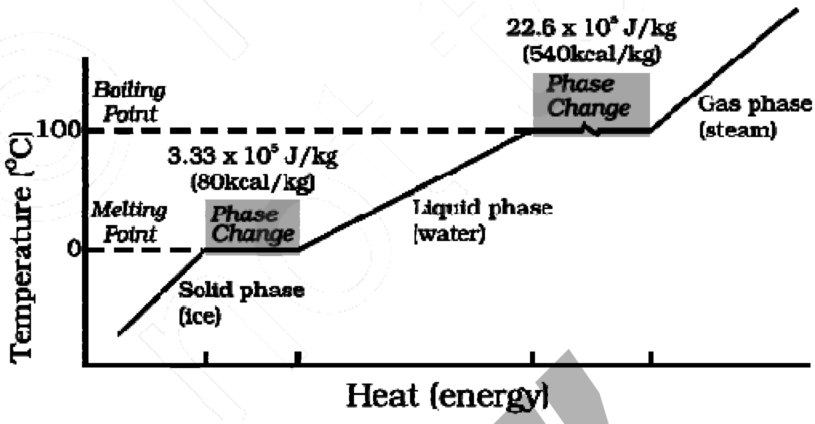
Note:- Principle of calorimetry follows the law of conservation of heat energy.

■ **Heating curve:-**
When a substance is given the heat, it undergoes rise in temperature and change of state at specific temperatures. The graph showing the change in temperature and state of the substance with time is called the heating curve.



- Specific heat (or thermal capacity) $\propto \frac{1}{\text{Slope of curve}}$
- Latent heat $\propto$ Length of horizontal line

■ **Heating curve of water:-**



(2) Modes of Heat transfer

■ Transfer of Heat

Heat is a form of energy which can be transferred from one part of the system to another part of system due to temperature difference.

■ **There are three modes of heat transfer**

(I) Conduction
(II) Convection
(III) Radiation

(I)Conduction:- In conduction, heat is transferred from one point to another point without the actual motion of heated particles.

Key points:-

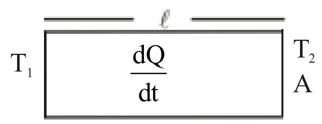
(i) Heat Transfer due to Temperature difference.
(ii) Heat transfer due to free electron or vibration of molecules.
(iii) Heat transfer in solid body (in mercury also)
(iv) It is slow process.
(v) In conduction irregular path follows

■ **Fourier law of heat Conduction-**

Rate of heat flow $Q = -\frac{kAdT}{dx}$

■ Thermal conductivity:-

Thermal conductivity of a solid is a measure of the ability of the solid to conduct heat through it. It is denoted by 'k'



Rate of heat flow $\frac{dQ}{dt} = \frac{-kAdT}{dx}$ or $\frac{Q}{t} = \frac{kA(T_1 - T_2)}{\ell}$

Thermal resistance $R_H = \frac{\ell}{kA}$

SI units of k are $\frac{\text{Joule/sec}}{m^2 k/m} = Wm^{-1}K^{-1}$

CGS unit of k are $cal^{-1}cm^{-1\circ}C^{-1}$

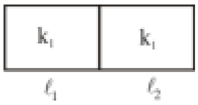
- The value of k is large in case of good conductors and it small in the case of insulator.
- $k_{solid} > k_{liquid} > k_{gas}$ (due to shape & size of matter or orientation of atoms in molecules)

■ Variation of Thermal conductivity with Temperature -

S. No.	Phase	Temperature	Effect	Cause
1.	Solid	Increase	Conductivity decrease	The crystal configuration of solid is affected and they start to occupy random positions.
2.	Liquid	Increase	Conductivity decrease	The liquids expand and try to attain gaseous configuration.
3.	Gas	Increase	Conductivity Increases	The gas molecules collide at higher rate and momentum transfer due to collision increases.

■ Applications:-

(a) Rods in series



Total equivalent thermal resistance (R) is equal to sum of individual thermal resistances i.e.

$$R_{eq} = R_1 + R_2 + R_3 + \ldots = \left(\frac{1}{k_1}\frac{\ell_1}{A_1}\right) + \left(\frac{1}{k_2}\frac{\ell_2}{A_2}\right) + \ldots\ldots$$

■ Rods in parallel:

In parallel, the total thermal resistance is given by -

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots. = \frac{1}{\left(\frac{1}{k_1}\frac{\ell_1}{A_1}\right)} + \frac{1}{\left(\frac{1}{k_2}\frac{\ell_2}{A_2}\right)} + \ldots\ldots$$

■ Ingen -Hausz experiment

The thermal conductivities of different materials are proportional to the square of the length of the melted wax of the rods of these materials in the steady state.

If ℓ_1, ℓ_2, ℓ_3 are the lengths of melted wax on the rods of different materials having coefficient of thermal conductivities-

$$\boxed{k_1 : k_2 : k_3 = \ell_1^2 : \ell_2^2 : \ell_3^2 \text{ or } \frac{k_1}{k_2} = \frac{\ell_1^2}{\ell_2^2}}$$

■ Formation of Ice on Lakes-

Time taken to form x thickness of ice on lake,

$$\boxed{t = \frac{\rho L}{2k\theta}x^2}$$

Where ρ density of ice

L = Latent heat of freezing of ice.

k = coefficient of thermal conductivity of ice.

θ = Temperature above take.

Time taken to increase the thickness of ice from x_1 to x_2.

$$\boxed{t = \frac{\rho L}{2k\theta}(x_2^2 - x_1^2)}$$

(II) Convection:- It is a mode of transfer of heat from one part of the medium to another part by the actual movement of heated particles of the medium.

Key points:-

(i) Heat transfer due to density difference

(ii) In convection actual motion of particles.

(iii) Heat transfer in fluids (Liquid & Gas)

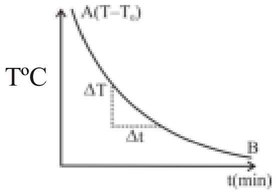
(iv) It is slow process.

(v) It follow irregular path.

■ Newton's law of cooling:-

It states that the rate of loss of heat of a body is directly proportional to the difference in temperature of the body and the surroundings, provided the difference in temperature is small, not more than 40ºC

$$\boxed{\frac{dT}{dt} = k(T_1 - T_0) \Rightarrow T = T0 + (T_1\text{-}T_0)e^{-kt}}$$



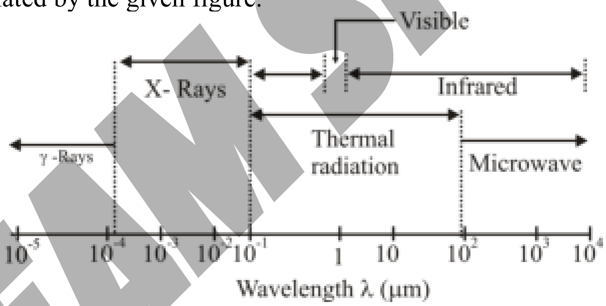
Where,
T = Temperature of body
T_0 = Temperature of surrounding
k = constant of proportionality
When a body cools from T_1 to T_2 in time 't' in a surrounding of temperature T_0 then,

$$\frac{T_1 - T_2}{t} = k\left(\frac{T_1 - T_2}{2} - T_0\right)$$

(III) Radiation:-Radiation is the energy emitted by the matter by the mechanism of photon emission or electromagnetic wave emission as result of changes in the electronic configuration of the atoms and molecules. Because the mechanism of transmission is photon emission, unlike conduction and convection, there is no need of intermediate matter to enable to transmission. In radiation, the internal energy of the object decreases.

Electromagnetic Spectrum:-

All the electromagnetic waves are classified in terms of the wave length and are propagated at the speed of light (c) i.e., 3×10^8 m/s. The difference between one form of radiation and another lies only in its frequency (f) and wavelength (λ) which is related by the given figure.



The emission of thermal radiation which (range are lies between wavelength of 10^{-5} μm and 10^4 μm) depends upon the nature, temperature and state of the emitting surface. However, with gases the dependence is also upon the thickness of the emitting layer and the pressure.

Key points:-

(i) Heat transfer without any medium.
(ii) It is an Electromagnetic radiation
(iii) Radiation acts in all form (Solid + liquid + Gas)
(iv) It is fast process (with speed of light $c=3\times10^8$ ms^{-1})
(v) Path of radiation is straight line like "light"

■ **Stefan's Boltzmann law:-**

Radiated energy emitted by a perfect black body per unit area second is directly proportional to fourth power of absolute temperature.

$$\boxed{E \propto T^4 \text{ or } E = \sigma T^4}$$

Where σ is a constant of proportionality and is called Stefan's constant. Its value is $5.67\times10^{-8}\ Wm^{-2}\ K^{-4}$

• If the body at temperature T is surrounded by a body at temperature T_0 then Stefan's law -

$$\boxed{E = \sigma\ (T_1^4\text{-}T_0^4)}$$

• **For a general body**: $\boxed{E = \ \varepsilon\sigma\ (T_1^4\text{-}T_0^4)}$

Here 'ε' is taken as emissivity of both the body and the enclosure.

■**Prevost's theory of heat exchange:-**

A body is simultaneously emitting radiations to its surrounding and absorbing radiations from the surroundings. If surrounding temperature is T_0 then

$$\boxed{E_{net} = \ \varepsilon\sigma\ (T_1^4\text{-}T_0^4)}$$

■ **Kirchhoff's Law**

- According to Kirchhoff's law, the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.
- If a and e represent absorptive and emissive power of a perfectly black body then $\frac{e}{a} = \frac{E}{A} =$ constant for all surface.
- For a perfectly black body A = 1 so, for any surface $\frac{e}{a} = E$ for the radiation of a particular wavelength

$$\boxed{\frac{e_\lambda}{a_\lambda} = \frac{E_\lambda}{A_\lambda} \Rightarrow \frac{e_\lambda}{a_\lambda} = E_\lambda}.$$

Since E_λ is constant at a given temperature hence according to this law, if a surface is a good absorber of a particular wavelength it is also a good absorber and bad emitter.

■ **Perfectly black body**

- A perfectly black body is that which absorbs completely all the radiation of all wavelengths incident on it.
- A perfectly Black body maintained at a suitable temperature emits radiations of all wavelengths.
- A perfectly black body neither reflects nor transmits any radiation, it will always appear black whatever be the colour of the incident radiation

■ **Absorptive Power:-**

Absorptive power of a surface is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time. It is represented by 'a'

$$a = \frac{(\Delta Q)_{absorbed}}{(\Delta Q)_{incident}}$$

- For a perfectly black body absorptive power is maximum and it is unity.
- It has no units and dimension.

■ **Emissive power:-**

- For a given surface it is defined as the radiant energy emitted per unit area of the surface.
- It is represented by 'e'
- Emissive power of a surface depends on its nature and temperature.
- Emissive power (e) of given wavelengths of radiations emitted range from 0 to ∞, $e = \int_0^\infty e_\lambda d\lambda$

'e' is measured in Joule sec^{-1} $metre^{-2}$ or wm^{-2}

■ **Rayleigh Jeans Law-** The Rayleigh- Jeans Law of radiation given us the intensity of radiation released by a black body. The law states that the intensity of the radiation emitted by a black body is directly proportional to the temperature and inversely proportional to the wavelength raised to a power of four.

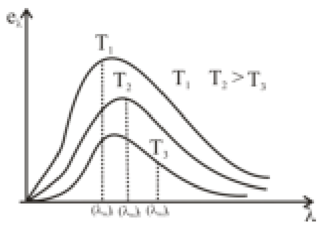
The Raleigh- Jeans law approximates the spectral radiance of a black body as a function of its wavelength at a given temperature. The Rayleigh- Jeans equation can be written as:

$$\boxed{B_\lambda(T) = \frac{2ck_BT}{\lambda^4}}$$

Where,
B = Spectral radiance,
k_B = Boltzmann constant
c = speed of light
T = Temperature in Kelvin
λ= wavelength $\left(\frac{c}{v}\right)$

■ **Wein's Displacement law:-**

Product of the wavelength λ_m of most intense radiation emitted by a black body and absolute temperature of the black body is a constant, $\lambda_m T = b = 2.89\times10^{-3}$mK
where b = Wein's constant.



- Area under $e_\lambda - \lambda$ graph = $\int_0^\infty e_\lambda d\lambda = \sigma T^4$

■ **Solar constant:-**

The solar radiant energy received per unit area per unit time by a black surface held at right angles to the sun's rays and placed at the mean distance of the earth (in the absence of atmosphere) is called solar constant. Solar constant is represented by 's'

$$s = \frac{P}{4\pi r^2} = \frac{4\pi R_s^2 \sigma T^4}{4\pi r^2}$$

$$= \left(\frac{R_S}{r}\right)^2 T^4$$

Where,
R_S = Radius of sun
r = average distance between Sun and Earth.
s = 2 cal cm^{-2} min = 1.4 kWm^{-2}
T = Temperature of sun ≈ 5800K

(3) Kinetic Theory of Gases

It relates the macroscopic properties of gases to the microscopic properties of gas molecule.

■ **Basic Postulates of kinetic theory of gases:-**

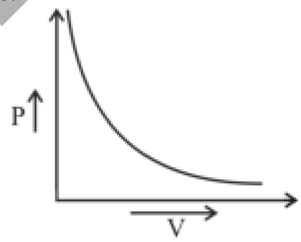
- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical rigid, and perfectly elastic point masses.
- The size is negligible in comparison to inter molecular distance.

■ **Assumptions of kinetic theory of Gases**

- The space occupied by the molecules is much smaller than the volume of the gas.
- There is no force of attraction among the molecules. The pressure of a gas is due to elastic collision of gas molecules with the walls of the container.
- The time of contact of a moving molecule with the walls of the container is negligible as compared to the time interval between two successive collisions.
- Gravitational attraction among the molecules is ineffective due to extremely small masses.

■ **Gas Laws:-**Through experiments, it was established that gases irrespective of their nature obey the following laws-

■ **Boyle's Law**:- According to Boyle's Law, At constant temperature the volume (V) of given mass of a gas is inversely proportional to its pressure (P) i.e.



$$P \propto \frac{1}{V} \Rightarrow PV = \text{constant.}$$

For a given gas,

$$P_1V_1 = P_2V_2$$

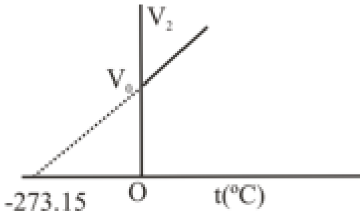
■ **Charles Law:**- According to this law, when pressure (P) of a certain mass of a gas is kept constant the volume 'V' occupied by the gas is directly proportional to the temperature T of the gas i.e when P is constant.

$$V \propto T \Rightarrow \frac{V}{T} = \text{constant.}$$

For a given gas,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

At constant pressure, the volume (V) of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its volume at 0°C for each 1°C rise or fall in temperature.



Volume of the gas at t °C,

$$V_t = V_0\left(1+\frac{t}{273.15}\right)$$

where V_0 is the volume of gas at 0ºC.

■ **Gay Lussac's Law or Renault's Law:-**

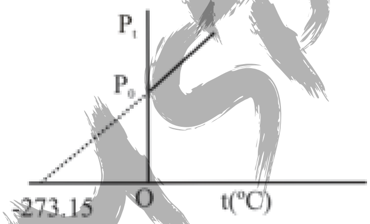
At constant volume, the pressure 'P' of a given mass of gas is directly proportional to its absolute temperature T i.e

$$P \propto T \Rightarrow \frac{P}{T} = \text{constant.}$$

For a given gas,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

At constant volume, the pressure P of a given mass of a gas increases or decreases by $\frac{1}{273.15}$ of its pressure at 0ºC for each 1ºC rise or fall in temperature.



Pressure of the gas at t °C,

$$P_t = P_0\left(1+\frac{t}{273.15}\right)$$

Where P_0 is the volume of gas at 0ºC.

■ **Avogadro's Law:-**

It states that equal volumes of all gases under identical conditions of temperature and pressure contain the same number of molecules this statement is called Avogadro's hypothesis. According to Avogadro's Law $N_1 = N_2$, where N_1 and N_2 are number of molecules in two gases respectively.

- The number of molecules present in 1 g mole of a gas is defined as Avogadro's number (N_A).
  $N_A = 6.023\times10^{23}$ per gram mole.
- At STP or NTP (T = 273K and P = 1atm) 22.4 L of each gas has 6.023×10^{23} molecules.
- One mole of any gas at STP occupies 22.4 *l* of volume.

■ **Dalton's Law of Partial Pressure:-**

It states that the total pressure exerted by a mixture of non-reactive ideal gases is equal to sum of the partial pressures which each would exert, if it alone occupied the same volume at the given temperature.

Hence,

$$P = P_1 + P_2 + P_3 + \ldots\ldots$$

■ **Ideal or Perfect Gas Equation**

Gases which obey all gas laws in all conditions of pressure and temperature are called perfect gases.

■ **Equation of Perfect gas**

$$PV = nRT$$

Where,
P = Pressure
V = Volume
T = Absolute temperature
R = Universal gas constant
n = Number of moles of a gas
$R = 8.31\ J\ mol^{-1}\ K^{-1}$

■ **Graham's law of diffusion:-**

It states that the rates of diffusion of two gases are inversely proportional to the square roots of their densities.

$$r \propto \frac{1}{\sqrt{\rho}}, \text{ which is Graham's law of diffusion}$$

For a given gas

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Where, r_1 & r_2 are the rates of diffusion and ρ_1 & ρ_2 are densities.

■ **Real Gases:-**

Real gases deviate slightly from ideal gas Laws because-

- Real gas molecules attract each other.
- Real gas molecules occupy a finite volume.

■ **Real or Vander Waal's Gas equation :-**

$$\boxed{\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT}$$

Where, a and b are called Vander Waal's constants.

- Dimension of $[a] = [ML^5T^{-2}]$ & $[b] = [L^3]$
- Units of $a = N - m^4 - mole^{-2}$ & $b = m^3$

Note:- Real gases obey ideal gas equation at high temperature and low pressure.

■ **Pressure of a gas:-**

The pressure exerted by the gas is due to the continuous bombardment of gas molecules against the walls of the container.

Pressure due to an ideal gas is given by

$$\boxed{P = \frac{1}{3}\frac{mn}{V}V_{rms}^2 = \frac{1}{3}\rho V_{rms}^2}$$

- For one mole of an ideal gas

$$P = \frac{1}{3}\frac{M}{V}V_{rms}^2$$

Where,

m = mass of one molecule

n = number of molecules

V = Volume of gas,

V_{rms} = root mean square (rms) velocity of the gas molecules

M = Molecular weight of the gas.

- If P is the pressure of the gas and E is the kinetic energy per unit volume then,

$$\boxed{P = \frac{2}{3}E}$$

■ **Kinetic energy of a Gas and speed of Gas molecules:-**

(i) Average kinetic energy of translation per molecules of a gas is given by

$$\boxed{E = \frac{3}{2}kT}$$

Where k = Boltzmann's constant.

(ii) Average kinetic energy of translation per mole of a gas is given by

$$\boxed{E = \frac{3}{2}RT}$$

where R = Universal gas constant.

(iii) For a given gas kinetic energy, $E \propto T$

$$\Rightarrow \boxed{\frac{E_1}{E_2} = \frac{T_1}{T_2}}$$

(iv) Root mean square velocity of the gas molecules is given by

$$\boxed{V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}}$$

(v) For a given gas, $V_{rms} \propto \sqrt{T}$

(vi) For different gases, $V_{rms} \propto \frac{1}{\sqrt{M}}$

(vii) Boltmann's constant, $k = \frac{R}{N_A}$

where R is an ideal gas constant

N_A = Avogadro Number

$k = 1.38 \times 10^{-23}$ J/k

(viii) The average speed of molecules of a gas is given by

$$\boxed{V_{avg} = \sqrt{\frac{8kT}{\pi M}} = \sqrt{\frac{8RT}{\pi M}}}$$

(ix) The most probable speed of molecules V_{mp} of a gas is given by

$$\boxed{V_{mp} = \sqrt{\frac{2KT}{m}} = \sqrt{\frac{2RT}{M}}}$$

$$\boxed{V_{rms} > V_{avg} > V_{mp}}$$

(x) With rise in temperature RMS speed of gas molecules increases as- $V_{rms} \propto \sqrt{T}$

(xi) With the increase in molecular weight rms speed of gas molecule decrease as $V_{rms} \propto \frac{1}{\sqrt{M}}$

(xii) RMS speed of gas molecules is of the order of km/s, e.g. at NTP for hydrogen gas

$$\boxed{V_{rms} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 273}{2 \times 10^3}} = 1.84}$$

(xiii) RMS speed of gas molecules does not depend on the pressure of gas (If temperature remains constant) because $P \propto \rho$ (Boyle's law). If pressure is increased n times, then density will also increase by n times but V_{rms} remains constant.

- **Degree of Freedom:-**

It is defined as the total number of co-ordinates or independent quantities required to describe completely the position and configuration of the system.

It is denoted by f or N.

Mathematically, Degree of freedom of a system is given by

f or N = 3A - R

Where A = Number of particles in the system

R = Number of independent relations among the particles.

- **Degree of freedom (DOF) for different atomic particles are given below.**

(i) Mono-atomic gases:-

Consists only of one atom (i.e. point mass) has 3, DOF (all translatory motion only) e.g. (Neon, argon, helium etc)

(ii) Di-atomic Gases:-

consists of two atoms and shows translatory motion (3 'DOF') and rotatory motion (2 'DOF') = 5 'DOF' e.g. (Hydrogen, Oxygen, Nitrogen etc)

(iii) Triatomic gas:-

•Linear triatomic gas = 7 DOF (3 translational, 3 rotational and 1 vibrational)

•Non-Linear triatomic gas = 6 DOF (3-translational, 3-rotational)

- **Specific heat of a gas:-**

(a) At constant volume, $C_v = \frac{f}{2}R$.

(b) At constant pressure $C_p = \left(\frac{f}{2}+1\right)R$.

(c) Ratio of specific heats of a gas all-constant pressure and at constant volume is given by

$$\boxed{\gamma = 1 + \frac{2}{f} = \frac{C_p}{C_v}}$$

Specific heat of solids, $C = 3R = 24.93$ Jmol^{-1}k^{-1}

Specific heat of water, $C = 9R = C = 74.97$ Jmol^{-1}k^{-1}

Where, $R = 3.31$ Jmol^{-1}k^{-1}

Nature of gas	$u=\frac{f}{2}RT$	$C_V=\frac{du}{dT}=\frac{f}{2}R$	$C_P=C_V+R$	$\gamma=\frac{C_P}{C_V}=1+\frac{2}{f}$
Monoatomic	$\frac{3}{2}RT$	$\frac{3}{2}R$	$\frac{5}{2}R$	1.67
Dia-and linear poly atomic	$\frac{5}{2}RT$	$\frac{5}{2}R$	$\frac{7}{2}R$	1.4
Non-linear polyatomic	3RT	3R	4R	1.33

■ **Law of Equipartition of energy:-**

According to this law, for any dynamical system in thermal equilibrium at temperature T (Kelvin), the total energy (translational, rotational and vibrational) is distributed equally amongst all the degrees of freedom, and energy associated with each molecule per degree of freedom is $\frac{1}{2}kT$, where k is Boltzmann constant.

■ **Mean Free Path:-**

The average distance travelled by a molecule between two successive collisions is called mean free path (λ)
Mean free path is given by

$$\boxed{\lambda = \frac{kT}{\sqrt{2}\pi\sigma^2 P}}$$

where σ = diameter of the molecule
P = pressure of gas
T = Temperature
k = Boltzmann constant
λ = Mean free path.

Note:-

(1) Mean free path, $\lambda \propto T$ and $\lambda \propto \frac{1}{P}$

(2) Use Ideal gas equation (PV = nRT) and $\frac{R}{N_A} = k$

$$\boxed{\lambda = \frac{1}{\sqrt{2}n\pi(\sigma)^2}}$$

■ **Kinetic interpretation of pressure:-**

$$\boxed{P \times V = \frac{1}{3}mNV_{rms}^2}$$

Where,
m = mass of molecule
N = no. of molecule.

■ **Kinetic interpretation of Temperature**

According to kinetic theory of gases, for 1 mole of an ideal gas

$$P = \frac{1}{3}\frac{M}{V}(V_{rms})^2$$

$$PV = \frac{1}{3}M(V_{rms})^2 \qquad \text{(here M = Total mass)}$$

So, $(V_{rms})^2 = \frac{3PV}{M} = \frac{3RT}{M}$ (PV = nRT , n = 1 mole)

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\boxed{V_{rms} \propto \sqrt{T}}$$

The absolute temperature of an ideal gas is directly proportional to mean square velocity of its molecule.

Also $\frac{1}{3}MV_{rms}^2 = RT \quad (n = 1)$

$$\Rightarrow \frac{1}{3}\frac{M}{N_0}V_{rms}^2 = \frac{R}{N_0}T$$

$$\Rightarrow \frac{1}{2}MV_{rms}^2 = \frac{3}{2}kT \ (k = \frac{R}{N_A} = \text{Boltzmann constant})$$

$$\Rightarrow \boxed{(K.E)_{Trans} = \frac{3}{2}kT}$$

$$\Rightarrow \boxed{(K.E)_{Trans} \propto T}$$

Where $\frac{1}{2}MV_{rms}^2$ is average translational energy per molecules.

■ **Critical Temperature:-**

The Maximum temperature below which a gas can be liquefied by pressure alone it is represented by 'T_C'

$$T_C = \frac{8a}{27Rb}$$

Where a & b are Vander Waal constant and R is Gas constant.

■ **Critical Volume:-**

The volume of one mole of a gas liquefied at critical temperature is known as critical volume.

It is represented by V_C.

Critical volume $V_C = 3b$

■ Critical Pressure is the maximum pressure required for the liquefaction of gases at the critical temperature. It is represented by P_C.

$$P_C = \frac{a}{27b^2}$$

Note:- For real gas $\frac{P_C V_C}{R T_C} = \frac{3}{8}$ is called critical coefficient and is same for all gases.

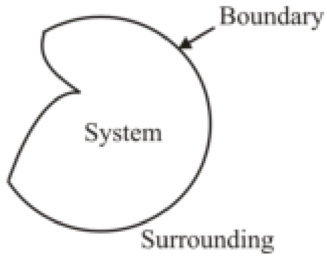
(4) Thermodynamics

It is the branch of physics which deals with conversion of thermal energy to mechanical energy or other forms of energy, and vice versa,

Thermodynamic Terms:-

In order to understand these transformations we need to understand the terms given below-

■ **Thermodynamics System:-** A thermodynamics system defined as finite quantity of matter of a prescribed region of space on which analysis is focused.



Example:- Quantity of steam mixture of vapour and gas in I.C. engine.

■ **Thermodynamic state Variables and Equation of state:-**

•Thermodynamic state variables of a system are the parameters which describe equilibrium states of the system.

•A thermodynamic system can be completely specified by the variables like pressure (P), volume (V), temperature (T), internal energy (u).

•The variables P, V, T and u whose knowledge specifies the state of a thermo-dynamical system, are called as thermodynamic variables /parameters.

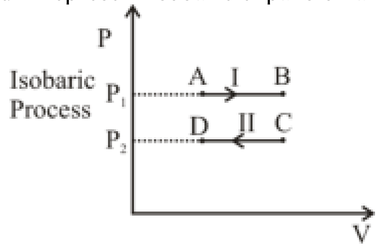
■ **Different Thermo dynamical Processes**

(1) Isobaric Process

(a) It is thermodynamic process in which pressure is kept constant i.e $\frac{V}{T}$ = constant

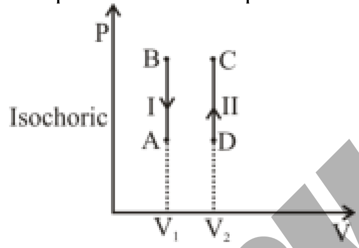
(b) The amount of heat energy transferred is given by $\Delta Q = n\, C_P\, \Delta T$ (n = number of moles)

(c) In the adjoining figure, graphs I and II represent Isobaric expansion and compression respectively.



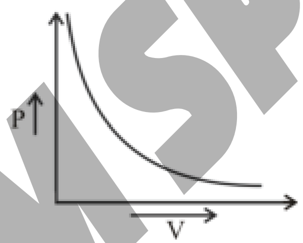
■ **Isochoric (Isometric) process:-**

(a) It is thermodynamic process in which the volume of the system remains constant (P/T = constant).
(b) The amount of heat energy transferred is given by $\Delta Q = n\, C_V\ \Delta T$ (n = number of moles)
(c) In the adjoining figure, graphs I and II represent Isochoric processes.



■ **Isothermal Process**:-

- It is a thermodynamic process in which the temperature of the system remains constant (PV = Constant).
- It is very slow process



■ **Adiabatic Process:-**

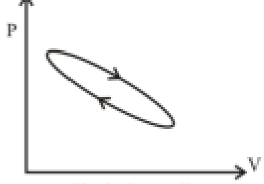
- It is a process taking place in a thermodynamic system for which there is no exchange of heat between the system and its surroundings.
- Adiabatic processes are very fast processes.
- This process follows Poisson's law, according to which $PV^{\gamma} = TV^{\gamma-1} = \frac{T^{\gamma}}{P^{\gamma} - 1} = \text{constant}$.
- From dQ = nCdT, C_{adi} = O as dQ = 0 i.e Molar heat capacity for Adiabatic process is Zero.

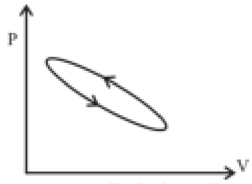
■ **Cyclic process:-**

When a thermodynamic system returns to its initial state after passing through several states, then it is called a cyclic process.
For cyclic process dU = o
or dQ = dW
Efficiency of the cycle is given by

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

work done by the cycle can be computed from area enclosed by cycle on P-V curve.





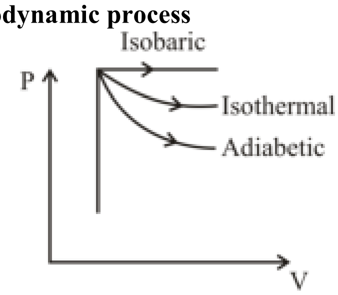
■ **Polytropic Process**

Polytropic is a thermodynamic process that obeys the relation
$PV^n = C$

The polytopic index will take any value between 0 to ∞, depending on the process

Process	Value of n ($PV^n = C$)
Isobaric ($dP = 0$)	$n = 0$
Isothermal ($dT = 0$)	$n = 1$
Polytropic	$n = n$
Adiabatic ($\delta Q = 0$)	$n = \gamma = 1.4$
Isochoric ($dV = 0$)	$n = \infty$

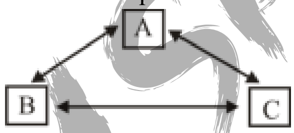
- **Graphical Comparison of thermodynamic process**



From the graph $\text{Workdone}_{\text{adiabetic}} < \text{Workdone}_{\text{Isothermal}} < \text{Workdone}_{\text{Isobaric}}$

■ **Zeroth law of thermodynamics**

When a body 'A' is in thermal equilibrium with another body 'b' and also separately in thermal equilibrium with a body 'C', then body 'B' and 'C' will also be in thermal equilibrium with each other.



- Temperature measurement is based on Zeroth law of thermodynamics

■ **First law of Thermodynamics:-**

(i) First law of thermodynamics is equivalent to law of conservation of energy.

(ii) According to this law, If heat ΔQ is added to a system then it will be used either as change in internal energy ΔU of the system or as work ΔW performed by the system i.e

$\boxed{\delta Q = dU + \delta W}$

$W = \int PdV$

$du = nC_v\Delta T$

■ **Sign Convention:-**

- Heat absorbed by the system $\rightarrow$ positive
- Heat rejected by the system $\rightarrow$ negative
- Work done by the system $\rightarrow$ positive
- Work done on the system $\rightarrow$ negative
- When temperature. rise, internal energy increases. $\rightarrow$ positive
- When temperature. fall, internal energy decreases $\rightarrow$ negative

■ **Applications of the First law of Thermodynamics**

- **For Isobaric Process:-**

$P = \text{constant} \Rightarrow V \propto T$

$Q = mC_P\Delta T,\ dU = mC_V\Delta T$

$W = P(V_2 - V_1) = mC_V\Delta T$

- **For Isothermal Process:-**

Temperature = Constant $\Rightarrow dU = 0$

The first law of thermodynamics

$\delta Q = dU + \delta W$

$\delta Q = 0 + \delta W = \delta W$

$\delta Q = \delta W$

i.e Heat supplied in an isothermal process is used entirely to do work against the external surroundings.

• **For Adiabatic process:-**

$\delta Q = 0$

The first law of thermodynamics

$\delta Q = dU + \delta W = 0$ or $dU = -\delta W$

• **For cyclic process:-**

$dU = 0$

The first law of thermodynamics

$\delta Q = dU + \delta W$

$\delta Q = \delta W$

• **For Melting process:-**

When a substance melts, the change in its volume (dv) is very small therefore, it can be neglected.

$\delta W = pdV = p\times 0 = 0$

First law of thermodynamics

$\delta Q = dU + \delta W$

$mL = dU + 0$ $\quad$ $(\therefore \delta Q = mL)$

Where L = Latent heat.

• **For Boiling Process:-**

Work done in expansion $\delta W = pdV = P(V_2 - V_1)$

$\delta Q = mL$

According to first law of thermodynamics.

$\delta Q = dU + \delta W$

$mL = dU + P(V_2 - V_1)$

Where L = Latent Heat.

■ **Second Law of The Thermodynamics:-**

The second law of thermodynamic can be stated in number of ways, out of which the following two forms are important-

• **Kelvin Planck Statement:-**

It is impossible for an engine operating in a cyclic process to extract heat from a reservoir and convert it completely into work.

• **Clausius Statement:-**

It is impossible for a self acting machine unaided by any external agency to transfer of heat from cold to hot reservoir i.e heat by itself cannot pass from a colder to hotter.

■ **Entropy:-**

Entropy is related to the disorder in the system. Greater the randomness or disorderness, greater the entropy. Change in entropy is given by $ds = \frac{dQ}{T}$

Where dQ = heat supplied to the system

T = absolute Temperature.

Note :-

(i) Entropy is a physical quantity that remains constant during a reversible adiabatic change.

(ii) Entropy of a system increases in an irreversible process.

(iii) Entropy of universe never decreases i.e. $ds \geq 0$.

■ **Clausius Equality & Inequality**

According to Clausius theorem for a cyclic process-

1. $\oint \frac{\partial Q}{T} = 0$ $\quad$ Reversible process
2. $\oint \frac{\partial Q}{T} < 0$ $\quad$ Irreversible process
3. $\oint \frac{\partial Q}{T} > 0$ $\quad$ Impossible

■ **Change in Entropy for solids and liquids**

•When heat is supplied to a solid and its state changes such that temperature remains constant, then change in entropy, $\Delta S = \frac{\Delta Q}{T}$

$dS = \frac{\Delta Q}{T} = \frac{\pm|mL|}{T}$

• Heat absorbed → Positive sign
• Heat rejected → Negative sign.
• When temperature of a substance changes from T_1 to T_2 then

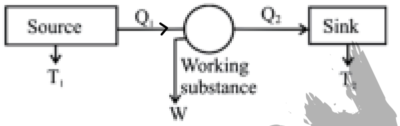
$$dS = \int \frac{dQ}{T} = mC\int_{T_1}^{T_2} \frac{dT}{T} = mC\ell n\left(\frac{T_2}{T_1}\right)$$

$$= 2.303\ mC\ \log_{10}\left(\frac{T_2}{T_1}\right)$$

Where M = mass of the substance
C = specific heat of the substance.

■ **Heat engine:-**

A heat engine is a device which converts heat energy into mechanical energy.
A heat engine essentially consists of three parts.
(1) Source of heat at higher temperature
(2) Working substance.
(3) Sink of heat at lower temperature.



$$\boxed{W = Q_1 - Q_2}$$

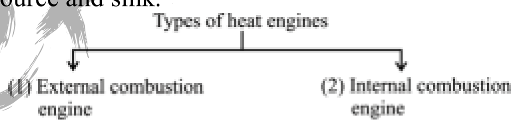
■ **Thermal efficiency of heat engine is given by-**

$$\eta = \frac{\text{Net work done / cycle (w)}}{\text{Total amount of heat absorbed / cycle }(Q_1)}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Q_1 = Heat absorbed from source.
Q_2 = Heat is rejected to the sink.
T_1 and T_2 are Temperatures of source and sink.



(1) External Combustion Engine:-

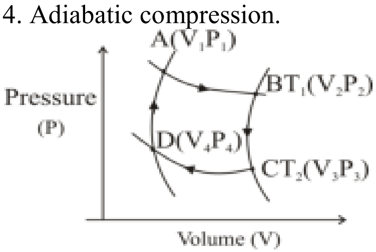
Heat is produced by burning fuel in a chamber outside the main body of the engine e.g. steam engine.
Note:- Thermal efficiency of a steam engine varies 12% to 16%.

(2) Internal Combustion Engine:-

Heat is produced by burning the fuel inside the main body of the engine e.g. Petrol engine, Diesel engine
Note:- The theoretical value of thermal efficiency of an internal combustion engine is about 52%

■ **Carnot's cycle:-**

Carnot devised an ideal cycle of operation for a heat engine called Carnot's cycle.
Carnot cycle consists of the following four stages.
1. Isothermal expansion
2. Adiabatic expansion
3. Isothermal compression
4. Adiabatic compression.



AB = Isothermal expansion (AB)
BC = Adiabatic expansion
CD = Isothermal compression
DA = Adiabatic compression
The network done per cycle by the engine is numerically equal to the area of the loop representing the Carnot's cycle.
From the Carnot cycle for doing the calculations for different processes,

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

Therefore, efficiency of the cycle is

$$\eta = 1 - \frac{T_2}{T_1} \text{ or } 1 - \frac{Q_2}{Q_1}$$

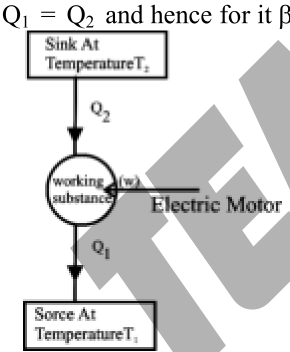
■ **Refrigerator or Heat pump:**

(1) A refrigerator or heat pump is basically a heat engine running in reverse direction. In it working substances takes heat Q_2 from a body at lower temperature T_2 has a net amount of work done on it by an external agent (usually compressor) and gives out a larger amount of heat Q_1 to a hot body at temperature T_1 usually atmosphere.
(2) A refrigerator or heat pump transfers heat from a cold to a hot body at the expense of mechanical energy supplied to it by an external agent.
The working substance here is called Refrigerant and works in cyclic process.
(3) **Efficient of Performance of refrigerator**
is defined as

$$\beta = \frac{\text{Heat extracted from the reservoir at low temperature } (T_2)}{\text{work done to transfer the heat}}$$

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \Rightarrow \frac{1}{\beta} = \frac{Q_1}{Q2} - 1$$

(4) A perfect refrigerator is one which transfers heat from cold to hot body without doing any work i.e w =0
So, that $Q_1 = Q_2$ and hence for it $\beta \to \infty$.



■ **Gibbs function (G)**

$$G = H - TS$$
$$G = U + pV - TS$$

For two equilibrium states at constant pressure and temperature,
$(G_1 - G_2) = (U_1 - U_2) + p(V_1 - V_2) - T(S_1 - S_2)$
$W_{max} = (G_1 - G_2)_{p,T=C}$
$W \geq (G_1 - G_2)_{p,T=C}$
It is applicable for open system.

■ **Helmholtz function (F) :**

$F = U - TS$
For two equilibrium states 1 and at constant (T),
$(F_1 - F_2)_{T=C} = (U_1 - U_2)_{T=C} - T(S_1 - S_2)_{T=C}$
$(W\ T = C)_{max} = (F_1 - F_2)_{T=C}$
$\therefore \quad W_{T=C} \leq (F_1 - F_2)_{T=C}$
It is applicable for closed system.
Note: Gibbs function (G) and Helmholtz function (F) are used in chemical reaction.

■ **Maxwell's thermo dynamical Relations**

It is based on first and second law of thermodynamics in the equilibrium conditions.

There are four-Maxwell's thermo dynamical relations.

(i) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ **(ii)** $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

(iii) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ **(iv)** $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

Two other Maxwell's relation can be given as,

(v) $\left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial S}{\partial V}\right)_P - \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial S}{\partial P}\right)_V = 1$

(vi) $\left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial V}{\partial S}\right)_T - \left(\frac{\partial P}{\partial S}\right)_T \left(\frac{\partial V}{\partial T}\right)_S = 1$

- **Clausius - clapeyron equation.**

The equation which deals with the change in the boiling point or the freezing point of a substance with increase in pressure is called clausius -clapeyron equation.
Mathematically-

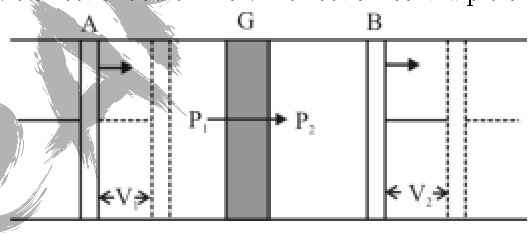
$$\boxed{\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}}$$

where P = Pressure
T = Temperature
L = Latent Heat
V = Volume.

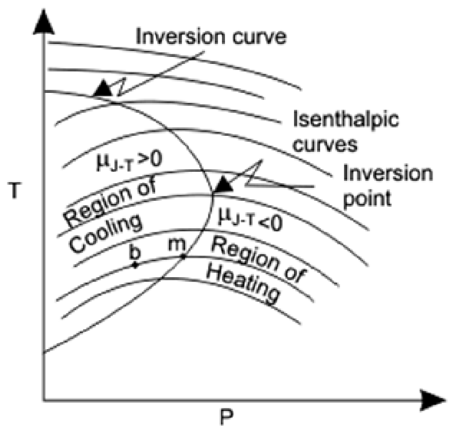
Note:- This equation use the concept of Carnot cycle and work on P-V diagram.

- **Joule- Thomson effect:-**

The phenomena of change of temperature produces when a gas is made to expand adiabatically from a region of high pressure to a region of low pressure is known as Joule Thomson effect.
It is also known as Kelvin - Joule effect or Joule - Kelvin effect or Isenthalpic effect.



AB = Cylindrical non conducting walls
A and B are Piston
$P_1 > P_2$
G = Porous Plug



Note:- (i) In Joule-Kelvin expansion, the enthalpy remains constant.

(ii) For an ideal gas there is no Joule Thomson effect.

(iii) Joule Thomson coefficient is given by

$$\mu \left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left[T\left(\frac{\partial v}{\partial T}\right)_P - V\right]$$

(iv) For Ideal gas $\mu_{JT} = 0$ because gases neither warm not cool upon being expanded at constant

(v) The necessary condition of the Joule-Kelvin expansion in the porous plug experiment is 'U+PV' must remain constant.

(vi) In Joule - Thomson coefficient porous plug expansion for cooling $\frac{\partial P}{\partial T}$ should be negative.

(vii) If 'μ' is positive then the temperature will fall during throttling.

(viii) If μ is negative then the temperature will rise during throttling.

■ **Inversion Temperature:-**

The temperature at which there is no change in temperature of gas during Joule - Thomson expansion of gases is called inversion temperature.

$$T_i = \frac{2a}{Rb}$$

- Bayle's temperature - $T_B = \frac{a}{Rb} \Rightarrow T_i = 2T_B$

For Real Gas -

$$\mu = \frac{1}{C_P}\left[\frac{2a}{RT} - b\right]$$

Case i : If $\mu = 0$ then there will be neither heating nor cooling.

$$0 = \frac{1}{C_P}\left[\frac{2a}{RT} - b\right]$$

$b = \frac{2a}{RT}$ (Where a and b are Vander Waal constant)

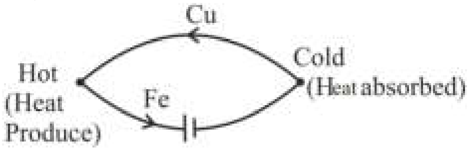
$\boxed{\frac{2a}{Rb} = T_i}$ Inversion temperature.

Case ii : If $\frac{2a}{RT} > b$ then μ = Positive $\rightarrow$ (gas cooling)

Case iii If $\frac{2a}{RT} < b$ then μ = Negative $\rightarrow$ (gas warm up)

■ **Peltier effect :-**

The Phenomenon of evolution or absorption of heat at Junction of thermo couple when current is made to pass through it, is called as Peltier effect.



(i) Heat is evolved or absorb

(ii) Heating or cooling is determined by direction of current.

Fe $\rightarrow$ Cu $\rightarrow$ Heat evolved $\rightarrow$ Hot junction

Cu $\rightarrow$ Fe $\rightarrow$ Heat absorbed $\rightarrow$ Cold Junction

■ **Peltier Coefficient:-**

The Peltier effect may be defined as the amount of heat evolved and absorbed per unit charge flowing across the Junction. It is represented by π

$Q \propto I$ (current)

$\propto t$ (time)

$Q = \pi It$

$$\boxed{\pi = \frac{Q}{It}}$$

Unit of $\pi = JA^{-1} S^{-1}$ or JC^{-1}

■ **Seebeck effect:-**

The Phenomenon of production of current in a thermocouple due to temperature difference between two Junction of thermocouple is called as See back effect.

■ **Seebeck Series:-**

Bi, Ni, Co, Pt, Cu………Fe, As, Sb

The amount of Emf generated in thermocouple depend upon on following terms,

Emf $\propto$ Separation between metal

$\propto$ nature of metals

$\propto$ Temperature difference of Junction

Key points

(1) A P-V diagram for a system is called an indicator diagram. Each dot in a P-V diagram represents a possible state of the system.

(2) A curve drawn between two points on the indicator diagram shows a thermodynamic process obeying some rule.

(3) The super fluidity is the characteristic property of a fluid with zero viscosity which therefore flows without any loss of kinetic energy.

(4) The super fluid component has zero viscosity and zero entropy.

(5) When P and V, the relation Pv^x = Constant. Where $x \neq 1$ or γ the process is called a polytropic one

In this process the molar heat capacity is–

$$C = C_v + \frac{R}{1-x} = \frac{R}{\gamma - 1} + \frac{R}{1-x}$$

(6) Enthalpy-: Four Quantities called "thermodynamic potentials" are useful in the chemical thermodynamics of reactions and non-cyclic processes.

They are internal energy, the enthalpy, he Helmholtz free energy and the Gibbs free energy.

Enthalpy is defined by-

$H = U + Pv$

Where P and V are the pressure and volume and U is internal energy.

(7) If temperature of a body become θ_1 to θ_2 in t time and it becomes θ_2 to θ_3 in next time then use

$$\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0} = \frac{\theta_3 - \theta_0}{\theta_2 - \theta_0} \qquad (\theta_0 = \text{temperature of environment.})$$

(8) Newton's law of cooling can be used to compare the specific heat of the two liquids.

(9) Radiations from sun take 8 min and 20 sec to reach earth.

(11) Suppose temperature of a body decreases θ_1ºC to θ_2ºC in time t_1 and θ_2ºC to θ_3ºC in time t_2 in the same environment.

if $(\theta_1 - \theta_2) \geq (\theta_2 - \theta_3)$ then $t_2 > t_1$

(12) Green glass is a good absorber of red light and a good reflection of green light. Consequently at lower temperature it is a good emitter of red light.

Hence,

Green $\rightleftharpoons$ Red

Also Yellow $\rightleftharpoons$ Blue

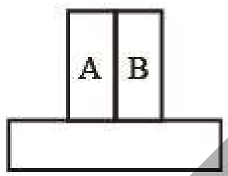
Exemplar Problems
Heat & Thermodynamics

Thermal Properties of Matter

1. **A solid metallic cube having total surface area 24 m2 is uniformly heated. If its temperature is increased by 0ºC, calculate the increase in volume of the cube (Given : $\alpha = 5.0 \times 10^{-4}$ºC^{-1})**
(a) 2.4×10^6 cm^3 (b) 1.2×10^5 cm^3
(c) 6.0×10^4cm^3 (d) 4.8×10^5 cm^3
JEE MAIN 2022

2. **A copper block of mass 5.0 kg is heated to a temperature of 500ºC and is placed on a large ice block. What is the maximum amount of ice that can melt? [Specific heat of copper: 0.39 J g^{-1} and latent heat of fusion of water : 335 J g^{-1}]**
(a) 1.5 kg (b) 5.8 kg
(c) 2.9 kg (d) 3.8 kg
JEE MAIN 2022

3. **A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold both, it will :**



(a) Bend towards the right
(b) Not bend but shrink
(c) Neither bend nor shrink
(d) Bend towards the left
JEE MAIN 2021

4. **A copper rod of 88 cm and an aluminium rod of unknown length have their increase in length independent of increase in temperature. The length of aluminium rod is (aCu = 1.7×10^{-5} K^{-1}, aAl = 2.2×10^{-5} K^{-1})**
(a) 68 cm (b) 6.8 cm
(c) 113.9 cm (d) 88 cm
NEET 2019

5. **The quantities of heat required to raise the temperature of two solid copper spheres of radii r_1 and r_2 ($r_1 = 1.5r_2$) through 1 K are in the ratio**
(a) $\frac{27}{8}$ (b) $\frac{9}{4}$
(c) $\frac{3}{2}$ (d) $\frac{5}{3}$
NEET 2020

6. **A calorimeter of thermal capacity 5 cal/^{0}C contains 10 gm of water at 25^{0}C. If 10 gm of ice at 0^{0}C is dropped into the calorimeter what will be the final temperature?**
[Latent heat of ice = 80 cal/gm]
(a) 0^{0}C (b) $\frac{50}{3}$ ^{0}C
(c) 12.5^{0}C (d) 25^{0}C
UPTGT Science - 2021

7. **At low temperature the lattice specific heat of solids varies as -**
(a) T^3 (b) $\frac{1}{T^3}$
(c) T (d) $\frac{1}{T}$
UPPSC Poly. Lect. 2021 Paper - II
UPPSC GDC - 2021
RPSC School Lect. 26.07.2016

8. **Which of the following is a reversible process?**
(a) Production of heat by the passage of current through an electrical resistance
(b) Conduction of heat from a hot body to a colder body
(c) The changes in the pressure and volume of the working substance take place at infinitely slow rate
(d) Forcing of a real gas or liquid through a valve of porous plug while keeping them insulated with the environment
UP PGT - 2021

9. **Which of the following is path dependent? Where, U = internal Energy, P = Pressure, V = Volume**
(a) U (b) PdV
(c) P (d) V
UP PGT - 2021

10. **Specific heat at constant pressure is C_p and at constant volume is C_v. Why is the ratio C_p/C_v greater than one?**
(a) For constant pressure external work has to be done
(b) C_p includes rotational energy
(c) C_p includes vibrational energy
(d) $C_p/C_v>1$ only for ideal gases, not for real ones
UPTGT Science - 2021

11. **The mechanical equivalent of heat**
(a) has same dimension as work
(b) has same dimension as heat
(c) has same dimension as rate of doing work
(d) Dimensionless
UPTGT Science - 2021

12. **An ideal material for making handle of cooking vessels must have :**
(a) Large heat capacity and small conductivity
(b) Small Heat capacity and large conductivity
(c) Small heat capacity and small conductivity
(d) Large heat capacity and large conductivity
UP TGT Physics 2016

13. **Which one of the following statements is not true?**
(a) Water freezes at 273 K.
(b) Ice melts above $0^{O}C$
(c) Water boils at 100 K.
(d) The normal temperature of healthy man is $37^{O}C$
UP LT Grade Science 2018 (Exam : 29.07.2018)

14. **A 10 KW drilling machine is used to drill a bore in a small aluminium block of mass 8 kg. Find the rise in temperature of the block in 2.5 minutes, assuming 50% power is used up in the heating machine itself or lost to the surroundings. (Specific heat of aluminium = 0.91 $Jg^{\circ}C^{-1}$) :**
(a) $103^{0}C$ (b) $150^{0}C$
(c) $206^{0}C$ (d) $155^{0}C$
UP TGT Physics 2016

15. **Under steady state, the temperature of body–**
(a) increases with time
(b) decreases with time
(c) does not change with time and is same at all points of body
(d) does not change with time but is different at different cross-section of the body
(UPPGT 2011)

16. **Which of the following is not a thermodynamical variable–**
(a) Pressure (b) Volume
(c) Temperature (d) Stress
(UPPGT 2011)

17. **A mercury thermometer has equally spaced markings from 0 to 100. In melting ice the mercury is at the 10th mark, while in boiling water it is t the 80th mark. What is the temperature in degrees centigrade when the mercury is at 45th mark?**
(a) $45^{0}C$ (b) $35^{0}C$
(c) $50^{0}C$ (d) $64^{0}C$
UPTGT Science - 2021

18. **At what temperature of a body, the Fahrenheit and Kelvin thermometer scales give equal numerical values?**
(a) 40° (b) 180°
(c) 435° (d) 574°
UP PGT - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)

19. **A faulty thermometer has its fixed points marked as 5ºC and 95ºC. The temperature of a body as measured by the faulty thermometer is 59ºC. What is the correct temperature of the body on Celsius scale?**
(a) 59ºC (b) 62.1ºC
(c) 56.05ºC (d) 60ºC
UPTGT Science - 2021

20. **On a new scale of temperature called W scale, the freezing and boiling points of water are $39^{0}W$ and $239^{0}W$ respectively. What will be the temperature on the new scale, corresponding to a temperature of $39^{0}C$?**
(a) $200^{0}W$ (b) $139^{0}W$
(c) $78^{0}W$ (d) $177^{0}W$
UP TGT Physics 2016

21. **Which thermometer is preferred for rapidly changing temperature ?**
(a) Liquid in glass
(b) Gas thermometer
(c) Resistance thermometer
(d) Thermo couple thermometer
UP TGT Physics 2016

22. **An object's temperature is raised by $150^{0}C$. The resulting increase in its absolute temperature is :**
(a) 32K (b) 150K
(c) 180K (d) 373K
UP TGT Physics 2016

23. **Two metallic blocks M1 and M2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be : [Assume steady state heat conduction]**

(a) 10K (b) 8 K
(c) 12.5 K (d) 2K
JEE MAIN 2022

24. **A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms–1. What will be the rise in the temperature of the nail if one fourth of energy of the hammer goes into heating the nail? [Specific heat capacity of iron = 0.42 Jg–1 °C–1]**
(a) 675°C (b) 1600°C
(c) 160.7°C (d) 6.75°C
JEE MAIN 2022

25. **A cup coffee cools from 90°C to 80°C in t minutes, when the room temperature is 20°C. The time taken by a similar cup of coffee to cool from 80°C to 60°C at a room temperature same at 20°C is**
(a) $\frac{5}{13}t$ (b) $\frac{13}{10}t$
(c) $\frac{13}{5}t$ (d) $\frac{10}{13}t$
NEET 2021

26. **Transport phenomena is gases represent the transport of certain quantities. Which of the following is not correctly matched?**

Transport Phenomenon	**Quantity**
(a) Viscosity	Mass
(b) Conduction	Momentum
(c) Diffusion	Energy
(d) None of the above	

UPPSC Poly. Lect. 2021 Paper - I

27. **Thermal conductivity of bad conductors is measured by**
(a) Searle's method
(b) Lee's disc method
(c) Callendar and Barne's method
(d) None of the above

UP PGT - 2021

28. **Transmission of heat by molecular collision is**
(a) Conduction (b) Convection
(c) Radiation (d) Scattering

UPTGT Science - 2021

29. **If σ and K are electrical and thermal conductivity of metals, respectively at absolute temperature T. Then correct Wiedemann-Franz law in term of Lorentz number L is**
(a) $\frac{K\sigma}{T}=L$ (b) $\frac{K}{\sigma T}=L$
(c) $\frac{T}{K\sigma}=L$ (d) $\frac{KT}{\sigma}=L$

RPSC School Lect. 08.01.2020

30. **Coefficient of thermal conductivity of a gas is**
(a) independent of the pressure of gas
(b) proportional to the pressure of gas
(c) inversely proportional to the pressure of gas
(d) proportional to the square of the pressure of gas

RPSC School Lect. 26.07.2016

31. **Which of the following has least surface area for a given volume?**
(a) Cube (b) Cone
(c) Sphere (d) Cylinder

UPPSC Poly. Lect. 2021 Paper - I

32. **A spherical body with radius 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, what would be the power radiated?**
(a) 900 W (b) 7200 W
(c) 3600 W (d) 1800 W

UP PGT - 2021

33. **The light from the sun has maximum intensity for the wavelength of 470 nm. Assuming that the surface of the sun emits as a blackbody, calculate the temperature of the sun.**
(Wien's constant = 0.288 cm. °K)
(a) 8000°K (b) 7000°K
(c) 6128°K (d) 5000°K

UP PGT - 2021

34. **A metal ball has a surface area of 200 cm^2 and temperature is 527°C. The ball is kept in a vessel at 27°C. If the emissivity of metal is 0.4, then rate of loss of heat is -**
(Given, $\sigma=5.67\times10^{-8}\,J/m^2secK^4$)
(a) 182 Watt (b) 282 Watt
(c) 242 Watt (d) None of the above

UPPSC Poly. Lect. 2021 Paper - I

35. **Surface temperature of an object is 34°C. What is the wavelength of most intense radiation from this object? Wien's constant is 2.898×10^{-3} mK.**
(a) 6.44μm (b) 7.44μm
(c) 8.44μm (d) 9.44μm

UPPSC Poly. Lect. 2021 Paper - I

36. **Assume that the surface of the Sun emits as a black body at the temperature of 6000 K. If the light from the Sun has maximum intensity for the wavelength 4800 Å, What is the value of Wien's constant?**
(a) 0.288 cm-K (b) 0.360 cm-K
(c) 0.480 cm-K (d) 0.600 cm-K

UPPSC GDC - 2021

37. **The wavelength of the radiations for which the energy is maximum in the spectrum is 490 nm. The effective temperature of the sun is**
(a) 6920 K (b) 6000 K
(c) 5914 K (d) 5820 K

UP LT Grade Science 2018 (Exam : 29.07.2018)

38. **Wien's displacement law is :**
(a) $\lambda_m T=$ constant (b) $\lambda_m^{1/3}=$ constant
(c) $\lambda_m T^{4/3}=$ constant (d) $\lambda_m^2 T=$ constant

UPPCS Pre 2010
UPPGT 2011

39. **The colour of a star predicts:**
(a) its weight (b) its size
(c) its temperature (d) its distance

UPPCS (Pre) 2006, 2003
(UPPGT 2013)
UPTGT 2013

40. **According to Rayleigh-Jeans law, the energy of wavelength λ radiated by a black-body at temperature T is proportional to**
(a) $\frac{T}{\lambda^3}$ (b) $\frac{T}{\lambda^4}$
(c) $\frac{T}{\lambda^5}$ (d) $\frac{T}{\lambda^2}$

(UPPGT 2005)

41. **A body at 300^{0}C radiates energy at the rate of 10^5 Watt/m^2. If the sun radiates energy at the rate of 10^9 Watt/m^2, its temperature is**
(a) 3517^{0}C (b) 5280^{0}C
(c) 5457^{0}C (d) 5730^{0}C

UPPSC Ashram Paddhati PGT - 2021

42. Two identical spheres A and E are at temperature 7^0C and 17^0C respectively. In what ratio will they emit thermal radiation?

(a) 1 : 1.143 (b) 7 : 17
(c) 49 : 289 (d) $7^4 : 17^4$

UPTGT Science - 2021

43. For emission of maximum radiation, the ratio of wavelengths of two stars A and B is 4:5. If temperature of star B is 6800 K, then temperature of star A will be :

(a) 8500 K (b) 7500 K
(c) 7200 K (d) 8550 K

(UPPGT 2004)

44. The Rayleigh scattering is proportional to :

(a) $\frac{1}{\lambda^4}$ (b) $\frac{1}{\lambda^3}$
(c) $\frac{1}{\lambda^2}$ (d) $\frac{1}{\lambda}$

(UPPGT 2013, 2009)

45. A body cools from 70^0 C to 50^0 C in 4 minutes, in a room at temperature 30^0C. The time taken by the body to cool from 50^0 C to 40^0 C will be

(a) 4 minutes (b) 8 minutes
(c) 6 minutes (d) 2 minutes

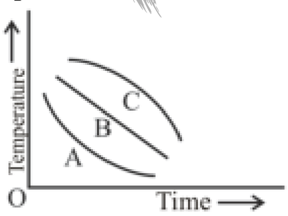
UPPSC Ashram Paddhati PGT - 2021

46. Newton's law of cooling is a special case of

(a) Stefan's law
(b) Kirchhoff's law
(c) Planck's law
(d) Wien's displacement law

UP PGT - 2021, 2013
UPPCS (Pre) 2007, 2004

47. A block of steel heated to 100°C is left in a room to cool. Which of the curves shown in figure, represents the correct behaviour?



(a) A (b) B
(c) C (d) None

UP PGT - 2021, 2000

48. Newton's Law of cooling is an approximate form of

(a) Wien displacement law
(b) Stefan's law
(c) Kirchoff's law
(d) Jeans's law

UPTGT Science - 2021

49. A body cools from 61^0C to 59^0C in 4 minutes and from 51^0C to 49^0C 6 minutes, then temperature of the room is :

(a) 30^0C (b) 25^0C
(c) 35^0C (d) 20^0C

UP TGT Physics 2016

50. The ratio of specific heats $\left(\frac{C_P}{C_V}\right)$ in terms of degree of freedom (f) is given by:

(a) $\left(1+\frac{f}{3}\right)$ (b) $\left(1+\frac{2}{f}\right)$
(c) $\left(1+\frac{f}{2}\right)$ (d) $\left(1+\frac{1}{f}\right)$

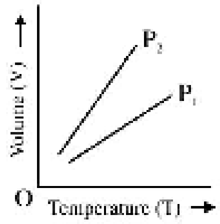
JEE MAIN 2022

51. A mixture of hydrogen and oxygen has volume 2000 cm^3, temperature 300 K, pressure 100 kPa and mass 0.76 g The ratio of number of moles of hydrogen to number of moles of oxygen in the mixture will be :

(a) $\frac{1}{3}$ (b) $\frac{3}{1}$ (c) $\frac{1}{16}$ (d) $\frac{16}{1}$

JEE MAIN 2022

52. For a perfect gas, two pressures P_1 and P_2 are shown in figure. The graph shows:



(a) $P_1 > P_2$
(b) $P_1 < P_2$
(c) $P_1 = P_2$
(d) Insufficient data to draw any conclusion

JEE MAIN 2022

53. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal. W_2. if the process is purely adiabatic and W_3 if the process is purely isobaric. Then, choose the coned option

(a) $W_1 < W_2 < W_3$ (b) $W_2 < W_3 < W_1$
(c) $W_3 < W_1 < W_2$ (d) $W_2 < W_1 < W_3$

JEE MAIN 2022

54. What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T? (k_B is Boltzmann constant)

(a) $\frac{1}{2}k_BT$ (b) $\frac{2}{3}k_BT$
(c) $\frac{3}{2}k_BT$ (d) k_BT

JEE MAIN 2021

55. **The volume occupied by the molecules contained in 4.5 kg water at STP, if the intermolecular forces vanish away is**

(a) $5.6\ m^3$ (b) $5.6\times10^{-6} m^3$

(c) $5.6\times10^{3} m^3$ (d) $5.6\times10^{-3} m^3$

NEET 2022

56. **The mean free path for a gas, with molecular diameter d and number density n can be expressed as :**

(a) $\frac{1}{\sqrt{2}n\pi d^2}$ (b) $\frac{1}{\sqrt{2}n^2\pi d^2}$

(c) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$ (d) $\frac{1}{\sqrt{2}n\pi d}$

NEET 2020

57. **The average thermal energy for a mono-atomic gas is : (k_B is Boltzmann constant and T, absolute temperature)**

(a) $\frac{3}{2}k_BT$ (b) $\frac{5}{2}k_BT$

(c) $\frac{7}{2}k_BT$ (d) $\frac{1}{2}k_BT$

NEET 2020

58. **Increase in temperature of a gas filled in a container would lead to**

(a) decrease in intermolecular distance
(b) increase in its mass
(c) increase in its kinetic energy
(d) decrease in its pressure

NEET 2019

59. **The gas law $\frac{PV}{T}$ = constant is true for :**

(a) Isothermal changes only
(b) Adiabatic changes only
(c) Both Isothermal and adiabatic changes
(d) None of these

UKPSC Lecturer (Mains) 2020

60. **The Kinetic energy per unit volume of a perfect gas is equal to**
Where P is pressure.

(a) $\frac{2}{3}P$ (b) $\frac{3}{2}P$

(c) $\frac{P}{3}$ (d) $\frac{1}{2}P$

UP PGT - 2021

61. **At constant pressure the temperature of gas T, at which root – mean – square velocity is**

(a) 819° K (b) 900° K

(c) 980° K (d) 819° C

UP PGT - 2021

62. **The mean Kinetic energy of one mole of gas per degree of freedom (on the basis of Kinetic theory of gases) is**

(a) $\frac{1}{2}KT$ (b) $\frac{3}{2}KT$

(c) $\frac{3}{2}RT$ (d) $\frac{1}{2}RT$

UP PGT - 2021

63. **The ratio of most probable speed and average speed of a gas enclosed in a vessels is**

(a) $\frac{\sqrt{\pi}}{4}$ (b) 1

(c) $\frac{\sqrt{\pi}}{2}$ (d) $\frac{2}{\sqrt{\pi}}$

UP PGT - 2021

64. **For a van der Waals gas the inversion temperature at low pressure is**

(a) $\frac{27a}{bR}$ (b) $\frac{a}{8Rb}$

(c) $\frac{ab}{8R}$ (d) $\frac{2a}{Rb}$

UP PGT - 2021

65. **The mean square speed of the molecules of a gas at absolute temperature T is proportional to :**

(a) $\sqrt{T}$ (b) T

(c) T^2 (d) $\frac{1}{T}$

UKPSC Lecturer (Mains) 2020

66. **For van der Waal's gases, the correct relation is**

(a) $C_p - C_v = R$ (b) $C_p - C_v < R$

(c) $C_p - C_v > R$ (d) $C_p - C_v = 0$

UPPSC Ashram Paddhati PGT - 2021

67. **According to law of equipartition of energy, the average kinetic energy per molecule associated with each degree of freedom is :**

(a) $\frac{3}{2}kT$ (b) $\frac{5}{2}kT$

(c) $\frac{1}{2}kT$ (d) kT

UKPSC Lecturer (Mains) 2020

68. **The relation between Pressure (P) of the gas and its total translational kinetic Energy (E) of the molecules per unit volume is:**

(a) $P=\frac{2}{3}E$ (b) $P=\frac{3}{2}E$

(c) $P=\frac{3}{4}E$ (d) $P=\frac{4}{3}E$

UKPSC Lecturer (Mains) 2020
UPPCS Pre 2010

69. **According to Maxwell, the value of mean free path is given by:**

(a) $\lambda = \frac{1}{\sqrt{\pi\sigma^2 n}}$ (b) $\lambda = \frac{1}{\sqrt{2\pi\sigma^2}}$

(c) $\lambda = \frac{1}{\sqrt{2(\pi\sigma^2 n)}}$ (d) $\lambda = \frac{1}{\pi\sigma^2 n}$

UKPSC Lecturer (Mains) 2020

70. **A gas of molecules, each having mass m, is at rest in thermal equilibrium at the absolute temperature T. If v_x and v_y are cartesian components of velocity of a molecule. what is the mean value of $(v_x + bv_y)^2$, where b, is a constant ?**

(a) $\frac{3kT}{m}(1+b^2)$ (b) $\frac{8kT}{\pi m}(1+b)^2$

(c) $\frac{kT}{m}(1+b)^2$ (d) $\frac{3kT}{m}$

RPSC School Lect. 08.01.2020

71. **If the root mean square velocity of hydrogen molecules at NTP is1.84km/sec, what will it be for oxygen molecules at NTP ?**

(a) 0.42 km/sec (b) 0.40 km/sec

(c) 0.38 km/sec (d) 0.46 km/sec

UP PGT Physics 2016 (Exam : 02.02.2019)

72. **The rms speed of oxygen molecule at a certain temperature is v. If the temperature is doubled and oxygen gas dissociates into atomic oxygen, the rms speed will become**

(a) v (b) $\frac{v}{2}$

(c) 2v (d) $\sqrt{2}v$

RPSC School Lect. 26.07.2016

73. **If the pressure in a clossed vessel is reduced by allowing some gas to escape, then the mean free path of the gas molecules in the vessel**

(a) decreases

(b) increases

(c) remains the same

(d) nothing can be predicted

RPSC School Lect. 26.07.2016

74. **Boyle's temperature T_B of a Vander Waals gas is given by :**

(a) $T_B = \frac{a}{bR}$ (b) $T_B = \frac{8a}{27Rb}$

(c) $T_B = \frac{apV}{bR}$ (d) $T_B = \frac{aR}{bpV}$

UPPCS Pre 2010

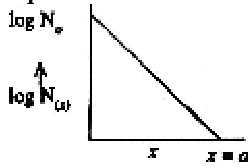
75. **The absolute temperature of a gas is increased three times. The r.m.s. velocity of its molecules will increase by a factor :**

(a) 3 (b) 9

(c) $\frac{1}{3}$ (d) $\sqrt{3}$

UPPCS Pre 2008

76. **In the figure N(x) is the number of particles whose mean free path exceeds a certain value of x. Mean free path λ will be :**



(a) $\log N_0/a$ (b) a

(c) 1/a (d) $\log N_0 - a$

UPPCS PRE 1996

77. **Critical temperature of carbondioxide gas is 31°C. If gas obeys Vander Waals equation of state. the Boyle temperature will be :**

(a) 1026°C (b) 753°C

(c) 300°C (d) 500°C

(UPPGT 2004)

78. **Mean free path of a molecule of a gas at pressure P and temperature T is 2 ×10^{-5}cm. At pressure *P*×10^{-6} N/m^2 and temperature T, the mean free path of the same molecule of gas will be :**

(a) 2 cm (b) 20 cm

(c) 3 cm (d) 10 cm

(UPPGT 2004)

79. **A vessel contains a mixture of different types of gases. Which of the following statements is correct?**

(a) On the average the heavier molecules have higher speed

(b) The average speed of different molecules is the same

(c) The average translational energy of different types of molecules is the same.

(d) On the average the heavier molecules have higher translational energy.

(UPPGT 2003)

80. **At a given temperature, mean free path of a gas is**

(a) Directly proportional to the pressure

(b) Independent of pressure

(c) Inversely proportional to the pressure

(d) None of the above

UPPSC Ashram Paddhati PGT - 2021

81. **At what temperature the oxygen molecules will have same r.m.s. velocity as the hydrogen molecule at 150^{0}C (molecular weight of O_2 = 32 and H_2 = 2)**

(a) 5500^{0}C (b) 5800^{0}C

(c) 6495^{0}C (d) 6400^{0}C

UPPCS Pre 2013

(UPPGT 2002)

82. **The mean free path of air molecules at N.T.P. is of the order:**
(a) 10^{-6}cm (b) 10cm
(c) 10^{-8}cm (d) 10^{-2}cm
UPPCS Pre 1995

83. **If $\bar{c}$ C and α represent the mean velocity, root mean square velocity and the most probable velocity respectively at a temperature T, then,**
(a) $\alpha > \bar{c} > C$ (b) $\alpha < C < \bar{c}$
(c) $\alpha > C > \bar{c}$ (d) $\alpha < \bar{c} < C$
UPPCS Pre 2006, 2008

84. **The diffusivity D and the mobility μ are related to each other by a relation,**
(Where symbols have their usual meanings)
(a) $\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{K_B T}{e}$ (b) $\frac{D_p}{\mu_p} \times \frac{D_n}{\mu_n} = \frac{K_B T}{e}$
(c) $\frac{D_p}{\mu_p} + \frac{D_n}{\mu_n} = \frac{K_B T}{e}$ (d) $\frac{D_p}{\mu_p} - \frac{D_n}{\mu_n} = \frac{K_B T}{e}$

85. **For hydrogen the value of Boyle temperature is:**
(a) 50^0 C (b) -164^0 C
(c) -102^0 C (d) -250^0 C
UPPCS Pre 2001

86. **In a gas the expression for the average speed of molecule is given by:**
(The terms have their usual meanings)
(a) $\sqrt{\frac{3KT}{m}}$ (b) $\sqrt{\frac{2KT}{m}}$
(c) $\sqrt{\frac{8KT}{\pi m}}$ (d) $\sqrt{\frac{3KT}{2m}}$
UPPCS Pre 2010, 2005, 2001

87. **The RMS speed of a molecule of hydrogen at STP will be [given ρ = 0.09 kg/m^3]**
(a) 4.84×10^3 m/s (b) 3.84×10^3 m/s
(c) 2.84×10^3 m/s (d) 1.84×10^3 m/s
UPPCS Pre 2007

88. **Which of the following physical quantities has same value for molecules of all gases at a given temperature?**
(a) Mass (b) Speed
(c) Momentum (d) Kinetic energy
UPPCS Pre 2005

89. **The rms speed of hydrogen molecules at room temperature is 2000 m/s. The rms speed of oxygen molecules at the same temperature will be:**
(a) 1000 m/s (b) 500 m/s
(c) 125 m/s (d) 31 m/s
UPPCS Pre 2005

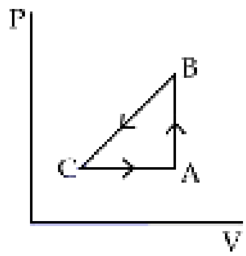
90. **On increasing the temperature of gas, the mean free path of molecules :**
(a) increases (b) decreases
(c) remains unchanged (d) nothing can be said
UPPCS Pre 2005

91. **The relation between the pressure P and translational kinetic energy E of an ideal gas, is given by :**
(a) $E = \frac{1}{2}PV$ (b) $E = \frac{3}{2}PV$
(c) $E = \frac{1}{2}\frac{PV}{T}$ (d) $E = \frac{3}{2}P^2V$
UPPCS Pre 2009

92. **The mean free path 'λ' of a gas molecule, as given by Maxwell, is related to its radius 'a' as :**
(a) $\lambda = \frac{1}{\sqrt{2}.4\pi a^2 n}$ (b) $\lambda = \frac{1}{\sqrt{2}.\pi a^2 n}$
(c) $\lambda = \frac{1}{\sqrt{2}.2\pi^2 a^2 n}$ (d) $\lambda = \frac{1}{\sqrt{2}.2\pi a^2 n}$
UPPCS Pre 1994

93. **In a Maxwellian gas, if v_{rms} is the root mean square velocity, then the most probable velocity v_p is :**
(a) $\sqrt{\frac{1}{3}}.v_{rms}$ (b) $\sqrt{\frac{2}{3}}.v_{rms}$
(c) $\sqrt{\frac{3}{2}}.v_{rms}$ (d) none of these
UPPCS Pre 1994

94. **A sample of an ideal gas is taken through the cyclic process ABCA as shown in figure. It absorbs, 40 J of heat during the part AB, no heat during BC and rejects 60J of heat during CA. A work 50J is done on the gas during the part BC. The internal energy of the gas at A is 1560J. The work done by the gas during the part CA is:**



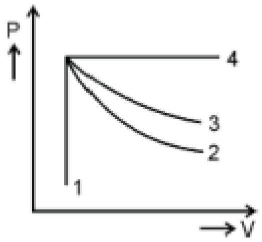
(a) 20 J (b) 30 J
(c) –30J (d) –60J
JEE MAIN 2022

95. **A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency ?**
(a) Increases by 18°C (b) Increase by 200°C
(c) Increase by 120°C (d) Increase by 73°
JEE MAIN 2022

96. **In thermodynamics, heat and work are :**
(a) Path functions
(b) Intensive thermodynamic state variables
(c) Extensive thermodynamic state variables
(d) Point functions

JEE MAIN 2021

97. **An ideal gas undergoes four different processes from the same initial state as shown in the figure below. Those processes are adiabatic, isothermal, isobaric and isochoric. The curve which represents the adiabatic process among 1, 2, 3 and 4 is**



(a) 4 (b) 1
(c) 2 (d) 3

NEET 2022

98. **A cylinder contains hydrogen gas at pressure of 249 kPa and temperature 27°C.**
Its density is : (R = 8.3 J mol^{-1} K^{-1})
(a) 0.2 kg/m^3 (b) 0.1 kg/m^3
(c) 0.02 kg/m^3 (d) 0.5 kg/m^3

NEET 2020

99. **A sample of 0.1 g of water at 100°C and normal pressure (1.013 × 105 N m^{-2}) requires 54 cal of heat energy to convert to steam at 100°C. If the volume of the steam produced is 167.1 cc, the change in internal energy of the sample, is**
(a) 1.4.3 J (b) 208.7 J
(c) 42.2 J (d) 84.5 J

NEET 2018

100. **Which of the following states Clausius theorem for a reversible Carnot cycle?**
(a) $\oint \frac{dQ}{T} > 0$ (b) $\oint \frac{dQ}{T} < 0$
(c) $\oint \frac{dQ}{T} = 0$ (d) $\oint \frac{dQ}{T} = \text{constant}$

UPPSC Poly. Lect. 2021 Paper - I
(UPPGT 2013)
UPPCS Pre 2008

101. **The internal energy of a system changes in -**
(a) Isothermal change (b) Adiabatic change
(c) Free expansion (d) Cyclic process

UPPSC Poly. Lect. 2021 Paper - I

102. **20 grams of ice at 0°C melts to water at 0°C. The entropy change in this process, is (Latent heat of water is 80 cal/gm)**
(a) 24.5 J/°K (b) 30.2 J/°K
(c) 35.7 J/°K (d) 49.2 J/°K

UP PGT - 2021

103. **Entropy of a system in reversible cycle -**
(a) Increases (b) Decreases
(c) Remain unchanged (d) Always remain zero

UPPSC Poly. Lect. 2021 Paper - I

104. **Super fluid has**
(a) Zero Entropy (b) Infinite Entropy
(c) Positive Entropy (d) Negative Entropy

UPPSC GDC - 2021

105. **An adiabatic process is also known as**
(a) isothermal (b) isobaric
(c) isentropic (d) isochoric

UP PGT - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)

106. **The concept of internal energy was first introduced by**
(a) First law of thermodynamics
(b) Second law of thermodynamics
(c) Stefan's law
(d) Wien's law

UPPSC Ashram Paddhati PGT - 2021
UPPCS (Pre) 2009, 2003

107. **The first law of thermodynamics for an adiabatic process is :**
(a) dU = δW (b) dU = 0
(c) dU = – δW (d) dU = –δQ + 2δW

UPTGT Science - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)
UP LT Grade Science 2018 (Exam : 29.07.2018)
UP TGT Physics 2016

108. **The first law of thermodynamics is equivalent to**
(a) Conservation of temperature
(b) Conservation of heat
(c) Conservation of energy
(d) Conservation of mass

UPPSC GDC - 2021
UPTGT Science - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)

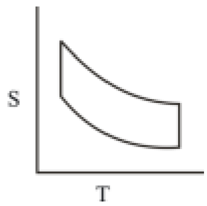
109. **A system absorbs 45 J of heat and in the process it does 11J of work. The system follows a different thermodynamic path to the same initial and final states and does 16 J work, the heat transferred in the process is**
(a) 45 J (b) 18 J
(c) 29 J (d) 50 J

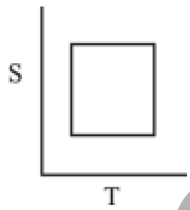
RPSC School Lect. 08.01.2020

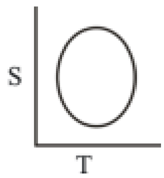
110. **A gas is kept in a rigid container. Heat equal to 500 J is supplied to the gas. Change in the internal energy of the gas is:**
(a) 50 J (b) 500 J
(c) 23.8 J (d) 11.9 J
UKPSC Lecturer (Mains) 2020

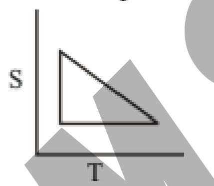
111. **The thermodynamical probability of a system in equilibrium should be**
(a) Zero
(b) Minimum
(c) Maximum
(d) Can be either maximum or minimum
RPSC School Lect. 26.07.2016

112. **A Carnot cycle is represented on an entropy-temperature diagram by which one of the following figures?**

(a) 

(b) 

(c) 

(d) 

UP PGT - 2021

113. **The efficiency of a Carnot engine is 25%, if the temperature of its source is 327°C, the temperature of the sink must be**
(a) 82°C (b) 127°C
(c) 177°C (d) 227°C
UP PGT - 2021

114. **The thermodynamic scale of temperature is based on -**
(a) Expansion of gas
(b) Change in pressure
(c) Carnot cycle
(d) Change in vapour pressure
UPPSC Poly. Lect. 2021 Paper - I (UPPGT 2011)

115. **The efficiency of a Carnot engine operating between 137^0 C and 17^0 C is**
(a) 27.6% (b) 29.3%
(c) 33.3% (d) 41.2%
UPPSC Ashram Paddhati PGT - 2021

116. **In a carnot engine, the temperature of sink is 27°C. If efficiency of engine is 40%, then the temperature of source will be -**
(a) 2770°C (b) 227 °C
(c) 453 °C (d) 108 °C
UPPSC Poly. Lect. 2021 Paper - I

117. **A system in contact with a heat reservoir can most suitably be represented by**
(a) Canonical ensemble
(b) Micro canonical ensemble
(c) Grand canonical ensemble
(d) Coulomb ensemble
UPPSC GDC - 2021

118. **A Carnot engine operates between 27^0C and 327 ^{0}C. If it does a net work of 800 joule, how much heat provided to the engine from source is :**
(a) 8800 joule (b) 1600 joule
(c) 873 joule (d) 800 joule
UP LT Grade Science 2018 (Exam : 29.07.2018)

119. **A heat engine is operating between termperatures 500 K and 400 K. What is the efficiency of the engine?**
(a) 0.20 (b) 0.80
(c) 1.25 (d) 0.50
UP LT Grade Science 2018 (Exam : 29.07.2018)

120. **Which of the following is NOT a Maxwell's thermodynamic relation?**
(a) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$ (b) $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$
(c) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ (d) $\left(\frac{\partial P}{\partial V}\right)_T = \left(\frac{\partial S}{\partial T}\right)_v$
UPPSC Ashram Paddhati PGT - 2021

121. **Which of the following is not Maxwell's thermodynamic relation?**
(a) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ (b) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$
(c) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ (d) $\left(\frac{\partial V}{\partial P}\right)_T = -\left(\frac{\partial S}{\partial T}\right)_P$
UPPSC Poly. Lect. 2021 Paper- I

122. **Gibbs paradox in Statistical Mechanics is related to the additive property of -**
(a) Energy (b) Entropy
(c) Momentum (d) Temperature
UPPSC Poly. Lect. 2021 Paper- I

123. **Which of the following is not a correct relation?**
(a) $dU = TdS - PdV$ (b) $dG = VdP - SdT$
(c) $dF = -PdV - SdT$ (d) $dH = TdS - VdP$
UPPSC Poly. Lect. 2021 Paper- I

124. **The first derivative of Gibbs function is continuous in**
(a) First order phase transition
(b) second order phase transition
(c) Both in first order and second order phase transition
(d) Neither in first order nor in second order phase transition
UPPSC GDC - 2021

125. **Choose the wrong Maxwell's relations for thermodynamical variables**

(a) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial P}{\partial T}\right)_V$

(c) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ (d) $-\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$

RPSC School Lect. 08.01.2020

126. **Choose wrong Maxwell's thermodynamic equation**

(a) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

(c) $\left(\frac{\partial T}{\partial P}\right)_S = -\left(\frac{\partial V}{\partial S}\right)_P$ (d) $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

RPSC School Lect. 26.07.2016

127. **Which one of the following is not a Maxwell's thermodynamic relation?**

(a) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

(c) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ (d) $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial T}{\partial V}\right)_P$

UP LT Grade Science 2018 (Exam : 29.07.2018)

128. **The Maxwell's relation in thermodynamics that can be derived from enthalpy is :**

(a) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ (b) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$

(c) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ (d) $\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$

UPPCS Pre 2009

129. **Which one is correct relation?**

(a) $dU = TdV - PdS$ (b) $dF = -PdV + SdT$

(c) $dH = VdT + SdV$ (d) $dG = VdP - SdT$

(UPPGT 2009)

130. **Gibbs function 'G' and Helmholtz function 'H' may be expressed as :**

(Symbols have their usual meanings)

(a) $G = \frac{PV}{H}$ (b) $G = \frac{H}{PV}$

(c) $G = H - PV$ (d) $G = H + PV$

UPPCS Pre 2009

131. **The thermo dynamical relation expressing the change of temperature with volume at constant entropy is :**

where symbols have their usual meaning.

(a) $\left(\frac{dT}{dV}\right)_S = -T\left(\frac{dP}{dQ}\right)_V$ (b) $\left(\frac{dT}{dV}\right)_S = T\left(\frac{dP}{dQ}\right)_V$

(c) $\left(\frac{dT}{dV}\right)_S = -V\left(\frac{dP}{dQ}\right)_T$ (d) $\left(\frac{dT}{dV}\right)_S = T\left(\frac{dP}{dQ}\right)_T$

UPPCS Pre 1995

132. **The enthalpy of unit mass for any system is :**

(a) $H = U + PV + S$ (b) $H = U + PV - S$

(c) $H = U + PV$ (d) None of these

UPPCS Pre 1994

133. **Which of the following is not a Maxwell's thermodynamic relation?**

(a) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ (b) $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$

(c) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ (d) $\left(\frac{\partial S}{\partial P}\right)_V = -\left(\frac{\partial V}{\partial T}\right)_V$

UPPCS Pre 2010

134. **In Joule-Kelvin expansion, the quantity remaining constant is**

(a) Internal energy (b) Entropy

(c) Enthalpy (d) Work

UPPSC Ashram Paddhati PGT - 2021

UP PGT Physics 2016 (Exam : 02.02.2019)

135. **The enthalpy of a gas before and after Joule-Thompson expansion are 80 cal and 120 cal respectively. If enthalpy of emergent liquid is 40 cal, then what fraction of gas will liquefy ?**

(a) 25% (b) 43%

(c) 50% (d) 63%

RPSC School Lect. 08.01.2020

136. **The enthalpies of certain gas before and after suffering Joule-Thomson expansion through a throttle valve are 77.2 cals and 106.8 cals respectively. The fraction of the gas liquefied, if the enthalpy of emerging liquid be 55.4 calories, is**

(a) 0.32 (b) 0.44

(c) 0.57 (d) 0.71

RPSC School Lect. 26.07.2016

137. **For an ideal gas Joule-Thomson coefficient is :**

(a) (+)ve

(b) (-) ve

(c) Zero

(d) Dependent upon molecular weight

TGT 2009

138. **In a Vander waals gas Joule-Thomson expansion produces heating only for :**

(a) $T > \frac{2a}{Rb}$ (b) $T < \frac{2a}{Rb}$

(c) $T = \frac{2a}{Rb}$ (d) $T = \frac{a}{Rb}$

UPPCS Pre 1996, 2009

139. **In Joule Thomson expansion, the inversion temperature is the temperature below which gas :**

(a) shows cooling (b) obeys Boyle's law

(c) shows heating (d) none of the above

UPPCS Pre 2010

140. **In Joule–Thormon porus plug expansion for cooling $\frac{\Delta P}{\Delta T}$ should be :**

(a) negative (b) positive

(c) zero (d) constant

UPPCS (Pre) 2004

141. If a current is allowed to pass through a circuit consisting of two dissimilar metals, there is either evolution or absorption of heat at the junctions; depending upon the direction of the current. The effect is known as–

(a) Seebeck effect (b) Joule's effect
(c) Peltier's effect (d) Thomson's effect

UPPCS (Pre) 1998

142. The value of Joule-Thomson coefficient during an isothermal expansion of an ideal gas is :

(a) > 1 (b) < 1
(c) 0 (d) none of these

(UPPGT 2013)

143. The Clausius-Clapeyron equation is :

(a) $\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$ (b) $\frac{dP}{dT} = \frac{T}{L(V_2 - V_1)}$

(c) $\frac{dP}{dT} = \frac{L^2}{T(V_2 - V_1)}$ (d) $\frac{dP}{dT} = \frac{L}{LT(V_2 - V_1)}$

UP LT Grade Science 2018 (Exam : 29.07.2018)

144. If C_1 and C_2 represent the specific heat of a liquid and its saturated vapour and L is its latent heat then $C_2–C_1$ is given by :

(a) $\frac{dL}{dT} - \frac{L}{T}$ (b) $\frac{dL}{dT} + \frac{L}{T}$

(c) $\frac{dL}{dT} + \left(\frac{L}{T}\right)^2$ (d) $\frac{dL}{dT} - \left(\frac{L}{T}\right)^2$

UPPCS Pre 2006, 2004, 2001

145. The clausius-clapeyron equation can be derived with which of the following relations? where P, V, T and S are pressure, volume, temperature and entropy respectively.

(a) $\left(\frac{\delta S}{\delta V}\right)_T = \left(\frac{\delta P}{\delta T}\right)_V$ (b) $\left(\frac{\delta S}{\delta P}\right)_T = -\left(\frac{\delta V}{\delta T}\right)_P$

(c) $\left(\frac{\delta T}{\delta P}\right)_S = \left(\frac{\delta V}{\delta S}\right)_P$ (d) $\left(\frac{\delta P}{\delta S}\right)_V = -\left(\frac{\delta T}{\delta V}\right)_S$

(UPPGT 2004)

146. In the 'throttling' process through a porous plug, the physical quantities which remains conserved, is

(a) Entropy (b) Internal energy
(c) Enthalpy (d) Temperature

UPPCS Pre 2008

147. According to third law of thermodynamics :

(a) Entropy of system is zero at absolute zero temperature
(b) Heat capacity of system is zero at absolute zero temperature
(c) Volume expansion coefficient of system is zero
(d) All of the above

(UPPGT 2004)

148. The physics underlying the operation of a refrigerator most closely resembles the physics underlying

(a) Heat engine
(b) Melting of ice
(c) The freezing of water
(d) The vaporisation of water

(UPPGT 2003)

149. Which of the following statements does not represent second law of thermodynamics

(a) It is impossible to obtain work by cooling a body below the coldest to its surroundings.
(b) It is impossible for a self acting machine to transfer heat from a body at a lower temperature to a body at a higher temperature without taking energy from an outside source.
(c) It is impossible to construct a perpetual motion machine, which takes heat from a system and converts into work in a cycle without producing any change in the system.
(d) Only such changes in nature take place in which entropy decrease.

(UPPGT 2000)

Answer Key

1. (b)	2. (c)	3. (d)	4. (a)	5. (a)	6. (a)	7. (a)	8. (c)	9. (b)	10. (a)
11. (d)	12. (a)	13. (c)	14. (a)	15. (d)	16. (d)	17. (c)	18. (d)	19. (d)	20. (d)
21. (d)	22. (b)	23. (b)	24. (c)	25. (c)	26. (d)	27. (b)	28. (a)	29. (b)	30. (a)
31. (c)	32. (d)	33. (c)	34. (a)	35. (d)	36. (a)	37. (c)	38. (a)	39. (c)	40. (b)
41. (c)	42. (a)	43. (a)	44. (a)	45. (a)	46. (a)	47. (a)	48. (b)	49. (a)	50. (b)
51. (b)	52. (a)	53. (d)	54. (a)	55. (a)	56. (a)	57. (a)	58. (c)	59. (c)	60. (b)
61. (d)	62. (d)	63. (c)	64. (d)	65. (b)	66. (c)	67. (c)	68. (a)	69. (c)	70. (c)
71. (d)	72. (c)	73. (b)	74. (a)	75. (d)	76. (a)	77. (b)	78. (b)	79. (c)	80. (c)
81. (c)	82. (a)	83. (d)	84. (a)	85. (b)	86. (c)	87. (d)	88. (d)	89. (b)	90. (a)
91. (b)	92. (a)	93. (b)	94. (b)	95. (b)	96. (a)	97. (c)	98. (a)	99. (b)	100. (c)
101. (b)	102. (a)	103. (c)	104. (a)	105. (c)	106. (a)	107. (c)	108. (c)	109. (d)	110. (b)
111. (c)	112. (b)	113. (c)	114. (c)	115. (b)	116. (b)	117. (a)	118. (b)	119. (a)	120. (d)
121. (d)	122. (b)	123. (d)	124. (b)	125. (b)	126. (c)	127. (d)	128. (b)	129. (d)	130. (d)
131. (a)	132. (c)	133. (d)	134. (c)	135. (c)	136. (c)	137. (c)	138. (a)	139. (a)	140. (a)
141. (c)	142. (c)	143. (a)	144. (a)	145. (a)	146. (c)	147. (d)	148. (c)	149. (d)	

03.

Ray Optics

(1) Light

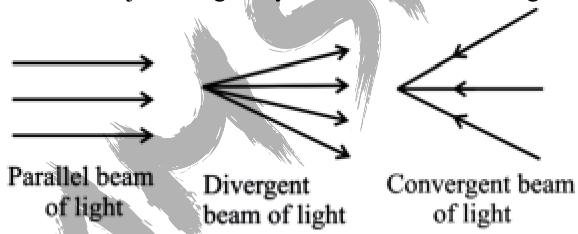
- Light is a form of energy which produces the sensation of sight in our eyes. Sources of light are of three types-thermal sources, gas discharge sources and luminescent sources.
- Photometry is a branch of ray optics which deals with the measurement of light energy.

■ Characteristic of light :-

- Light waves are electromagnetic waves whose nature is transverse. The speed of light in vacuum is 3×10^8 m/s but it is different in different medium. The speed and wavelength of light changes when it travel from one medium to another but its frequency remains unchanged.

■ Important terms:-

- **Luminous object**:-The object which emits its own light are called luminous objects e.g.- sun, other stars, an oil lamp etc.
- **Non-luminous object:-** The object which does not emits its own light but become visible due to the reflection of light falling on them are called non-luminous object e.g.- moon, table, chair, bat etc.
- **Ray of light**- A straight line drawn in the direction of propagation of light is called a ray of light.
- **Beam of light**- A bundle of the adjacent light rays is called a beam of light.

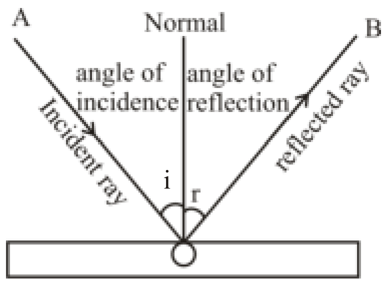


- **Image:-** If light ray coming from an object meets or appear to meet at a point after reflection or refraction, then this point is called image of the object.
- **Real Image:-** The image obtained by the actual meeting of light rays is called a real image. Real image can be obtained on a screen.
- **Virtual Image:-** The Image obtained when light rays are not actually meeting but appears to meet only.

(2) Reflection of light

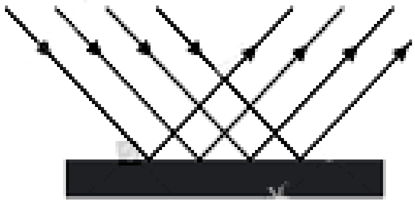
The re-bouncing back of light rays into the same medium on striking the surface of any object is called reflection. In reflection, path of light rays changes without any change in medium.

■ Laws of Reflection of light:-



- **First law**- The incident ray, the reflected ray and the normal (at the point of incidence) all lies in the same plane.
- **Second law:-** The angle of reflection (r) is always equal to the angle of incidence (i). It means that a ray of light incident normally on the mirror, retraces its path on reflection.

■ Types of reflection:-

Regular reflection	Irregular reflection
	
• When the reflecting surface is smooth and well polished, the parallel rays falling on it are reflected parallel to one another. The reflected light goes in one particluar direction. This is regular reflection.	• When the reflecting surface is rough, the parallel rays falling on it are reflected in different directions. Such a reflection is known as diffuse reflection or irregular reflection or even scattering of light.

(3) Mirrors

A Smooth and highly polished reflecting surface is called a mirror.

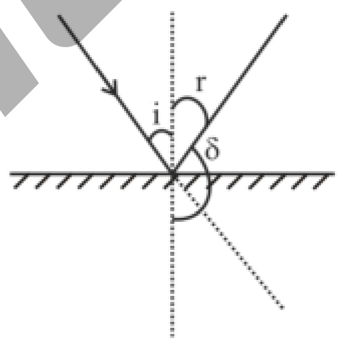
■ Types of Mirrors :-

(1) Plane mirror:- A highly polished plane surface is called a plane mirror.

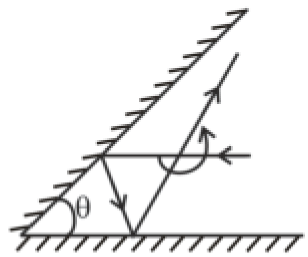
Different properties of Image formed by plane mirror are given below:-

- Size of image = size of object
- Magnification = unity
- Distance of image from the mirror= Distance of object from the mirror
- A plane mirror always forms a virtual image of a real object.
- A man may see his full image in a mirror of half height of man.
- Focal length as well as radius of curvature of a plane mirror is infinity.
- Power of a plane mirror is zero.

Deviation (δ) produced by a plane mirror and by two inclined plane mirrors.



$\delta=(180-2i)$
(A) SingleReflection

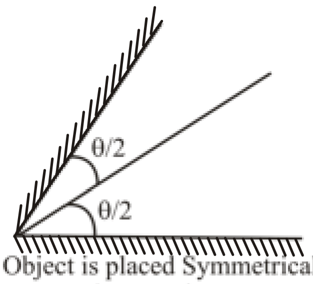


$\delta=(360-2\theta)$
(B) Double Reflection

Images formed by two inclined plane mirrors:-

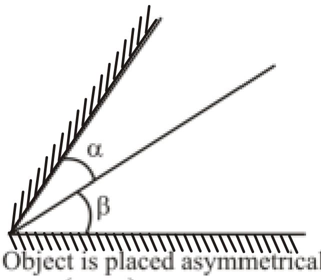
- When two plane mirrors are inclined to each other at an angle θ, then number of images (n) formed of an object which is kept between them.
- $n=\left(\dfrac{360^\circ}{\theta}-1\right)$; if $\dfrac{360^\circ}{\theta}=$ even integer

- If $\frac{360^\circ}{\theta}$ = odd integer then there are two possibilities-



(A) Object is placed Symmetrically

$n = \left(\frac{360^\circ}{\theta} - 1\right)$

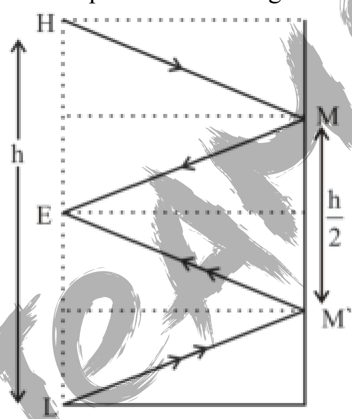


(B) Object is placed asymmetrically

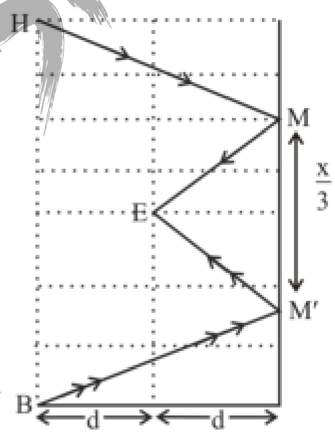
$n = \left(\frac{360^\circ}{\theta}\right)$

Important points: -

- When the object moves with speed u towards (or away) from the plane mirror then image also move towards (or away) with speed u, but relative speed of image with respect to object is 2u.
- When mirror moves towards the stationary object with speed u, the image will move with speed 2u in same direction as that of mirror.
- A man of height h requires a mirror of length at least equal to h/2 to see his own complete image.
- To see complete wall behind himself a person requires a plane mirror of at least one third the height of wall. It should be noted that person is standing in the middle of the room.



(A)

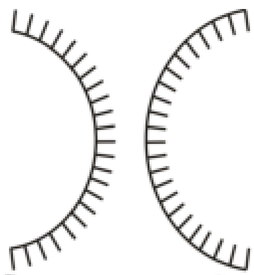


(B)

(2) Spherical mirror

A highly polished curved surface whose reflecting surface is a cut part of a hollow glass sphere is called a spherical mirror. Spherical mirrors are of two types.

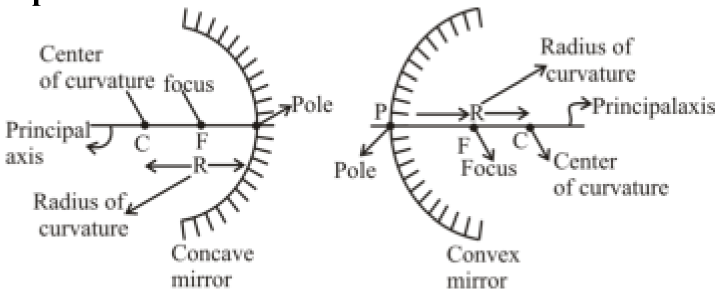
- **Concave mirror-** A spherical mirror whose bent in surface is reflecting surface, is called concave mirror.
- **Convex mirror** - A spherical mirror whose bulging out surface is reflecting surface is called a convex mirror.



Concave mirror

Convex mirror

■ Terms related to spherical mirrors: –



- **Centre of curvature(C)** – It is the centre of the sphere of which the mirror is a part.
- **Radius of curvature (R)** – The radius of the hollow sphere of which the mirror is a part.
- **Pole (P) –** The central point of the spherical mirror is called its pole (P).
- **Focus (F) –** When a parallel beam of light rays is incident on a spherical mirror. Then after reflection it meets or appears to meet at a point on principal axis which is called focus of the spherical mirror.
- **Focal length (f) –** The distance between the pole and focus is called focal length, $(f) = \frac{R}{2}$
- **Power** - The power of a mirror is given as $P = \frac{1}{f}$

■ Mirror Formula and Magnification

For a spherical mirror if u = distance of object from pole, v = distance of image from pole, f = focal length, R = radius of curvature and O = size of object, I = size of image

- **Mirror formula :-** $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

- **Lateral or Linear magnifications:-** When an object is placed perpendicular to the principal axis then it is called linear or lateral or transverse magnification.

$$m = \frac{I}{O} = \frac{-v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

- **Axial magnification:-** When small sized object placed along the principal axis then it is called axial magnification.

$$m = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f-u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

- **Areal magnification:-** If a 2D-object is placed with its plane perpendicular to Principal axis then the ratio of area of image to the area of object is called its areal magnification.

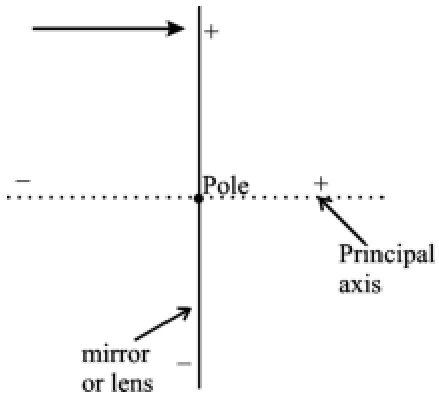
$$m_s = \frac{\text{Area of image}(A_i)}{\text{Area of object}(A_0)} = \frac{v^2}{u^2}$$

- **Newton's formula for a concave mirror** –

$$f = \sqrt{x_1 x_2} \Rightarrow (f^2) = x_1 x_2$$

Where, x_1 and x_2 are the distance of object and image from the focus.

■ Sign Conventions for spherical mirrors

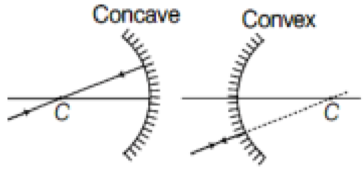


- All distances are measured from the pole of the mirror.
- Distance measured in the direction of incident rays are taken as positive while in the direction opposite of incident rays are taken negative.
- Distance measured above the principle axis is taken positive and below the principal axis are taken negative.

  Always use sign convention while solving the problems.

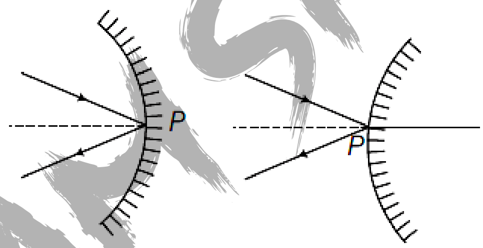
■ Rules for image Formation in Spherical Mirrors

- A ray passing through center of curvature retraces its path



- A ray parallel to principal axis after reflection passes through the principal focus F or appears to diverge from it and vice-versa.

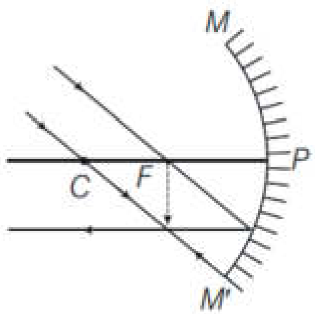
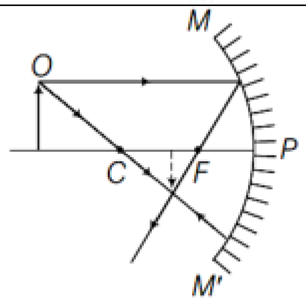
- A ray striking at pole P is reflected symmetrically back in the opposite side.

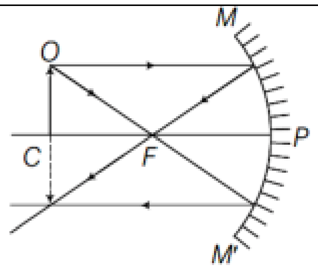
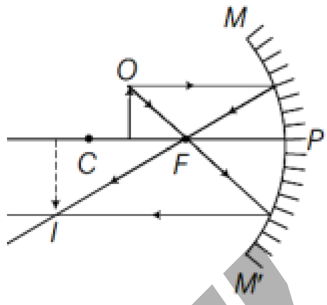
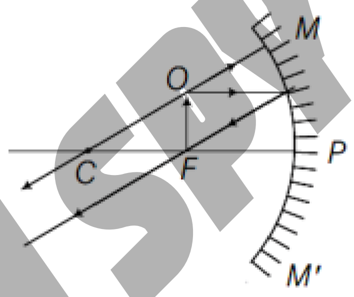
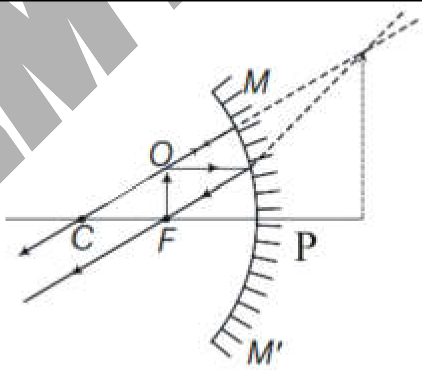


■ Image Formation by Spherical Mirrors

1. Image formation by Concave Mirror

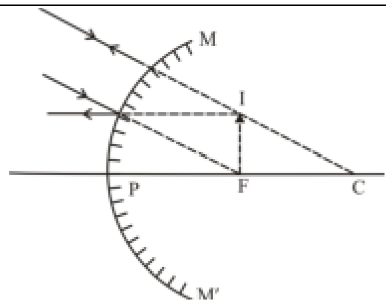
In case of a concave mirror, the image is erect and virtual when the object is placed between F and P. In all other positions of object, the image is real and inverted. All the cases are given below-

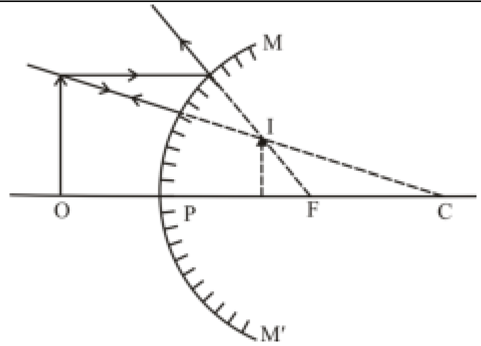
Sr. No.	Position of Object	Ray Diagram	Properties of Image
1.	At infinity		Real, inverted, very small at F (point image)
2.	Between infinity and C		Real, inverted, diminished between F and C

3.	At C		Real, inverted, equal in size at C
4.	Between F and C		Real, inverted and very large beyond C
5.	At F		Real, inverted, very large at infinity
6.	Between F and P		Virtual, erect, large in size behind the mirror

2. Image formation by convex mirror

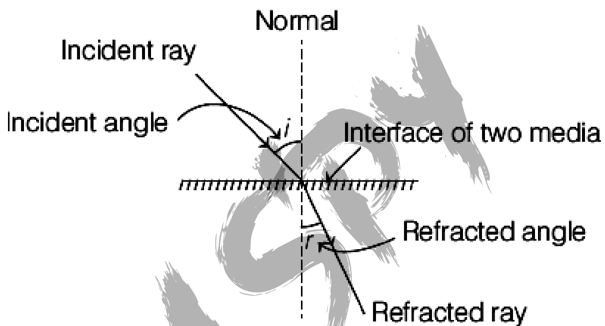
Image formed by convex is always virtual, erect and diminished no matter where the objects. All the images formed by this mirror will be between pole and focus as shown in the table given below.

Sr. No.	Position of Object	Ray Diagram	Details of Image
1.	At infinity		Virtual, erect, very small in size at F

2.	In front of mirror		Virtual, erect, diminished between P and F

(4) Refraction of Light

- The deviation of light rays while passing through one transparent medium to another transparent medium is called refraction of light.



- When a ray of light goes from a denser medium to a rarer medium, it bends away from the normal.
- When a ray of light goes from a rarer medium to a denser medium, it bends towards the normal.
- A medium in which the speed of light is more then it is known as optically rarer medium and a medium, in which the speed of light is less, is known as optically denser medium.
- Greater the difference in the speeds of light in the two medium, greater will be the amount of refraction.

■ Laws of Refraction

• Snell's law

The ratio of sine of the angle of incidence to sine of angle of refraction (r) is a constant called refractive index

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

Where ${}_1\mu_2$ is called refractive index of second medium with respect to first medium.

• The incident ray, the refracted ray and the normal at the point of incidence, all three lies in the same plane.

■ Refractive Index

- The ratio of speed of light in vacuum (c) to the speed of light in any medium (v) is called refractive index of the medium. Refractive index of a medium, $\mu = \frac{c}{v}$
- Refractive index of a medium decides speed of light in it.
- It is scalar, unit less and dimension-less quantity.
- Distance x travelled by light in a medium of refractive index μ is equal to distance (μx) travelled in vacuum.
- Time taken by light to traverse a thickness x = $\frac{\mu x}{c}$

Refractive Index of some substance :-

Substance	Refractive Index	Substance	Refractive Index
Diamond	2.419	Water	1.33
Fluorite (CaF_2)	1.434	Glycerin	1.47
Crown Glass	1.52	Air	1.00
Glass flint	1.66	ICE/ polystyrene	1.309/1.49

Dependence of Refractive index

- Nature of the medium of incidence and refraction.
- Colour of light or wavelength of light.
- Inversely on temperature of the medium.

■ Stoke's law

- When light is reflected by a denser medium, phase difference of π radian or path difference of $\lambda/2$ or time difference T/2 is produced.

■ Relative refractive index

- The refractive index of the second medium with respect to the first medium. i.e., ${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

  Where, v_1 and v_2 are the speed of light in medium 1 and 2 respectively.

■ Cauchy's Formula

- According to this formula the refractive index of a media is inversely proportional to λ^2. It is given by -

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots.$$

  Where, λ = wavelength of light and A, B, C..... are constants.

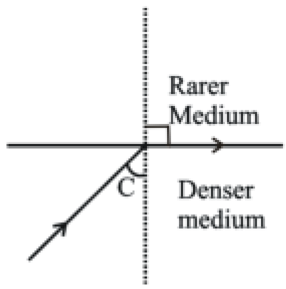
■ Refraction through a Glass Slab

- When a glass slab is placed in the path of a light ray it produces a shift in the position of object when viewed through it. If t is the thickness of glass slab then shift = $\left(1 - \frac{1}{n}\right)$t.
- When object is in denser medium and seen from rarer medium normally through the plane surface, then apparent depth of object = $\left(1 - \frac{1}{n}\right) \times$ actual depth of object in denser medium.

■ Critical Angle

- The angle of incidence in a denser medium for which the angle of refraction in rarer medium becomes 90° is called critical angle (C).

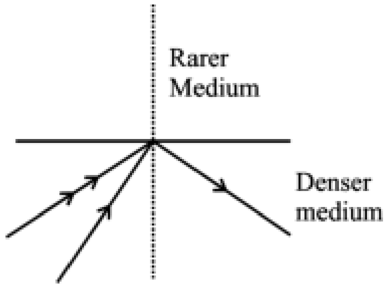
Critical angle for diamond = 24°
Critical angle for glass= 42°
Critical angle for water = 48°



- Refractive index of denser medium, $\mu = \frac{1}{\sin C}$
- Critical angle increases with temperature.
- Since refractive index depends on the wavelength of light, the critical angle for a given pair of media is different for different wavelengths of light.

■ Total Internal Reflection (TIR)

When a light ray travelling from a denser medium towards a rarer medium is incident at the interface at an angle of incidence greater than critical angle, then light rays are totally reflected back in to the denser medium. This phenomenon is called TIR.

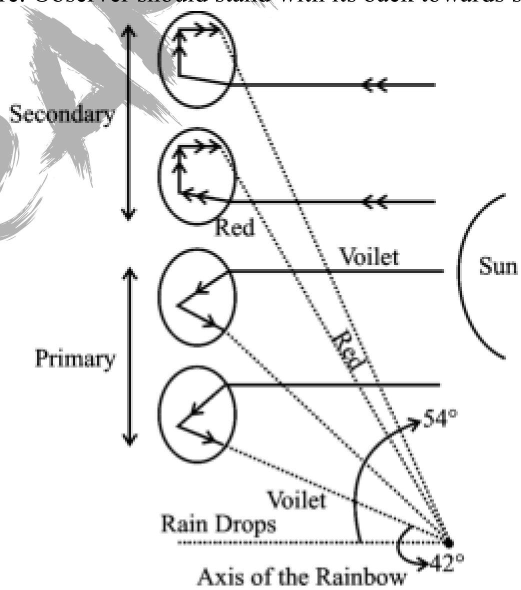


For total internal reflection to take place following conditions must be obeyed-

- The ray must travel from denser medium to rarer medium.
- The angle of incidence ($\angle i$) must be greater than critical angle ($\angle C$).

Examples of TIR

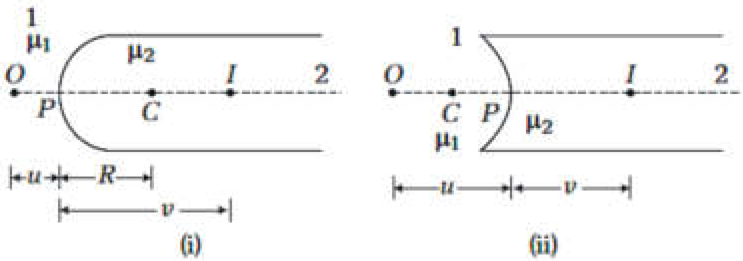
- **Mirage:-** is an optional illusion observed in deserts and roads on a hot day which is a application of TIR.
- **Looming:-** An optical illusion in cold countries.
- **Brilliance of diamond:-** Due to repeated internal reflections diamond sparkles.
- **Optical fibers:-** Optical fibers are also based on the phenomenon of total internal reflection. Optical fiber consists of several thousands of very long fine quality fibers of glass or quartz. The diameter of each fiber is of the order of 10^{-4} cm with refractive index of material being of the order of 1.5.
  These fibers are fabricated in such a way that light reflected at one side of the inner surface strikes the other at an angle larger than critical angle. Even if fiber is bent, light can easily travel along the length. Thus these are used in transmission and receiving of electrical signals by converting them first into light signals.
- **Rain bow**:- Rainbow is formed due to the dispersion of light suffering refraction and TIR in the droplets presents in the atmosphere. Observer should stand with its back towards sun to observe rainbow.



Primary rain bow:-	**Secondary rain bow:-**
• Two refractions and one TIR. • Innermost arc is violet and outermost is red. • Subtends an angle of 42° at the eye of the observer. • More bright.	• Two refraction and two TIR • Innermost arc is red and outermost is violet • It subtends an angle of 52.5° at the eye of the observer • Comparatively least bright.

■ Refraction at spherical surface :-

- When two transparent media are separated by a spherical surface light incident on the surface from one side get refracted into the medium on the other side. Spherical surfaces are of two types as shown.



Spherical surface (i) convex (ii) concave

For both surfaces refraction formula is –

$$\frac{\mu}{v} - \frac{1}{u} = \frac{(\mu - 1)}{R}$$

Where, μ_1, μ_2 are refractive indexes of given media and $\mu = \frac{\mu_2}{\mu_1}$, u= distance of object, v = distance of image from pole, R= radius of curvature of spherical surface.

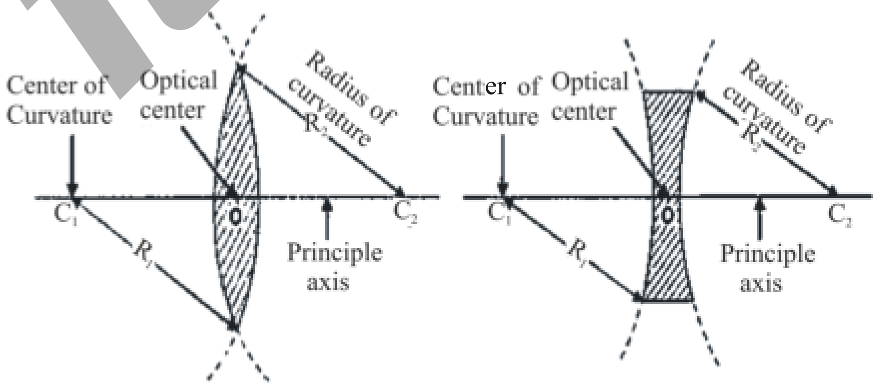
(5) Lens

A lens is a uniform transparent medium bounded between two spherical or one spherical and one plane surface.

- **Convex Lens**- A lens which is thinner at edges and thicker at middle is called a convex or converging lens.
- **Concave Lens-** A lens which is thicker at edges and thinner at middle is called a concave or diverging lens.

Convex Lens Concave Lens

■ Terms related to Lens :-



Convex Lens Concave Lens

- **Optical Centre :-** The point on the principle axis at the centre of the lens is called optical centre.
- **Centre of curvature :-** A lens has two spherical surfaces, these two spherical surface form a part of sphere. The centre of these spheres is known as the centre of curvature.
- **Principle axis :-** The principle axis is an imaginary line passing through the centers of curvature and the pole.

- **Aperture :-** The area of the lens suitable for refraction is called aperture. The aperture of the lens is the effective diameter of its light-transmitting area.
- **Focus :-** Focus is the point onto which collimated light parallel to the axis is focused.
- **Focal length :-** The focal length is the distance between the optical centre and the focal point or focus of the lens.

■ Lens Formula and magnification -

- **Lens formula :-** $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

  Where, f = focal length of the lens, u = distance of object and v = distance of image.

- **Lens Maker's formula :-** $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

  where, μ = refractive index of the material of the lens and R_1 and R_2 are radii of curvature of the lens

- **Power of a Lens :-** The reciprocal of the focal length of a lens, when it is measured in metre is called power of a lens.

  Power of lens, (P) = $\frac{1}{f(\text{metre})}$

  Its unit is diopter (D).

  The power of a convex (converging) lens is positive and for a concave (diverging) lens it is negative.

- **Lateral or Linear Magnification**

  The lateral magnification m is the ratio of the image height to the object height

$$m=\frac{I}{O}=\frac{v}{u}=\frac{h_i}{h_o}$$

- **Axial Magnification**

  If a small size object placed linearly along the principal axis.

$$\frac{dv}{du}=\left(\frac{v}{u}\right)^2=\left(\frac{f}{f-u}\right)^2=\left(\frac{f-v}{f}\right)^2$$

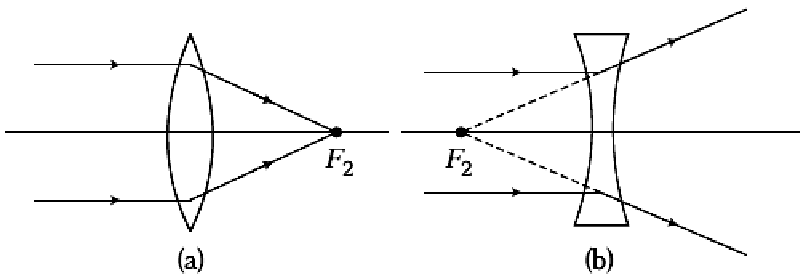
■ Sign Conventions of Lens - Sign convention used for lens are similar to that used for spherical mirrors.

■ Laws of Formation of Images by Lens

The position and nature of the image by lens, in any case can be obtained either from a ray diagram or by calculation.

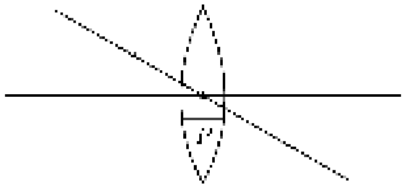
To construct the image of a small object perpendicular to the axis of a lens, two of the following three rays are drawn from the top of the object.

- A ray parallel to the principal axis after refraction passes through the principal focus or appears to diverge from it.



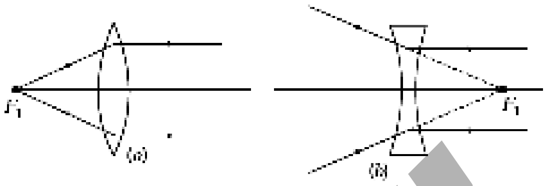
Path of incident ray parallel to principal axis for
(a) convex lens (b) concave lens

- A ray through, the optical centre P passes undeviated because the middle of the lens acts like a thin parallel sided slab.



Path of incident ray passing through centre for convex lens

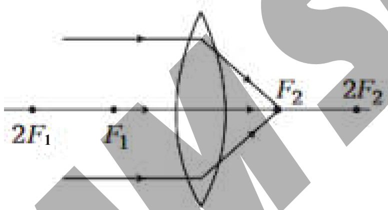
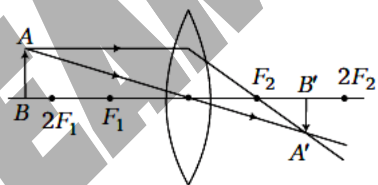
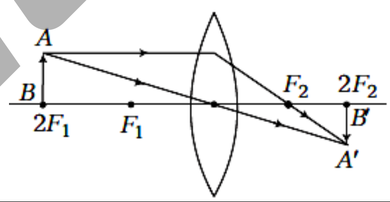
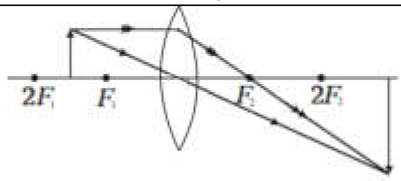
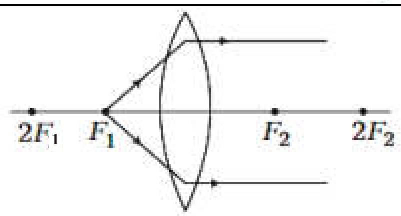
- A ray passing through, the first focus F_1 become parallel to the principal axis after refraction.

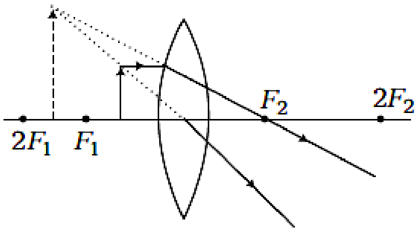


Path incident ray passing through the focus for
(a) convex lens (b) concave lens

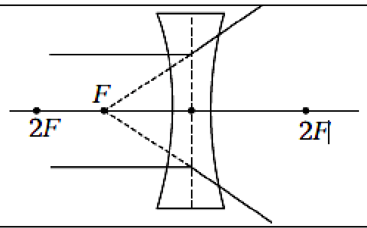
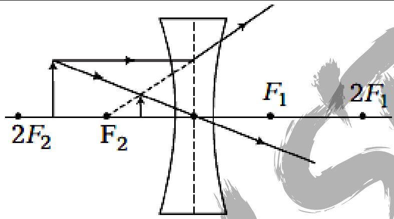
■ Image Formation by Lens

1. Image Formation by Convex lens:- The image formed by convex lens depends on the position of object-

Sr. No.	Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
1.	At infinity		At the principal focus (F_2) or in the focal plane	Real, inverted and extremely diminished
2.	Beyond $2F_1$		Between F_2 and $2F_2$	Real, inverted and diminished
3.	At $2F_1$		At $2F_2$	Real, inverted and of same size as the object
4.	Between F_1 and $2F_1$		Beyond $2F_2$	Real, inverted and highly magnified
5	At F_1		At infinity	Real, inverted and highly magnified

6	Between F_1 and optical centre		On the same side as the object	Virtual, erect and magnified

2. Image Formation by Concave lens :- The image formed by a concave lens is always virtual, erect and diminished (like a convex mirror).

Sr. No.	Position of Object	Ray Diagram	Position of Image	Nature and Size of Image
1.	At infinity		At the focus	Virtual erect and point size
2.	Anywhere except on the principal axis		Between the optical centre and F_2	Virtual, erect, diminished

■ Combination of Lens:-

- For a system of lenses, the net power, net focal length and magnification are given as follows.

$$P = P_1 + P_2 + P_3 \ldots\ldots\ldots\ldots$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots\ldots$$

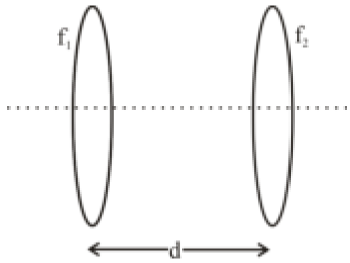
$$m = m_1 \times m_2 \times m_3 \times \ldots\ldots$$

- In case when two thin lens are in contact, combination will behave as a lens which has more power or lesser focal length

$$\frac{dv}{du} = \left(\frac{v}{u}\right)^2 = \left(\frac{f}{f+u}\right)^2 = \left(\frac{f-v}{f}\right)^2$$

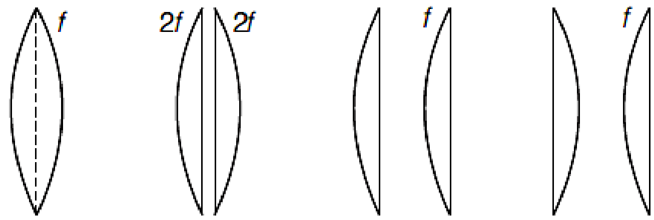
- If two lens of equal focal length but of opposite nature are in contact then combination will behave as a plane glass plate and $F_{combination} = \infty$
- When two lenses are placed at a distance d from each other then equivalent focal length (F).

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \text{ and } P = P_1 + P_2 - dP_1P_2$$

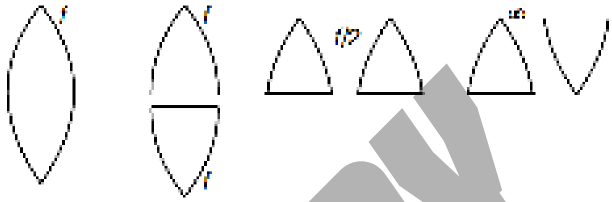


■ Cutting of Lens:-

- If a symmetric lens is cut along optical axis in two equal part. Intensity of image formed by each part will be same and focal length is doubled the original for each part.
- If the two parts are joined as shown in figure, the focal length of combination is again f.



- If a symmetric lens is cut along principle axis in two equal parts intensity of image formed by each part will be less compared as that of complete lens (aperture of each part is $\frac{1}{\sqrt{2}}$ times that of complete lens). Focal length remains same for each part.
- If these two parts are joined with curved ends on one side, focal length of the combination is f/2. But on joining the two parts in opposite sense, the net focal length becomes ∞.



■ Aberration of lenses

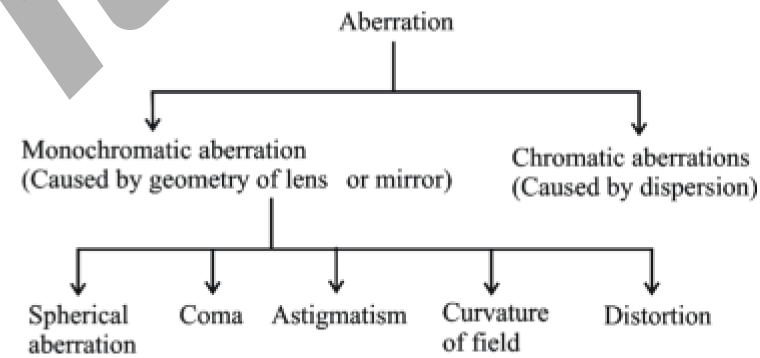
- The departure of real image from the ideal image, in respect of the actual size, shape, and position are called aberrations.
- Aberrations are only due to inherent shortcomings of a lens and not caused by faulty construction of the lens.
- The condition for minimum spherical aberration is that the distance between the two lenses is equal to the difference in their focal i.e. $d = f_1 - f_2$
- If dispersive power of an achromatic doublet component are ω_1 and ω_2 and focal length of a convex lens is f_1 then focal length of the concave lens,

$$f_2 = -\frac{\omega_2}{\omega_1} f_1$$

- An achromatic combination of two thin lens the separation between lenses is the average of the focal length. i.e.

$$f_{avg} = \frac{f_1 + f_2}{2}$$

■ Types of Aberration :-



■ Types of Aberration, character and their correction required is given below:-

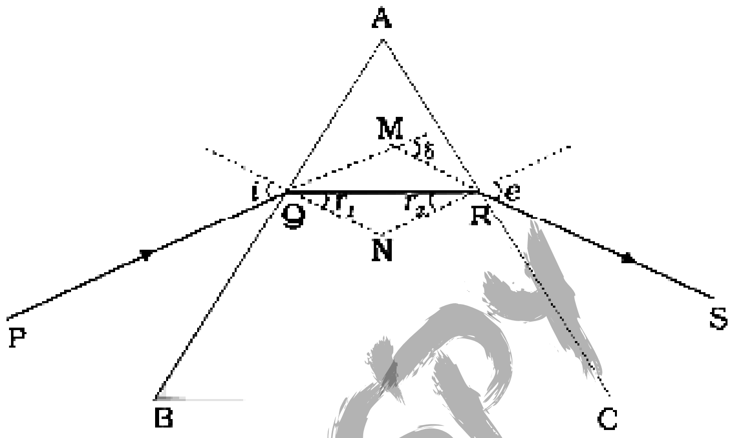
Aberration	Character	Correction
Chromatic Aberration	White light, on & off axis, image blur	Contact doublet, spaced doublet, crystal fluorite
Spherical Aberration	Monochromatic light, on & off axis, image blur	Diaphragm, stoppers
COMA	Monochromatic light, off axis only, comet shaped image	Spaced doublet, stoppers

Field Curvature	Monochromatic light, off axis only,	Flat field objective, spaced doublet
Distortion	Monochromatic light, off axis only, distorted image	Spaced doublet
Astigmatism	Monochromatic light, off axis, image blur	Cylindrical lens

(6) Prism and Scattering of light

Prism

- Prism is a transparent medium bounded by refracting surface such that the incident surfaces (on which light ray is incident) and emergent surfaces (from which light ray's emerges) are plane and non parallel.



Where,
i = Angle of incidence
e= angle of emergence
A=Angle of prism or refracting angle of prism = $r_1 + r_2$
r_1 and r_2 – Angle of refraction.
δ= angle of deviation.
A= r_1+r_2

■ **Angle of Deviation**

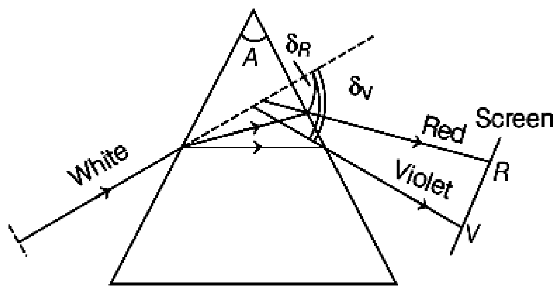
- The angle subtended between the direction of incident light ray and emergent light ray from a prism is called angle of deviation. $\delta = i + e - A$

Maximum deviation		Minimum deviation	
•	Condition of maximum deviation is $\angle$ i=90° ⇒ r_1=C, r_2 = A – C and from Snell's law on emergent surface -	•	It is observed if $\angle i = \angle e$ and $\angle r_1 = \angle r_2 = \angle r$, deviation produced is minimum -
(a)	$e = \sin^{-1}\left[\frac{\sin(A-C)}{\sin C}\right]$	(a)	$r = \frac{A}{2}$ and $i = \frac{A+\delta_{min}}{2}$
(b)	$\delta_{max} = \frac{\pi}{2} + \sin^{-1}\left[\frac{\sin(A-C)}{\sin C}\right] - A$	(b)	$\mu = \frac{\sin i}{\sin r}$ or $\mu = \frac{\sin\frac{A+\delta_{min}}{2}}{\sin A/2}$

■ **Dispersion Through a Prism-**

The splitting of white light into it's constituent colors is called dispersion of light.

The refractive index $\mu_V > \mu_R$, therefore violet colour deviates most and red colour deviates least, i.e. $\delta_V > \delta_R$.



- **Angular dispersion (θ) :-** Angular separation between extreme colours is 'θ' which depends upon μ and A.

$$\theta = \delta_v - \delta_R = (\mu_v - \mu_R)A.$$

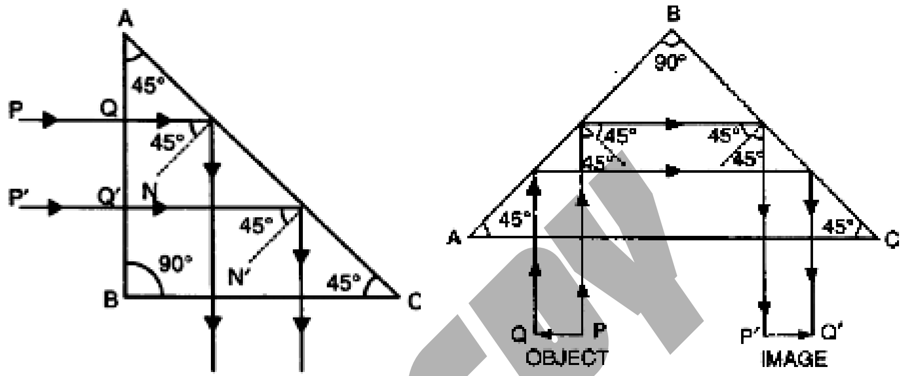
- **Dispersive power (ω) :-**

$$\omega = \frac{\theta}{\delta_y} = \frac{\mu_v - \mu_R}{\mu_y - 1} \text{ where } \left\{\mu_y = \frac{\mu_v + \mu_R}{2}\right\}$$

It depends only upon the material of the prism i.e. μ and it does not depends upon angle of prism A.

■ **Porro Prism:-**

- A right angled isosceles prism, which is used in periscopes or binoculars. It is used to deviate light rays through 90°and 180° and also to correct the image.



Deviation through 90° *Deviation through 180°*

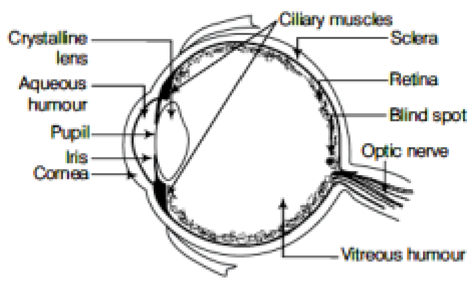
■ **Scattering of light**

- `When light passes through a medium in which particles are suspended whose size is of the order of wavelength of light, then light on striking these particles, deviated in different directions. These phenomena is called scattering of light.
- According to Rayleigh, intensity of scattered light $\propto \frac{1}{\lambda^4}$
- **Elastic Scattering-** When the wavelength of radiation remains unchanged the scattering is called elastic.
- **Inelastic scattering (Raman's effect):-** Under specific condition, light can also suffer inelastic scattering from molecules in which it's wavelength changes.

Some phenomenon based on scattering-

(i) Sky looks blue due to scattering.
(ii) At the time of sunrise or sunset sun looks reddish.
(iii) Danger signals are made of red colour.

(7) Human Eye



Human eye is similar to an optical instrument which forms real image of the objects on retina.

A human eye has the following main parts-

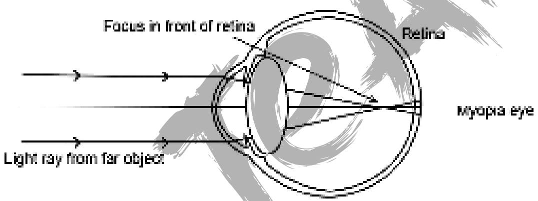
- **Cornea-** It is the transparent spherical membrane covering the front of the eye. Light enters the eye through this membrane.

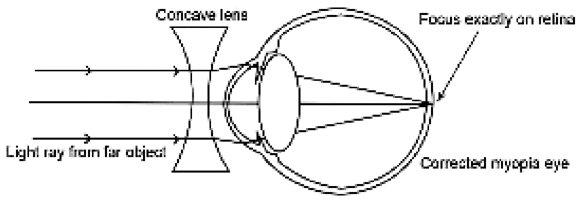
- **Crystalline lens-** The eye lens is a convex lens made of a transparent, soft and flexible material like a jelly made of proteins.
- **Iris**- It is a dark muscular diaphragm between the cornea and the lens. It controls the size of the pupil.
- **Pupil**- It is a small hole between the iris through which light enters modifying the curvature of the lens.
- **Retina-** It is the light sensitive surface of eye of which the image is formed. It contains light sensitive cells rods and cones.
- **Optic nerve-** It transmits visual information from the **retina** to the **brain**.
- **Sclera-** It is an opaque, fibrous, protective, outer layer of an eye containing collagen and elastic fibre. It is also the white portion of the eye.
- **Blind spot-** It is the point at which the optic nerve leaves the eye. It contains no rods and cones, so an image formed at this point is not sent to the brain.
- **Aqueous humour-** Behind the cornea, we have a space filled with a transparent liquid called the aqueous humour.
- **Vitreous humour-** The space between eye lens and retina is filled with another liquid called vitreous humour.
- **Accommodation of eye-** It is the ability of eye lens, to change its focal length to form sharp images of objects at different positions from the eye on the retina of the eye.
- **Range of vision-** It is the distance between near point and the far point of an eye. For normal eye, the range of vision is 25 cm to infinity.
- **Near point-** It is the nearest position of an object from human eye, so that its sharp images is formed on the retina.

■ Defects in vision of human eye

(1) Myopia (short sightedness)-

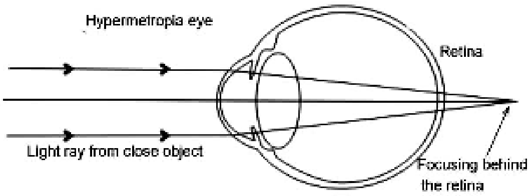
- A short- sighted eye can see only nearer objects. Distance objects are not seen clearly.
- In this defect image is formed before the retina and far point comes closer.
- In this defect focal length or radii of curvature of lens reduces or power of lens increases or distance between eye retina increases.
- This defect can be removed by using a concave lens of suitable focal length.
- If defected, far point is at a distance d from eye then focal length of used lens.
  f= –d = – (defected far point)

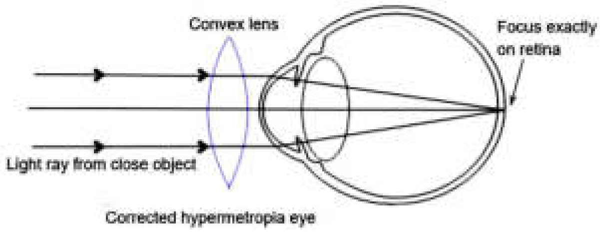




(2) Hypermetropia (long sightedness)

- A long-sighted eye can see distant objects clearly but nearer object are not clearly visible.
- Image formed behind the retina and near point moves away.





- In this defect focal length or radii of curvature of lens increases or power of lens decreases or distance between eye lens and retina decreases.
- This defect can be removed by using convex lens.
- If a person can't see before distance d but wants to see to object placed at distance D from eye so.

$$f = \frac{dD}{d - D} \text{ and power of the lens } P = \frac{d - D}{dD}$$

(3) **Presbyopia :-** In this defect both near and far objects are not clearly visible it is an old age disease and it is due to the loss of power of accommodation. It can be removed by using bifocal lens.

(4) **Astigmatism :-** In this defect, a person cannot focus on horizontal and vertical lines at the same distance at the same time. This defect can be removed by using suitable cylindrical lenses.

(5) **Colour Blindness:-** In this defect, a person is unable to distinguish between few colours. The reason of this defect is the absence of cone cells sensitive for few colours.

(6) **Cataract** :- In this defect, an opaque white membrane is developed on cornea due to which person loses power of vision partially or completely. This defect can be removed by removing this membrane through surgery.

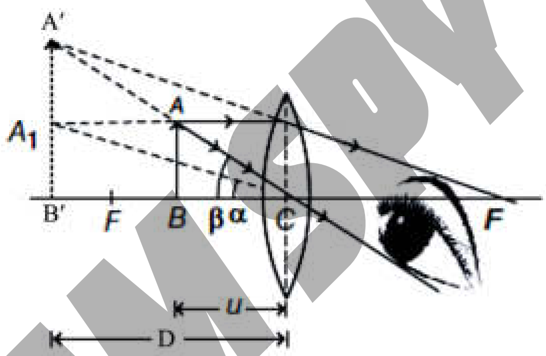
(8) Optical Instruments

■ Microscope

- It is an optical instrument used to see very small objects. Its magnifying power is given by

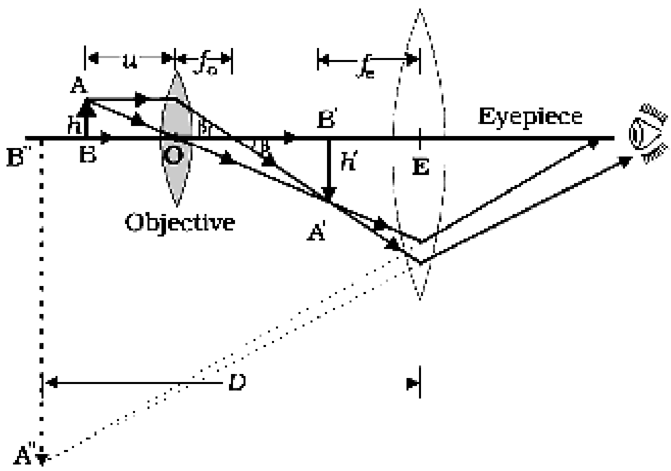
$$M = \frac{\text{visual angle with instrument } (\beta)}{\text{visual angle when object is Placed at least distance of distinct vision } (\alpha)}$$

(1) Simple microscope



- It is used for observing magnified images of objects. It consists of a converging lens of small focal length.
- It is a single convex lens of lesser focal length.
- When final image is formed at least distance of distinct vision (D), then $M = 1 + \frac{D}{f}$ Where, f = focal length of the lens.
- When final image is formed at infinity, then $M = \frac{D}{f}$.

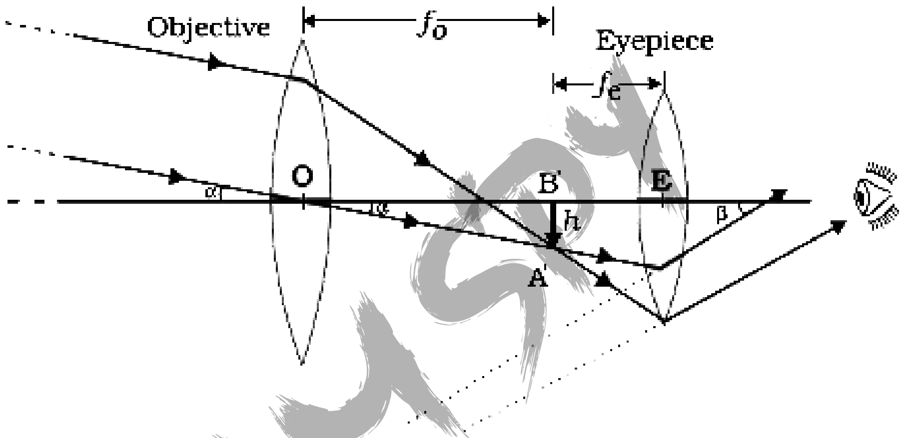
(2) Compound microscope:-



- Consists of two converging lenses called objective and eye lens.
- $f_{eyelens} > f_{objective}$.
- Intermediate image is real and enlarged.
- Final image is magnified, virtual and inverted
- When final image is formed at least distance of distinct vision (D), then $M = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$

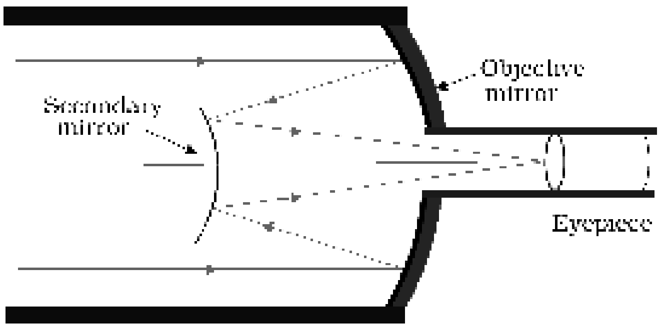
  where, v = distance of image formed by objective lens
  and u = distance of object from the objective lens.
- When final image is formed at infinity, then, $M = \frac{v}{u}.\left(\frac{D}{f_e}\right)$

■ Astronomical Telescope



- It is also a combination of two lenses called objective lens and eyepiece, separated by a distance. It is used for observing distinct images of heavenly bodies like stars, planets etc.
- $f_{objective} > f_{eyelens}$
- Intermediate image is real, inverted and small.
- Final image is virtual, inverted and small.
- When final image is formed at least distance of distinct vision (D), then $M = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$ where f_0 and f_e are focal lengths of objective and eyepiece respectively and length of the telescope (L) = $(f_0 + u_e)$ where, u_e = distance of object from the eyepiece.
- When final image is formed at infinity, then $M = \frac{f_0}{f_e}$ and Length of the telescope, (L) = $f_0 + f_e$

■ Reflecting Telescope



- Reflecting telescopes and based upon the same principle except that the formation of images takes place by reflection instead of by refraction. It consists of concave mirror of large aperture and large focal length (objective). A plane mirror is placed between the concave mirror and its focus. A small convex lens works as eye-piece.
- It is focal length of the concave spherical mirror used as objective and f_e, the focal length of the eye-piece, the magnifying power of the reflecting telescope is given by $m = \frac{f_0}{f_e}$

■ Limit of Resolution

- The minimum distance between two nearby objects which can be just resolved by the instrument is called it's limit of resolution (Δd).

■ Resolving Power

The ability of an optical instrument to produce separate and clear images of two nearby objects is called its resolving power.

- Resolving power of a microscope $= \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$
- Resolving power of a telescope $= \frac{1}{\Delta\theta} = \frac{d}{1.22\lambda}$

Where, Δd = limit of resolution, λ = wavelength of light used and d = diameter of aperture of objective.
μ = refractive index of the medium between the objects and objective lens and θ = half of the cone angle.

■ Objective and Eyepiece

- The magnifying power of simple microscope can be increased by decreasing the focal length of the lens. However, the focal length of a lens cannot be decreased beyond a certain limit.
- To increase the magnifying power, two separate lenses are used. The lens near the object is called the objective, which forms a real image of an object under examination
- The lens used to enlarge this image further to form a final image and which is then viewed by the eye is called an eyepiece or ocular.

■ Comparison of Ramsden eyepiece with Huygens eyepiece

S.NO.	Ramsden Eyepiece	Huygens Eyepiece
1.	It is Positive eyepiece	It is negative eyepiece.
2.	Image formed in front of the field lens. Therefore, cross-wires can be used.	Image formed between the two lenses. Therefore, cross-wires cannot be used.
3.	Minimum spherical aberration is not satisfied.	Minimum spherical aberration is satisfied.
4.	It does not satisfy the condition for achromatism but can be made achromatic be using an achromatic doublet as the eye lens. (It is achromatic for only two chosen colours.)	It satisfies the condition for achromatism. (It is achromatic all colours.)
5.	The other types of aberration are better eliminated. Coma is absent and distortion is 5% higher.	Other aberrations like pincushion distortion are not eliminated.
6.	The eye clearance is 5% higher.	The eye clearance is too small and less comfortable.
7.	It is used for quantitative purposes in microscopes and telescopes.	It is used for qualitative purposes in microscopes and telescopes.
8.	Its power is positive.	Its power is positive.
9.	The two principal planes are crossed.	The two principal planes are crossed.
10.	It can be used as a simple microscope because the first principal plane lies to the left of the field lens and the focal plane is real.	It cannot be used as simple microscope because the first focal plane lies to the right of the field lens and the focal plane is virtual.
11.	The nodal points coincide with the principal points.	The nodal points coincide with the principal points.

Exemplar Problems

Ray optics

1. **The speed of light in media 'A' and 'B' are 2.0 × 10^{10} cm/s and 1.5 × 10^{10} cm/s respectively. A ray of light enters from the medium B to A at an incident angle 'θ'. If the ray suffers total internal reflection, then**

(a) $\theta = \sin^{-1}\left(\frac{3}{4}\right)$ (b) $\theta > \sin^{-1}\left(\frac{2}{3}\right)$

(c) $\theta < \sin^{-1}\left(\frac{3}{4}\right)$ (d) $\theta > \sin^{-1}\left(\frac{3}{4}\right)$

JEE MAIN 2022

2. **Consider a light ray travelling in air is incident into a medium of refractive index $\sqrt{2n}$. The incident angle is twice that of refracting angle. Then, the angle of incidence will be : 2n**

(a) $\sin^{-1}\left(\sqrt{n}\right)$ (b) $\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

(c) $\sin^{-1}\left(\sqrt{2n}\right)$ (d) $2\cos^{-1}\left(\sqrt{\frac{n}{2}}\right)$

JEE MAIN 2022

3. **If a wave gets refracted into a denser medium, then which of the following is true?**

(a) wavelength speed and frequency decreases.
(b) wavelength increases, speed decreases and frequency remains constant.
(c) wavelength and speed decreases but frequency remains constant.
(d) wavelength, speed and frequency increases.

JEE MAIN 2022

4. **Red light differs from blue light as they have :**

(a) Different frequencies and different wavelengths
(b) Different frequencies and same wavelengths
(c) Same frequencies and same wavelengths
(d) Same frequencies and different wavelengths

JEE MAIN 2021

5. **A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is**

(a) 0.16 m (b) 1.60 m
(c) 0.32 m (d) 0.24 m

JEE MAIN 2019

6. **The plane mirrors (M_1 and M_2) are inclined to each other such that a ray of light incident on mirror M1 and parallel to the mirror M_2 is reflected from mirror M_2 parallel to the mirror M_1. The angle between the two mirror is**

(a) 45º (b) 75º
(c) 90º (d) 60º

JEE MAIN 2019

7. **When a ray of light enters a glass slab from air**

(a) its wavelength decreases
(b) its wavelength increases
(c) its frequency increases
(d) neither its wavelength nor its frequency changes

IIT 1980

8. **A light wave enters glass from air. What does not change?**

(a) Frequency (b) Wavelength
(c) Velocity (d) All the above change

UP PGT Physics 2016 (Exam : 02.02.2019)

9. **A student performed the experiment of determination of focal length of a concave mirror by *u*-*v* method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (*u*, *v*) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are)**

(a) (42, 56) (b) (48, 48)
(c) (66, 33) (d) (78, 39)

IIT 2009

10. **An object is placed at a distance of 40 cm from a concave mirror of focal length 15 cm. If the object is displaced through a distance of 20 cm towards the mirror, the displacement of the image will be**

(a) 30 cm away from the mirror
(b) 36 cm away from the mirror
(c) 30 cm towards the mirror
(d) 36 cm towards the mirror.

NEET 2018

11. **Diameter of a Plano convex lens is 6 cm and thickness at the centre is 3 mm. The speed of light in material of lens is 2 × 10^8 m/s, the focal length of the lens is :**

(a) 20 cm (b) 30 cm
(c) 10 cm (d) 15 cm

UKPSC Lecturer (Mains) 2020

12. **Quartz is a**

(a) Negative bi-axial crystal
(b) Positive bi-axial crystal
(c) Negative uni-axial crystal
(d) Positive uni-axial crystal

UKPSC Lecturer (Mains) 2020

13. A ray of light making an angle 10º with the horizontal is incident on a plane mirror making an angle θ with the horizontal. What should be the value of θ so that the reflected ray goes vertically upwards?

(a) 20º (b) 30º
(c) 40º (d) 50º

RPSC School Lect. 26.07.2016

14. A convex mirror forms an enlarged image of an object

(a) For no values of object distance
(b) When object distance is less than twice the focal length
(c) When object distance is more than twice the focal length
(d) For all values of object distance

UP TGT Physics 2016

15. Which mirror is best suited to focus parallel rays at a point?

(a) Spherical convex (b) Spherical concave
(c) Hyperbolic (d) Parabolic

UPTGT Science - 2021

16. Two plane mirrors are placed at 60^0 angle to each other. How many images will be seen if the object is placed between them?

(a) 2 (b)3
(c) 5 (d) Infinite

UPTGT Science - 2021

17. A person of height 2 m stands in front of a plane mirror. What must be the minimum vertical height of the mirror so he can see his full image?

(a) 2 m (b) 1 m
(c) 0.5 m (d) 0.25 m

UPTGT Science - 2021

18. For which one of the following concave mirror is not used?

(a) Shaving glass
(b) Reflector in search lights
(c) Inspection internal pasts of ear
(d) Rear-view mirror in cars

UPTGT Science - 2021

19. A green light is incident from the water to the air-water interface at the critical angle (q). Select the correct statement.

(a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium
(d) The entire spectrum of visible light will come out of the water at various angles to the normal

IIT 2014

20. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of the virtual image from the surface of the sphere is

(a) 2 cm (b) 4 cm
(c) 6 cm (d) 12 cm

IIT 2004

21. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence 45°. The ray undergoes total internal reflection. If *n* is the refractive index of the medium with respect to air, select the possible value (s) of *n* from the following

(a) 1.3 (b) 1.4
(c) 1.5 (d) 1.6

IIT 1998

22. A light ray falls on a glass surface of refractive index $\sqrt{3}$ at an angle 60°. The angle between the refracted and reflected rays would be

(a) 120º (b) 30º
(c) 60º (d) 90º

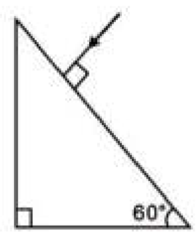
NEET 2022

23. Two transparent media A and B are separated by a plane boundary. The speed of light in those media are 1.5 × 108 m/s and 2.0×10^8 m/s, respectively. The critical angle for a ray of light for these two media is

(a) $\tan^{-1}(0.750)$ (b) $\sin^{-1}(0.500)$
(c) $\sin^{-1}(0.750)$ (d) $\tan^{-1}(0.500)$

NEET 2022

24. Find the value of the angle of emergence from the prism. Refractive index of the glass is $\sqrt{3}$.



(a) 90° (b) 60°
(c) 30° (d) 45°

NEET 2021

25. In total internal reflection when the angle of incidence is equal to the critical angle for the pair of media in contact, what will be angle of refraction?

(a) 90º (b) 180º
(c) 0º (d) equal to angle of incidence

NEET 2019

26. **The critical angle is maximum when light travels from**
(a) water to air (b) glass to air
(c) glass to water (d) air to water
UP PGT - 2021

27. **Index of refraction for ordinary glass relative to air is 1.5. What will be the critical angle?**
(a) $\sin^{-1}\left(\frac{1}{1.5}\right)$ (b) $\sin^{-1}(1.5)$
(c) $\cos^{-1}\left(\frac{1}{1.5}\right)$ (d) $\cos^{-1}(1.5)$
UPPSC GDC - 2021

28. **A prism of material having refractive index $\mu=\frac{4}{3}$ has refracting angle 15^0. The angle of minimum deviation of this prism is**
(a) Zero (b) 5^0
(c) 7.5^0 (d) 15^0
UPTGT Science - 2021

29. **If the refractive index of glass is 1.5, the speed of light in glass is**
(a) 2.3×10^8 m/s (b) 3.0×10^8 m/s
(c) 1.5×10^8 m/s (d) 2.0×10^8 m/s
UPTGT Science - 2021

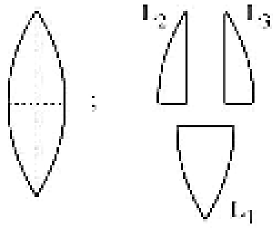
30. **The refractive index of water $_a\mu_w = \frac{4}{3}$ and refractive index of glass $_a\mu_g = \frac{3}{2}$. A lens placed in air has focal length 10 cm. What will be its focal length if placed inside water?**
(a) 10 cm (b) 15 cm
(c) 50 cm (d) 40 cm
UPTGT Science - 2021

31. **In any two media the reversibility of light rays obeys :**
(a) ${}^1n_2 \times {}^2n_1 = 1$ (b) ${}^1n_2 = {}^2n_1$
(c) $({}^1n_2) = ({}^2n_1)^2$ (d) None of these
UKPSC Lecturer (Mains) 2020

32. **Refractive index of the material of a prism having an angle A = 60°, which produces a minimum deviation of 30°, is:**
(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\sqrt{5}$ (d) $\frac{1}{\sqrt{2}}$
UKPSC Lecturer (Mains) 2020
UPPGT Physics 2016 (Exam : 02.02.2019)

33. **The refractive index of diamond is 2.42. The critical angle for total internal reflection for light passing from diamond to air is, about**
(a) 30^0 (b) 24.4^0
(c) 35^0 (d) 40^0
UP LT Grade Science 2018 (Exam : 29.07.2018)

34. **A convex lens has power P. It is cut into two halves along its principal axis. Further one piece (out of the two halves) is cut into two halves perpendicular to the principal axis (as shown in figure). Choose the incorrect option for the reported pieces.**



(a) Power of $L_1 = \frac{P}{2}$ (b) Power of $L_2 = \frac{P}{2}$
(c) Power of $L_3 = \frac{P}{2}$ (d) Power of $L_1 = P$
JEE MAIN 2022

35. **The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? (Assume the radii of curvature of the faces of lens are R1 and R2 respectively)**
(a) 1 (b) Infinite
(c) $\frac{R_1R_2}{R_1 - R_2}$ (d) Zero
JEE MAIN 2021

36. **A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 (x_1 x_2) > from the lens. The ratio of x_1 and x_2 is**
(a) 5 : 3 (b) 2 : 1
(c) 4 : 3 (d) 3 :1
JEE MAIN 2019

37. **The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image.**
(a) 1.25 cm (b) 2.5 cm
(c) 1.05 cm (d) 2 cm
IIT 2003

38. **A biconvex lens has radii of curvature, 20 cm each. If the refractive index of the material of the lens is 1.5, the power of the lens is**
(a) Infinity (b) +2 D
(c) +20 D (d) +5 D
NEET 2022

39. **A lens of large focal length and large aperture is best suited as an objective of an astronomical telescope since**

(a) A large aperture contributes to the quality and visibility of the images.
(b) A large area of the objective ensures better light gathering power.
(c) A large aperture provides a better resolution.
(d) All of the above

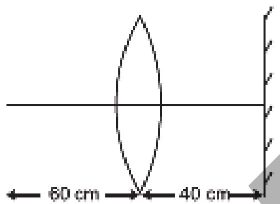
NEET 2021

40. A convex lens 'A' of focal length 20 cm and a concave lens 'B' of focal length 5 cm are kept along the same axis with a distance 'd' between them. If a parallel beam of light falling on 'A' leaves 'B' as a parallel beam, then the distance 'd' in cm will be

(a) 30 (b) 25
(c) 15 (d) 50

NEET 2021

41. A point object is placed at a distance of 60 cm from a convex lens of focal length 30 cm. If a plane mirror were put perpendicular to the principal axis of the lens and at a distance of 40 cm from it, the final image would be formed at a distance of



(a) 20 cm from the plane mirror, it would be a virtual image
(b) 20 cm from the lens, it would be a real image
(c) 30 cm from the lens, it would be a real image
(d) 30 cm from the plane mirror, it would be a virtual image

NEET 2021

42. An achromatic combination of two thin lenses of focal lengths f_1 and f_2 is achieved if separation between them is equal to

(a) $\frac{f_1}{2}$ (b) $\frac{1}{2}(f_1+f_2)$
(c) $\frac{f_2}{2}$ (d) $\frac{1}{2}(f_1-f_2)$

UP PGT - 2021
UP PSC Pre 2001

43. To minimize spherical aberration, two lenses of focal lengths f_1 and f_2 are placed at a distance equal to

(a) f_1-f_2 (b) f_1+f_2
(c) $\frac{f_1-f_2}{2}$ (d) $\frac{f_1+f_2}{2}$

UPPSC Ashram Paddhati PGT - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)
TGT 2010

44. Given the dispersive powers of the crown and flint glasses are 0.02 and 0.04 respectively, the focal lengths of the two components of an achromatic doublet of focal length 20 cm, will be:

(a) – 30 cm and + 15 cm
(b) – 40 cm and + 20 cm
(c) – 20 cm and + 10 cm
(d) – 15 cm and + 15 cm

UPPCS (Pre) 2002

45. Dispersive power of an achromatic doublet components are 0.021 and 0.042 and focal length of a convex lens is +75 cm. Focal length of the concave is:

(a) – 50 cm (b) – 75 cm
(c) – 150 cm (d) – 175 cm

UPPCS (Pre) 1999

46. Chromatic aberration is due to

(a) Reflection (b) Refraction
(c) Diffraction (d) Dispersion

(UPPGT 2000)

47. Chromatic aberration and spherical aberration both are minimum for two convex lenses of focal length f_1 and f_2 placed at a distance d apart from each other. The value of $\frac{f_1}{f_2}$ will be

(a) 2:1 (b) 3:1
(c) 2:3 (d) 4:1

(UPPGT 2010)

48. The dispersive powers of lenses used in an achromatic combination are in ratio 5 : 3. If focal length of the concave lens is 15 cm, the nature and focal length of other lens will be–

(a) Convex, 9 cm (b) Concave, 9cm
(c) Convex, 25cm (d) Concave, 25 cm

(UPPGT 2011)

49. The Brewsters angle i_b for an interface should be

(a) $30° < i_b < 45°$ (b) $45° < i_b < 90°$
(c) $i_b = 90°$ (d) $0° < i_b < 30°$

NEET 2020

50. What is the number of nodal points in a coaxial lens system?

(a) Zero (b) One
(c) Two (d) Any number

UPPSC Poly. Lect. 2021 Paper - I

51. A lens of refractive index μ_2 is immersed in a medium of refractive index μ_1. The radii of curvature of the surfaces of the lens are R_1 and R_2. The focal length (f) will be

(a) $\frac{1}{f}=(\mu_2-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(b) $\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(c) $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(d) $\frac{1}{f} = \left(\frac{\mu_1}{\mu_2} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

UP PGT Physics 2016 (Exam : 02.02.2019)

52. A concave lens (diverging lens) made of a material of refractive index μ_g is immersed in a medium of refractive index μ_l. Then lens behave as convergent lens when

(a) $\mu_l = \mu_g$ (b) $\mu_l > \mu_g$
(c) $\mu_l < \mu_g$ (d) It is not possible

RPSC School Lect. 26.07.2016

53. Two thin lenses, of focal lengths +15 cm and –12 cm, are placed in contact. The focal length of the combination lens will be

(a) +3 cm (b) +15 cm
(c) –12 cm (d) –60 cm

UP PGT - 2021

54. Two lenses have dispersive powers in ratio 2 : 3. These glasses are used to make achromatic lens of focal length 20 cm. The value of focal lengths of two lenses are

(a) $f_1 = 5$ cm, $f_2 = -10$ cm
(b) $f_1 = -10$ cm, $f_2 = 10$ cm
(c) $f_1 = 6.67$ cm, $f_2 = -10$ cm
(d) $f_1 = 10$ cm, $f_2 = -10$ cm

UP PGT - 2021

55. A telescope has an objective lens of focal length 150 cm and an eye piece of focal length 5 cm. If a 50 m tall tower at a distance of 1km is observed through this telescope (in normal setting), the angle formed by the image of the tower is θ, then θ is close to :

(a) 1° (b) 15°
(c) 30° (d) 56°

UKPSC Lecturer (Mains) 2020

56. Newton's formula for the focal length of a thin lens is

(a) $xx' = -ff'$ (b) $xx' = ff'$
(c) $xx' = -f^2f'$ (d) $xx' = f/f'$

UP PGT Physics 2016 (Exam : 02.02.2019)

57. When two lenses are in contact, the condition for achromatism is

(a) $\frac{\omega_1}{f_1} = \frac{\omega_2}{f_2}$ (b) $\omega_1 f_1 = \omega_2 f_2$

(c) $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ (d) $\omega_1 f_1 = -\omega_2 f_2$

UP PGT Physics 2016 (Exam : 02.02.2019)
UPPCS (Pre) 2002
UPPCS Pre 2007
UPPCS Pre 2006

58. Two lenses of powers +4D and –2D are kept in contact. The combinations focal length will be

(a) 100cm (b) 75cm
(c) 50cm (d) 25cm

UP PGT Physics 2016 (Exam : 02.02.2019)

59. For a thin convex lens, if object is at distance of x_1 from its first focus and image at a distance of x_2 from the second focus, if f_1 and f_2 are respectively the first and second focal lengths of the lens, the condition which is satisfied is

(a) $x_1 f_1 = x_2 f_2$ (b) $x_1 f_2 = x_2 f_1$
(c) $x_2 - x_1 = f_2 - f_1$ (d) $x_1 x_2 = f_1 f_2$

UP LT Grade Science 2018 (Exam : 29.07.2018)

60. A converging lenses of focal length f is placed in contact with a diverging lens of focal length 3f. The combination is

(a) A diverging lens of focal length 3f/2
(b) A converging lens of focal length 2f
(c) A diverging lens of focal length 3f
(d) A converging lens of focal length 3f/2

UP TGT Physics 2016

61. For a system of two coaxially separated lenses of focal lengths f_1 & f_2 having equal dispersive power to be achromatic, their separation should be

(a) $f_1 + f_2$ (b) $\frac{f_1 + f_2}{2}$

(c) $f_1 - f_2$ (d) $\frac{f_1 - f_2}{2}$

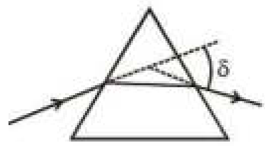
(UPPGT 2013)
UPPCS (Pre) 2004, 2008

62. The refracting angle of a prism is A and refractive index of the material of the prism is cot (A/2). Then the angle of minimum deviation will be -

(a) 180 – 2A (b) 90 – A
(c) 180 + 2A (d) 180 – 3A

JEE MAIN 2022

63. The angle of deviation through a prism is minimum when



(A) Incident ray and emergent ray are symmetric to the prism
(B) The refracted ray inside the prism becomes parallel to its base
(C) Angle of incidence is equal to that of the angle of emergence
(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below:

(a) Statements (A), (B) and (C) are true
(b) Only statement (D) is true
(c) Only statements (A) and (B) are true
(d) Statement (B) and (C) are true

JEE MAIN 2021

64. A ray is incident at an angle of incidence i on one surface of a small angle prism (with angle of prism A) and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, then the angle of incidence is nearly equal to :

(a) $\frac{2A}{\mu}$ (b) μA
(c) $\frac{\mu A}{2}$ (d) $\frac{A}{2\mu}$

NEET 2020

65. A thin prism having refracting angle 10° is made of glass of refractive index 1.42. This prism is combined with another thin prism of glass of refractive index 1.7. This combination produces dispersion without deviation. The refracting angle of second prism should be

(a) 6° (b) 8°
(c) 10° (d) 4°

NEET 2017

66. With the decrease in the prism angle, the angular dispersion between red and violet rays-

(a) Increases
(b) Decreases
(c) Does not change
(d) Depends on the incidence angle

UPPSC Poly. Lect. 2021 Paper - I

67. Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as:

(a) Astigmatism
(b) Myopia with Astigmatism
(c) Presbyopia with Astigmatism
(d) Myopia and hypermetropia

JEE MAIN 2021

68. An eye specialist prescribes spectacles having combination of convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination in diopters is

(a) + 1.5 (b) –1.5
(c) + 6.67 (d) – 6.67

IIT 1997

69. A person can see clearly objects only when they lie between 50 cm and 400 cm from his eyes. In order to increase the maximum distance of distinct vision to infinity, the type and power of the correcting lens, the person has to use, will be

(a) convex, + 2.25 dioptre
(b) concave, – 0.25 dioptre
(c) concave, – 0.2 dioptre
(d) convex, + 0.15 dioptre.

NEET - II 2016

70. Which defect of vision is corrected using cylindrical lenses in spectacles?

(a) Myopia
(b) Hypermetropia
(c) Astigmatism
(d) Loss of accommodation

UPTGT Science - 2021

71. To remove hypermetropia lens used is

(a) Concave (b) Convex
(c) Cylindrical (d) Plano-convex

(UPPGT 2003)

72. Of the following human eye are most sensitive to

(a) red light (b) violet light
(c) blue light (d) green light

TGT 2004 (Cancelled)

73. The amount of light entering the eye is controlled by the

(a) pupil (b) iris
(c) cornea (d) eye lens

TGT 2004 (Cancelled)

74. A myopic eye can be corrected by using a:

(a) convex lens (b) cylindrical lens
(c) plano–convex lens (d) concave lens

TGT 2013
UPPCS (Pre) 2004
(UPPGT 2013)

75. A man cannot see objects distant more than 40 cm. What is the power of the lens needed to allow him to see very distant objects?

(a) – 3.5 D (b) + 3.5 D
(c) – 2.5 D (d) + 2.5 D

UP PGT Physics 2016 (Exam : 02.02.2019)

76. A short-sighted man can clearly see the objects up to a distance is 1.5 m. The power of the lens of spectacles necessary for the remedy of this defect is

(a) -0.67D (b) -1.50D
(c) +0.67D (d) +1.50D

UP LT Grade Science 2018 (Exam : 29.07.2018)

77. The aperture of the objective is 24.4 cm. The resolving power of this telescope. If a light of wavelength 2440 Å is used to see the object will be

(a) 8.1×10^{6} (b) 10.0×10^{7}
(c) 8.2×10^{5} (d) 1.0×10^{-8}

JEE MAIN 2022

78. **Assume that light of wavelength 600 nm is coming from a star. The limit of resolution of telescope whose objective has a diameter of 2 m is :**
(a) 1.83×10^{-7} rad
(b) 7.32×10^{-7} rad
(c) 6.00×10^{-7} rad
(d) 3.66×10^{-7} rad
NEET 2020

79. **Ratio of focal lengths of two lenses of Huygens eyepiece is -**
(a) 1 : 1
(b) 2 : 1
(c) 3 : 1
(d) None of the above
UPPSC Poly. Lect. 2021 Paper - I

80. **An astronomical telescope has a large aperture to :**
(a) Reduce spherical aberration
(b) Have high resolution
(c) Increase span of observation
(d) Have low dispersion
UKPSC Lecturer (Mains) 2020

81. **If d_0, d_e, f_o, f_e are respectively diameters, focal lengths of objective and eyepiece in a telescope, the optical ratio is**
(a) $\frac{d_o}{d_e} = \frac{f_e}{f_o}$
(b) $\frac{d_e}{d_o} = \frac{f_o}{f_e}$
(c) $\frac{d_o}{d_e} = \frac{f_e}{f_o} + 1$
(d) $\frac{d_o}{d_e} = \frac{f_e}{f_o} - 1$
TGT 2003

82. **A magnifying glass comprises a simple**
(a) Convex lens
(b) Convex mirror
(c) Concave lens
(d) Concave mirror
TGT 2004 (Cancelled)

83. **If the focal length of two convex lenses in an eyepiece made of the same material be 4.0 and 2.0 cm and this combination be achromatic, the distance between them will be (in cms) :**
(a) 2
(b) 3
(c) 4
(d) 8
UPPCS PRE 1996

84. **Focal length of the objective and eyepiece of a telescope are 100 cm and 10 cm. respectively. Magnification of the telescope, when final image is formed at infinity, is :**
(a) infinity
(b) 100
(c) 0.1
(d) 10
UPPCS PRE 1996

85. **In Huygens eyepiece, the focal lengths of an eye lens and field lens are respectively.**
(a) f and 3f
(b) f and 2f
(c) 2f and 3f
(d) 3f and 2f
UPPCS Pre 2010

86. **In eye piece, by using two lenses, instead of a single lens–**
(a) effective focal length always increases
(b) power remains unchanged
(c) field of view decreases
(d) defects of chromatic and spherical aberrations are removed in image and the field of view increases
(UPPGT 2011)

87. **The focal length of one of the lenses in a Ramsden eyepiece is f_1. The eyepiece will have focal length :**
(a) $2f_1/3$
(b) $3f_1/4$
(c) f_1
(d) $5f_1/4$
UPPCS PRE 1996

88. **The distance between two plane convex lenses in Huygen's eyepiece is:**
(a) f
(b) 2f
(c) $\frac{f}{2}$
(d) 4f
UPPCS Pre 1994

89. **Ramsden eyepiece consists of two plano convex lenses of focal lengths f each. The lenses are separated by distance:**
(a) 2f
(b) f
(c) $\frac{2}{3}f$
(d) $\frac{3}{4}f$
UPPCS (Pre) 2003

90. **The wave fronts of a light wave travelling in vacuum are given by x + y + z = c. The angle made by the direction of propagation of light with X-axis is-**
(a) 0^0
(b) 45^0
(c) 90^0
(d) $\cos^{-1}\sqrt{\frac{1}{3}}$
UPPCS (Pre) 2004
(UPPGT 2013)

Answer Key

1. (d)	2. (d)	3. (c)	4. (a)	5. (c)	6. (d)	7. (a)	8. (a)	9. (c & d)	10. (b)
11. (b)	12. (d)	13. (c)	14. (a)	15. (d)	16. (c)	17. (b)	18. (d)	19. (d)	20. (c)
21. (c&d)	22. (d)	23. (c)	24. (b)	25. (a)	26. (c)	27. (a)	28. (b)	29. (d)	30. (d)
31. (a)	32. (b)	33. (b)	34. (a)	35. (b)	36. (d)	37. (b)	38. (d)	39. (d)	40. (c)
41. (a)	42. (b)	43. (a)	44. (c)	45. (c)	46. (d)	47. (b)	48. (a)	49. (b)	50. (c)
51. (b)	52. (b)	53. (d)	54. (c)	55. (d)	56. (b)	57. (c)	58. (c)	59. (d)	60. (d)
61. (b)	62. (a)	63. (a)	64. (b)	65. (a)	66. (d)	67. (b)	68. (b)	69. (b)	70. (c)
71. (b)	72. (d)	73. (a)	74. (d)	75. (c)	76. (a)	77. (c)	78. (d)	79. (c)	80. (b)
81. (b)	82. (a)	83. (b)	84. (d)	85. (a)	86. (d)	87. (b)	88. (b)	89. (c)	90. (d)

04.

Wave Optics

(1) Introduction

■ Wave Optics :-

- The concept of wave nature of light was first introduced in 1678, by the Dutch Physicist **Christian Huygens.** Wave optics describes the connection between wave and rays of light.
- Light consists of very small invisible elastic particles called corpuscles, which travels in vacuum with a speed of 3×10^8 m/s.
- The size of corpuscles of different colours of light are different, the theory could explain reflection and refraction.

■ Newton's Corpuscular theory:-

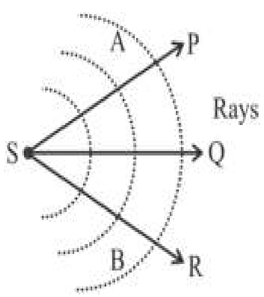
- Newton thought that light is made up of tiny, light and elastic particles called corpuscles which are emitted by luminous body.
- The corpuscles travel with speed equal to the speed of light in all directions in straight line.
- The corpuscular theory explains that light carry energy and momentum, light travels in a straight line, light propogate in vacuum, laws of reflection and refraction.
- The corpuscular theory fails to explain interference, diffraction and polarization.
- The corpuscles of different colour are of different sizes (red corpuscles larger than blue corpuscles).

■ Huygen's wave theory:-

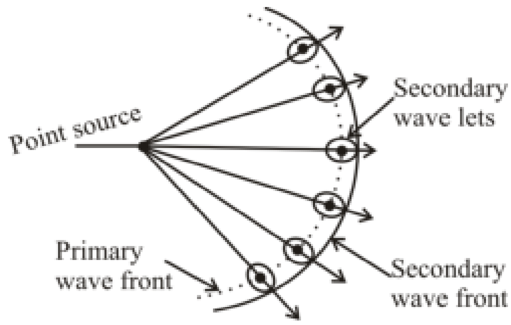
- According to this, a luminous body is a source of disturbance in hypothetical medium **ether.** This medium pervades all space.
- The wave carries energy and momentum. Huygen assumed that the waves were longitudinal. Further when polarization was discovered, then to explain it, light waves were assumed to be transverse in nature by Fresnel.
- This theory explains successfully, the phenomenon of interference and diffraction apart from other properties of light.
- The Huygen's theory fails to explain photo-electric effect, Compton's effect etc.
- The wave theory introduces the concept of wave front.

■ Wave Front :-

A wave front is defined as the continuous locus of all the particles of a medium, which are vibrating in the same phase.

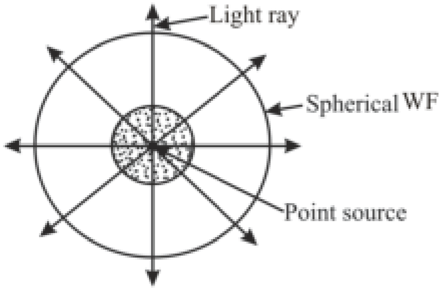
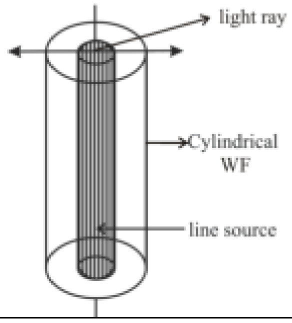
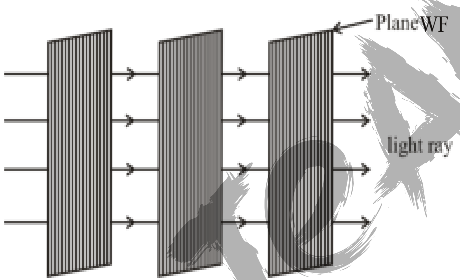


Where -
S = source of light
AB = wave front and
SP, SQ and SR are rays of light

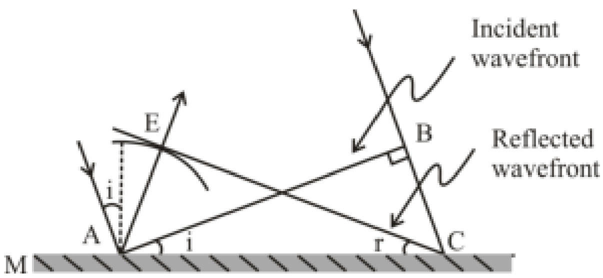
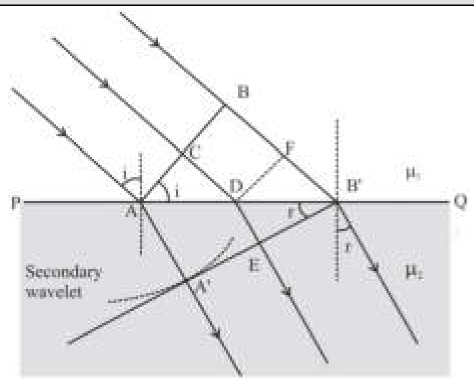


- The direction of propagation of light is perpendicular to the wave front.
- Every point on the given wave front acts as a source of new disturbance called secondary wavelets which travel in all directions with velocity of light in the medium.
- A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new wave front at that instant. This is called secondary wave front

Different types of wave fronts:-

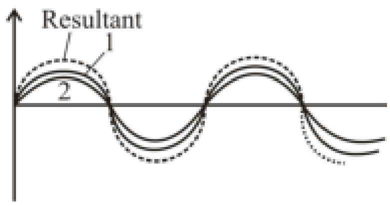
Type of wave fronts (WF)	Intensity	Amplitude
Spherical– when source of light is a point source  	$I \propto \frac{1}{r^2}$	$A \propto \frac{1}{r}$
Cylindrical - when source of light is linear  	$I \propto \frac{1}{r}$	$A \propto \frac{1}{\sqrt{r}}$
Plane – When the source of light is very far off  	$I \propto r^o$	$A \propto r^o$

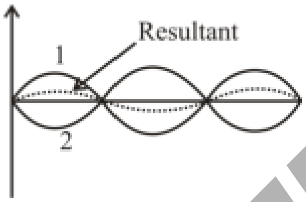
■ Reflection and Refraction of wave front –

Reflection	Refraction
 • EC = AB • $\angle$ **i** = $\angle$ **r**	 • $\frac{BB'}{AA'} = \frac{v_1}{v_2} = \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ where, v_1 and v_2 are velocity of wavefront in medium 1 and 2

(2) Superposition of Waves

When two or more than two wave superimpose over each other at a common particle of the medium then the resultant displacement (y) of the particle is equal to the vector sum of the displacements (y_1 and y_2) produced by individual waves, i. e. $y = y_1+y_2$.





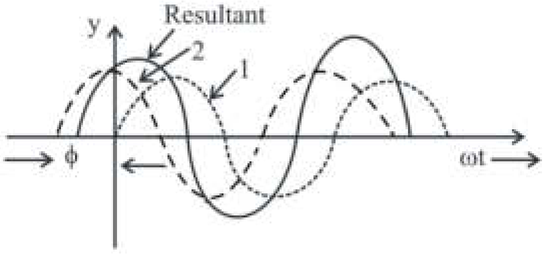
■ Important Terms:–

- **Phase –** The argument of sine or cosine in the expression for displacement of a wave is defined as the phase. For displacement $y = a \sin \omega t$ where, ωt = Phase or instantaneous phase.
- **Phase difference (ϕ) :–**
  The difference between the phase of two waves at a point is called phase difference, i.e. if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t + \phi)$ so phase difference = ϕ
- **Path difference (Δ)**
  The difference in path lengths of two waves meeting at a point is called path difference between the waves at that point .It is given as -
  $$\Delta = \frac{\lambda}{2\pi} \times \phi$$
- **Time difference (T. D)**
  Time difference between the waves meeting at a point is –
  $$T.D = \frac{T}{2\pi} \times \phi$$

■ Resultant Amplitude and Intensity –

Let us consider two waves that have the same frequency but have a certain fixed (Constant) phase difference between them.

Their super position is shown below –



Let the two waves are be -

$y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t + \phi)$

where, a_1, a_2 = Individual amplitudes

ϕ = Phase difference between the waves at an instant when they are meeting at a point.

- **Resultant amplitude:–**
  The resultant wave can be written as
  $y = A \sin (\omega t + \phi)$
  Where A = resultant amplitude = $\sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$

- **Resultant intensity:–**
  As we know,
  Intensity $\propto$ (Amplitude)^2
  $I_1 = K a_1^2$, $I_2 = K a_2^2$
  $I = K A^2$ (K is a proportionality constant)
  Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$
  For two identical source $I_1 = I_2 = I_0$
  $I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\phi$
  $I = 2I_0 + 2I_0\cos\phi = 2I_0(1+\cos\phi) = 4I_0\cos^2\dfrac{\phi}{2}$

(3) Coherence

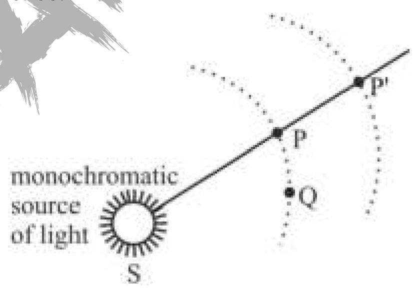
The phase relationship between two light waves can vary from time to time and from point to point in space. The property of definite phase relationship is called coherence.

1. **Temporal Coherence:–** In a light source a light wave (photon) is produced when an excited atom goes to the ground state and emits light.

- The duration of this transition is about 10^{-9} to 10^{-10} sec thus the emitted wave remains sinusoidal for this time is known as coherence time (τ_c)
- Definite phase relationship is maintained for a length $L = c\,\tau_c$, called coherence length ($c = 3\times10^8$ m/s).
  For neon $\lambda = 6328$ Å, $\tau_c \approx 10^{-10}$ sec and L = 0.03m.
  For cadmium $\lambda = 6438$ Å, $\tau_c = 10^{-9}$ sec and L = 0.3m
  For Laser $\tau_c = 10^{-5}$ sec and L = 3 km
- The spectral line width $\Delta\lambda$ is related to coherence length L and coherence time τ_c.

$$\Delta\lambda = \frac{\lambda^2}{c\lambda_c} \quad \text{or} \quad \Delta\lambda = \frac{\lambda^2}{L}$$

(2) **Spatial coherence:–** Two points in space are said to be spatially coherence if the waves reaching there maintains a constant phase difference.



Points P and Q are at the same distance from S, they will always be having the same phase. Length between point P and P' is much less than the coherence length i.e. $PP' \ll c\,\tau_c$

Methods of obtaining coherent sources:–

Two coherent sources are produced from a single source of light by two methods-

Division of wave front	Division of amplitude
• The wave front emitted by a narrow source is divided in two parts by reflection, refraction or diffraction. • The coherent sources so obtained are imaginary they are produced in Fresnel's bi-prism and young's double slit experiment etc.	• In this arrangement light wave is partly reflected (50%) and partly transmitted (50%) to produce two light rays. • The amplitude of wave emitted by an extended source of light is divided in two parts by partial reflection and partial refraction. • The Coherent sources obtained are real and are obtained in Newton's rings, Michelson's interferometer, colours in thin films.

(4) Interference of light

When two waves of exactly same frequency (coming from two coherent sources) travels in a medium, in the same direction simultaneously then due to their superposition, at some point's intensity of light is maximum while at some other point's intensity is minimum. This phenomenon is called interference of light.

It is of following two types:–

1. **Constructive interference:–** When the waves meet a point with same phase, constructive interference is obtained at that point (i.e. maximum light).

- Phase difference between the waves at the point of observation, $\phi = 0°$ or $2n\pi$
- Path difference between the waves at the point of observation, $\Delta = n\lambda$ (i.e. even multiple of $\frac{\lambda}{2}$)
- Resultant amplitude at the point of observation will be maximum

$$A_{max} = a_1 + a_2, \text{ if } a_1 = a_2 = a_0 \Rightarrow A_{max} = 2a_0$$

- Resultant intensity at the point of observation will be maximum

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$\text{if } I_1 = I_2 = I_0 \Rightarrow I_{max} = 4\,I_0$$

2. **Destructive interference:–** When the waves meet at a point with opposite phase, destructive interference is obtained at the point (i.e. minimum light).

- Phase difference $\phi = 180°$
  or $(2n - 1)\pi$; $n = 1, 2, \ldots\ldots\ldots$
  or $(2n + 1)\pi$; $n = 0, 1, 2\ldots\ldots\ldots$
- Path difference $\Delta = (2n - 1)\frac{\lambda}{2}$ (i.e. odd multiple of $\frac{\lambda}{2}$)
- Resultant amplitude at the point of observation will be minimum,

$$A_{min} = a_1 - a_2, \text{ If } a_1 = a_2 \Rightarrow A_{min} = 0$$

- Resultant intensity at the point of observation will be minimum,

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

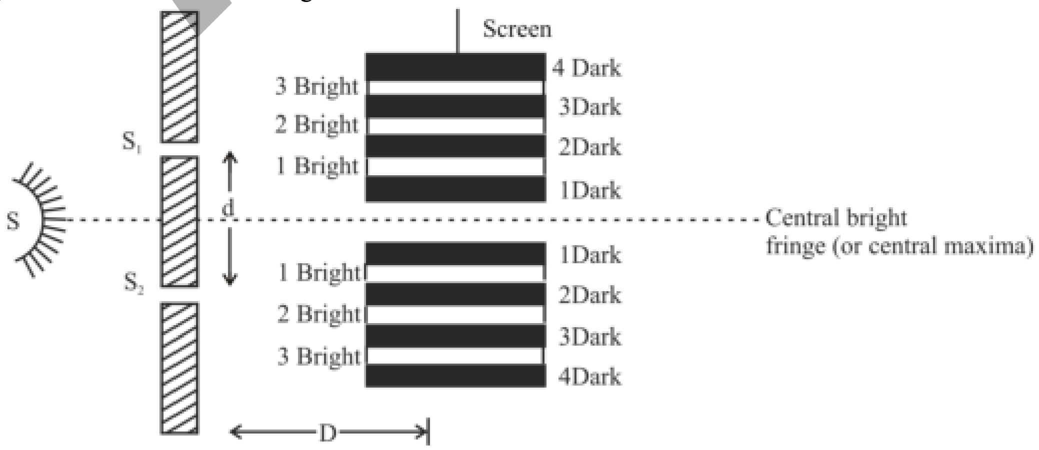
$$\text{If } I_1 = I_2 = I_0 \Rightarrow I_{min} = 0$$

3. Super position of waves of random phase difference – When two waves (or more waves) having random phase difference between them super impose, then no interference pattern is produced. Then the resultant intensity is just the sum of the two intensities –

$$I = I_1 + I_2$$

■ Young's Double Slit Experiment (YDSE) –

Monochromatic light (single wavelength) falling on two narrow slits S_1 and S_2 which are very close together acts as two coherent sources, when waves coming from two coherent sources (S_1, S_2) superimpose on each other, on screen then alternate bright and dark bands are obtained on the screen. These bands are called Fringes.



d = Distance between slits

D = Distance between slits and screen

λ = wavelength of monochromatic light emitted from source.

- Central fringe is always bright, because at central position $\phi = 0°$ or $\Delta = 0$
- The fringe pattern obtained due to a slit is brighter than that due to a point.
- If the slit widths are unequal, the minima will not be complete dark. For very large width uniform illumination occurs.
- If one slit is illuminated with red light and the other slit is illuminated with blue light, no interference pattern is observed on the screen.
- If the two coherent sources consist of object and it's reflected image, the central fringes is dark instead of bright one.

Useful results

- **Path difference**:–

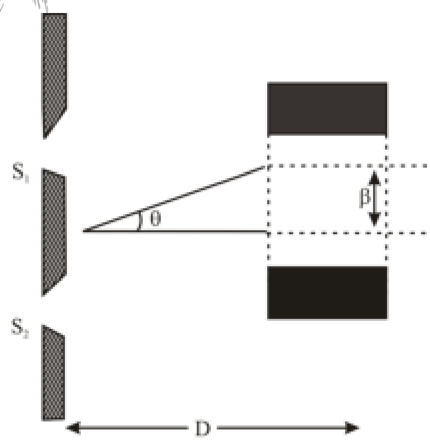
Path difference between the interfering waves meeting at a point P on the screen is given by $\Delta = \Delta_i + \Delta_f$,where Δ_i = initial path difference between the wave before the slit and Δ_f = Path difference between the waves after emerging from the slit.

In case $\Delta_i = 0$ (commonly used Condition) $\Rightarrow \Delta = \Delta_f = \dfrac{xd}{D} = d \sin\theta$

where x is the position of point P from central maxima.

	Bright fringe	**Dark fringe**
Path difference (Δ)	$\Delta = n\lambda$	$\Delta = \left(\dfrac{2n-1}{2}\right)\lambda$
Location of n^{th} fringe from (x_n) Central maxima	$x_n = \dfrac{n\lambda D}{d} = n\beta$	$x_n = \left(\dfrac{2n-1}{2}\right)\dfrac{\lambda D}{d} = \left(\dfrac{2n-1}{2}\right)\beta$
No. of fringe (n)	$n = 0, \pm, 1, \pm 2$,........	$n = \pm 1, \pm 2$,........
Fringe width (β)	$\beta = \dfrac{\lambda D}{d}$	
Angular fringe width (θ)	$\theta = \dfrac{\lambda}{d} = \dfrac{\beta}{D}$	

- **Fringe width (β) :-**



The separation between any two consecutive bright or dark fringes is called fringe width. In YDSE all fringes are of width $\beta = \dfrac{\lambda D}{d}$ and angular fringe width $\theta = \dfrac{\lambda}{d} = \dfrac{\beta}{D}$

- In YDSE, if n_1 fringes are visible in a field of view with light of wave length λ_1, while n_2 with light of wave length λ_2 in the same field, then–

$$n_1 \lambda_1 = n_2 \lambda_2$$

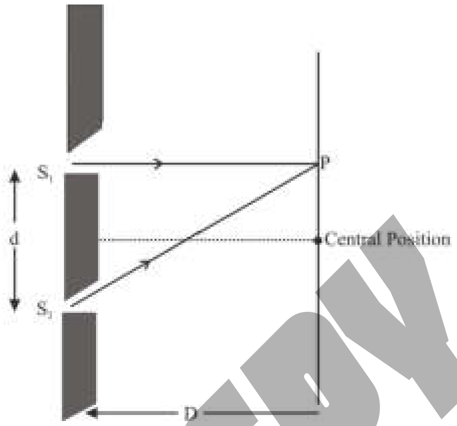
- **Fringe visibility (V):-**With the help of visibility, Knowledge about coherence, fringe contrast an interference pattern is obtained.

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 2\frac{\sqrt{I_1 I_2}}{(I_1 + I_2)}$$

If $I_{min} = 0$, V = 1 (maximum) i.e, fringe visibility will be best.

Also if $I_{max} = 0$, V = –1 and if $I_{max} = I_{min}$, V = 0

- **Missing wave length in front of one slit in (Y D S E) –**



Suppose P is a point of observation in front of slit S_1 as shown then missing wavelength at P is-

$$\lambda = \frac{d^2}{(2n-1)D}$$

By putting n = 1, 2, 3, ... missing wavelengths are,

$$\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}$$

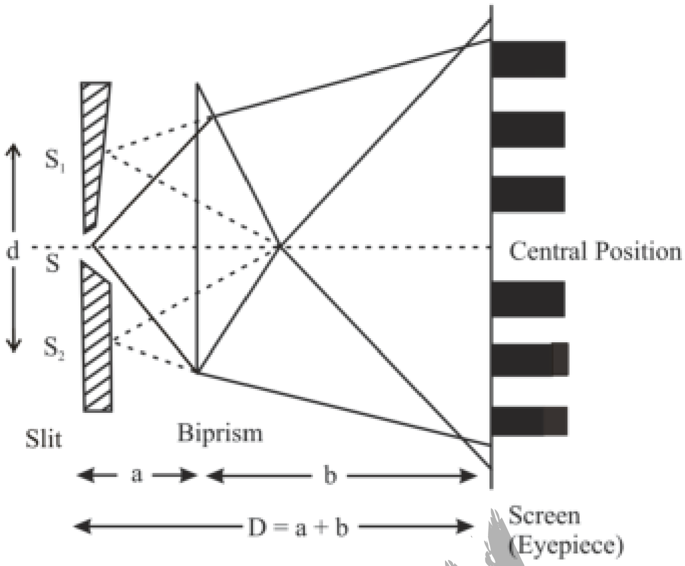
Special case:

- When a transparent sheet having refractive index μ and thickness t, introduced in the path of one ray of double slit experiment then, shifting of fringe is given by $x = (\mu - 1)\frac{Dt}{d}$
- When complete setup submerged in medium μ then $\lambda' = \frac{\lambda}{\mu}$ & $w' = \frac{w}{\mu}$

■ Fresnel's Biprism –

- In Fresnel's biprism experiment, two prism which are connected through their bases are used.
- It is an optical device of producing interference of light Fresnel's biprism is made by Joining base to base two thin prism of very small angle.
- Acute angle of Prism is about α= $\left(\frac{1}{2}\right)^{\circ}$ or 30' and obtuse angle of Prism is about 179°. When a monochromatic light source is kept in front of biprism, two coherent virtual sources S_1 and S_2 are produced then interference fringes are found on the screen placed behind the biprism. Interference fringes are formed in the limited region which can be observed with the help of eye piece.
- Fringe width is measured by a micrometer attached to the eye piece.
- Fringes are equal width and its value is $\beta = \frac{\lambda D}{d}$

Experimental Arrangement-



- Let the separation between S_1 and S_2 be d and its distance of slits and the screen from the biprism be a and b respectively. i.e. $D = (a + b)$

  If angle of prism is α and refractive index is μ then,

$$d = 2a(\mu - 1)\alpha$$

$$\lambda = \frac{\beta\left[2a(\mu-1)\alpha\right]}{(a+b)}$$

$$\beta = \frac{\lambda D}{d} = \frac{(a+b)\lambda}{2a(\mu-1)\alpha}$$

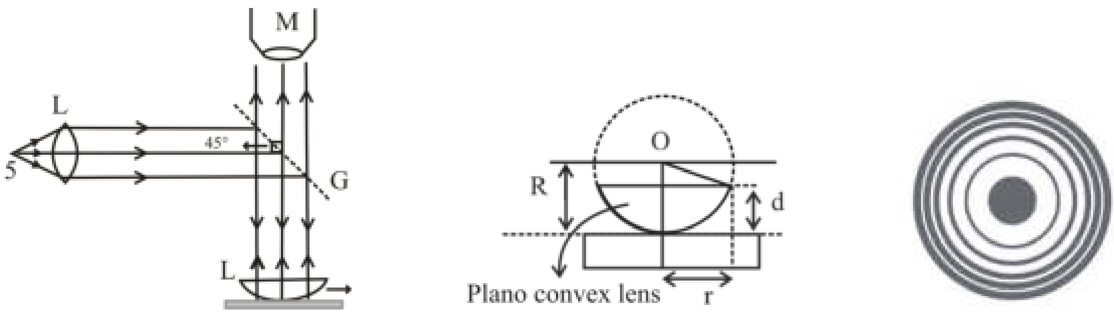
- If a convex lens is mounted between the biprism and eye piece there will be two positions of lens when the sharp images of coherent sources will be observed in the eyepiece.

  The Separation of the images in the two positions are measured.

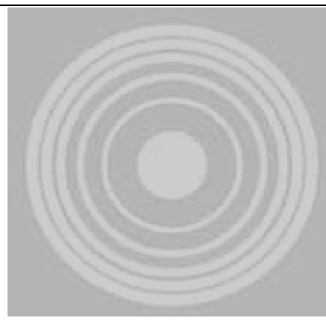
  Let these be d_1 and d_2 then

$$d = \sqrt{d_1 d_2} \Rightarrow \lambda = \frac{\beta d}{D} = \frac{\beta\sqrt{d_1 d_2}}{(a+b)}$$

■ Newton's Rings:–



- If we place a plano–convex lens on a plane glass surface, a wedge shaped thin film of air is formed between the curved surface of the lens and surface of plane glass.
- If we allow monochromatic light to fall normally on the surface of lens then circular interference fringes of radius r can be seen in the reflected light.These circular fringes are called Newton's rings.
- These Newton's rings are formed by reflected rays and also by transmitted rays.
- The central fringes is a dark spot then there are alternate bright and dark fringes (Ring shape).

	Reflected rays	Transmitted rays
Bright rings n = 1, 2, 3…….	$r_n = \sqrt{2n-1}\sqrt{\dfrac{\lambda R}{2\mu}}$	$r_n = \sqrt{\dfrac{nR\lambda}{\mu}}$
Dark rings n = 0,1,2,3……	$r_n = \sqrt{\dfrac{rn\lambda}{\mu}}$	$r_n = \sqrt{2n-1}\sqrt{\dfrac{\lambda R}{2\mu}}$
Newton rings		
	at center minima (dark)	at center maxima (Bright)
	'n' increases → spacing decrease	'n' increases → spacing increases
Intensity	Higher, so Prefer for study	lower

- Newton's ring arrangement is used for determining the wavelength of monochromatic light. For this the diameter of n^{th} dark ring (D_n) and $(n + p)^{th}$ dark Ring (D_n + p) are measured then–

$$D_n^2 = 4n\lambda R ----(i)$$

Let us Consider n^{th} and $(n+p)^{th}$ dark rings, then

$$D_{n+p}^2 = 4(n+P)\lambda R ---(ii)$$

Subtracting Equation (i) & (ii) we get

$$D_{n+p}^2 - D_n^2 = 4P\lambda R$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$$

- Spacing between Newton's rings $= \left(\sqrt{n} - \sqrt{n-1}\right)\lambda R$

(5) Diffraction of light

- The Phenomenon of diffraction was first discovered by Grimaldi its experimental study was done by Newton and young. The theoretical explanation was first given by Fresnel.
- The phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wave length of light is called diffraction.
- The phenomenon resulting from the Superposition of secondary wavelets originating from different parts of the same wave front is defined as diffraction of light.
- Diffraction is the characteristic of all types of waves.
- Greater the wavelength of wave, higher will be its degree of diffraction.
- As $\lambda_{sound} > \lambda_{light}$ diffraction is more easily observed in sound as compared to light.

■ Diffraction at Single Slit (Fraunhofer Diffraction)

(i) Distance of n^{th} Secondary minima (Dark) from central maxima

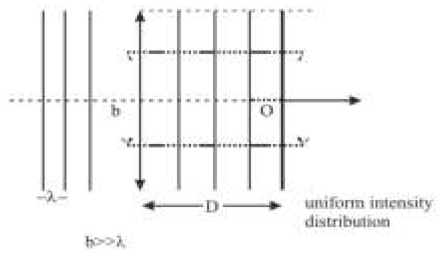
$$X_n = D.\theta = \frac{n\lambda D}{b} = \frac{n\lambda f}{b},$$

Where D = Distance between slit and screen; $f \approx D$ = Focal length of Converging lens.

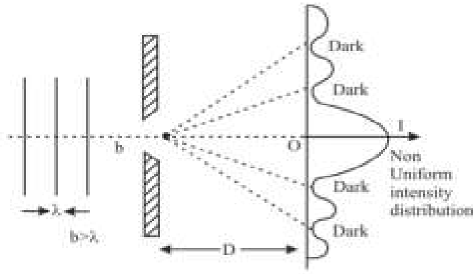
$$\left(\because \sin\theta \approx \theta = \frac{n\lambda}{d}\right)$$

(ii) Distance of n^{th} Secondary maxima bright from central maxima

$$X_n = D.\theta = \frac{(2n-1)\lambda D}{2b} = \frac{(2n-1)\lambda f}{2b}$$



(A) Size of the slit is very large compared to wave length.



(B) Size of the slit is comparable to wavelength.

■ Types of Diffraction:–

- **Fresnel diffraction:–** If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel diffraction.
- **Fraunhofer diffraction:–** In this case both source and screen are effectively at finite distance from the diffracting device.

(6) Polarisation & Polaroids

- **Polarisation of light:-** Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. Generally light described as electric field oscillations.
- **Unpolarised light:–** An ordinary light having vibration along all possible direction perpendicular to the direction of propagation of light is called an unpolarised light
- **Plane polarised light:–** When ordinary light is passed through a tourmaline, calcite or quartz crystal the transmitted light will have electric vectors in a particular direction parallel to the axis of crystal. This light is then known as plane polarised light with intensity variation having zero minima.

  A plane containing the vibrations of polarised light is called plane of vibration. A plane perpendicular to the plane of vibration is called plane of polarisation.

■ Optical Activity – The ability of a substance to rotate the plane of polarisation of a beam of light that is passed through it. In plane-polarized light, the vibrations of the electric field are confined to a single plane.

Optical activity can be found out from the magnitude of the rotation of the plane of polarized light.

Magnitude of rotation of optical activity depends on-

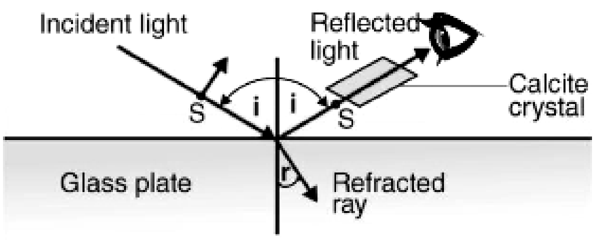
1. Nature of the substance.
2. Length of the liquid column through which the light passes.
3. The temperature of the solution.
4. The wavelength of light.
5. The concentration of the solution.

Polarisation can take place only in transverse waves.

- **Nicol Prism** –

  Nicol Prism is a convenient optical device for producing and analysing the plane polarised light. It was invented by William Nicol in 1826.

  The Nicol prism consists of two calcite crystal cut at 68° with its principal axis joined by a glue called Canada Balsam.
- **Brewster's Law –**



- When unpolarised light is incident at an angle of polarisation (i_p) on the interface separating air from a medium of refractive index μ, then reflected light becomes fully polarized.
- The relation between the angle of polarisation (i_p) and refractive index (μ) of the medium is given as,

$$\mu = \tan i_p$$
$$i_p + r = 90^\circ$$

If θ be the angle between the reflected and refracted light beams then

$$i_p + \theta + r = 180^\circ$$
$$\theta = 180^\circ - (i_p + r) = 180^\circ - 90^\circ = 90^\circ$$

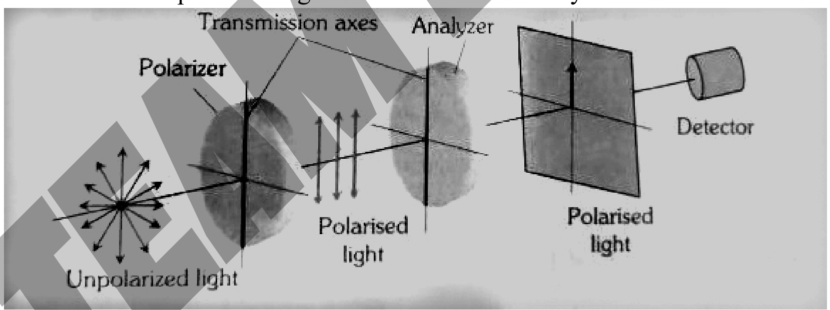
■ **Rayleigh Scattering:-** According to Rayleigh Scattering law, the intensity of scattering light is inversely proportional to the fourth power of wavelength.

- Scattering intensity $I_s \propto \frac{1}{\lambda^4}$
- Light intensity of different wavelengths ratio. $\frac{I_1}{I_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$

- Rayleigh scattering is proportional to (ν^4) where $\nu \to$ frequency.
- Rayleigh scattering explained the phenomenon of sun appears red at sunrise or sunset.

■ **Polaroids:–**

- It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal.
- It is a thin film of ultramicroscopic crystals of quinine iodosulphate with their optical axis parallel to each other.
- Polaroids allow the light oscillations parallel to the transmission axis, to pass through them.
- The crystal or polaroid on which unpolarised light is incident is called polariser.
- Crystal or polaroid on which polarised light incident is called analyser.



- In the above figure transmission axis of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser.
- If transmission axis of the analyser is perpendicular to the polariser then no light passes through the analyser.

Uses of polaroids :–

- Polaroids are used in sun glasses. They protect the eyes from glare.
- The pictures taken by a stereoscopic camera are with the help of polaroid spectacles. It helps in creating three dimensional effect.
- The wind shield of an automobile is made of polaroid. Such a wind shield protects the eyes of the driver of the automobile from the dazzling light of the approaching vehicles.

■ **Zone plate :-** A zone plate is a device used to focus light or other things exhibiting wave character.

- Zone plate has many focii. It acts like a multi focii converging lens and forms a real image.
- Zone plate behaves like a convex lens and produces an image of a source of light on the screen placed at a suitable distance.
- Focal length of zone plate depends on wavelength of light used.
- In case of zone plate the image is formed by the diffraction phenomenon.
- A zone plate can be used over a wide range of wavelengths from micro waves to X - rays.
- Radii of zones are $r_n = \sqrt{nb\lambda}$

- Focal length of the zone plate is given by $f_n = b = \frac{r_n^2}{n\lambda}$ or $\frac{r_n^2}{(2p-1)n\lambda}$

Where λ = wavelength of light
b = position of the screen say b cm from the zone plate.
p = Number of focal length

■ Quarter Wave plate (QWP)

It is a plate of doubly refracting crystal cut parallel to optical axis of the crystal to introduce a path difference of $\lambda/4$ or a phase difference of $\pi/2$ between extra ordinary and ordinary components of the polarized light.

For quarter wave plate, the path difference = $\frac{\lambda}{4}$

$$(\mu_0 - \mu_e)\, t = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

μ_0 = Refractive index of ordinary ray

μ_e = Refractive index of extra ordinary ray.

■ Half wave plate:-

It is a doubly refracting crystal with face parallel to the optical axis.

It introduces a path difference of $\frac{\lambda}{2}$ or a phase difference of π between ordinary and extra ordinary components of light.

For Half wave plate, thickness of the plate (t)

$$t = \frac{\lambda}{2(\mu_0 - \mu_e)}$$

Where λ = wavelength of incident light.

■ Dispersive power of plane

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wave length between two spectral line.

The diffraction of the n^{th} order principal maximum for a wavelength λ is given by

$$(a + b)\sin\theta = n\lambda$$

Dispersive power is given by $\frac{dQ}{d\lambda}$

$$\frac{dQ}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

where,
n = order of spectrum
Q = Angle of diffraction for the n^{th} order principal maximum of wavelength λ
(a + b) = Grating constant (Unit is in cm.)

Note- Deviation produced in a plane transmission grating does not depends on refractive index of grating

$$d\theta = \frac{n}{(a+b)\cos\theta}\, d\lambda$$

■ Birefringence

A single incident ray of unpolarized light entering an anisotropic medium splits into two rays (ordinary and extra ordinary), each travelling in a different direction (mutually orthogonal planes). It is also called double refraction.

• The process of double refraction occurs only in some crystals are calcite, quartz .

■ Malus law:–

- This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.

- $I = I_0 \cos^2\theta$ and $A^2 = A^2_o \cos^2\theta \Rightarrow A = A_0 \cos\theta$
  If $\theta = 0^\circ$, $I = I_0$, $A = A_0$
  If $\theta = 90^\circ$, $I = 0$, $A = 0$
- If I_i = Intensity of unpolarised light, $S_0\, I_0 = \frac{I_i}{2}$, i.e. if an unpolarised light is converted into plane polarised light (say by Passing it through a polaroid or a nicol-prism), its intensity becomes half i.e. $I = \frac{I_i}{2}\cos^2\theta$

■ Key Points :–

- In interference redistribution of energy takes place in the form of maxima and minima.
- Average intensity: $I_{av} = \frac{I_{max} + I_{min}}{2} \Rightarrow I_{av} = I_1 + I_2 = a_1^2 + a_2^2$
- Ratio of maximum and minimum intensities –

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1}\right)^2$$

$$\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{a_1}{a_2} = \left[\frac{\sqrt{\frac{I_{max}}{I_{min}}} + 1}{\sqrt{\frac{I_{max}}{I_{min}}} - 1}\right]$$

- If two waves having equal intensity ($I_1 = I_2 = I_0$) meet at two locations P and Q with path difference Δ_1 and Δ_2 respectively then the ratio of resultant intensity at point P and Q will be

$$\frac{I_P}{I_Q} = \frac{\cos^2\frac{\phi_1}{2}}{\cos^2\frac{\phi_2}{2}} = \frac{\cos^2\left(\frac{\pi\Delta_1}{\lambda}\right)}{\cos^2\left(\frac{\pi\Delta_2}{\lambda}\right)}$$

- The angular thickness of fringe width is defined as $\delta = \frac{\beta}{D} = \frac{\lambda}{d}$, which is independent of the screen distance D.
- Central maxima means the maxima formed with zero optical path difference. It may be formed anywhere on the screen.
- All the wavelengths produce their central maxima at the same position.
- The wave with smaller wavelength forms its maxima before the wave with longer wavelength.
- The first maxima of violet colour is closest and that for the red colour is farthest.
- Fringes with red light are thicker than those with blue light ($\because \lambda_{red} > \lambda_{blue}$)
- In an interference pattern, whatever energy disappears at the minimum appears at the maximum.
- In YDSE, the n^{th} maxima always comes before the n^{th} minima.
- In YDSE, the ratio $\frac{I_{max}}{I_{min}}$ is maximum when both the sources have same intensity.
- For two interfering waves initial phase difference between them is ϕ_0 and phase difference due to path difference between them is ϕ then total phase difference will be –

$$\phi = \phi_0 + \phi' = \phi_0 + \frac{2\pi}{\lambda}\Delta$$

- Sometimes maximum number of maximas or minimas are asked in the questions which can be obtained on the screen for this we use the fact that value of $\sin\theta$ (or $\cos\theta$) cannot be greater than 1.
  For example–

  $\sin\theta = \frac{n\lambda}{d}$ (for maximum intensity)

  $\because \sin\theta \ngtr 1$ $\qquad\qquad$ $\therefore \frac{n\lambda}{d} \ngtr 1$ or $n \ngtr \frac{d}{\lambda}$

  Suppose in some question d/λ comes out say 4:6 then total number of maximas on the screen will be 9.
  Corresponding to, $n = 0, \pm1, \pm2, \pm3$ and ±4

Exemplar Problems

Wave optics

1. **Using Young's double slit experiment, a monochromatic light of wavelength 5000 Å produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000Å is used and the separation between the slits is doubled, then the new fringe width will be :**

(a) 0.5 mm (b) 1.0 mm
(c) 0.6 mm (d) 0.3 mm

JEE MAIN 2022

2. **In young's double slit experiment performed using a monochromatic light of wavelength λ, when a glass plate (μ = 1.5) of thickness xλ is introduced in the path of the one of the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:**

(a) 3 (b) 2
(c) 1.5 (d) 0.5

JEE MAIN 2022

3. **Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The ratio of the intensity of maxima and minima will be :**

(a) 2 : 3 (b) 16 : 81
(c) 25 : 169 (d) 25 : 1

JEE MAIN 2022

4. **In young's double slit arrangement, slits are separated by a gap of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is:-**

(a) 1178×10^{-9} m (b) 1178×10^{-6} m
(c) 1178×10^{-12} m (d) 5890×10^{-7} m

JEE MAIN 2021

5. **In a Young's double slit experiment, a student observes 8 fringes in a certain segment of screen when a monochromatic light of 600 nm wavelength is used. If the wavelength of light is changed to 400 nm, then the number of fringes he would observe in the same region of the screen is**

(a) 12 (b) 6
(c) 8 (d) 9

NEET 2022

6. **In Young's double slit experiment, if the separation between coherent sources is halved and the distance of the screen from the coherent sources is doubled, then the fringe width becomes :**

(a) half (b) four times
(c) one-fourth (d) double

NEET 2020

7. **In Young's double slit experiment, if the separation between coherent sources is halved and the distance of the screen from the coherent sources is doubled, then the fringe width becomes**

(a) double (b) half
(c) four times (d) one-fourth

NEET 2020

8. **In a double slit experiment, when light of wavelength 400 nm was used, the angular width of the first minima formed on a screen placed 1 m away, was found to be 0.2°. What will be the angular width of the first minima, if the entire experimental apparatus is immersed in water? (μ_{water} = 4/3)**

(a) 0.1° (b) 0.266°
(c) 0.15° (d) 0.0°

NEET 2019

9. **In Young's double slit experiment the separation *d* between the slits is 2 mm, the wavelength l of the light used is 5896 Å and distance *D* between the screen and slits is 100 cm. It is found that the angular width of the fringes is 0.20°. To increase the fringe angular width to 0.21° (with same l and *D*) the separation between the slits needs to be changed to**

(a) 1.8 mm (b) 1.9 mm
(c) 2.1 mm (d) 1.7 mm

NEET 2018

10. **In a Young' double slit experiment if there is no initial phase difference between the light from the two slits, a point on the screen corresponding to the fifth minimum has path difference.**

(a) $5\frac{\lambda}{2}$ (b) $10\frac{\lambda}{2}$
(c) $9\frac{\lambda}{2}$ (d) $11\frac{\lambda}{2}$

Odisha NEET 2019

11. **Young's double slit experiment is first performed in air and then in a medium other than air. It is found that 8th bright fringe in the medium lies where 5th dark fringe lies in air. The refractive index of the medium is nearly**

(a) 1.59 (b) 1.69
(c) 1.78 (d) 1.25

NEET 2017

12. **When the movable mirror of Michelson interferometer moves a distance of 100 μm. a shift of 500 fringes is observed. What will be the wavelength of used light?**

(a) 7000Å (b) 5500Å
(c) 4000Å (d) 4500Å

UPPSC Ashram Paddhati PGT - 2021

13. **Ratio of intensities of two waves is 1:9. What will be the ratio of maximum intensity to minimum intensity if these waves interface?**
(a) 1 : 3 (b) 4 : 1
(c) 2 : 6 (d) 1 : 4
UPPSC Poly. Lect. 2021 Paper-I

14. **Two interfering waves have intensities in the ratio 9 : 1. The ratio of maximum and minimum intensities, in interference pattern, will be**
(a) 5 : 4 (b) 2 : 1
(c) 25 : 16 (d) 4 : 1
UP PGT - 2021
UPPCS Pre 2001, 03, 05

15. **Michelson-Morley experiment was based upon**
(a) Refraction of light
(b) Interference of light
(c) Diffraction of light
(d) Polarization of light
UPPSC Ashram Paddhati PGT - 2021

16. **Fiber optics works on the principle of**
(a) Snell's law
(b) Total internal reflection
(c) (a) and (b) both
(d) Population inversion
UP PGT - 2021

17. **Two coherent sources of intensity ratio 100 : 1 interfere. The ratio intensity between maxima and minima is**
(a) 100 : 1 (b) 10 : 1
(c) 11 : 9 (d) 121 : 81
UP PGT - 2021
UPPGT 2005

18. **In Michelson interferometer with monochromatic light, when mirrors are exactly perpendicular, the fringes are -**
(a) Straight line (b) Circular
(c) Parabolic (d) Elliptical
UPPSC Poly. Lect. 2021 Paper- I

19. **Oil floating on the surface of water appears coloured in white light. The expected thickness of oil film is -**
(a) 100 Å (b) 10,000 Å
(c) 1mm (d) 1cm
UPPSC Poly. Lect. 2021 Paper- I

20. **The films of oil and soapy water have their brilliant colours, because of :**
(a) Dispersion of light
(b) Interference of light
(c) Diffraction of light
(d) Polarization of light
UKPSC Lecturer (Mains) 2020

21. **When a thin glass plate (n = 1.5) is introduced in one of the arm of Michelson Interferometer using light of wavelength 5890 Å, there is a shift of 10fringes. The thickness of plate is**
(a) 2.95 μm (b) 5.89 μm
(c) 8.85 μm (d) 11.8 μm
RPSC School Lect. 08.01.2020
UPPCS Pre 2002

22. **In an interference pattern, at a point we observe the 12th order maxima for $\lambda_1 = 6000\,\mathring{A}$. What order will be visible here if the source is replaced by light of wavelength $\lambda_2 = 4800\,\mathring{A}$.**
(a) 15th (b) 10th
(c) 18th (d) 16th
RPSC School Lect. 26.07.2016

23. **The ratio of intensities of two light waves is 25 : 16. If they interfere, the ratio of intensities of maxima and minima of resultant pattern is**
(a) 5 : 4 (b) 9 : 1
(c) 49 : 9 (d) 81 : 1
UP PGT Physics 2016 (Exam : 02.02.2019)

24. **Two coherent monochromatic light beam of intensities I and 4I are superposed. The ratio of the maximum and minimum possible intensities in the resulting beam is**
(a) 5:1 (b) 5:3
(c) 9:1 (d) 3:1
UPPCS Pre 2010, 2008, 2006

25. **A ray of light is incident on the surface of a glass plate of refractive index 1.5 at polarizing angle. What is the angle of polarization?**
(a) 56.3° (b) 33.7°
(c) 90° (d) None of the above
UPPSC Poly. Lect. 2021 Paper- I
UPPGT 2021, 2000

26. **The ratio of intensities of D_1 and D_2 lines of sodium at high temperature is**
(a) 1:1 (b) 2:3
(c) 1:3 (d) 1:2
UPPSC GDC - 2021

27. **Which property of light shows it is a transverse wave?**
(a) Refraction (b) Interference
(c) Diffraction (d) Polarization
UPTGT Science - 2021

28. **Which of the following statement is correct about optically active substances?**
(a) Optically active substances produce polarized light.
(b) Optically active substances rotate the plane of polarization of polarized light.
(c) Optically active substances produce double refraction.
(d) None of the above
UPPSC Poly. Lect. 2021 Paper- I

29. **The Clausius–Mossotti relation holds best for -**
(a) Solids
(b) Polar molecules
(c) Gases and dilute solutions
(d) Concentrate solutions

UPPSC Poly. Lect. 2021 Paper - II

30. **The first diffraction minimum due to single slit diffraction is at 30^0 for a light of wavelength λ. If width of slit is $1.0\,\mu m$, the wavelength λ is.**
(a) 1250 Å (b) 4000 Å
(c) 4500 Å (d) 5000 Å

UPPSC Ashram Paddhati PGT - 2021

31. **When a thin transparent plate is introduced in the path of interfering beams, the fringe width will**
(a) increase (b) decrease
(c) become zero (d) remains the same

UPPSC Ashram Paddhati PGT - 2021

32. **Sunlight is reflected from a lake is 100% polarized at any instant. The angle between the sun and horizon at that instant is (Given $\tan^{-1}(1.33) = 53.06°$, μ of water = 1.33)**
(a) 53.06° (b) 143.06°
(c) 36°54' (d) 126°54'

UP PGT - 2021

33. **In Young's double slit experiment the slit separation is 0.12 mm, the wavelength of light used is 5893 Å and interference pattern is observed on a screen 1 m away. The separation between successive bright fringes will be**
(a) 6.50 mm (b) 5.50mm
(c) 4.91 mm (d) 4.50mm

UP PGT - 2021

34. **For incidence at Brewster's angle, the angle between reflected and refracted rays, is**
(a) 120° (b) 90°
(c) 70° (d) 60°

UP PGT - 2021

35. **A parallel beam of light of wavelength 5460 Å is incident at an angle of 30^o on a plane transmission grating which has 6000 lines/cm. the highest order spectrum that can be seen is -**
(a) 2 (b) 3
(c) 4 (d) 5

UPPSC Poly. Lect. 2021 Paper -I

36. **The total scattering cross section can be written in terms of the phase shifts, as**
(a) $\sigma = \dfrac{4\pi}{K^2}\sum_{l=0}^{\infty}(2l+1)\tan^2\delta_l$
(b) $\sigma = \dfrac{4\pi}{K^2}\sum_{l=0}^{\infty}(2l+1)\cot^2\delta_l$
(c) $\sigma = \dfrac{4\pi}{K^2}\sum_{l=0}^{\infty}(2l+1)\cos^2\delta_l$
(d) $\sigma = \dfrac{4\pi}{K^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l$

UPPSC GDC - 2021

37. **Polaroid A and Polaroid B are in crossed position. On Polaroid A unpolarized light of intensity 100 W/m^2 is incident. By rotating Polaroid B by 60° the intensity of light coming out of B will be ;**
(a) 75 W/m^2 (b) 37.5 W/m^2
(c) 1000 W/m^2 (d) $\dfrac{50}{\sqrt{3}}\,W/m^2$

UKPSC Lecturer (Mains) 2020

38. **When the angle of incidence on a material is 60^o, the reflected light is completely polarized. The velocity of the refracted ray inside the material is (in ms^{-1})**
(a) 3×10^8 (b) $\left(\dfrac{3}{\sqrt{2}}\right)\times10^8$
(c) $\sqrt{3}\times10^8$ (d) $\dfrac{1}{2}\times10^8$

RPSC School Lect. 08.01.2020

39. **In Young's double slit experiment, if the screen is moved away from the plane of slits, then**
(a) Angular fringe width will decrease
(b) Angular fringe width will increase
(c) Angular fringe width will remain constant
(d) Angular fringe width first increases then decreases

RPSC School Lect. 08.01.2020

40. **Light is incident on a plane refracting surface at Brewster's angle, the refracted light ray will be :**
(a) Unpolarized
(b) Plane polarized
(c) Circularly polarized
(d) Elliptically polarized

UKPSC Lecturer (Mains) 2020

41. **In an experiment of single slit diffraction pattern, I^{st} minimum of red light coincides with I^{st} maximum of some other wavelength. If $\lambda_R = 6000\,\overset{o}{A}$, then wavelength of I^{st} maximum will be :**
(a) 3300 Å (b) 4000 Å
(c) 5500 Å (d) 6600 Å

UKPSC Lecturer (Mains) 2020

42. **In double slit experiment the shape of the fringes is :**
(a) Circular (b) Straight lines
(c) Hyperbolic (d) None of these

UKPSC Lecturer (Mains) 2020

43. **If we repeat Young's double slit experiment in water by maintaining all other conditions same. then the fringe-width :**
(a) will be increased
(b) will be decreased
(c) will be unchanged
(d) Both (a) and (b) above are possible

UKPSC Lecturer (Mains) 2020

44. **Monochromatic X-ray beam falls on a crystal surface. Diffraction pattern will be formed if beam falls :**
(a) Normal to the surface
(b) Parallel to the surface
(c) At a particular angle to the surface
(d) None of these

UKPSC Lecturer (Mains) 2020

45. **X-rays of λ = 0.3 Å are incident on a crystal with lattice spacing 0.5 Å. The angle at which second Bragg's diffraction maxima observed is**
(a) $\sin^{-1}\left(\frac{3}{5}\right)$ (b) $\cos^{-1}\left(\frac{3}{5}\right)$
(c) $\sin^{-1}\left(\frac{3}{10}\right)$ (d) $\cos^{-1}\left(\frac{3}{10}\right)$

RPSC School Lect. 28.01.2020

46. **A ray of light is incident at polarising angle on a transparent plate of refractive index $\sqrt{3}$. What is the angle of refraction?**
(a) 30^0 (b) 60^0
(c) 90^0 (d) 120^0

UP PGT Physics 2016 (Exam : 02.02.2019)

47. **If the polarizing angle for a medium is 60^0, what will the critical angle be?**
(a) 60^0 (b) $\cos^{-1}\frac{1}{\sqrt{3}}$
(c) $\sin^{-1}\frac{1}{\sqrt{3}}$ (d) $\tan^{-1}\frac{1}{\sqrt{3}}$ A

UP PGT Physics 2016 (Exam : 02.02.2019)

48. **What does diffraction indicate?**
(a) Particle nature
(b) Wave nature
(c) None of the above
(d) Bother particle and wave nature

UP PGT Physics 2016 (Exam : 02.02.2019)

49. **Two polaroids are placed in such a way that their axis makes an angle of 30° with each other. If unpolarised light is incident on the system of this polaroids, the fraction of incident light transmitted is**
(a) Zero (b) $\frac{1}{2}$
(c) $\frac{3}{8}$ (d) $\frac{2}{3}$

RPSC School Lect. 26.07.2016

50. **Which of the following are coherent sources?**
(a) A 60 Watt and a 100 Watt bulbs
(b) Two bulbs of 60W each
(c) Two halves of a 60 Watt bulbs
(d) Two virtual sources obtained by biprism

UPPSC Ashram Paddhati PGT - 2021

51. **If air film is formed in Newton's Ring experiment, the ratio of diameter of first four bright rings are**
(a) 1 : 2 : 3 : 4 (b) $\sqrt{1}:\sqrt{2}:\sqrt{3}:\sqrt{4}$
(c) 1 : 3 : 5 : 7 (d) $1:\sqrt{3}:\sqrt{5}:\sqrt{7}$

RPSC School Lect. 08.01.2020

52. **In a Newton's ring experiment the air in the interspaces is replaced by a liquid of refractive index 1.33, ratio of diameter of rings will be**
(a) 1.33 (b) $\sqrt{1.33}$
(c) $\frac{1}{1.33}$ (d) $\frac{1}{\sqrt{1.33}}$

RPSC School Lect. 26.07.2016

53. **The angular dispersion for a lens is 0.0178 and deviation for yellow light is 0.5170. Then the dispersive power is**
(a) 0.034 (b) 0.033
(c) 0.031 (d) 0.035

(UPPGT 2010)

54. **Newton's rings are usually studied in the reflected light and not in the transmitted light because of**
(a) convenience in the experimental set up
(b) higher intensity in the reflected light
(c) higher coherence in the reflected light
(d) better visibility of fringes in the reflected light

UPPCS Pre 2005, 2003

55. **Fraunhofer spectrum is -**
(a) Line absorption spectrum
(b) Band absorption spectrum
(c) Line emission spectrum
(d) Band emission spectrum

UPPSC Poly. Lect. 2021 Paper -I
PGT Physics 2003

56. **Which elements was discovered through the Fraunhofer lines in the solar radiation?**
(a) Hydrogen (b) Helium
(c) Lithium (d) Uranium

UPTGT Science - 2021

57. **In case of diffraction due to an opaque circular disc, the centre of the shadow is always**
(a) bright (b) dark
(c) coloured (d) none of the above

UPPCS Pre 2008, 2006

58. **In the diffraction of a plane wave due to a circular obstacle (disc) the axial point is always.**
(a) dark (b) bright
(c) may be bright of dark (d) None of the above

UPPCS Pre 2010

59. **Fraunhofer spectrum is a**
(a) Line absorption spectrum
(b) Band absorption spectrum
(c) Line emission spectrum
(d) Band emission spectrum
(UPPGT 2003)

60. **Select the correct statement from the following.**
(a) In Fresnel diffraction light source and screen are at infinite distance from diffracting object.
(b) In Fraunhofer diffraction light source and screen are at infinite distances from the diffracting object
(c) In Fresnel diffraction only light sources is at infinite distance from diffracting object
(d) In Fresnel diffraction only screen is at infinite distance from diffracting object
(UPPGT 2000)

61. **The essential condition for Fraunhofer class diffraction is that–**
(a) The incident wave front must be plane
(b) The incident wave front must be spherical
(c) Both the incident and diffracted wave fronts be plane
(d) All the above
(UPPGT 2011)

62. **The maximum scattering angle for equal masses in laboratory system is**
(a) Zero (b) 90º
(c) 180º (d) 360º
RPSC School Lect. 26.07.2016

63. **The method of partial wave is suitable for**
(a) low energy scattering
(b) medium-high energy scattering
(c) high energy scattering
(d) for all energy scattering
RPSC School Lect. 26.07.2016

64. **Scattering cross-section σ(θ) for Coulomb potential using Born approximation is**
(a) proportional to $\sin^4\theta/2$.
(b) inversely proportional to $\sin^4\theta/2$
(c) proportional to $\sin^2\theta/2$
(d) inversely proportional $\sin^2\theta/2$
RPSC School Lect. 26.07.2016

65. **The mean separation of two points on moon that can be resolved by a 500 cm telescope aperture is (distance of the moon is 4×10^5 km, eye is most sensitive to wavelength 5500Å)**
(a) 40 m (b) 43 m
(c) 53.6 m (d) 50.6 m
UP PGT - 2021

66. **The resolving power of a prism depends on**
(a) The width of the base of the prism
(b) The rate of change of refractive index with respect to wavelength
(c) Both (a) and (b)
(d) None of the above
UPPSC Ashram Paddhati PGT - 2021

67. **Wavelength of light used in an optical instrument are $\lambda_1 = 4000\text{Å}$ and $\lambda_2 = 5000\text{Å}$ The ratio of their respective resolving power is-**
(a) 1:9 (b) 5:4
(c) 16:5 (d) 9:1
UPPSC Ashram Paddhati PGT - 2021
UPPSC Poly. Lect. 2021 Paper - I

68. **The instrument used to measure optical rotation is known as**
(a) Spectrometer (b) Microscope
(c) Polarimeter (d) Interferometer
UP PGT - 2021

69. **Which of the following gives the resolving power of a telescope whose objective diameter is 'd'?**
(a) $\frac{1.22\lambda}{d}$ (b) $\frac{\lambda}{d}$
(c) $\frac{d}{1.22\lambda}$ (d) $\frac{d}{\lambda}$
UPPSC Poly. Lect. 2021 Paper-I
UPPCS (Pre) 2003

70. **What should be the minimum number of lines in grating to resolve sodium doublet (5890 Å and 5896 Å) in third spectral order?**
(a) 1312 (b) 984
(c) 656 (d) 328
UPPSC Poly. Lect. 2021 Paper-I

71. **If the symbols have their usual meaning, then the resolving power of a plane transmission grating is -**
(a) nN (b) $\frac{1}{nN}$
(c) $\frac{n}{N}$ (d) $\frac{N}{n}$
UPPSC Poly. Lect. 2021 Paper-I

72. **The diameter of the objective of a telescope is d and light of wavelength λ is being used. What will be the limit of resolution?**
(a) $\frac{1.22\lambda}{d}$ (b) $\frac{\lambda}{d}$
(c) $\frac{d}{1.22\lambda}$ (d) $\frac{1.22d}{\lambda}$
UP PGT Physics 2016 (Exam : 02.02.2019)

73. **The dispersive power of a plane grating is given by**
(symbols have their usual meanings)
(a) $\frac{n}{(a+b)\cos\theta}$ (b) $\frac{n\cos\theta}{(a+b)}$
(c) $\frac{(a+b)\sin\theta}{nN}$ (d) $\frac{(a+b)\cos\theta}{nN}$
UPPSC Ashram Paddhati PGT - 2021

74. **Total number of rulings in a diffraction grating needed to just resolve sodium doublet (589 nm and 589.6 nm) in the first order is :**
(a) 589 (b) 590
(c) 950 (d) 982
UKPSC Lecturer (Mains) 2020

75. **r is radius of first circle in zone plate and λ is wavelength of light used. First focal length of zone plate will be**
(a) $r^2\lambda$ (b) $\frac{\lambda}{r^2}$
(c) $\frac{r^2}{\lambda}$ (d) $\frac{2r^2}{\lambda}$
RPSC School Lect. 08.01.2020

76. **A zone plate behaves like a**
(a) Concave lens (b) Convex lens
(c) Plane mirror (d) Glass plate
UKPSC Lecturer (Mains) 2020

77. **Which of the following is incorrect?**
(a) Zone plate has many focii
(b) Zone plate is similar to convex lens
(c) Focal length of zone plate depends on wavelength of light used
(d) None of the above
UP PGT Physics 2016 (Exam : 02.02.2019)

78. **A zone plate is to be constructed with focal length of 50cm for $\lambda = 5.0\times10^{-5}$cm. Its first radius will be:**
(a) 0.40 mm (b) 0.50 mm
(c) 0.60 mm (d) 0.75 mm
UP LT Grade Science 2018 (Exam : 29.07.2018)

79. **A Nicol prism is based on the principle of–**
(a) Refraction (b) Scattering
(c) Dichroism (d) Double refraction
(UPPGT 2011)

80. **In a doubly refracting crystal, optic axis is a direction along which–**
(a) a plane polarised beam does not suffer deviation
(b) any beam of light does not suffer any deviation
(c) double refraction does not take place
(d) ordinary and extraordinary rays undergo maximum deviation.
(UPPGT 2011)

81. **Double refracting crystals are negative if :**
UPPCS Pre 2010
(a) $\mu_e > \mu_0$ (b) $\mu_0 > \mu_e$
(c) $\mu_0 = \mu_e$ (d) None of these

82. **A beam of circularly polarized light is passed through a quarter wave plate. The emergent light is**
(a) Plane polarized
(b) Circularly polarized
(c) Elliptically polarized
(d) Partially polarized
UPPSC Poly. Lect. 2021 Paper-I

83. **A half-wave plate produces a phase difference between O (ordinary) and E (extraordinary) waves equal to**
(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$
(c) π (d) π/4
UP PGT - 2021
UPPCS (Pre) 1998

84. **For μ_0 = 1.544, μ_i = 1.558 and λ = 6500Å, the thickness of a quarter wave plate is.**
Here notations carry their usual meaning.
(a) 10.0 μm (b) 11.6 μm
(c) 0.25 μm (d) 0.28 μm
UP PGT - 2021

85. **What will be the thickness of a quarter wave plate made of quartz for wavelength λ = 5000Å and refractive index μ_0 = 1.553 and μ_e = 1.543 ?**
(a) 5.00×10^{-3}cm (b) 3.75×10^{-3}cm
(c) 2.50×10^{-3}cm (d) 1.25×10^{-3}cm
UPPSC Poly. Lect. 2021 Paper-I

86. **For calcite, μ_0 = 1.658 and μ_E = 1.486 and λ = 5893 Å for sodium yellow light. The thickness of the thinnest quarter-wave plate of calcite for this light, is**
(a) 0.70×10^{-4}cm (b) 0.86×10^{-4}cm
(c) 0.95×10^{-4}cm (d) 0.15×10^{-3}cm
UP LT Grade Science 2018 (Exam : 29.07.2018)

87. **In Helium-Neon laser, helium atoms**
(a) Emit laser radiation
(b) Impart energy to the Neon atoms
(c) Act as catalytic atom
(d) Act as quenching agents
UPPSC Ashram Paddhati PGT - 2021
UPPCS (Pre) 2004, 2008
UPPCS Pre 2001

88. **The population inversion between energy states in a Ruby laser is achieved through**
(a) raising of temperature
(b) lowering of temperature
(c) optical pumping
(d) vacuum pumping
UP PGT - 2021

89. **A laser beam has pulse power 10^{12} Watt. It is focused on an object of area 10^{-4}cm^2. What will be the energy flux?**
(a) 10^{20} erg / m^2 (b) 10^{16} Watt / cm^2
(c) 10^{8} Watt / cm^2 (d) 10^{4} Watt / cm^2
UPPSC Poly. Lect. 2021 Paper-I

90. Working of Laser is based on -
(a) Spontaneous emission of radiation
(b) Stimulated absorption of radiation
(c) Stimulated emission of radiation
(d) None of the above

UPPSC Poly. Lect. 2021 Paper-I

91. Helium - Neon laser is -
(a) Two level laser (b) Three level laser
(c) Four level laser (d) Five level laser

UPPSC Poly. Lect. 2021 Paper-I

92. Calculate coherence length for a laser beam for which band width is Δr=3000 H_3 .
(a) 100 km (b) 50km
(c) 20km (d) 10km

UPPSC GDC - 2021

93. The coherence time of laser beam of wavelength 7400 Å is 4 × 10^{-5} second . The coherence length will be :
(a) 10 km (b) 5 km
(c) 11.2 km (d) 12 km

UKPSC Lecturer (Mains) 2020

94. Which of the following is not necessary for performing LASER action ?
(a) Population inversion
(b) Meta-energy level
(c) Spontaneous emission
(d) Stimulated emission

UKPSC Lecturer (Mains) 2020

95. Which statement is false about LASER ?
(a) In the absence of resonator cavity there would be no amplification of light
(b) In laser the active medium is the amplifying medium
(c) Laser is a light source and analogous to an electronic oscillator
(d) In laser the rate of spontaneous emission is higher than stimulated (induced) emission

RPSC School Lect. 08.01.2020

96. The pumping method in Ruby Laser is
(a) optical pumping
(b) electron impact
(c) chemical pumping
(d) gas dynamic pumping

RPSC School Lect. 08.01.2020

97. The population inversion in He-Ne laser is produced by
(a) Photon excitation
(b) electron excitation
(c) inelastic atomic collisions
(d) chemical reaction

RPSC School Lect. 26.07.2016

98. For holography which statement is false :
(a) Laser's light source is used
(b) 3-D image is developed
(c) It demonstrate both depth and parallax
(d) It records only intensity variation and not phase variation

RPSC School Lect. 26.07.2016

99. What is the fundamental basis of a laser?
(a) Absorption
(b) Spontaneous emission
(c) Stimulated emission
(d) Scattering

UP PGT Physics 2016 (Exam : 02.02.2019)

100. With reference to laser, which of the following statements (s) is/are correct?
(a) It is used in surgery.
(b) It is used in radar
(c) It is used in the study of moon's surface
(d) It is used in the study of distance objects.

Select the correct answer using the code given below :
(a) 1only (b) 1 and 2 only
(c) 1, 2 and 3 (d) 1, 2, 3 and 4

UP LT Grade Science 2018 (Exam : 29.07.2018)

Answer Key

1. (d)	2. (b)	3. (d)	4. (b)	5. (a)	6. (b)	7. (c)	8. (c)	9. (b)	10. (c)
11. (c)	12. (c)	13. (b)	14. (d)	15. (b)	16. (b)	17. (d)	18. (b)	19. (b)	20. (b)
21.(b)	22. (a)	23. (d)	24. (c)	25. (a)	26. (d)	27. (d)	28. (b)	29. (b)	30. (d)
31. (d)	32. (c)	33. (c)	34. (b)	35. (a)	36. (d)	37. (b)	38. (c)	39. (c)	40. (b)
41. (b)	42. (c)	43. (b)	44. (c)	45. (a)	46. (a)	47. (c)	48. (b)	49. (c)	50. (d)
51. (d)	52. (b)	53. (a)	54. (d)	55. (a)	56. (b)	57. (a)	58. (b)	59. (a)	60. (b)
61. (c)	62. (b)	63. (a)	64. (b)	65. (c)	66. (c)	67. (b)	68. (c)	69. (c)	70. (d)
71. (a)	72. (a)	73. (a)	74. (d)	75. (c)	76. (b)	77. (d)	78. (b)	79. (d)	80. (c)
81. (b)	82. (a)	83. (c)	84. (b)	85. (d)	86. (b)	87. (b)	88. (c)	89. (b)	90. (c)
91. (c)	92. (a)	93. (d)	94. (c)	95. (d)	96. (a)	97. (b)	98. (d)	99. (c)	100.(d)

05.

Wave & Oscillations

(1) Simple Harmonic Motion

■ Periodic Motion:

- Any motion, which repeats itself after regular interval of time (i.e. time period) is called periodic motion or harmonic motion.
  Examples- Motion of planets around the sun, Motion of the pendulum of wall clock.
- Every SHM is periodic but every periodic motion is not SHM.

■ Oscillatory Motion:-

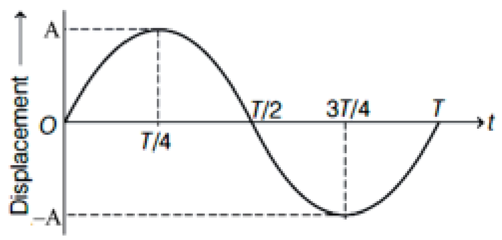
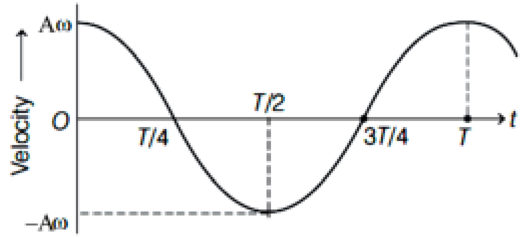
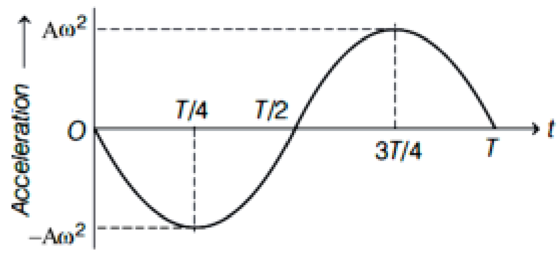
- The motion of body is said to be oscillatory or vibratory motion If it moves back and forth (to and fro) about a fixed point after regular interval of time. The fixed point about which the body oscillates is called mean position or equilibrium position.
  Examples - Vibration of Wire of Sitar, Oscillation of the mass suspend from Spring.
- Every Oscillatory motion is periodic but every periodic motion is not oscillatory.

■ Simple Harmonic Motion (SHM):

- Simple harmonic motion (SHM) is that type of oscillatory motion in which the particle moves to and fro repeatedly about a fixed point under a restoring force, whose magnitude is directly proportional to its displacement
- Simple harmonic motion is said to be simple harmonic if it can be expressed in terms of:

$$x(t) = A \sin(\omega t + \phi) \text{ or } x(t) = A \cos(\omega t + \phi)$$

■ Graphical Representation of SHM:-

Displacement - Time Graph	When $x(t) = A \sin(\omega t + \phi)$, $\phi = 0$ At mean position $x = 0$ At extreme position $x = \pm A$	
Velocity - Time Graph	When $V(t) = \dfrac{dx}{dt} = A\omega \cos\omega t$ At mean position, $V_{max} = A\omega$ At extreme position, $V_{min} = 0$	
Acceleration - Time Graph	When $a(t) = \dfrac{dV}{dt} = \dfrac{d^2x}{dt^2}$ $= -A\omega^2 \sin\omega t = -\omega^2 x$ At mean position, $A_{min} = 0$ At extreme position, $A_{max} = A\omega^2$	

■ Important terms related to S.H.M.

- **Displacement** – A physical quantity which represents change in position with respect to mean position or equilibrium position is called displacement.

- **Mean Position:-** The point at which the restoring force on the particle is zero and potential energy is minimum is known as its Mean position.
- **Restoring force:-** The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force. Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- **Amplitude:-** The Maximum (Positive and Negative) Value of displacement of particle from mean position is defined as amplitude.
- **Time period:-**Time taken by the body to complete one oscillation is known as time period. It is denoted by T.
- **Frequency (ν)**– It is defined as the number of oscillations executed by body per Second. S.I. unit frequency is hertz (Hz).

$$\text{Frequency} = \frac{1}{\text{Time Period}}$$

- **Angular frequency (ω):** Angular frequency is defined as angular displacement of a particle per unit time in a periodic motion. It is given by,

$$\omega = 2\pi\nu \Rightarrow \frac{\omega}{2\pi} = \nu = \frac{1}{T}$$

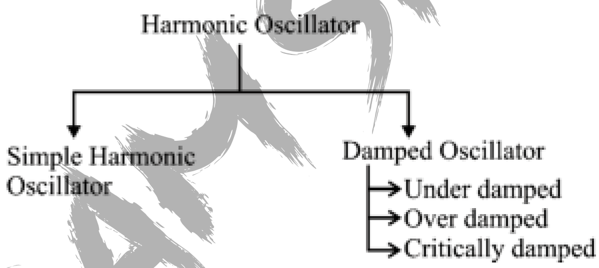
Its S.I. unit is radian per second.

- **Phase** – A physical quantity which express the position and direction of motion of an oscillating particle is called phase. It is denoted by (ϕ).

Harmonic Oscillator:-

A harmonic oscillator is a system that, then displaced from to equilibrium particle experiences a restoring force F proportional to displacement k :

$$\vec{F} = -kx$$



Force Law in SHM:-

- The force acting on a particle of mass m is SHM is given by -

$$F = ma = -m\omega^2 x$$

We know that, $F = -kx$

$$\Rightarrow k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

Where, k = force constant of spring used in simple harmonic oscillator.

Energy in SHM:-

- **Kinetic Energy :-** The K.E of the particle is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi) = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

The K.E is maximum at the centre (x = 0) and zero at the extreme point of oscillation (x = ± A)

- **Potential Energy :-** The potential energy of the particle is given by -

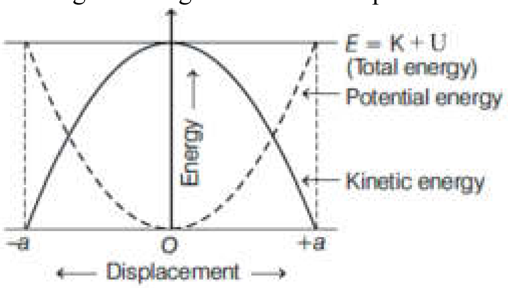
$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2(\omega t + \phi) = \frac{1}{2}m\omega^2x^2$$

- **Total energy of the S.H.M.**

$$E = K + U = \frac{1}{2}m\left(A^2 - x^2\right)\omega^2 + \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2$$

where, A = amplitude,
m = mass of particle executing SHM
and ω = angular frequency.

- Change in kinetic and potential energies during oscillations is represented in the graph -



■ Key points related to energy of the particle executing S.H.M.–

- Total energy of a particle in SHM is same at all instant and at all displacement and is independent of time.
- Total energy depends upon mass, amplitude and frequency of Vibration of particle executing SHM
- The frequency of kinetic energy or potential energy of a particle executing SHM is double than of the frequency in SHM.
- The frequency of total energy of particles executing SHM is zero as total energy in SHM remain constant at all positions.
- At extreme position, the total energy of the particle in SHM is in the form of its potential energy

■ Tabular form of different physical quantities vary with time or displacement are listed below-

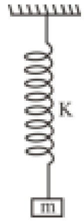
S. N.	Name of the equation	Expression of the equation	Remarks
1.	Displacement -time	$x = A\sin(\omega t + \phi)$	x varies between A and -A
2.	Velocity -time $\left(V = \frac{dx}{dt}\right)$	$V = A\omega\cos(\omega t + \phi)$	V varies Between + Aω and -Aω
3.	Acceleration - time $a = \frac{dv}{dt}$	$a = -A\omega^2 \sin(\omega t + \phi)$	a varies between + $A\omega^2$ and – $A\omega^2$
4.	Kinetic energy - Time $\left(K = \frac{1}{2}mv^2\right)$	$K = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \phi)$	K varies between 0 and $\frac{1}{2}mA^2\omega^2$
5.	Potential energy-time $U = \frac{1}{2}m\omega^2 x^2$	$U = \frac{-1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$	U varies between $\frac{1}{2}m\omega^2 A^2$ and 0
6.	Total energy time	$E = \frac{1}{2}m\omega^2 A^2$	E is Constant
7.	Velocity -Displacement	$V = \omega\sqrt{A^2 - x^2}$	v = 0 at x = ± A and at x = 0 v = ± ωA
8.	Acceleration-displacement	$a = -\omega^2 x$	a = 0 at x = 0 and a = ± ω^2 A at x = ± Ax
9.	Kinetic energy - displacement	$K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$	k =0 at x = ± A and $K = \frac{1}{2}m\omega^2 A^2$at $x = 0$
10.	Potential Energy-Displacement	$U = \frac{1}{2}m\omega^2 x^2$	U = 0 at x = 0 and $U = \frac{1}{2}m\omega^2 A^2$ at $x = \pm A$
11.	Total energy displacement	$E = \frac{1}{2}m\omega^2 A^2$	E is constant.

■ Some Systems executing SHM

- **Spring Pendulum :-**A point mass suspended from a mass less (or light) spring constitutes a spring pendulum. If the mass is once pulled downwards so as to stretch the spring and then released, the system oscillated up and down about its mean position simple harmonically. Time period is given by

$$T = 2\pi\sqrt{\frac{m}{k}}\ ,\ \text{frequency}\,(\nu) = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Where, m = mass of block
k = Spring constant



- **When the Spring is not light but has a definite mass m_s, time period is given by -**

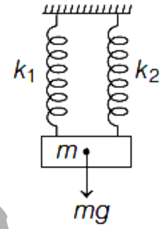
$$T = 2\pi\sqrt{\frac{M + m_s/3}{k}} \text{ or } \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{m_s}{3}}}$$

- **When two springs of force constants k_1 and k_2 are connected in parallel to mass m as shown in fig. then-**

  Effective force constant of the spring combination-

  $k_{effective} = k_1 + k_2$

  Time period is given by $T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$



- **When two springs of force constant k_1 and k_2 are connected in Series to mass m as shown in fig. then**

  Effective force constant of the spring combination

  $- \frac{1}{k_{effective}} = \frac{1}{k_1} + \frac{1}{k_2}$ $k_{effective} = \frac{k_1 k_2}{k_1 + k_2}$

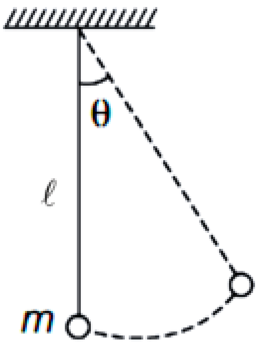
  Time period is given by $T = 2\pi\sqrt{\frac{m}{k_{effective}}}$

■ **Simple Pendulum:-**

A Simple pendulum consists of heavy point mass suspended from a rigid support by means of an elastic inextensible string.

- Time period of Simple Pendulum , $T = 2\pi\sqrt{\frac{\ell}{g}}$
- For a simple pendulum of length equal to radius of earth $(\ell = R)$ $T = 2\pi\sqrt{\frac{R}{2g}} = 60$ min
- For infinite length ($(\ell >> R)$, the time period of Simple pendulum is given by.

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6\,\text{min}$$



- Time period of Simple pendulum is independent of mass of bob.
- Time period depend upon the length of the pendulum and acceleration due to gravity at the given place.

■ **Factors affecting to the time period of simple Pendulum:-**

- **Effect of Temperature**

  If $d\theta$ is change in temperature then effective length $\ell' = \ell(1 + \alpha d\theta)$ and $T' = 2\pi\sqrt{\frac{\ell(1+\alpha d\theta)}{g}}$

- **Effect of density**

  If $\rho_0 < \rho$ (the density of the bob) then time period of Simple pendulum get increased.

$$g_{effective} = g\left(1 - \frac{\rho_0}{\rho}\right)$$

Where, ρ_0 = density of fluid

ρ = density of bob

$$T' = T \times \sqrt{\frac{\rho}{\rho - \rho_0}}$$

Where, T' = new time period.

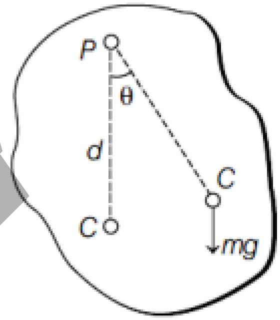
■ **Second Pendulum:-**

- It is that simple Pendulum whose time period of vibrations is two seconds, $T = 2\pi\sqrt{\frac{\ell}{g}} = 2$ sec.
- The effective length of second's pendulum is $\ell = 99.3\,\text{cm} \approx 1\,\text{m}\left(\text{on earth's surface}\right)$.
- The length of second's pendulum changes with the changes in the value of acceleration due to gravity from place to place.

■ **Physical Pendulum:-**

A Physical Pendulum is an extended body pivoted about point O, which is at a distance 'd' from its centre of mass.

- The time period of Simple Pendulum is same as that of a physical Pendulum also known as an equivalent Simple Pendulum.



$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\ell}{g}}$$

Where, I = moment of inertia

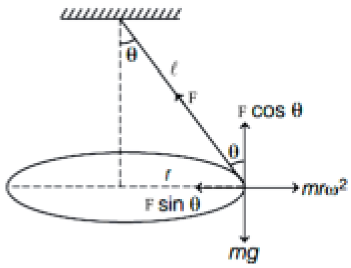
g = gravity

- The length of an equivalent Simple Pendulum is given by,

$$\ell = \frac{I}{md}$$

■ **Conical Pendulum:-**

- A tiny mass (bob) Connected to long, flexible, mass less, inextensible string suspended from rigid support is known as conical pendulum.
- Instead of swinging back and forth the bob of a conical pendulum moves at a constant speed is a circle with the string tracing out a cone.



- In equilibrium, $F\sin\theta = mr\omega^2$, $F\cos\theta = mg$

Its time period, $T = 2\pi\sqrt{\frac{mr}{F\sin\theta}} = \sqrt{\frac{\ell\cos\theta}{g}}$

Where, r = radius of circle

F = Tension in String

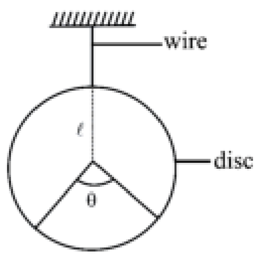
m = Mass of bob.

- Tension in the string ,

$$F = mg\sqrt{1 + \left(\frac{r}{\ell\cos\theta}\right)^2}$$

■ **Torsional Pendulum :-**

It consists of a disc (or some other object) suspended from a wire attached to a rigid support, which is then twisted and released, resulting in oscillatory motion.



Time period of torsional pendulum is given by

$$T = = 2\pi\sqrt{\frac{I}{C}}$$

Where, I = Moment of Inertia of the body about the axis of rotation.

C = Torsional costant of wire = $\frac{\pi\eta r^4}{2\ell}$

η = Modulus of rigidity of wire

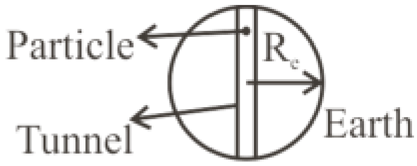
r = Radius of wire

ℓ = length of wire

- **SHM of a particle tuned inside across the diameter the earth.**

Time period inside the tunnel is given by

$$T = 2\pi\sqrt{\frac{R_e}{g}}$$



Where,
g = Gravity of earth at Surface.
But at inside the earth (through the tunnel) g vary through

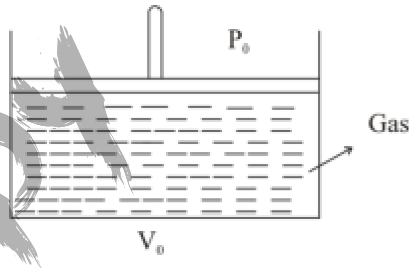
$$g_d = g\left(1-\frac{d}{R_e}\right)$$

Where, g_d = Gravity at depth,
g = Gravity at Surface

- **SHM of gas- piston system**

Here elastic force is developed due to bulk elasticity of the gas.

$$\Delta P = \gamma P_0 \frac{\Delta V}{V_0}$$



Where P = net pressure
P_0 = atmospheric pressure
V_0 = Volume of container
$\Delta V = Ax$ = volume displacement
γ = adiabatic index

F_{net} force on piston , $F = -A^2\frac{\gamma P_0}{V_0}x$

$$F = -kx \Rightarrow k = A^2\frac{\gamma P_0}{V_0}$$

Time period is given by ,$T = 2\pi\sqrt{\frac{V_0 m}{A^2\gamma P_0}}$

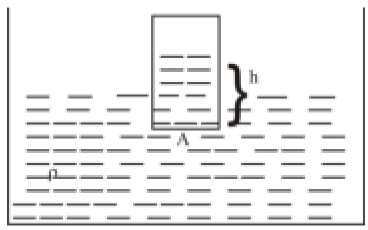
- **SHM of Floating Body**

Restoring force $\rightarrow$ Thrust Force
$Mg = \rho Ahg \rightarrow$ Vertical Equilibrium
Restoring force is given by

$F = -(\rho Ag)x \Rightarrow K = \rho Ag$

Time period is given by

$$T = 2\pi\sqrt{\frac{m}{\rho Ag}} = 2\pi\sqrt{\frac{\rho Ah}{\rho Ag}} = 2\pi\sqrt{\frac{h}{g}}$$

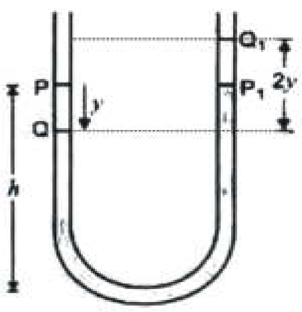


Where, h = Height of body dipped into the liquid of density (ρ)

- **SHM in U - Tube**

If a liquid is filled up to height h in both limbs of a U- tube and now liquid is depressed up to a small distance y in one limb and then released, then liquid column in U- tube start executing SHM

The time period of oscillation is given by, $T = 2\pi\sqrt{\frac{h}{g}}$



The period in U- tube is independent of the following terms-
(i) Area of cross section of U-tube
(ii) Density of liquid in U- tube
But, The time period depends upon the length of liquid column and acceleration due to gravity.

■ **Key points-**

- Suppose a body of mass m vibrates Separately with two different Springs (of Spring Constants k_1 and k_2) with time period T_1 and T_2 respectively

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}} \text{ and } T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

- If the same body vibrates with series Combination of these two springs then for the system time period

$$T = \sqrt{T_1^2 + T_2^2}$$

- If the same of body vibrates with Parallel Combination of these two Springs then time period of the system

$$T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}}$$

- The Pendulum clock runs slow due to increase in its time period whereas it becomes fast due to decrease in time period.
- Percentage change in time period with ℓ and g if g is constant and length varies by n% then % change in time period $\frac{\Delta T}{T} \times 100 = \frac{n}{2}\%$ if ℓ is constant and g varies by n% Then % Change in time period $\frac{\Delta T}{T} \times 100 = \frac{n}{2}\%$ (Valid only for Percentage change less than 10%)
- If a Particle Performs S.H.M whose velocity is V_1 at x_1 distance from mean position and velocity v_2 distance x_2

$$\omega = \sqrt{\frac{V_1^2 - V_2^2}{x_2^2 - x_1^2}} \; ; \; T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{V_1^2 - V_2^2}}$$

$$a = \sqrt{\frac{V_1^2 x_2^2 - V_2^2 x_1^2}{V_1^2 - V_2^2}} \; ; \; V_{max} = \sqrt{\frac{V_1^2 x_2^2 - V_2^2 x_1^2}{x_2^2 - x_1^2}}$$

- If $y_1 = a \sin\omega t$ and $y_2 = b \cos\omega t$ are two S.H.M then by the Superposition of these two S.H.M we get

$$\vec{y} = \vec{y_1} + \vec{y_2}$$

$\Rightarrow$ $Y = a \sin\omega t + b \cos\omega t \Rightarrow y = A \sin(\omega t + \phi)$ this is also the equation of S.H.M ; where

$$A = \sqrt{a^2 + b^2} \text{ and } \phi = \tan^{-1}(b/a).$$

- In the absence of resistive force the work done by a Simple pendulum in one complete Oscillation is zero.

(2) Free, Forced, Damped Oscillation & Resonance

■ **Undamped Simple Harmonic Oscillations**

When a simple harmonic system oscillates with a constant amplitude which does not change with time, its oscillations are called undamped simple harmonic oscillation. The total energy of the system executing undamped SHM is given by

$$E_0 = \frac{1}{2} k x_0^2$$

Where, k = Force constant or Spring constant.
x_0 = Extended length in string.

■ **Damped Simple Harmonic Oscillations**

- The oscillation of a body whose amplitude goes on decreasing with time is defined as a damped oscillations.
  Example- (i) Guitar string stops oscillating a few second after being plucked because of the friction in the air.
  (ii) Shock absorbers in automobile.
- In damping, the amplitude of such oscillations decreases exponentially with time.
- Due to decrease in Amplitude, the energy of the oscillator also goes on decreasing exponentially.
- The force, producing a resistance to the oscillations is called damping force. It always act at opposite to the direction of velocity.
- The forces acting on the body undergoing damped harmonic motion are-
  (a) Restoring force $F_{Res} = -kx$
  (b) Damping force $F_{damp} = -b\frac{dx}{dt}$
  Where b is known as damping constant

- The total force on the body is the sum of the restoring force (F_{Res}) and damping force (F_{damp})–

$$F_{net} = F_{Res} + F_{damp} = -kx - b\frac{dx}{dt}$$

Using Newton's second law–

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0 \text{ ....(i)}$$

Where, $2\gamma = \frac{b}{m}$, is known as damping coefficient

$\omega_0 = \sqrt{\frac{k}{m}}$ = natural frequency of undamped oscillator

Roots of the equation are $\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$

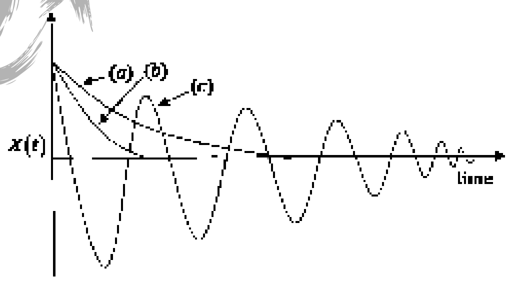
The solution of the equation (i) is given as –

$x(t) = Ae^{-\gamma t}\, e^{\pm\sqrt{\gamma^2-\omega_0^2}\,t}$

where, $\omega' = \sqrt{\omega_0^2 - \gamma^2}$ or $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m}}$

- There are three different possibilities in damped oscillation.

Over damped	Critically damped	Under damped case
If $\gamma > \omega_0$ ω' will be imaginary. Hence, x(t) will be a negative exponential function. It is shown by curve a in the given figure.	**If $\omega_0 = \gamma$** Then the square root vanishes. In this case, the solution is again a negative exponential function. The response of Critically damped system if faster than over damped i.e. it touches the zero value faster shown by curve b in the given figure.	**If $\omega_0 > \gamma$** Then the quantity under the square root is + ve and we have a real no. for ω'. The solution for 'x' is then- $x(t) = Ae^{-\gamma t}\sin(\omega' t + \phi)$ This is the solution of the damped harmonic oscillator. The oscillatory motion shown by curve c in the given figure.



- **Time period:** The time period of the damped harmonic oscillator is-

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\omega_0^2 - \gamma^2}}$$

This shows that due to damping the time period slightly increased.

- **Logarithmic Decrement:** This measure the rate at which the amplitude decay

$$\lambda = \gamma T' = \frac{bT'}{2m}$$

- **Mean Life time (τ_m)** : It is the time taken for the amplitude to decay to $\frac{1}{e}$ of the initial value.

When $t = \tau_m = \frac{1}{\gamma}$, Amplitude $= \frac{A}{e}$ $\quad (\because Ae^{-\gamma t})$

■ Energy of the Damped Oscillator

(i) Kinetic energy (K)

The displacement of a damped harmonic oscillator is

$x(t) = Ae^{-\gamma t}\sin(\omega' t + \phi)$

$$v = \frac{dx}{dt} = Ae^{-\gamma t}\left[-\gamma\sin(\omega' t+\phi)+\omega'\cos(\omega' t+\phi)\right] \approx Ae^{-\gamma t}\omega'\cos(\omega' t+\phi)$$

The approximation is done as $\gamma << \omega_0$

Thus, kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2e^{-2\gamma t}\omega'^2\cos^2(\omega' t+\phi)$

(ii) Potential Energy (U)

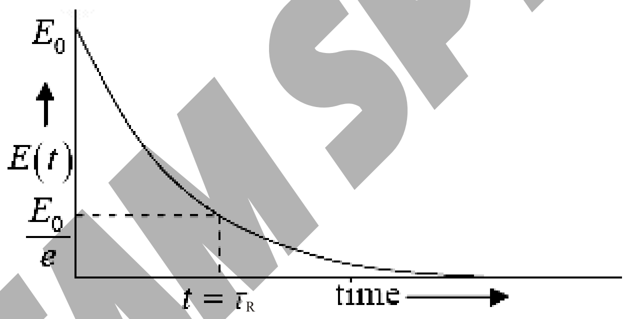
$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-2\gamma t}\sin^2(\omega' t+\phi)$$

(iii) Total energy (E)

$$E = K + U = \frac{1}{2}mA^2e^{-2\gamma t}\omega'^2\cos^2(\omega' t+\phi)+\frac{1}{2}kA^2e^{-2\gamma t}\sin^2(\omega' t+\phi)$$

$\Rightarrow E = \frac{1}{2}kA^2e^{-2\gamma t}$ Since $\gamma << \omega_0$

This shows that the energy of the oscillator decreases with time, the exponential decay of energy is shown below.



- **Power Dissipation :**

  It is the rate at which the energy is lost

  $$P = -\frac{dE}{dt} = 2\gamma\left(\frac{1}{2}kA^2e^{-2\gamma t}\right) = 2\gamma E$$

- **Relaxation Time:** It is the time taken for the total energy to decay to $\frac{1}{e}$ of its initial value E_0 . If τ_R is the relaxation time, then at $t = \tau_R$, we shall have $E = \frac{E_0}{\tau_R}$.

  Thus $\frac{E_0}{\tau_R} = E_0e^{-2\gamma t}$. This gives relaxation time $\tau_R = \frac{1}{2\gamma}$

  Thus we can also write power dissipated as $P = \frac{E}{\tau_R}$ and energy can be expressed as $E(t) = E_0e^{-\frac{t}{\tau_R}}$

- **Quality Factor (Q)** :

  It is defined as the 2π times the ratio of the energy stored in the system to the energy lost per period.

  $$Q = 2\pi\frac{\text{energy stored in system}}{\text{energy loss per period}}$$

  This shows that, lower the damping, higher the value of Q.

Now energy can be written in term of Q as $E(t) = E_0 e^{-\frac{\omega_0 t}{Q}}$

It means that Q is related to the number of oscillation over which the energy fall to $\frac{1}{e}$ of its original value E_0, which is also called the relaxation time. This happens in time, $t = \tau_R$, where τ_R is given by

$$\frac{\omega_0 \tau_R}{Q} = 1 \Rightarrow \tau_R = \frac{Q}{\omega} = \frac{TQ}{2\pi}$$

In one period (T) number of oscillation is = 1

In time τ_R the number of oscillation = n, then $n = \frac{\tau_R}{T} = \frac{Q}{2\pi}\sigma_X$

Thus the energy falls to $\frac{1}{2}$ of its original value after $n = \frac{Q}{2\pi}$ cycle of free oscillation.

- **Relation between relaxation time, mean time and Quality factor:**

Let N is the number of oscillation in time $\tau_m = \frac{1}{\gamma}$

While *n* is the number of oscillation in time $\tau_R = \frac{1}{2\gamma}$

Thus, relation between mean and relaxation time is $\tau_m = 2\tau_R$
In one period (T) number of oscillation is = 1
In time τ_m the number of oscillation = N

$$N = \frac{\tau_m}{T} = \frac{2\tau_R}{T} = 2\frac{Q}{2\pi} \text{ or } N = 2\frac{Q}{2\pi} = 2n$$

Forced Oscillations

When a body is made to oscillate under external periodic force, then in the beginning the body tries to oscillate with its natural frequency, but very soon these oscillation dies out and the body oscillate with the frequency of the applied force. Such oscillation is called forced oscillation.

Example:
1) Person swinging in a swing without anyone pushing → Free or damped oscillation
2) Someone pushes the swing periodically → Forced or driven oscillations
When system oscillates freely, natural angular frequency is ω_0.
When system oscillates under the influence of external force then forced angular frequency is p

- **Resonance**

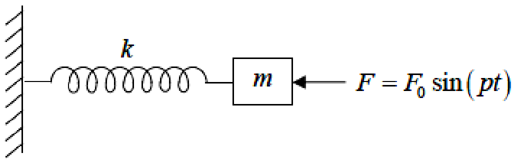
When the frequency of the external force is same as the natural frequency of the body, the amplitude of the oscillation is maximum. This phenomenon is called resonance or when the frequency of external force is equal to the natural frequency of the Oscillator is known as Resonance.

- **Transient Effect**

In the presence of external periodic force, initially body tries to oscillate with its natural frequency, while external force tries to impose its own frequency. Thus there is a hussle between external force and the body during which the amplitude rises and falls alternatively. This is the transient effect which soon dies out and body start oscillating with external frequency.

- **Equation of Forced Oscillation**

Consider a mass *m* oscillating under external periodic force *F*. let *x* be the displacement at any instant. The forces acting on the mass are



(i) Restoring force is proportional to the displacement $\Rightarrow F = -kx$

(ii) A Damping force is proportional to the velocity $\Rightarrow F_{damp} = -b\frac{dx}{dt}$

(iii) An external periodic force $\Rightarrow F = F_0 \sin(pt)$

Thus the total force on the body is $F = -kx - b\frac{dx}{dt} + F_0 \sin(pt)$

Using Newton's second law, we can write $\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = f_0 \sin(pt)$

Where $2\gamma = \frac{b}{m}$, $\omega_0^2 = \frac{k}{m}$ and $f_0 = \frac{F_0}{m}$

This is the differential equation of the forced harmonic oscillator.

- **General solution of the differential equation:**

Solution of differential equation:

$$x = A \sin(pt - \phi)$$

Amplitude:

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2) + 4\gamma^2 p^2}}$$

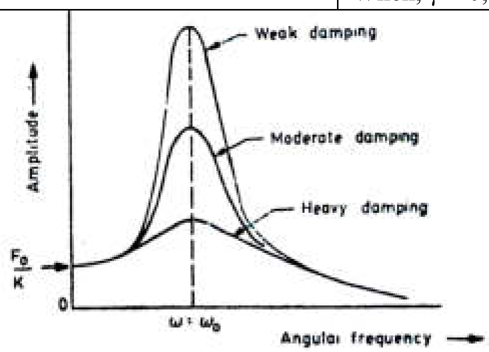
Phase difference: $\tan\phi = = \frac{2\gamma p}{\omega_0^2 - p^2}$

Solution of the forced oscillator: $x = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4\gamma^2 p^2}} \sin(pt - \phi)$

- **Amplitude Resonance**

The amplitude A of the forced oscillator depends on the constant f_0 and p of the driving force and the constant ω_0 and γ of the oscillator. At certain driving frequency amplitude A becomes maximum that is called amplitude resonance. Three different cases arise

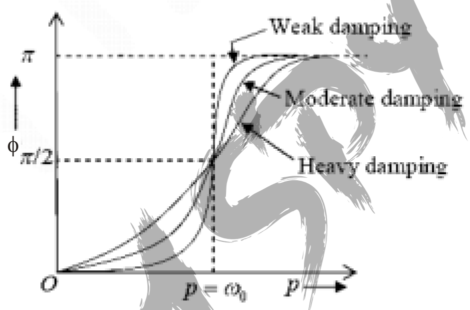
Case : 1	Case : 2	Case : 3
At ($p \ll \omega_0$) The amplitude A turns to be $A = \frac{f_0}{\omega_0^2} \cong \frac{F_0}{k}$ This shows that the amplitude depends only upon force constant and independent of mass, damping constant and driving frequency.	**At ($p \gg \omega_0$),** we get $A = \frac{f_0}{p^2} \cong \frac{F_0}{mp^2}$ This shows that amplitude now depends upon mass and driving frequency.	Amplitude will be maximum when the denominator $\sqrt{(\omega_0^2 - p^2) + 4\gamma^2 p^2}$ in A is minimum. So that its first derivative coefficient will be zero $\frac{d}{dp}\left[\omega_0^2 - p^2 + 4\gamma^2 p^2\right] = 0$ This gives the corresponding driving frequency i.e. amplitude resonance frequency at which A is maximum is $p = \sqrt{(\omega_0^2 - 2\gamma^2)}$ At this frequency the amplitudes is $A_{max} = \frac{f_0}{2\gamma\sqrt{\gamma^2 + p^2}} = \frac{f_0}{2\gamma\sqrt{\omega_0^2 - \gamma^2}}$ This shows that maximum amplitude depends upon the damping constant γ, smaller the damping larger is the amplitude. When, $\gamma = 0$, $A_{max} \to \infty$.



- **Dependence of the phase of displacement on the frequency of driving force**

In steady state condition, the force equation and displacement equation is $F = F_0 \sin(pt)$ and $x = A \sin(pt - \phi)$, the phase factor is $\tan\phi = \dfrac{2\gamma p}{\omega_0^2 - p^2}$ This indicates that the displacement of the forced oscillation lags behind the driving force $F_0 \sin(pt)$ by an angle ϕ. The phase difference ϕ depends upon the damping γ and also on the difference between natural driving frequency. Three different cases arise-

Case 1	Case 2	Case 3
If $p \ll \omega_0$ Then $\tan\phi$ is positive and ϕ lies between 0 and $\frac{\pi}{2}$. In this condition (when $\gamma \approx 0$), ϕ is nearly zero i.e. displacement is in phase with driving force	**If $p = \omega_0$** Then ϕ is infinite and $\phi = \frac{\pi}{2}$. This is the resonance condition, displacement always large behind the force by $\frac{\pi}{2}$. It means that at resonance the displacement is minimum when the driving force is maximum and vice versa.	**If $p \gg \omega_0$** Then ϕ is negative and ϕ lies between $\frac{\pi}{2}$ and π. In this condition, ϕ is nearly π i.e. displacement almost opposite in phase with the driving force. The variation of ϕ with driving frequency p is given below.



- **Velocity Resonance**

Velocity of the body also depends on the constant f_0 and p of the driving force and the constant ω_0 and γ of the oscillator. At certain driving frequency velocity amplitude becomes maximum that is called velocity resonance. The instantaneous velocity of the body is

$$u = \frac{dx}{dt} = \frac{f_0 p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}} \cos(pt - \phi)$$

The velocity will be maximum when $\cos(pt - \phi) = 1$. The maximum value is known as "velocity amplitude" u_0. Thus

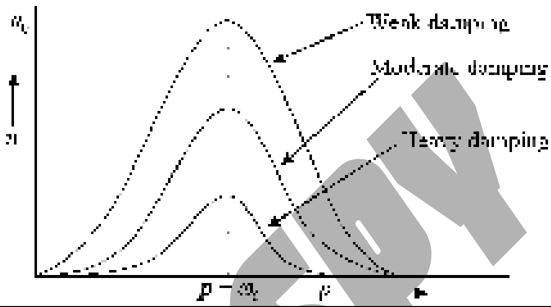
$$\frac{f_0 p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}}$$

Three different cases arise–

Case 1	Case 2	Case 3
At ($p \ll \omega_0$). The velocity amplitude u_0 turns to be $u_0 = \frac{f_0 p}{\omega_0^2} = \frac{F_0 p}{k}$ This indicates that the velocity amplitude depends on the spring constant k.	**At ($p \gg \omega_0$)** We get $u_0 = \frac{f_0}{p} = \frac{F_0}{mp}$ It shows that velocity amplitude depends on mass as well as driving frequency.	At certain frequency the velocity amplitude becomes maximum, that frequency is called the velocity resonance frequency. The velocity amplitude can be written as $u_0 = \frac{f_0 p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}} = \frac{f_0}{\sqrt{\left(\frac{\omega_0^2 - p}{p}\right)^2 + 4\gamma^2}}$ The velocity amplitude u_0 will be maximum when the denominator $\sqrt{\left(\frac{\omega_0^2 - p^2}{p}\right)^2 + 4\gamma^2}$ is minimum i.e.

<table>
<tr><td></td><td></td><td>when
$$\left(\frac{\omega_0^2 - p^2}{p}\right) = 0 \Rightarrow p = \omega_0$$
This shows that the velocity resonance irrespective of damping value, always maximum when driving frequency is equal to the natural undamped frequency of the body.
The velocity amplitude at the resonance is
$$u_0 = \frac{f_0}{2\gamma} = \frac{F_0}{b}$$
This shows that velocity amplitude at resonance only depends on the damping constant.</td></tr>
</table>

Dependence of the velocity of a forced oscillator on the driving frequency is shown as below

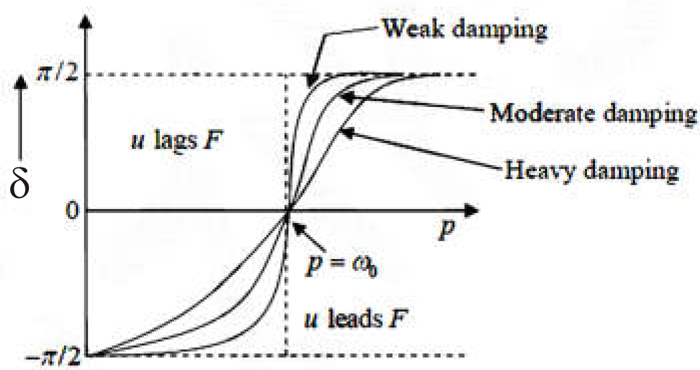


- **Dependence of the phase of velocity on the frequency of driving force:**
  In steady state condition, the force equation and velocity equation is
  $F = F_0 \sin(pt)$

  $$u = u_0 \cos(pt - \phi) = u_0 \sin\left(pt - \left(\phi - \frac{\pi}{2}\right)\right) u_0 \sin(pt - \delta) \Rightarrow \delta = \phi - \frac{\pi}{2}$$

  $$\Rightarrow \tan\delta = \cot\phi = \frac{\omega_0^2 - p^2}{2\gamma p} \quad \because \tan\phi = \frac{2\gamma p}{\omega_0^2 - p^2}$$

Case 1	Case 2	Case 3
If $p \ll \omega_0$ Than $\tan\phi$ is positive and ϕ lies between 0 and $\frac{\pi}{2}$. In this condition (when $\gamma \approx 0$), ϕ is nearly zero. Thus $\delta = -\frac{\pi}{2}$, this means velocity leads the driving force.	**If $p = \omega_0$** Than $\tan\phi$ is infinite and $\phi = \frac{\pi}{2}$, and $\delta = 0$, this means that at resonance, the velocity is always in phase with the driving force.	**If $p \gg \omega_0$** Than $\tan\phi$ is negative and is ϕ lies between $\frac{\pi}{2}$ and π, so that the δ is positive. This means that velocity lags behind the driving force. The dependence of phase of velocity of the body on the frequency of the driving force is shown below.



- **Power supplied to the oscillator by the driving force : Average power absorbed by oscillator**

  An oscillator absorbed energy from the driving force which is dissipated in doing work against the damping force present.

  The instantaneous power *P* (i.e. rate at which work is done) absorbed by the oscillator is equal to energy per unit time.

$$P_{in} = \frac{\text{energy}}{\text{time}} = \frac{F.dx}{dt} = F.\frac{dx}{dt}$$

Thus P_{in} is equal to the product of the instantaneous driving force and the instantaneous velocity.

Thus $P_{in} = F_0 \sin(pt)\dfrac{dx}{dt} = \dfrac{mf_0^2 p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma P^2}} \sin(pt).\cos(pt - \delta)$

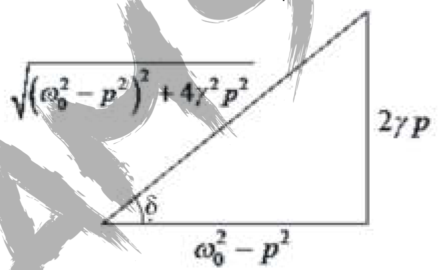
$$= \frac{mf_0^2 p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}} \quad \sin pt \cos pt \cos\delta + \sin^2 pt \sin\delta$$

Now the average power absorbed is $P_{in} = \dfrac{mf_0^2 p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}}\left(\dfrac{1}{2}\sin\delta\right)$

The average values of the periodic function for the one time period $T = \dfrac{2\pi}{p}$ is $\dfrac{1}{T}\int_0^T \sin pt \cos pt\, dt = 0$ and

$\dfrac{1}{T}\int_0^T \sin^2 pt dt = \dfrac{1}{2}$

Since, $\tan\phi = \dfrac{2\gamma p}{\omega_0^2 - p^2}$ thus using the vector model $\sin\delta = \dfrac{2\gamma p}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}}$



Therefore, the average power absorbed by the oscillator (average power supplied by the driving force) is

$$P_{in} = \frac{mf_0^2 \gamma p^2}{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}$$

The power supplied by the driving force is not stored in the system, but is dissipated as work done in moving the system against the force of friction.

- **Power Dissipated through frictional force**

  The instantaneous power dissipated through friction is given by–

  P_{dis} = instantaneous frictional force × instantaneous velocity

$$= b\frac{dx}{dt} \times \frac{dx}{dt} = 2m\gamma\left(\frac{dx}{dt}\right)^2 = 2m\gamma \frac{f_0^2 p^2}{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 P^2}\cos^2(pt - \delta)$$

Now the average of $\cos^2(pt-\delta)$ for one full period is $\dfrac{1}{2}$. Therefore the average power dissipated is

$$P_{dis} = \frac{mf_0^2 \gamma p^2}{\left(\omega_0^2 - p^2\right)^2 + 4\gamma^2 p^2}$$

Thus $\qquad P_{in} = P_{dis}$

This shows that in the steady state the average power supplied by the driving force is equal to the average power dissipated by the frictional force.

The instantaneous input power is not equal to the instantaneous power dissipated. Therefore at any instant of time the power stored in the oscillator is not constant.

- **Maximum power absorption**

The average power can also be written as $P_{in} = \dfrac{mf_0^2\gamma}{\left(\dfrac{\omega_0^2 - p^2}{p}\right)^2 + 4\gamma^2}$

P_{in} will be maximum when the denominator $\left(\dfrac{\omega_0^2 - p^2}{p}\right)^2 + 4\gamma^2$ is a minimum, this occurs when $\omega_0^2 - p^2 = 0$ or

$p = \omega_0$

this is the condition of velocity resonance. Hence the power transferred from the driving force is maximum at the frequency of velocity resonance. The maximum power is

$$P_{in(maximum)} = \frac{mf_0^2}{4\gamma}$$

Sharpness of Resonance and Bandwidth

- **Sharpness of Resonance**

The rapidity with which the power falls from its resonant value with change in a driving frequency is known as sharpness of resonance.

The average power is maximum at certain frequency. As the driving frequency deviates either way from its resonant value, the power falls from its maximum value. If the fall in power with change in driving frequency from the resonant value is large, the resonance is said to be sharp, on the other hand, the fall is small, the resonant is said to be flat. It is defined in terms of quality factor.

Quality Factor

The quality factor is defined as $Q = \dfrac{\omega_0}{\omega_2 - \omega_1} = \dfrac{\text{Resonance frequency}}{\text{bandwidth}} = \dfrac{\omega_0}{2\gamma} = \omega_0\tau_R$

Quality factor also defined as, $Q = 2\pi\dfrac{\text{average energy stored in one period}}{\text{average energy lost in one period}} = 2\pi\dfrac{E}{P_{dis}T}$

Where $P_{dis} \times T$ is the average energy lost in one period. On solving for energy we get the following expression of the quality factor.

$$Q = \frac{p^2 + \omega_0^2}{4\gamma p} = \frac{1}{2}\left(\frac{p^2 + \omega_0^2}{p^2}\right)(p\tau_R)$$

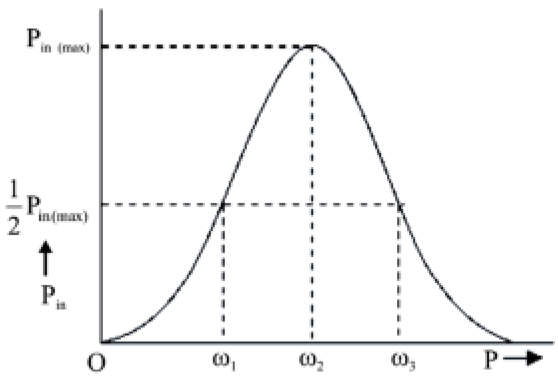
This is the exact expression for the quality factor of forced oscillator.

Near resonance, $P = \omega_0$, so we get $Q = \omega_0\tau_R$

- **Bandwidth of Resonance**

The difference in values of the driving frequency, at which the average power absorbed drops to half of its maximum value, is called the band width of the resonance.

Bandwidth = $\omega_2 - \omega_1$



The average power absorbed is $P_{in} = \dfrac{mf_0^2\gamma p^2}{(\omega_0^2 - p^2)^2 + 4\gamma^2 p^2}$

The maximum average power is $P_{in(maximum)} = \dfrac{mf_0^2}{4\gamma}$

The variation of the average power with driving frequency is shown in above figure also shows the half power frequencies.

The value of p at which the power goes half the maximum value obtained as

$$\langle P_{in}\rangle = \frac{1}{2}P_{in(maximum)} \Rightarrow \frac{mf_0^2\gamma^2p^2}{\left(\omega_0^2-p^2\right)^2+4\gamma^2p^2} = \frac{1}{2}\frac{mf_0^2}{4\gamma}$$

$$\text{or } \left(\omega_0^2-p^2\right)^2+4\gamma^2p^2 = 8\gamma^2p^2 \Rightarrow \left(\omega_0^2-p^2\right)^2 = 4\gamma^2p^2$$

$$\text{or } \omega_0^2-p^2 = \pm 2\gamma p \Rightarrow p^2 = \omega_0^2 \pm 2\gamma p$$

These are two quadratic equation in p,

$$p^2+2\gamma p-\omega_0^2 = 0 \text{ and } p^2-2\gamma p-\omega_0^2 = 0$$

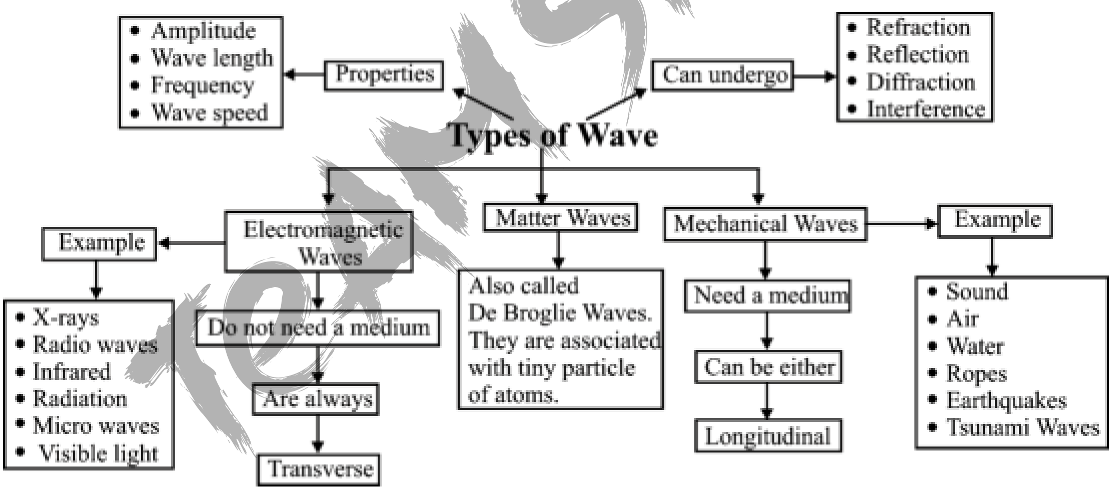
each has two roots one positive and other negative. Since negative frequencies are not allowed, thus the allowed positive roots are $p_1(=\omega_1) = -\gamma+\sqrt{\gamma^2+2\gamma\omega_0^2}$ and $p_2(=\omega_1) = \gamma+\sqrt{\gamma^2+2\gamma\omega_0^2}$ The frequency difference between two half power points i.e. bandwidth is

Bandwidth $= \omega_2 - \omega_1 = 2\gamma = \dfrac{1}{\tau}$ which indicates that smaller the bandwidth, sharper is the resonance.

(3) Travelling waves

▪ Wave

A wave is a disturbance in a medium that carries energy without a net movement of particles.



▪ Difference between Pulse and wave -

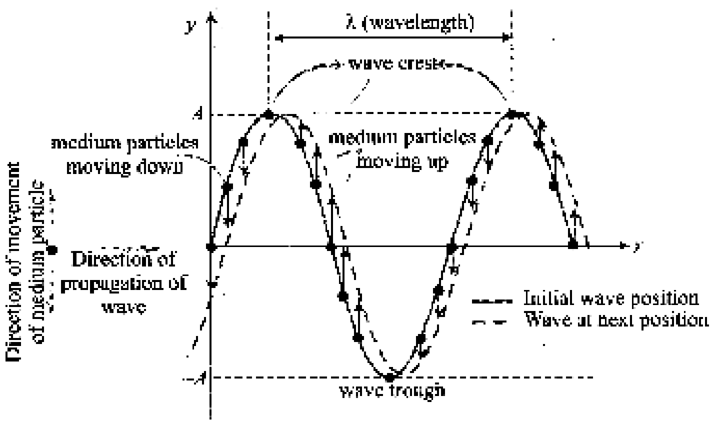
Pulse	wave
• It is a short duration disturbance	It is a continuous disturbance
• It does not repeat	It repeats periodically
• It is formed in small portion	It is formed continuously due to disturbance
• Disturbance is momentary	Disturbance exists for long period.

▪ Nature of Waves-

(i) Transverse waves:

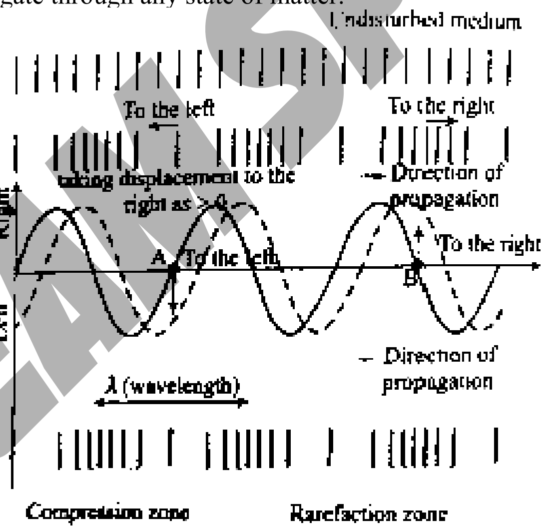
- A wave in which the particle of the medium vibrate at right angles to the direction of propagation of wave, is called transverse waves.
- A particle either at the crest or at the trough has a tendency to move towards the mean position
- A particle at the crest or the trough has zero velocity and the distance of the particle from the mean position is termed as amplitude of the wave.
- Distance between two consecutive crests/trough is equal to the wavelength of the wave.

- Distance between a consecutive pair of crest and trough is half the wavelength of wave.



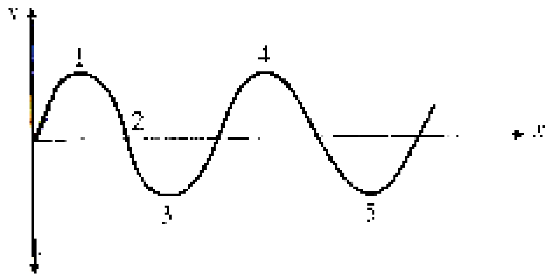
(ii) Longitudinal wave-

- A wave motion in which the individual particles of a medium executes periodic motion about their mean position along the direction of propagation of wave e.g. sound wave
- A longitudinal waves moves by the phenomena of compression and rarefaction in the medium.
- All particle in compression phase are in phase (at a given instant of time)
- All particle in rarefaction phase are out of phase (at a given instant of time)
- Distance between two consecutive compressions or rarefaction is equal to the wavelength of the wave. Longitudinal waves can propagate through any state of matter.



▪ Some Important terms related to wave Motion:

- **Phase**: It defines the position (in terms of distance from mean position) and velocity of a particle oscillating under the influence of wave.



(a) Oscillations of the particles 1 and 4 are in phase.
(b) Oscillations of the particles 1 and 2 are not in phase
(c) Oscillations of the particles 3 and 5 are in phase.
(d) Oscillation of the particle 4 and 5 are out of phase.

- **Amplitude:-** It is the maximum displacement of the particles from its mean position on either side.
- **Wavelength:-** The distance between two nearest points which are in the same phase of vibration is called the wavelength. It is denoted by 'λ'
- **Time period:-** Time taken to complete one oscillation is called time period (T)
- **Frequency**: The number of vibrations completed in one second is called frequency of wave.

$$\text{Frequency } f = \frac{1}{\text{Time period}(T)}$$

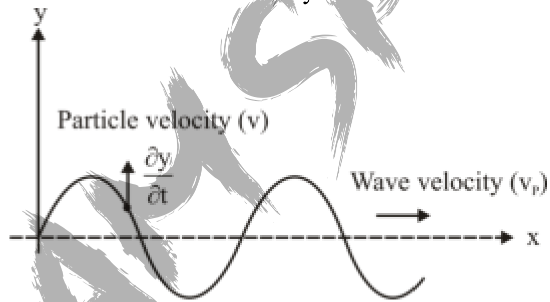
Its S.I unit is Hertz.

Angular frequency, $\omega = \frac{2\pi}{T}$ rad-s^{-1}

Relation among velocity, frequency and wavelength of wave, is given by

$$v = f\lambda$$

- **Angular wave Number:-**
  The number of wavelength per unit distance is called angular wave number.
  It is denoted by k.
  $\therefore k = \frac{2\pi}{\lambda}$
  SI unit is rad/m
- **Particle velocity in wave motion: -** The instantaneous velocity of an oscillation particle of the medium through which a wave is travelling is known as a 'Particle Velocity.'



- **Wave Velocity :-** The velocity with which the disturbance, travel thorugh the medium is called wave (or phase) velocity.

■ **Relation between particle velocity and wave velocity:-**

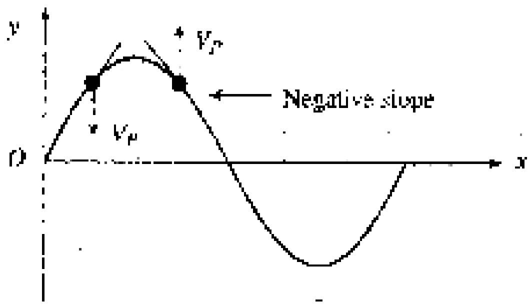
Wave equation, $y = A\sin(\omega t - kx)$

Particle velocity, $v = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$

Wave velocity , $v_P = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}$

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k\cos(\omega t - kx) = \frac{1}{v_P}\frac{\partial y}{\partial t}$$

$$\Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_P}\frac{\partial y}{\partial t} \Rightarrow v = -v_P\frac{\partial y}{\partial x}$$



$\frac{\partial y}{\partial x}$ represents the slope of the string (wave) at the point x.

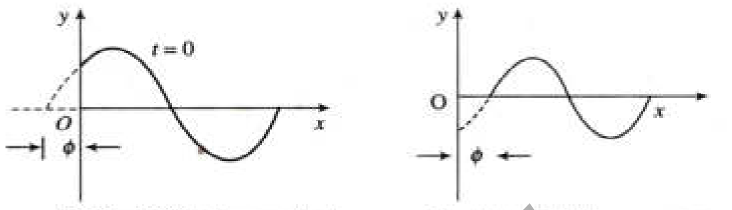
Particle velocity at a given position and time is equal to negative of the product of wave velocity, with slope of wave at that point at that instant.

- **Wave functions: -** Mathematical functions which describe the motion of a wave pulse are called wave functions.

■ **Plane Progressive wave-**

A wave which travels in a given direction with constant amplitude is known as a progressive wave.

- **General Expression for a plane progressive Wave** :
  The general equation of simple harmonic progressive wave is given by.
  $y = A \sin 2\pi \left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$
  If an initial phase ϕ exists at origination point of wave, equation is changed to
  $$y = A\sin\left[2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right) + \phi\right]$$
- **Positive and Negative Initial Phase constant**:



Positive initial phase constant
$y = A\sin(kx - \omega t + \phi)$
The sine curve starts from the left of the origin.

Negative initial phase constant
$y = A\sin(kx - \omega t - \phi)$
The sine curve starts from the right of the origin.

- **Some Important Relation in Wave**:
  For the wave travelling in positive x-direction, the relation between phase difference and time difference is given by-
  $\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{T} \times (t_2 - t_1) = \frac{2\pi}{T} \times \Delta t$
  Phase difference = $\frac{2\pi}{T}$ × Time difference
  Relation between variation of phase with Distance – $\Delta\phi = \frac{2\pi}{T}\Delta x$
  Phase difference = $\frac{2\pi}{T}$ × Path difference
  In general, the expression of linear wave equation as it applies to wave on strings is given by –
  $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$
  Where, v = wave speed,
  (a) For waves on strings, y represent the vertical position of elements of the string.
  (b) For sound waves, y corresponds to longitudinal position of elements of air from equilibrium,
  (c) For electromagnetic waves, y corresponds to electric & magnetic field components.
- **Superposition Principles -**
  The resultant displacement of a particles at any point of the medium at any instant of time is the vector sum of the displacements caused to the particles by the individual waves.
  If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...$ are the displacement of particles of a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$
- **Speed of wave on strings –**
  $v = \sqrt{\frac{T}{\mu}}$; Where T = Tension on string, μ = Linear mass density = $\frac{m}{L}$,
- **Rate of energy transfer by sinusoidal wave on strings–**
  Total energy = Potential energy + Kinetic energy.
  $E_\lambda = U_\lambda + K_\lambda = \frac{1}{4}\mu^2 A^2 \lambda + \frac{1}{2}\mu\omega^2 A^2 \lambda$
- **Power of a wave–**
  $P = \frac{1}{4}\mu\omega^2 A^2 v$

- **Intensity of the wave–**

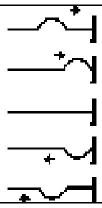
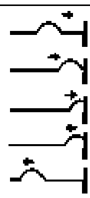
$$I = \frac{\text{Power}}{\text{area of cross-section}}$$

$$I = \frac{1}{2}\rho\omega^2 A^2 v$$

- **Frequency of vibration in P^{th} harmonic on string**

$$n = \frac{P}{2\ell}\sqrt{\frac{T}{m}}$$

■ Reflection of Waves at fixed end and free End

Reflection of Waves– Fixed End	Reflection of Waves– Free End
• When the wave reaches the fixed end and it exerts an upwards pull on the end. According to Newton's third law, the fixed point exerts an equal and opposite force downward on the string. Thus, there is phase change of π for the reflected wave, the wave is inverted as shown in the Fig. below • Whenever a travelling wave reaches a boundary, some or all of the wave is reflected • When it is reflected from a fixed end, the wave is inverted.	• If the end of the string is free to move vertically, the free and end overshoots twice the amplitude as shown in fig. below. • When a travelling wave reaches a boundary, all part of it is reflected. • When reflected from a free end, the pulse is not inverted.
	

Incident wave	Reflected wave from free boundary, $\phi = 0$	Reflected wave from fixed boundary, $\phi = \pi$
• $y = A \sin(kx - \omega t)$ • $y = A \cos(kx - \omega t)$	$y = A \sin(kx - \omega t + 0) = A \sin(kx - \omega t)$ $y = A \cos(kx - \omega t + 0) = A \cos(kx - \omega t)$	$y = A \sin(kx - \omega t + \pi) = -A \sin(kx - \omega t)$ $y = A \cos(kx - \omega t + \pi) = -A \cos(kx - \omega t)$

(4) Standing Waves

■ Different between Standing waves and Travelling waves-

Standing waves	Travelling waves
• Wave amplitude is constant at all points along the wave.	Wave amplitude depends upon the position along the wave.
• Energy is always transferred.	Energy is not transferred.
• The wavelength is twice the distance between consecutive nodes.	The wavelength is the distance between consecutive crests.
• Phase varies continuously along the wave.	Phase is constant between consecutive crests.

■ Stationary or standing waves–

When two similar waves propagate in a bounded medium in opposite direction, then due to their superposition a new type of wave is obtained, which appears stationary in the medium. This wave is called stationary or standing wave.

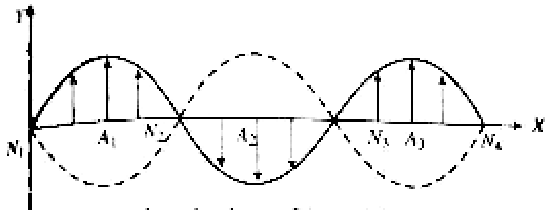
Equation of a stationary wave,

$$y = 2a \sin\frac{2\pi t}{T}\cos\frac{2\pi x}{\lambda}$$

■ Characteristics of standing wave–

- In a stationary wave, the disturbance does not move in any direction. The conditions of crests and troughs merely appear and disappear in fixed positions to be followed by opposite condition after every T/2 sec.
- All the particles of the medium except those at nodes, execute simple harmonic motion with the particle of the wave about their mean position.
- During the formation of a stationary wave, the medium is broken into loops between equally spaced points called nodes remain at rest and in between them are points of maximum displacement called antinodes.

- The amplitude of the particles are different at different points. The amplitude varies gradually from zero at the nodes to the maximum at the antinodes.



Opposite phases of the particles in consecutive segments

- The maximum velocity is different at different points. Its value is zero at the nodes and gradually increases towards the antinodes. All the particles attain their maximum velocities simultaneously when they pass through their mean positions.
- All the particles in a particular segment between two nodes vibrate in the same phase but the particles in the neighboring segments vibrate in opposite phase, as shown in above figure.
- The energy becomes alternatively wholly potential and wholly kinetic twice in each cycle. It is wholly potential when particles are at their positions of maximum displacement and wholly kinetic when the particle pass through the mean positions.
- A stationary wave has the same wavelength and time period as the component waves.
- The distance between two consecutive nodes and antinodes is $\lambda/2$. The distance between nodes and next antinodes is $\lambda/4$.
- **Position of nodes:** Nodes are the points on the string where the amplitude of oscillation of constituents is zero. i.e.,

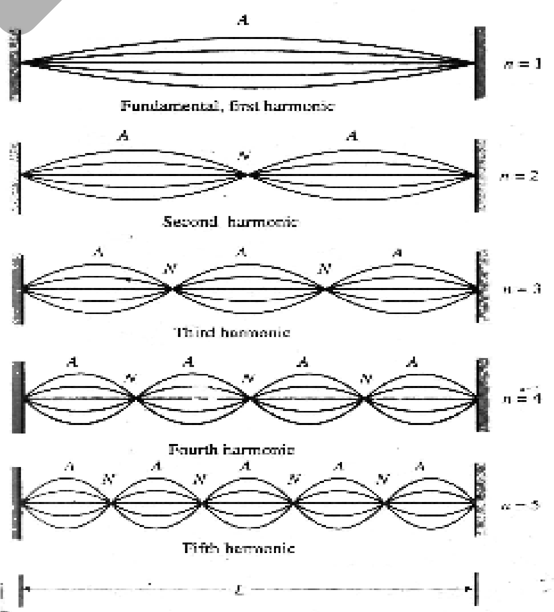
$$\sin kx = 0$$
$$kx = n\pi, \quad n = 0, 1, 2, 3, \ldots\ldots$$
$$\frac{2\pi}{\lambda}x = n\pi \Rightarrow x = \frac{n\lambda}{2}$$

At x = L (L is length of string), $L = \frac{n\lambda}{2}$

- **Position of Antinodes:** Antinodes are the points where the amplitude of oscillation of the constituents is maximum. For maximum amplitude

$$\sin kx = \pm 1$$
$$kx = (2n+1)\frac{\pi}{2} \quad \text{where, } n = 0,1,2\ldots$$
$$\frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\lambda}{2}$$

▪ Standing waves in a string fixed at both Ends–



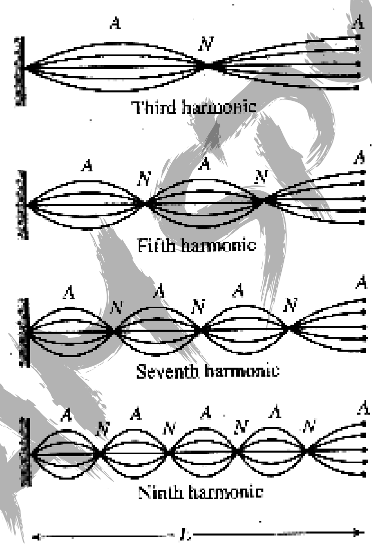
- Velocity, $v = \sqrt{\frac{T}{\mu}}$

  Where T = Tension in string

  μ = Linear mass density
- The lowest possible natural frequency of a system is called its fundamental mode or first harmonics. It is given by $f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$
- Frequency of first overtone or second harmonic $f_2 = 2\frac{v}{2L} = 2f_1$
- Frequency of second overtone or third harmonic $f_3 = 3\frac{v}{2L} = 3f_1$

  $\Rightarrow$ $f_1 : f_2 : f_3 : .... = 1:2:3: ....$
- Frequency of nth overtone is $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = n\frac{v}{2L}$

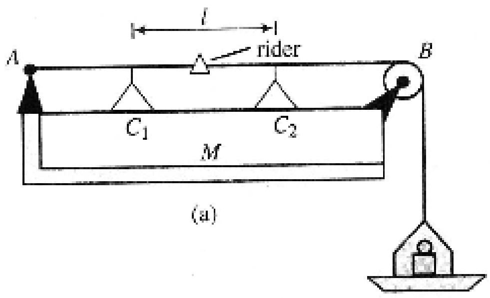
■ **String Fixed at one end and free from other End –**



- The resonance frequencies are given by $f_n = n\frac{v}{4L} = nf_1$ where, n = 1, 3, 5

  Where, $f_1 = v/4L$ (fundamental frequency)
- The natural frequency occur in the rations: 1: 3: 5: 7

■ **Sonometer**:
- It is a device used to measure velocity of transverse mechanical wave in a stretched metal wire. The principle of sonometer is based on resonance of string vibration.
- If tension in wire is T and f_0 is the frequency of turning fork, the wavelength of wave in wire is –



(a)

$\lambda = \frac{v}{f_0} = \frac{1}{f_0}\sqrt{\frac{T}{\mu}}$ $(\lambda = 2\ell)$

▪ Laws of String

- **Law of length** : If T and m are constant then

$$f \propto \frac{1}{\ell}, \quad f\ell = \text{Constant} \Rightarrow f_1\ell_1 = f_2\ell_2$$

- **Law of mass** : if T and ℓ are constant then

$$f \propto \frac{1}{\sqrt{m}} \quad \Rightarrow f\sqrt{m} = \text{Const.}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}}$$

- **Law of density** : If T , ℓ and r are Const.

$$\text{then } f \propto \frac{1}{\sqrt{d}} \quad \Rightarrow f\sqrt{d} = \text{const.} \quad \Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{d_2}{d_1}}$$

Where, r = radius of string
d = density of string

- **Law of tension** : if ℓ and m are const. then

$$f \propto \sqrt{T} \Rightarrow \frac{f}{\sqrt{T}} = \text{const.}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$$

▪ Melde's Experiment–

This is an experiment of demonstration of transverse stationary wave in stretched string.

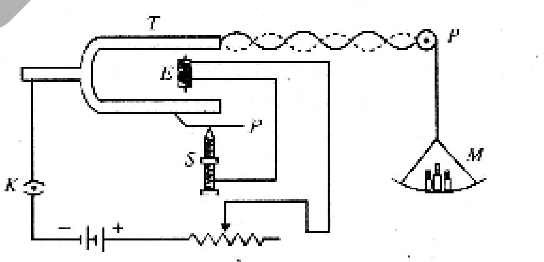
In Melde's experiment, the one end of the string is connected to the prong of an electrically oscillated tuning fork. The other end of the string is connected to the scale pan. The string passes over a smooth frictionless pulley. The distance between tuning fork and pulley can be adjusted. There are two different ways in which oscillations can be established in the string.

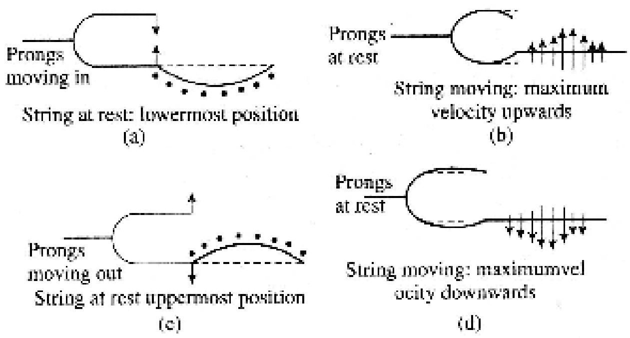
Case 1: Transverse Mode of Vibration

- In this case tuning fork vibrates right angle to the length of the string.
- In this case the frequency of vibration of string is equal to the frequency of tuning fork.
- No. of loops are double than that of longitudinal.
- The frequency of the oscillating string can be given as –

$$f_f = f_s = \frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$$

where, f_f = frequency of oscillation of tuning fork.
f_s = frequency of string.
p = no. of loops





Case 2: Longitudinal Mode of Vibrations:

- In this case the vibrations of tuning fork are along the length of the string.
- Frequency of fork is twice the frequency of string

$f_s = \frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$

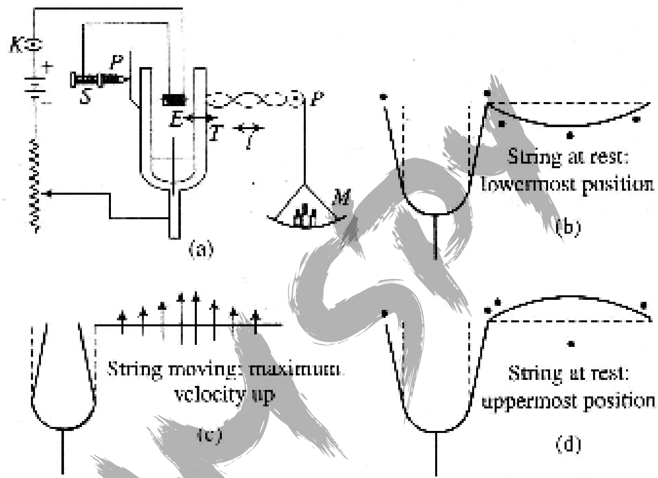
$f_f = 2f_s = \frac{p}{\ell} \times \sqrt{\frac{T}{\mu}}$

- If the frequency of the tuning fork remains constant then–

$$p\sqrt{T} = \text{constant}$$

Where, p = no of loops

μ = constant



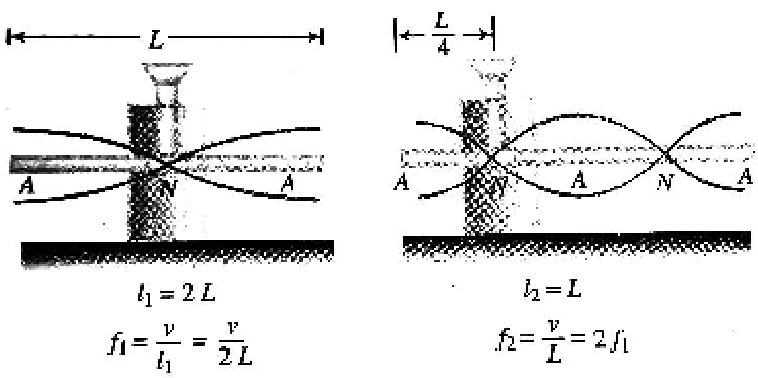
Standing Waves in Air Columns

Standing waves are the result of interference between longitudinal sound waves travelling in opposite directions.

- In a pipe closed at one end, the closed end is a displacement node because the rigid barrier at this end does not allow longitudinal motion of the air. Because the pressure wave is 90° out of phase with the displacement wave, the closed end of an air column corresponds to a pressure antinode (that is, a point of maximum pressure variation)
- The open end of an air column is approximately a displacement antinode and a pressure node. The pressure at this end must remain constant at atmospheric pressure.

Standing waves in rods

It is also possible to set up transverse standing waves in rods included triangles, marimbas, xylophones, glockenspiels, chimes, and vibraphones. Other devices that make sounds from vibrating bars include music boxes and wind chimes.



- **Resonance**

  If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal the system frequency. This phenomenon, Known as resonance.

- **Lissajou's figure :-**

  If two SHMs are acting in mutually perpendicular directions, then due to their superpositions the resultant motion in general is a curve /loop. The shape of the curve depends on the frequency ratio of two SHMs and initial phase difference between them such figures are called Lissajou's figures. Let two SHMs be of same frequency (eg. x = a, sin ωt and y = a_2 sin ($\omega t + \phi$), then the general equation of resultant motion is found to be

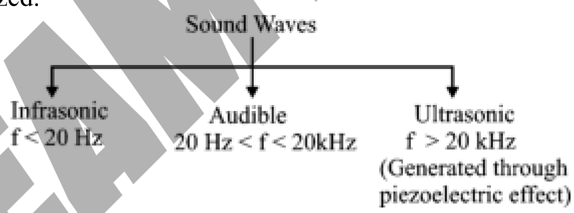
$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - \frac{2xy}{a_1 a_2}\cos\phi = \sin^2\phi$$

Lissajous		Phase difference
(I)	Straight line	When $\phi = 0^\circ, 180^\circ$
(II)	Ellipse	When $0^\circ < \phi < 90^\circ, 90^\circ < \phi < 180^\circ$
(III)	Circle	When $\phi = 90^\circ, 270^\circ$
(IV)	Oblique ellipse	When $\phi = 45^\circ, 135^\circ$

(5) Sound Waves and Doppler Effect

Introduction:-

- Sound is a form of energy which produces a sensation of hearing in our ears.
- Sound waves are longitudinal in nature.
- It travel through any material medium with a speed that depends on the property of the medium.
- It cannot propagate through vacuum.
- Sound waves (longitudinal waves) can reflect, refract, interfere and diffract but cannot be polarized as only transverse waves can polarized.



- **Characteristics of Sound Wave:** Sound wave has three characteristics-

  - **Intensity or Loudness:** Intensity of sound is energy transmitted by sound waves per second per unit area by sound waves. Its SI unit is watt/mtere^2. It is given by -

    $I = \frac{P}{A}$ Where, P = sound power, A = area

    Intensity is measured in decibel (dB).

    Sound Intensity level (I_L) = $10\log_{10}\frac{I}{I_0}$ Where, I_0 = reference intensity.

  - **Pitch or Frequency:** Pitch of sound directly depends on frequency.

    A shrill and sharp sound has higher pitch and a grave and dull sound has lower pitch.

  - **Quality or Timbre:** Quality is the characteristic of sound that differentiates between two sounds of same intensity and same frequency. Quality depends on harmonics and their relative order and intensity.

- **Velocity of Sound Waves**

  - In the case of water wave of longer wavelength the speed depends only in the acceleration due to gravity. In shallow water, the speed of water decrease. A wave approaching the shore gradually slows down to zero speed. The velocity of waves in a canal is given by $V = \sqrt{gh}$. Where, g is acceleration due to gravity and h is height of the waves.
  - The velocity of sound is maximum in solids, intermediate in liquids and minimum in gases.
  - Newton's Assumed that when sound wave travel in a gas, the changes in volume and pressure are isothermal.

- Newton's formula for the velocity of a longitudinal wave in a homogeneous medium is $V = \sqrt{E/d}$. Where, E is the modulus of elasticity for a particular type of strain set up and d is the density of the medium.

  In solids, $V = \sqrt{\frac{Y}{d}}$ Where, Y = Young's modulus

  In liquids, $V = \sqrt{\frac{B}{d}}$ Where, B = Bulk modulus

  In gases, $V = \sqrt{\frac{P}{d}}$ Where, P = Pressure of the gas

  For air at NTP:- $P = 1.013\times10^5\ N/m^2$ & $\rho = 1.29\ kg/m^3$

  By Newton's formula the speed of sound at one atmosphere is

$$v = \sqrt{\frac{1.013\times10^5}{1.29}} \simeq 280 m/s$$

  This value is less than experimental value of 332 m/s. Hence Newton's formula requires some correction, which was made by Laplace in 1816.

- **Laplace's Correction**

  French scientist Laplace pointed out that when sound propagates in air the heat of the medium remains constant instead of its temperature. So he replaced isothermal elasticity by adiabatic elasticity B_{ad}. The corrected formula is

$$v = \sqrt{\frac{B_{ad}}{\rho}}, \quad B_{ad} = \gamma P$$

  For air $\gamma = 1.41$, so the speed of sound in air at STP will be

$$v = \sqrt{\gamma}\sqrt{\frac{P}{\rho}} = \sqrt{1.41}\times 280 = 332\ m/s$$

  This value is in very close agreement with the experimental value.

- Velocity of sound in a gas in given by $V = \sqrt{\frac{\gamma RT}{M}}$

■ Factors Affecting Speed of Sound in Gas

- **Effect of Pressure**

$$\frac{P}{\rho} = \text{constant}$$

  With the change in pressure, the density also changes in same proportion, so that P/ρ remains constant. Hence pressure has no effect on the speed of sound in a gas.

- **Effect of Density**

$$v = \sqrt{\frac{\gamma P}{d}} \Rightarrow v \propto \frac{1}{\sqrt{d}}$$

  For two gases of densities d_1 and d_2 at same pressure with ratios of specific heats γ_1 and γ_2

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2}\times\frac{d_2}{d_1}}$$

- **Effect of Temperature**

  We have,

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\text{Clearly, } v \propto \sqrt{T} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{273+t_1\,^\circ C}{273+t_2\,^\circ C}}$$

  Hence the speed of sound in a gas is proportional to the square root of its absolute temperature.

- **Effect of Humidity**

  With the increase in humidity, the density of air decreases.

$$v \propto \frac{1}{\sqrt{d}}$$

  Hence, the speed of sound will increase with increase in humidity.

- **Effect of Frequency**:-

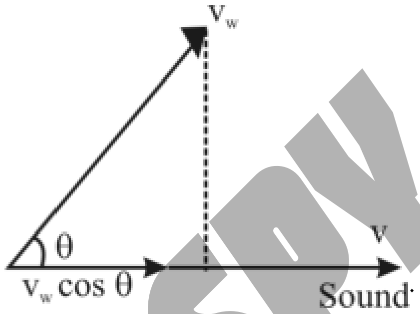
  With the change in frequency of the sound wave, wavelength also changes, so that

$$f\lambda = v \text{ (constant)}$$

  Thus, the speed of sound is independent of its frequency.

- **Effect of Wind-**

  As the sound is carried by air, so its speed is affected by the wind velocity. Suppose the wind is blowing with a velocity v_w at an angle θ with the direction of propagation of the sound.



  When wind blows in the direction of sound ($\theta = 0$), the resultant velocity is $v + v_w$.

  When wind blows in the opposite direction of sound ($\theta = 180^\circ$), the resultant velocity is $= v - v_w$

▪ Organ Pipes

Organ pipes are those cylindrical pipes which are used for producing musical (longitudinal) sounds. Organ pipes are of two types

(i) Open Organ Pipe cylindrical pipes open at both ends.

(ii) Closed Organ Pipe Cylindrical pipes open at one end and closed at other end.

- **Terms Related to Vibrating Air Columns/Strings**

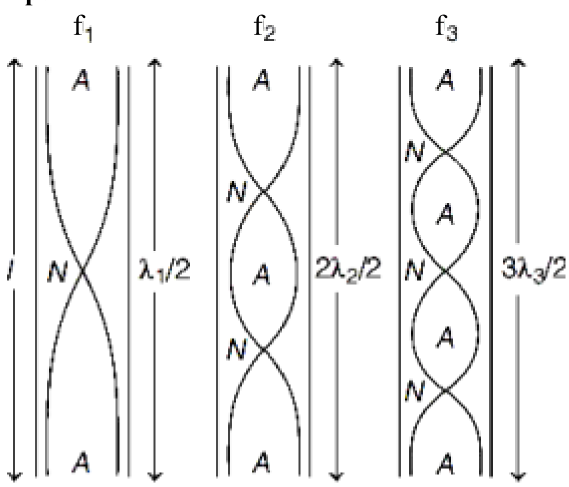
  Fundamental Note: It is the sound of lowest frequency produced in fundamental note of vibration of a system.

  Overtones: Tones having frequencies greater than the fundamental note are called overtones.

  Harmonics: When the frequencies of overtone are integral multiples of the fundamental, then they are known as harmonics. Thus the note of lowest frequency f is called fundamental note or harmonic

  First harmonics: The node of frequency 2f is called **second harmonic** or **first overtone**.

- **Vibrations in open Organ Pipe**



Fundamental frequency or frequency of first harmonic $f_1 = \frac{v}{2\ell}$

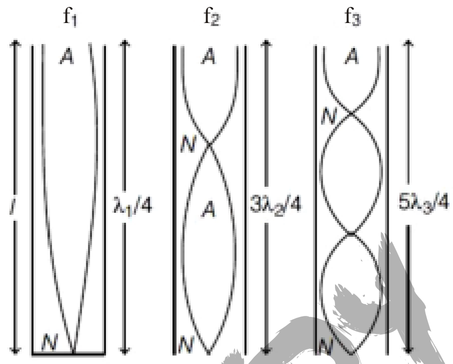
Frequency of first overtone or second harmonic $f_2 = 2\frac{v}{2\ell} = 2f_1$

Frequency of second overtone or third harmonic $f_3 = 3\frac{v}{2\ell} = 3f_1$

$\therefore \quad f_1 : f_2 : f_3 : ...... = 1 : 2 : 3 : .....$

Therefore, even and odd harmonics are produced by an open organ pipe.

Vibrations in Closed Organ Pipe



Fundamental frequency or frequency of first harmonic $f_1 = \frac{v}{4\ell}$

Frequency of first overtone or third harmonic $f_2 = \frac{3v}{4\ell} = 3f_1$

Frequency of second harmonic or fifth harmonic $f_3 = 5.\frac{v}{4\ell} = 5f_1$

$\therefore \quad f_1 : f_2 : f_3 : ..... = 1 : 3 : 5 :......$

Therefore only odd harmonics are produced by a closed organ pipe.

- **End Correction**

  Antinode is not obtained at exact open end but slightly above it. The distance between open end and antinode is called end correction.

  It is denoted by e.

  Effective length of an open organ pipe $\ell' = (\ell + 2e)$

  Effective length of a closed organ pipe $\ell' = (\ell + e)$

  If r is the radius of organ pipe, then e = 0.6r

  With end correction fundamental frequency of closed pipe $f = \frac{v}{4(\ell + 0.6r)}$

  Fundamental frequency of open pipe $f = \frac{v}{2(\ell + 1.2r)}$

- **Factors Affecting frequency of wave in a pipe**

  (i) Length of air column, $n \propto \frac{1}{\ell}$

  (ii) Radius of air column, $n \propto \frac{1}{r}$

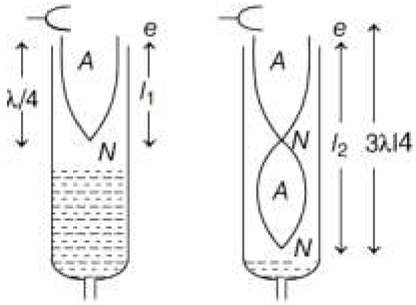
  (iii) Temperature of air column, $n \propto \sqrt{T}$

  (iv) Pressure of air inside air column, $n \propto \sqrt{T}$

  (v) Density of air, $n \propto = \frac{1}{\sqrt{\rho}}$

  (vi) Velocity of sound in air column, $n \propto v$

- **Resonance Tube**
  Resonance tube is a closed organ pipe in which length of air column can be changed by changing height of liquid column in it.



For first resonance, $\frac{\lambda}{4} = \ell_1 + e$

Second resonance, $\frac{3\lambda}{4} = \ell_2 + e$

$(\ell_2 - \ell_1) = \frac{\lambda}{2} \Rightarrow \lambda = 2\,(\ell_2 - \ell_1)$

Velocity of sound $v = f\lambda = 2f\,(\ell_2 - \ell_1)$

End correction, $e = \frac{\ell_2 - 3\ell_1}{2}$

Beats

- When two sound waves of nearly different frequencies are produced simultaneously, then intensity of the resultant sound produced by their superposition increases and decreases alternately with time. This rise and fall intensity of sound is called beats.
- The difference of frequencies heard in one second is called beats frequency.
- If simple harmonic progressive waves of frequencies f_1 and f_2 travelling in same direction superimpose, the resultant waves is represented by $y = \left\{(a_1 + a_2)\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right\}\sin 2\pi\left(\frac{f_1 - f_2}{2}\right)t$
- The frequency of resultant wave is $\left(\frac{f_1 - f_2}{2}\right)$
- The difference of frequencies should not be more than 10. Sound persists on human ear drums for 0.1 second. hence, beats will not be heard if the frequency difference exceeds 10.
- Number of beats heard per second = $n_1 - n_2$ = difference of frequencies of two waves.
- Maximum amplitude = $(a_1 + a_2)$
- Maximum intensity = $(\text{Maximum amplitude})^2 = (a_1 + a_2)^2$
- For loudness, time intervals are $\frac{1}{n_1 - n_2}, \frac{2}{n_1 - n_2} \ldots$
- If two sound waves f wavelength λ_1 and λ_2 product a beats per second , then velocity of sound can be determined by n = $n = \frac{v}{\lambda_1} + \frac{v}{\lambda_2}$ or $V = \frac{\lambda_1\lambda_2}{(\lambda_2 - \lambda_1)}$
- Beats can be produced by taking two identical tuning fork and loading or filling either of then and vibrating them together.

ECHO :-

- When a sound is produced and a listener hears it after reflection from an obstacle, the reflected sound is called an ECHO.
- If d is the distance between source and the reflecting surface and t is the time taken to hear echo after sound is produced, $t = \frac{2d}{V}$ where, V is velocity of sound.
- The distance between source and reflecting surface d = Vt/ 2.
- The minimum distance between sources and the reflecting surface to hear a clear is V/20. It is equal to 16.5 m if V = 300 m/s.
- If a person standing between two parallel hills fires a gun and hears echo after t_1 sec and the second echo after t_2 sec, the distance between two hills is $d = \frac{V(t_1 + t_2)}{2}$.
- In the above case, the third echo will be heard after $(t_1 + t_2)$ sec.
- In the above case, if echos are head at regular intervals of time is t sec, the distance between tow hills is d = V.t or d = (3/2)V.t.
- If a motor car approaching a cliff with a velocity 'u' sounds the horn and the echo is heard after 't' sec, then the distance between the cliff and the original position of car is $d = \left(\frac{v+u}{2}\right)t$.
- In the above case, the distance between the cliff and the point where the echo is heard is $d = \left(\frac{v-u}{2}\right)t$.
- A road runs paralleled to a long vertical line of hills. If a motorist moving with a speed 'u' sounds the horn and hears the echo after 't' sec, then the distance between the road the cliff is $d = \frac{1}{2}\sqrt{v^2 - u^2}$.
- A road runs midways between two parallel rows of hills. If a motorist moving with a speed 'u' sounds the horn and hears the echo after 't' sec, then the distance between two rows of hills $d = \sqrt{v^2 - u^2}$.

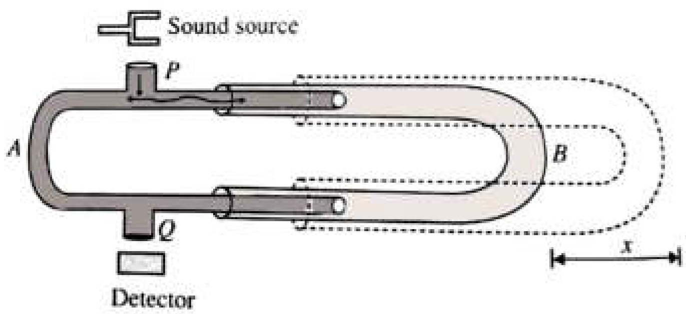
Threshold of Audibility :-

- The minimum intensity of sound at which it is audible is called threshold of audibility.
- The weakest audible sound has intensity of 10^{-12} w/m^2.
- An ordinary talk has intensity of sound 10^{-6} w/m^2.
- The lowest changes of intensity level that could be detected by human ear is 1dB.

Quink's Tube –

This is an apparatus to demonstrate the phenomena of interference and also used to measure velocity of sound in air

This is made up of two U-tube A and B as shown in fig.



Velocity of sound , $V = f_0\lambda = 2f_0x$

Where, f_0 = frequency of the source

V = velocity of sound

λ = path difference

x = B is pulled by a distance (x)

■ Doppler Effect–

- The phenomena of apparent change in frequency of source due to a relative motion between the source and observer is called Doppler's Effect. Let,

  V_s = Source velocity

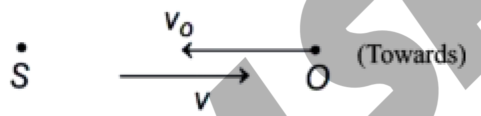
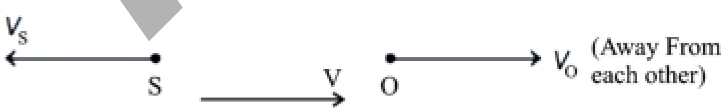
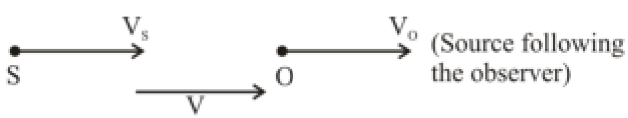
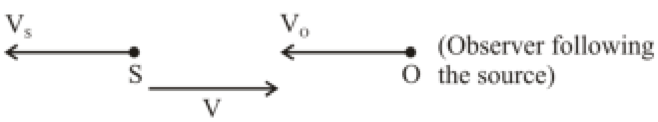
  v = velocity of sound

  v_0 = observer velocity

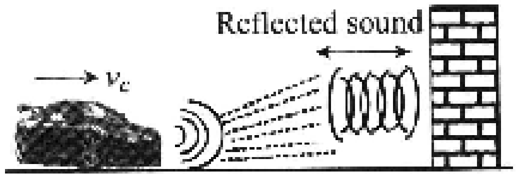
  f ' = apparent frequency

  f = frequency

■ Some Important Cases of Doppler Effect

S.No.	Situation	Apparent frequency
•	When source is moving and observer is at rest	
(a)		$f' = f\left(\frac{v}{v - v_s}\right)$
(b)		$f' = f\left(\frac{v}{v + v_s}\right)$
•	When source is at rest and observer is moving	
(a)		$f' = f\left(\frac{v + v_0}{v}\right)$
(b)		$f' = f\left(\frac{v - v_0}{v}\right)$
•	When source and observer both over moving	
(a)		$f' = f\left(\frac{v + v_0}{v - v_s}\right)$
(b)		$f' = f\left(\frac{v - v_0}{v + v_s}\right)$
•	When source and observer both are moving and they are following each other	
(a)		$f' = f\left(\frac{v - v_0}{v - v_s}\right)$
(b)		$f' = f\left(\frac{v + v_0}{v + v_s}\right)$

- **Doppler Effect in Reflected Sound**:

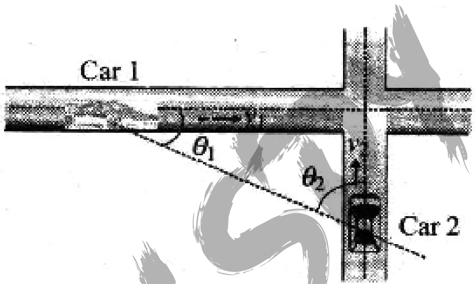


$$f = f'\left[\frac{v + v_c}{v - v_c}\right]$$

Where, v = velocity of sound
v_c = velocity of car
f ' = frequency of source

- **Doppler's effect when source and observer are not in same line of motion-**

The apparent frequency of sound heared by car 2 can be given as–



$$f = f'\left[\frac{v + v_2\cos\theta_2}{v - v_1\cos\theta_1}\right]$$

■ **Doppler effect in light-**

Doppler effect in light is the apparent change in the frequency of the light observed by on observer due to the relative motion between the observer and source of light.

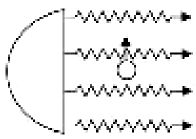
- **Transverse Doppler Effect in Light.**

The observer is moving perpendicular to a line between him and the light source. The proper time between ticks is
$t_0 = 1/v_0$, so between one tick and the next time

$$t = t_0/\sqrt{1 - v^2/c^2}$$

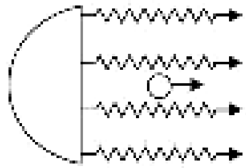
elapses in the reference frame of the observers. The frequency is given by

$$\nu(\text{transverse}) = \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0}$$



- **Observer and Source Moving Apart**

The observer is receding from the light source. Now the observer travels the distance *vt* away from the source between ticks, which mean that the light wave from a given tick takes *vt* / *c* longer to reach him than the previous one. Hence the total time between the arrivals of successive waves is

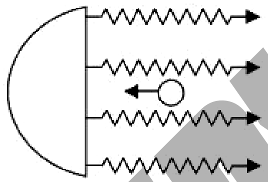


$$T = t + \frac{vt}{c} = t_0 \frac{1+v/c}{\sqrt{1-v^2/c^2}} = t_0 \frac{\sqrt{1+v/c}\sqrt{1+v/c}}{\sqrt{1+v/c}\sqrt{1-v/c}} = t_0\sqrt{\frac{1+v/c}{1-v/c}}$$

The observed frequency v is always less than the source frequency v_0.

- **Observer and Source Moving Together**

  The observer is approaching the light source. The observer here travels the distance $v\ \ t$ toward the source between ticks, so each light wave takes $v\ \ t/c$ less time to arrive than the previous one. In this case $T = t - v\ \ t/c$ and the result is



$$v(\text{approaching}) = v_0\sqrt{\frac{1+v/c}{1-v/c}}$$

  The observed frequency is higher than the source frequency. Again, the same formula holds for motion of the source toward the observer.

- **Doppler's frequency shift :**

  Doppler's frequency shift describes the changes in the frequency of any light and sound waves which is produced by moving source with respect to an observer on vice versa.
  It is given by,

$$\Delta f = V\left(\frac{f}{c}\right)$$

  Where, Δf = Doppler frequency shift
  f = frequency of source which is not moving
  c = Speed of light or sound
  V = Velocity of source

- **Applications of Doppler's effect :**

  The measurement from Doppler effect has been used–
  (i) By police to check over speeding of vehicles.
  (ii) At airports to guide the aircraft.
  (iii) To study heart beats and blood flow in different parts of the body.
  (iv) By astrophysicist to measure the velocities of planets and starts.

■ Key points:

- Doppler shift in frequency does not depends on distance between the source and the observer.
- When the distance between the source and listener is decreasing the apparent frequency increases.
- All harmonics are overtones but all overtones are not harmonics.
- Ultrasonic waves can be produced by utilizing piezoelectric effect.
- Doppler effect gives information regarding the change in frequency only. it does not say about intensity of sound.
- Doppler effect in sound is asymmetric but in light it is symmetric.

Exemplar Problems

Wave & Oscillations

Simple Harmonic Motions

1. **The displacement of simple harmonic oscillator after 3 seconds starting from its mean position is equal to half of its amplitude. The time period of harmonic motion is :**

(a) 6 s (b) 8 s
(c) 12s (d) 36 s

JEE MAIN 2022

2. **The motion of a simple pendulum executing S.H.M. is represented by following equation.**

$Y = A \sin(\pi t + \phi)$, where time is measured in second.

The length of pendulum is :

(a) 97.23 cm (b) 25.3 cm
(c) 99.4 cm (d) 406.1 cm

JEE MAIN 2022

3. **The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{\ell}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1 mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to the nearest integer is:-**

(a) 2% (b) 3%
(c) 5% (d) 4%

JEE MAIN 2021

4. **Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be :**

(a) $\sqrt{3}T$ (b) $\frac{T}{\sqrt{3}}$
(c) $\sqrt{\frac{3}{2}}T$ (d) $\sqrt{\frac{2}{3}}T$

JEE MAIN 2021

5. **A rod of mass '*M*' and length '2*L*' is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of '*m*' are attached at distance '*L*/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio *m*/*M* is close to**

(a) 0.57 (b) 0.37
(c) 0.77 (d) 0.17

JEE MAIN 2019

6. **A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number = 6 02 × 10^{23} . g mol^{-1})**

(a) 5.5 N/m (b) 6.4 N/m
(c) 7.1 N/m (d) 2.2 N/m

JEE MAIN 2018

7. **Two pendulums of length 121 cm and 100 cm start vibrating in phase. At some instant, the two are at their mean position in the same phase. The minimum number of vibrations of the shorter pendulum after which the two are again in phase at the mean position is:**

(a) 8 (b) 11
(c) 9 (d) 10

NEET 2022

8. **A body is executing simple harmonic motion with frequency 'n', the frequency of its potential energy is**

(a) 4n (b) n
(c) 2n (d) 3n

NEET 2021

9. **The phase difference between displacement and acceleration of a particle in a simple harmonic motion is :**

(a) $\frac{3\pi}{2}$ rad (b) $\frac{\pi}{2}$ rad
(c) zero (d) π rad

NEET 2020

10. **The displacement equation of oscillations of a particle executing simple harmonic motion is $x = 5 \sin(0.2\pi t + 0.5\pi)$ The time period of particle is**

(a) 10.0 s (b) 1.0 s
(c) 0.8 s (d) 0.5 s

UP PGT - 2021

11. **If the effective length of a pendulum is infinite, its time period will be**

(a) 1 hour (b) 0/0
(c) ∞ / ∞ (d) 84.6 min.

UPPSC Ashram Paddhati PGT - 2021

12. **What will be the value of following ratio for simple harmonic oscillator?**

$$\frac{\text{Average Kinetic energy for one time period}}{\text{Average potential energy for one time period}}$$

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$
(c) $\frac{2}{3}$ (d) 1

UPPSC Ashram Paddhati PGT - 2021

13. The potential energy U(x) of a particle executing S.H.M. is
(a) U(x) constant (b) $U(x) = Ae^{-bx}$
(c) $U(x) = \frac{K}{2}(x-a)^2$ (d) $U(x) = K_1x + K_2x^2$

UP PGT - 2021

14. If the symbols have their usual meaning, then which of the following gives the frequency of tensional oscillations?
(a) $2\pi\sqrt{\frac{c}{I}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{c}{I}}$
(c) $2\pi\sqrt{\frac{I}{C}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{I}{C}}$

UPPSC Poly. Lect. 2021 Paper - I

15. Which one of the following is a simple harmonic motion?
(a) Ball bouncing between two rigid vertical walls
(b) Particle moving in a circle with uniform speed
(c) Wave moving through a string fixed at both ends
(d) Earth spinning about its own axis

UPPSC GDC - 2021

16. A particle is executing simple harmonic motion governed by equation $\vec{F} = -k\vec{r}$. Which quantity is always negative?
(a) $\vec{F}.\frac{d^2\vec{r}}{dt^2}$ (b) $\vec{r}.\frac{d^2\vec{r}}{dt}$
(c) $\vec{r}.\frac{d^2\vec{r}}{dt^2}$ (d) None of the above

UPTGT Science - 2021

17. What is the percentage change in time period of a simple pendulum, if the length of simple pendulum increase by 3%?
(a) 3% (b) 6%
(c) 3.5% (d) 1.5%

UKPSC Lecturer (Mains) 2020

18. The length of two strings measured by a scale are l_1 = 20 cm ± 0.5 cm and l_2 = 50 cm ± 0.5 cm. The length difference and the error therein would be
(a) 30 cm ± 0 cm (b) 30 cm ± 0.5 cm
(c) 30 cm ± 1 cm (d) 30 cm ± 0.25 cm

RPSC School Lect. 08.01.2020

19. If the earth were a homogeneous sphere of radius R and a straight hole were bored in it through its centre, then a particle, dropped into the hole, will execute an SHM with its time period :
(a) $T = 2\pi\sqrt{\frac{R}{g}}$ (b) $T = 2\pi\sqrt{\frac{g}{R}}$
(c) $T = 2\pi\sqrt{\frac{2R}{g}}$ (d) $T = 2\pi\sqrt{\frac{R}{2g}}$

UP LT Grade Science 2018 (Exam : 29.07.2018)

20. A body of mass m that is undergoing simple harmonic motion passes through its equilibrium position, its velocity is
(a) Zero
(b) Maximum
(c) Half of its maximum value
(d) None of the above

UP TGT Physics 2016

21. Which of the following is not essential for the simple harmonic motion?
(a) Inertia (b) Restoring force
(c) Material medium (d) Gravity

UP TGT Physics 2016

22. What remains constant during simple harmonic motion?
(a) Restoring force (b) Kinetic energy
(c) Potential energy (d) Time period

UP PGT Physics 2016 (Exam : 02.02.2019)

23. A particle of mass m executes a simple harmonic motion of amplitude a and frequency n. The maximum kinetic energy is
(a) $mn^2\pi^2a^2$ (b) $4mn^2\pi^2a^2$
(c) $4mn^2\pi^2a$ (d) $2mn^2\pi^2a^2$

UP LT Grade Science 2018 (Exam : 29.07.2018)

Free, Forced, Damped Oscillations & Resonance

24. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value ? (ln 2 = 0.693)
(a) 34.65 s (b) 17.32 s
(c) 0.034 s (d) 15.01 s

JEE MAIN 2021

25. An external force F_0 sinpt is applied on a particle, executing simple harmonic motion with natural frequency ω_0. In steady state, the frequency of oscillator will be
(a) p (b) ω_0
(c) $\omega_0 - p$ (d) $p - \omega_0$

UPPSC Ashram Paddhati PGT - 2021

26. At small velocities, the damping force is
(a) Directly proportional to the square of velocity
(b) Directly proportional to the square root of velocity
(c) Directly proportional to velocity
(d) Inversely proportional to velocity

UPPSC Ashram Paddhati PGT - 2021

27. **Harmonic oscillator energies are given by $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$ where $n = 0,1,2,3.........$ It is in thermal equilibrium with some heat reservoir at absolute temperature T. what will be the partition function?**

(a) $e^{\frac{-\hbar\omega}{2kT}}\left[1-e^{\frac{-\hbar\omega}{kT}}\right]^{-2}$ (b) $e^{\frac{-\hbar\omega}{2kT}}\left[1-e^{\frac{-\hbar\omega}{kT}}\right]^{-1}$

(c) $e^{\frac{-\hbar\omega}{2kT}}\left[1+e^{\frac{-\hbar\omega}{kT}}\right]^{-1}$ (d) None of the above

UPPSC Poly. Lect. 2021 Paper - I

28. **Potential energy versus displacement curve of a Harmonic Oscillator is**

(a) Straight line (b) Circle
(c) Parabola (d) Ellipse

UPPSC GDC - 2021

29. **A cantilever of uniform cross-section and of length *l* shows a deprssion δ at the loaded end. The depression at a distance *l*/2 from the fixed end is**

(a) $\frac{1}{16}\delta$ (b) $\frac{5}{16}\delta$

(c) $\frac{5}{48}\delta$ (d) $\frac{3}{16}\delta$

UP LT Grade Science 2018 (Exam : 29.07.2018)

30. **A tuning fork of frequency 500 Hz is vibrated with a sonometer wire and 8 beats per second are heard. The beat frequency reduces if tension in the wire is slightly increased. The original frequency of the wire is**

(a) 516 Hz (b) 508 Hz
(c) 500 Hz (d) 492 Hz

UP LT Grade Science 2018 (Exam : 29.07.2018)

31. **The power absorbed in a driven harmonic oscillator is maximum at the**

(a) highest possible driven frequency
(b) amplitude resonance
(c) velocity resonance
(d) frequency where the amplitude drops to 1/e of its maximum value

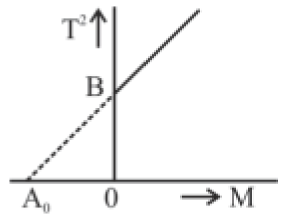
UP LT Grade Science 2018 (Exam : 29.07.2018)

32. **The solution of the differential equation $\frac{d^2y}{dt^2}+\omega^2 y=0$ is :**

(a) $y = a \tan \omega t$ (b) $y = a \cot \omega$
(c) $y = a \sec \omega$ (d) $y = a \sin (\omega t + \theta)$

UPPCS Pre 1994

33. **A graph plotted between T^2 and M. T being the periodic time and M the mass suspended, for a mass spring system with small damping is shown in figure. It can be started that :**



(a) Mass of spring is negligible
(b) A_0 represents 1/3 (Mass of the spring)
(c) A_0 represents 2/3 (Mass of the spring)
(d) A_0 represents 1/2 (Mass of the spring)

UPPCS PRE 1996

34. **The differential equation of motion of a damped mechanical oscillator is–**

$$m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=0$$

If the motion is oscillatory, the angular frequency is given by–
(symbols have their usual meanings)

(a) $\omega=\sqrt{\frac{k}{m}}$ (b) $\omega=\sqrt{\frac{k}{m}+\frac{r^2}{4m^2}}$

(c) $\omega=\sqrt{\frac{k}{m}-\frac{r^2}{4m^2}}$ (d) $\omega=\sqrt{\frac{k}{m}-\frac{r^2}{2m^2}}$

UPPCS (Pre) 1998, 2000

35. **The differential equation of a damped mechanical oscillator is :**
(where letters have their usual meanings

(a) $m\frac{d^2x}{dt^2}+kx=0$

(b) $m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=0$

(c) $m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=F_0 \sin wt$

(d) $m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=F_0 \cos wt$

UPPCS (Pre) 1997

36. **In damped harmonic motion the amplitude of vibration decreases with time as**

(a) $\frac{1}{kt}$ (b) e^{-kt}

(c) $e^{-\frac{1}{kt}}$ (d) e^{-kt^2}

UPPCS Pre 2006

37. **Which one of the following instruments work on the principle of damped oscillation?**

(a) Ammeter
(b) Voltmeter
(c) Helmholtz galvanometer
(d) Ballistic galvanometer

UPPCS Pre 2008,2007

38. **The energy of a damped harmonic oscillator**

(a) increases exponentially with time
(b) increases linearly with time
(c) decreases exponentially with time
(d) decreases linearly with time

UPPCS Pre 2008

39. **A damped vibrating system, starting from rest has an initial amplitude of 506 mm which reduces to 160 mm is 50 complete oscillations each of period 2.3 seconds. The damping coefficient is**
(a) 0.01 (b) 0.1
(c) 0.005 (d) 0.001
UPPCS Pre 2005

40. **In ballistic galvanometer damping is kept small so that**
(a) it may oscillate
(b) it may remains stable
(c) amplitude of first is small
(d) amplitude of first wing is large
UPPCS Pre 2009

41. **The differential equation that describes damped mechanical oscillations, is**
(a) $M\frac{d^2x}{dt^2}+R_m\frac{dx}{dt}=0$
(b) $M\frac{d^2x}{dt^2}+R_m\frac{dx}{dt}+Sx=0$
(c) $\frac{d^2x}{dt^2}+Sx=0$
(d) $R_m\frac{d^2x}{dt^2}+\frac{S}{M}x=\frac{1}{M}\frac{dx}{dt}$
UPPCS Pre 2009

Travelling Waves

42. **A longitudinal wave is represented by $x = 10 \sin 2\pi\left(nt-\frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to :**
(a) 2π (b) 5π
(c) π (d) $\frac{5\pi}{2}$
JEE MAIN 2022

43. **The equations of two waves are given by :**
$y_1 = 5\sin 2\pi(x-vt)$ **cm**
$y_2 = 3\sin 2\pi(x-vt+1.5)$ **cm**
These waves are simultaneously passing through a string. The amplitude of the resulting wave is
(a) 2 cm (b) 4 cm
(c) 5.8 cm (d) 8 cm
JEE MAIN 2022

44. **A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :**
(a) 300 (b) 400
(c) 200 (d) 100
JEE MAIN 2021

45. **Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is**
(a) 5 N (b) 12.5 N
(c) 7.5 N (d) 10 N
JEE MAIN 2019

46. **If the initial tension on a stretched string is doubled, then the ratio of the initial and final speeds of a transverse wave along the string is**
(a) 1 : 2 (b) 1 : 1
(c) $\sqrt{2}:1$ (d) $1:\sqrt{2}$
NEET 2022

47. **Two travelling waves**
$Y_1 = 0.65 \sin(0.4x - 800t)$ m and
$Y_2 = -0.65 \sin(0.4x + 800t)$ m are
superimposed in a medium, for the resultant wave at the point x = 2.5πm, maximum displacement is
(a) zero (b) 1.30m
(c) 0.65m (d) 0.91m
UPPSC Ashram Paddhati PGT - 2021

48. **Which of the following cannot represent an appropriate wave function, A & α are constants?**
(a) $A\sin\alpha x$ (b) $A\cos\alpha x$
(c) $ae^{-i\alpha x}$ (d) $Ae^{\alpha x}$
UPPSC Poly. Lect. 2021 Paper - II

49. **A transverse wave is represented by**
y = 2sin (62.8t - 0.314x + 3. 3)
Where x and y are in centimeters. its frequency is
(a) 100 Hz (b) 50 Hz
(c) 31.4 Hz (d) 10 Hz
UP LT Grade Science 2018 (Exam : 29.07.2018)

50. **The frequency of transverse vibrations of a string is 100 cycles/sec. If the tension of the string is increased to its four times, the frequency will be :**
(a) 25 cycles/sec (b) 100 cycles/sec
(c) 200 cycles/sec (d) 50 cycles/sec
UPPCS Pre 1995

51. **Which one of the following is not true for progressive wave given by**
$$y = 4\sin 2\pi\left(\frac{t}{0.02}-\frac{x}{100}\right)$$
Where t and x are represented in second and cm respectively.
(a) Its amplitude is 4cm.
(b) Its wavelength is 100cm.
(c) Its frequency is 50 cycles/sec.
(d) Its velocity of propagation is 5×10^4 cm/sec.
UPPCS Pre 1995

52. A plane wave is represented, in usual symbols, by the equation, $y = a \sin \pi \left(200t - \frac{x}{150}\right)$. Its frequency is :

(a) 2 Hz (b) 100 Hz
(c) 150 Hz (d) 200 Hz

UPPCS Pre 1994

53. The equation, which represents wave travelling along negative Y-axis is

(a) $x = A \cos(-ky + \omega t)$
(b) $x = A \sin(ky + \omega t)$
(c) $y = A \sin ky \cos \omega t$
(d) $y = A \cos ky \sin \omega t$

UPPCS Pre 2009

54. A wave travelling along a string is describe by $y(x, t) = 0.008 \sin(62.80x - 3.0\,t)$ in which numerical constants are in SI units (0.008 m, 62.80 $radm^{-1}$ and 3.0 $rads^{-1}$), What would be the wavelength of the wave ?

(a) 0.001 m (b) 0.1 m
(c) 0.0785 m (d) 0.1256 m

RPSC School Lect. 08.01.2020

55. The correct relationship between the phase velocity V_p and the group velocity V_g is

(a) $V_g = V_p - \lambda \frac{dV_p}{d\lambda}$ (b) $V_p = V_g - \lambda \frac{dV_g}{d\lambda}$

(c) $V_g = V_p + \lambda \frac{dV_p}{d\lambda}$ (d) $V_p = V_g - \frac{1}{\lambda} \frac{dV_p}{d\lambda}$

RPSC School Lect. 08.01.2020

56. A wave travelling along a string is describe by $y(x, t) = 0.008 \sin(62.80x - 3.0\,t)$ in which numerical constants are in SI units (0.008 m, 62.80 $radm^{-1}$ and 3.0 $rads^{-1}$), What would be the wavelength of the wave ?

(a) 0.001 m (b) 0.1 m
(c) 0.0785 m (d) 0.1256 m

RPSC School Lect. 08.01.2020

57. From a wave equation $Y = 0.5 \sin \frac{2\pi}{3.2}(64t - x)$ the frequency of the wave is

$Y = 0.5 \sin \frac{2\pi}{3.2}(64t - x)$

(a) 5 Hz (b) 15 Hz
(c) 20 Hz (d) 25 Hz

TGT 2009

Standing Waves

58. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

(a) 16.6 cm (b) 33.3 cm
(c) 10.0 cm (d) 20.0 cm

JEE MAIN 2019

59. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. The lowest resonant frequency for this string is

(a) 10.5 Hz (b) 105 Hz
(c) 155 Hz (d) 205 Hz

NEET 2015

60. Neglecting the effect of surface tension and finite depth, the wave velocity of water wave of wavelength λ is given by $v = \sqrt{\frac{g\lambda}{2\pi}}$. The relation between group velocity V_g and wave velocity v is given by

(a) $v_g = \frac{1}{2}v$ (b) $v_g = v$

(c) $v_g = \sqrt{v}$ (d) $v_g = \sqrt{v}/2$

UPPSC Ashram Paddhati PGT - 2021

61. Two rectangular SHMs are represented by $x = a\sin(2\,\omega t + \phi)$ and $y = b \sin \omega t$. The resultant of superposition of two represents

(a) a parabola (b) the figure of 8
(c) an ellipse (d) a straight line

UP LT Grade Science 2018 (Exam : 29.07.2018)

62. 1 dB corresponds in change in power level by (Given $10^{0.1} \simeq 1.26$) -

(a) 50% (b) 35%
(c) 26% (d) 12%

UPPSC Poly. Lect. 2021 Paper - II
UPPSC Ashram Paddhati PGT - 2021

63. In a waveguide which condition will always hold good?

(a) Phase velocity = c (b) Phase velocity > c
(c) Phase velocity < c (d) Group velocity = c

UPPSC Poly. Lect. 2021 Paper - II

64. If a particle has wave function $\psi = ax$ between $x = 0$ and $x = 1$. Find the expectation value $\langle x \rangle$ or the particle position -

(a) a^2 (b) $\frac{a}{4}$

(c) $\frac{a^2}{2}$ (d) $\frac{a^2}{4}$

UPPSC Poly. Lect. 2021 Paper - II

65. The internal radius of a 1 m long resonance tube is 3 cm. With a tuning fork of frequency 2000 Hz, the first resonating position is 4.6 cm and the second resonating position is 14.0 cm. Speed of sound at the room temperature is

(a) 336 m/s (b) 376 m/s
(c) 332 m/s (d) 340 m/s

UP PGT - 2021

66. **Find the expectation value of position of a particle whose normalized wave function is –**

$\psi(x) = Ne^{-}(x^2/2a^2) + ikx$

(a) $< x >= a^2$ (b) $< x >= k^2$

(c) $< x >= hk$ (d) $< x >= 0$

UPPSC Poly. Lect. 2021 Paper - II

67. **Two waves $y_1 = A_1$ sink $(x – \upsilon t)$ and $y_2 = A_2$ sink $(x – \upsilon t + x_0)$, where $k = 2\pi$ cm^{-1} and $x_0 = 1.50$ cm are superposed. If $A_1 = 9.00$ mm and $A_2 = 7.00$ mm, what is the amplitude of the resulting wave?**

(a) 16.0 mm (b) 8.0 mm

(c) 2.0 mm (d) 1.0 mm

UP PGT - 2021

68. **Phase velocity of ripples on a liquid surface is $\sqrt{\frac{2\pi T}{\lambda\rho}}$, where 'T' is surface tension and ρ is the density of liquid. What will be the group velocity?**

(a) $\sqrt{\frac{2\pi T}{\lambda\rho}}$ (b) $\frac{1}{2}\sqrt{\frac{2\pi T}{\lambda\rho}}$

(c) $\frac{3}{2}\sqrt{\frac{2\pi T}{\lambda\rho}}$ (d) None of the above

UPPSC Poly. Lect. 2021 Paper - I

69. **The speed of wave in a medium is 960 m/s. If 3600 waves are passing through a point in one minute, the wavelength will be :**

(a) 32 m (b) 64 m

(c) 14 m (d) 16 m

UKPSC Lecturer (Mains) 2020

70. **The expression for the velocity of longitudinal waves in gases is expressed as :**

(a) $v = \sqrt{E\rho}$ (b) $v = E/\rho$

(c) $v = \sqrt{E/\rho}$ (d) $v = E\rho$

UKPSC Lecturer (Mains) 2020

71. **For formation of standing waves, the medium should**

(a) have infinite extension

(b) be bounded

(c) be a gas only

(d) be having zero inertia and zero elasticity

RPSC School Lect. 26.07.2016

72. **Velocity of sound in hydrogen and oxygen gas at a given temperature will be in the ratio**

(a) 4 : 1 (b) 1 : 4

(c) $2\sqrt{2}:1$ (d) $c^2/\sqrt{2}K^2$

RPSC School Lect. 26.07.2016

73. **In a standing wave of frequency 1000 cycles per second the distance between a node and the adjacent antinode is 10 cm. The velocity of the wave is :**

(a) 200m/s (b) 400m/s

(c) 50m/s (d) 100m/s

UPPCS Pre 1995

74. **A metal string is fixed between rigid supports It is initially at negligible tension. Its Young's modulus is Y, density is ρ and coefficient of thermal expansion is α. If it is now cooled through a temperature t, transverse wave will move along it with speed**

(a) $\sqrt{\alpha t/\rho}$ (b) $\sqrt{Y\alpha t/\rho}$

(c) $\alpha t\sqrt{Y/\rho}$ (d) $t\sqrt{\frac{Y\alpha}{\rho}}$

UPPCS Pre 2005

75. **A stationary wave is represented by**

$y = 10\cos\frac{2\pi}{5}x\sin 80\pi t$

Where y and x are in meters and t in seconds. The wavelength of the wave is

(a) 80 metres (b) 10 metres

(c) 5 metres (d) 2 metres

UPPCS Pre 2009

76. **The vibration of a string fixed at both ends are described by the equation**

$y = 5\sin\frac{\pi x}{2}\sin 100\pi t$

If the length of the string is 16 cm, the number of loops formed in vibration shall be

(a) 4 (b) 8

(c) 12 (d) 16

UPPCS Pre 2009

77. **In a gas, two waves of wavelengths 1.00 m and 1.01 m respectively produce 4 beats in 1 second. Then the velocity of sound in that gas will be**

(a) 400 m/sec (b) 404 m/sec

(c) 410 m/sec (d) 330m/sec

(UPPGT 2010)

78. **A tuning fork of frequency 260 Hz is vibrated with a sonometer wire and 5 beats are heard. If the tension in the wire is slightly increase, the beat frequency also decreases. The original frequency of the sonometer is**

(a) 265 Hz (b) 260 Hz

(c) 255 Hz (d) 250 Hz

UP PGT - 2021

Sound Waves and Doppler's Effect

79. **An observer moves towards a stationary source of sound with a velocity equal to one-fifth of the velocity of sound. The percentage change in the frequency will be :**

(a) 20% (b) 10%

(c) 5% (d) 0%

JEE MAIN 2022

80. **The velocity of sound in a gas. in which two wavelengths 4.08m and 4.16m produce 40 beats in 12s, will be :**

(a) 2.82.8 ms^{-1} (b) 175.5 ms^{-1}
(c) 353.6 ms^{-1} (d) 707.2 ms^{-1}

JEE MAIN 2022

81. **A submarine *A* travelling at 18 km/h is being chased along the line of its velocity by another submarine *B* travelling at 27 km/h. *B* sends a sonar signal of 500 Hz to detect *A* and receives a reflected sound of frequency n. The value of n is close to (Speed of sound in water = 1500 ms^{-1})**

(a) 504 Hz (b) 507 Hz
(c) 499 Hz (d) 502 Hz

JEE MAIN 2019

82 **A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz)**

(a) 7 (b) 4
(c) 5 (d) 6

JEE MAIN 2019

83. **A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 1 ms- , the mass of the string is**

(a) 5 g (b) 10 g
(c) 20 g (d) 40 g

IIT 2010

84. **A tuning fork with frequency 800 Hz produces resonance in a resonance column tube with upper end open and lower end closed by water surface. Successive resonance are observed at length 9.75 cm, 31.25 cm and 52.75 cm. The speed of sound in air is**

(a) 500 m/s (b) 156 m/s
(c) 344 m/s (d) 172 m/s

Odisha NEET 2019

85. **A tuning fork is used to produce resonance in a glass tube. The length of the air column in this tube can be adjusted by a variable piston. At room temperature of 27°C two successive resonances are produced at 20 cm and 73 cm of column length. If the frequency of the tuning fork is 320 Hz, the velocity of sound in air at 27°C is**

(a) 330 m/s^{-1} (b) 339 m/s^{-1}
(c) 350 m/s^{-1} (d) 300 m/s^{-1}

NEET 2018

86. **An organ pipe of length 0.5 m is open at both ends. What is the frequency of its fundamental note? (Velocity of sound in air = 350 m/s)**

(a) 350 Hz (b) 175 Hz
(c) 700 Hz (d) 450 Hz

UP PGT Physics 2016 (Exam : 02.02.2019)

87. **The two nearest harmonics of a tube closed at one end and open at other end are 220 Hz and 260 Hz. What is the fundamental frequency of the system?**

(a) 20 Hz (b) 30 Hz
(c) 40 Hz (d) 10 Hz

NEET 2017

88. **The second overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe *L* metre long. The length of the open pipe will be**

(a) Lj (b) 2L
(c) $\frac{L}{2}$ (d) 4L

NEET - II 2016

89. **A sound wave of unknown frequency gives 10 beats with a wave of frequency 300 Hz and 15 beats with a wave of frequency 325 Hz. The frequency of unknown wave is**

(a) 290 Hz (b) 310 Hz
(c) 340 Hz (d) None of the above

UPPSC Ashram Paddhati PGT - 2021

90. **If a 1 m long wire of mass 2 gram is stretched by a weight of 400 kg. Then what will be the frequency of fundamental note of the wire ?**

(a) 500 Hz (b) 700 Hz
(c) 50 Hz (d) 100 Hz

UKPSC Lecturer (Mains) 2020

91. **The volume of hollow hall is 2000 m^3 and total absorption coefficient is 165 unit. Then reverberation time of hall is**

(a) 4s (b) 2s
(c) 1s (d) 0.5s

RPSC School Lect. 08.01.2020

92. **The musical interval between any two consecutive tones in a 12 – ET tempered musical scale is**

(a) $2^{1/6}$ (b) $2^{1/12}$
(c) $2^{1/13}$ (d) $2^{1/11}$

RPSC School Lect. 08.01.2020

93. **Using light of wavelength 6000Å in Young's experiment, 64 fringes were observed in the field of view. How many fringes will be observed if light of wavelength 4800Å is used**

(a) 50 (b) 64
(c) 80 (d) 120

UP PGT Physics 2016 (Exam : 02.02.2019)

94. **What is the relation between phase difference (Δϕ) and path difference (Δx)?**

(a) $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ (b) $\Delta\phi = \frac{\lambda}{2\pi}\Delta x$
(c) $\Delta\phi = \frac{\lambda}{2\pi\Delta x}$ (d) $\Delta\phi = \frac{\pi}{\lambda}\Delta x$

UP PGT Physics 2016 (Exam : 02.02.2019)

95. **For which of the following, Lissajous figure will be circle?**
(a) $x = \cos 2\omega t : y = \cos \omega t$
(b) $x = \cos 2\omega t : y = \cos\left(2\omega t - \frac{\pi}{4}\right)$
(c) $x = \cos 2\omega t : y = \sin 2\omega t$
(d) None of the above

UPPSC Poly. Lect. 2021 Paper - I

96. **Sound waves in hydrogen gas are of which type?**
(a) Transverse (b) Longitudinal
(c) Stationary (d) Electromagnetic

UP PGT Physics 2016 (Exam : 02.02.2019)

97. **For diffraction at a single slit, the width of the central maximum is_________ the wavelength of light**
(a) Proportional to
(b) Proportional to the square of
(c) Inversely proportional
(d) Independent of

UP PGT Physics 2016 (Exam : 02.02.2019)

98. **A source of unknown frequency produces 8 beats with a source of 250 Hz and 12 beats with a source of 270 Hz. The frequency of the unknown source is**
(a) 258 Hz (b) 242 Hz
(c) 262 Hz (d) 282 Hz

(UPPGT 2010)

99. **The number of beats produced by two waves $y_1 = a \sin (2000\pi t)$ and $y_2 = a \sin (2010\pi t)$ per second will be**
(a) four (b) five
(c) eight (d) ten

(UPPGT 2010)

100. **Select correct statement in which Doppler effect is not applicable-**
(a) Discovery of twin stars
(b) To determine the velocity of milky way
(c) To determine the radius of earth
(d) To determine the velocity of submarine

UPPSC Poly. Lect. 2021 Paper - I

101. **A car moving at 30 m/s is approaching a factory, whose whistle has a frequency of 500 Hz. If the speed of sound in air is 340 m/s, what is the apparent frequency of whistle as heard by the car driver?**
(a) 480 Hz (b) 500 Hz
(c) 544 Hz (d) 600 Hz

UP PGT - 2021

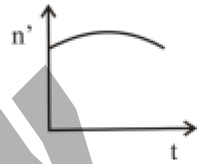
102. **The shape of the Lissajous Figures depends on**
(a) The amplitude of the superposing waves
(b) Frequencies of superposing waves
(c) The phase difference between the superposing waves
(d) All of the above

UP PGT - 2021

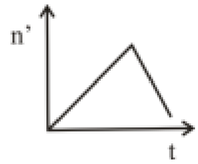
103. **Which of the following gives Doppler's frequency shift Δv?**
(a) $V\frac{\nu}{c}$ (b) $V\frac{c}{\nu}$
(c) $V\sqrt{1-\left(\frac{\nu}{c}\right)^2}$ (d) $V\nu c$

UPPSC Poly. Lect. 2021 Paper - I

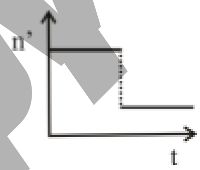
104. **A train whistling at constant frequency is moving towards station at a constant speed V. The train goes past a stationary observer on the station. The frequency n' of the sound as heard by observer can be expressed by which of the expected curve as a function of time t ?**



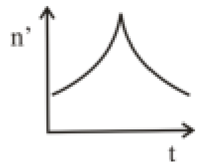
(a)



(b)



(c)



(d)

RPSC School Lect. 08.01.2020

105. **When engine of train moves away the observer, then observer observe its frequency $\frac{6}{7}$ of its actual frequency. If velocity of sound in air is 330 m/s, what will be the speed of engine ?**
(a) 50 m/s (b) 55 m/s
(c) 30 m/s (d) 330 m/s

UKPSC Lecturer (Mains) 2020

106. **A plane is flying towards east at a velocity of 500 km/h at a height of 2000 m. A man standing on the ground hears a sound coming vertically above him. Plane at this instant in its line of motion is at a distance of (velocity of sound 300 m/s):**
(a) 825.9 m (b) 925.9 m
(c) 625.9 m (d) 725.9 m

UKPSC Lecturer (Mains) 2020

107. **In light the shift of wavelength in Doppler's effect is expressed as (for non-relativistic velocities)**
(a) $\Delta\lambda = \frac{vc}{\lambda}$ (b) $\Delta\lambda = \frac{v}{c}\lambda$
(c) $\Delta\lambda = \frac{v^2c^2}{\lambda}\lambda$ (d) $\Delta\lambda = \frac{v^2}{c^2}\lambda$

UKPSC Lecturer (Mains) 2020

108. **Doppler shift in frequency does not depends on**
(a) speed of observer
(b) speed of source
(c) frequency of wave
(d) distance between the source and the observer
RPSC School Lect. 26.07.2016

109. **A sound source of frequency 440 Hz is tied to the end of a 1.5 m long string and rotated in a horizontal plane at an angular velocity of 20 radians/sec. What is the frequency range of the sound heard by a listener standing for away? (Velocity of sound in air = 330 m/s)**
(a) 440 – 480 Hz (b) 403 – 484 Hz
(c) 400 – 500 Hz (d) 408 – 490 Hz
UP PGT Physics 2016 (Exam : 02.02.2019)

110. **A sound source is moving towards a stationary listener with $\frac{1}{10}$ speed of sound. The ratio of real frequency as heard by the listener is**
(a) $\frac{9}{10}$ (b) $\frac{10}{11}$
(c) $\left(\frac{10}{11}\right)^2$ (d) $\left(\frac{10}{9}\right)^2$
(UPPGT 2010)

111. **The power of a small loudspeaker is 10 W. The intensity of sound obtained at a distance of 3 m from it is 2 Wm^{-2}. If the power of the loudspeaker is doubled, the intensity at 6 m will be :**
(a) 4 Wm^{-2} (b) 2 Wm^{-2}
(c) 1 Wm^{-2} (d) 0.5 Wm^{-2}
UPPCS Pre 2010

112. **Two sound sources have equations y_1 = 0.25 Sin 316 t and y_2 = 0.25 Sin 310 t respectively. If the listener hears both together how many beats will be hear per second?**
(a) 6 (b) $\frac{6}{\pi}$
(c) $\frac{3}{\pi}$ (d) 3π
UP PGT Physics 2016 (Exam : 02.02.2019)

113. **If we take $E = \frac{1}{2}m_0v^2$ and $P = m_0v$, then the phase velocity of the corresponding wave packet is**
(a) $\frac{v}{2}$ (b) v
(c) $\frac{c^2}{v}$ (d) $\frac{v^2}{c}$
UP LT Grade Science 2018 (Exam : 29.07.2018)

114. **A star emitting light of wavelength 5000 Å is coming towards the earth with speed equal to 1.5×10^6 m/sec. The change in wavelength of the radiation received on the earth will be**
(a) 2.5 Å (b) 100 Å
(c) Zero (d) 25 Å
(UPPGT 2009, 2002)

115. **If light of wavelength λ = 5000 Å coming from a star be observed as λ = 5200 Å :**
A. The star is moving away from earth
B. Its speed is 1.2×10^7 m/s
(a) A, B both are false
(b) B is true, A is false
(c) A is true, B is false
(d) A, B both are true
UPPCS PRE 1996

Answer Key

1. (d)	2. (c)	3. (b)	4. (d)	5. (b)	6. (c)	7. (b)	8. (c)	9. (d)	10. (a)
11. (d)	12. (d)	13. (c)	14. (b)	15. (c)	16. (c)	17. (d)	18. (c)	19. (a)	20. (b)
21. (d)	22. (d)	23. (d)	24. (a)	25. (a)	26. (c)	27. (b)	28. (c)	29. (b)	30. (d)
31. (c)	32. (d)	33. (b)	34. (c)	35. (b)	36. (b)	37. (d)	38. (c)	39. (c)	40. (d)
41. (b)	42. (b)	43. (a)	44. (d)	45. (b)	46. (d)	47. (b)	48. (d)	49. (d)	50. (c)
51. (d)	52. (b)	53. (b)	54. (b)	55. (a)	56. (b)	57. (c)	58. (d)	59. (b)	60. (a)
61. (c)	62. (c)	63. (b)	64. (d)	65. (b)	66. (d)	67. (c)	68. (c)	69. (d)	70. (c)
71. (b)	72. (a)	73. (b)	74. (b)	75. (c)	76. (b)	77. (b)	78. (c)	79. (a)	80. (d)
81. (d)	82. (d)	83. (b)	84. (c)	85. (b)	86. (a)	87. (a)	88. (b)	89. (b)	90. (b)
91. (b)	92. (b)	93. (c)	94. (a)	95. (d)	96. (b)	97. (a)	98. (a)	99. (b)	100. (c)
101. (c)	102. (d)	103. (a)	104. (c)	105. (b)	106. (b)	107. (b)	108. (d)	109. (b)	110. (a)
111. (c)	112. (c)	113. (a)	114. (d)	115. (d)					

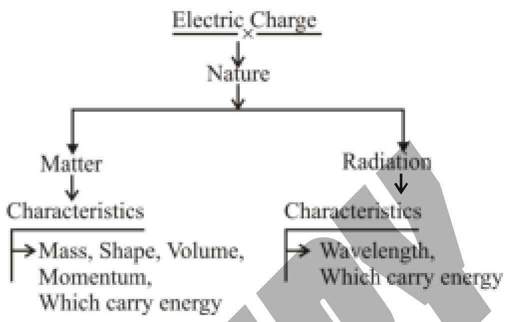
06

Electricity &Magnetism

(1) Electrostatics

■ Introduction:

- Electrostatics is that branch of electricity in which properties of static (non-moving) electrical charges are studied.



- Matter can apply force due to its two characteristics-
  (i) Mass (For applying gravitational force).
  (ii) Electric charge (For applying electrostatics force).

■ **Electric Charge:-** Electric charge is a physical property carried by a matter due to which it can apply force on other matter particle. A charged matter can apply force and this force is found to be of two types to explain it we have to discover that charges are of two type -
(i) Positive Charge
(ii) Negative Charge
Like charges repel and unlike charges attract each other.

Basic properties of Electric Charge –

- **Additive:** Charges can be added by simple rule of Algebra.
- **Quantization of Charge** : Electric charges on a body is not a continuous quantity but it is integral multiple of charge on one electron.

$$\boxed{Q = \pm ne}$$

- **Charge is conserved** : The total charge of isolated system is always conserved, it may get redistributed. Isolated electric charge can neither be created nor destroyed, it can only be transferred.

$$^{238}U = ^{234}Th + ^{4}He \qquad \text{(Active decay)}$$

The charges in pair can be created or destroyed.
Annihilation:-

$$e^- + e^+ \longrightarrow \gamma + \gamma \qquad \text{(Radiation)}$$

↓

Gamma rays

Pair Production:–

$$Pt + 2\gamma \rightarrow e^- + e^+ \qquad \text{(Positron)}$$

↓ (Heavy metal) ↓ gamma

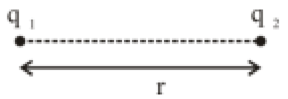
Charging a body -

- **By rubbing: -** Electrons get entangled and are transferred to other.
- **By conduction :-**An uncharged body can be charged by contact of surfaces.
- **By Electrostatic Induction :-** When a charged body is brought near an uncharged body, the charges in uncharged body get polarised. This phenomenon is called electrostatic Induction.

Conductors and Insulators -

- Conductors are those substances which allow charges to move within it. e.g. metal, electrolytes, tap water, human body etc.
- Insulators are those substances which do not allow charges to move through it. e.g. Non-Metals, Plastic, Gases, Pure water etc.

■ Coulomb's law of electrostatics -



- The law states that if two point charges q_1, q_2 are separated by a distance r in vacuum the magnitude of the force (F) between them is given by -

$$F = \frac{Kq_1q_2}{r^2}$$

Where $K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}$ (ε_0 = Permittivity of free space = $8.85\times10^{-12}\ C^2N^{-1}m^{-2}$)

- The law is applicable only for static and point charges.
- Moving charges may result in magnetic interaction.
- If charges are extended, Induction may charge the charge distribution.
- K is arbitrary (we choose any positive value)
- The choice of K determines the size of the unit of charge.
- The Separation between the charges must be greater than nuclear distance.

Permittivity and relative permittivity :-

- Permittivity is a characteristics of a medium between two charges which affect the force between them.
  It is represented by 'ε' (Epsilon).
  Permittivity of free space is represented by 'ε_0'. Its dimension is $[M^{-1}L^{-3}T^4A^2]$.
- Relative permittivity is the ratio of absolute permittivity of medium to the permittivity of free space.

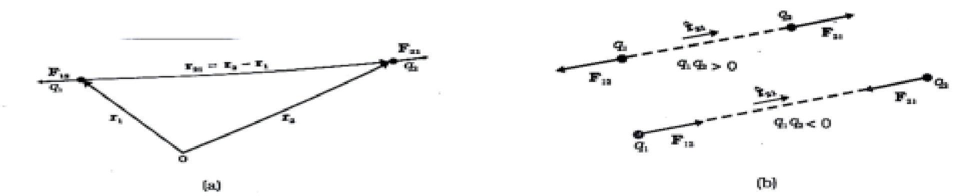
$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

It is also given by -

$$(\varepsilon_r)_m = \frac{\text{Electrostatics Force in free space}}{\text{Electrostatics Force in Medium}} = \frac{\text{Electric field in free space}}{\text{Electric field in medium}}$$

It is also equal to dielectric constant k.
(i) (ε_r) Free space = 1.
(ii) Any medium has permittivity > 1.
(iii) For water 'k' = 81
(v) For metal k = ∞.

Vector form of Coulomb's law :-



- It tell us about both magnitude and direction.
- Force on charge q_2 due to q_1 -

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^2}\hat{r}_{21}$$

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{21}^3}\vec{r}_{21} \qquad \because \left(\hat{r}_{21} = \frac{\vec{r}_{21}}{|r_{21}|}\right)$$

$$F_{21} = \frac{q_1q_2}{4\pi\varepsilon_0}\frac{\vec{r}_2 - \vec{r}_1}{|r_2 - r_1|^3}$$

- Similarly, the force on charge q_1 due to q_2 -

$$F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|^3}$$

- Force on q_1 due to q_2 = – Force on q_2 due to q_1 or $F_{12} = -F_{21}$.
- The forces due to two point charges are parallel to the line joining the point charges; such force are called central forces and so electrostatic forces are conservative forces.

Force between Multiple charges : Superposition principle -

- According to the principle of Superposition total force on a given charge due to number of charge is the vector sum of individual forces acting on that charge due to the presence of other charges.
  Let the charges be $q_1, q_2, q_3, \ldots\ldots q_n$ exert force $F_{12}, F_{13}, \ldots\ldots F_{1n}$ on charge q_1. The total force on charge q_1 is given by

$$F_1 = F_{12} + F_{13} + \ldots\ldots F_{1n} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_2}{r_{13}^2}\hat{r}_{13} + \ldots\ldots \frac{q_1 q_2}{r_{1n}^2}\hat{r}_{1n}\right) = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

Distribution of Continuous Electric Charge ;-

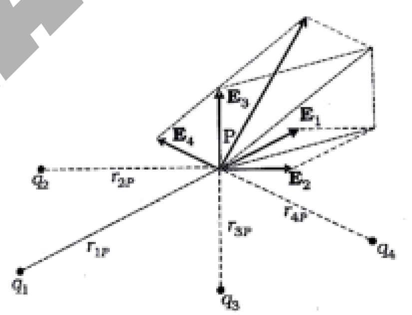
- The region in which charges are closely spaced is said to have continuous distribution of charge. It is of three types :-

Linear Charge Density (λ)	Surface Charge Density (σ)	Volume Charge Density (δ)
• It is defined as the charge per unit length. • Its unit is Coulomb/meter. • $\lambda = \frac{dq}{dL}$.	• It is defined as the charge per unit surface area. • Its unit is Coulomb/meter^2. • $\sigma = \frac{dq}{dS}$.	• It is defined as the charge per unit volume. • Its unit is Coulomb/meter^3. • $\delta = \frac{dq}{dV}$.

■ Electric Field :-

- Electric field is space around a charge where its effect can be experienced, the effect can be strong or weak, we call it intensity of electric field. It is represented by 'E'.
- Mathematically, Electric field produced by a charge q at a point 'r' is given by $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}|^2}\hat{r} = \frac{\vec{F}}{q_0}$.
- Unit of 'E' = NC^{-1}, Dimension = $MLA^{-1}T^{-3}$.

Electric Field due to a system of Charges :-



- If we place a unit test charge q_0 near n point charges $q_1, q_2, \ldots\ldots q_n$ then the net force $\vec{F}$ from the n point charges acting on the test charge is $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots + \vec{F}_n$.

  Then net electric field at the position on the test charge is-

$$E = E_1 + E_2 + E_3 + \ldots\ldots E_n = \frac{\vec{F}_1}{q_0} + \frac{\vec{F}_2}{q_0} + \frac{\vec{F}_3}{q_0} + \ldots\ldots + \frac{F_n}{q_0}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}^2}\hat{r}_{3P} + \ldots\ldots$$

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

Physical Significance of Electric field -

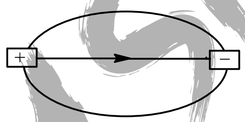
- Electric field is a characteristics of the system of charges. It is independent of the test charge that you place at a point to find the field.
- Since force is a vector quantity so electric field is a also vector quantity.
- Electric field can be detected only by its effects (forces) on charges. It has an independent dynamics of a force on q_2. Effect of any motion of q_1 and charge q_2 cannot arise instantaneously. The accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed of light c reach q_2 and causes a force on q_2.

Electric field lines :-

- Electric field lines are imaginary smooth curves drawn in sphere to represent electric field over there.

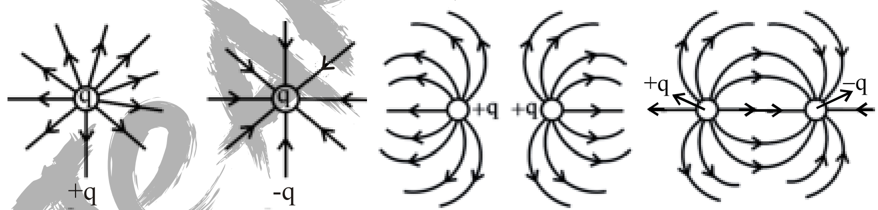
Characteristic of electric field lines:-

- Start from positive charge terminate at negative charge. These are discontinuous curve.
- Field lines are always smooth curves. Tangent at any point gives direction of electric field at that point.
- Two lines never cross each other because if they cross that will indicate two directions at that point which is not possible.
- Crowding of Electric field lines indicate relative strength of electric field.
- Electric field lines are always at right angle to the Surface.
- Number of lines assumed to be linked with a unit charge are $\frac{1}{\varepsilon_0} = \frac{1}{8.85\times10^{-12}}$.
- Lines behave like stretched string trying to decrease their strength. this explains force of attraction between positive and negative charges.



- The lines laterally repel each other (normal to line) that indicate the force of repulsion between similar charges.

Electric field lines due to different distribution -



(a) (b) (c) (d)

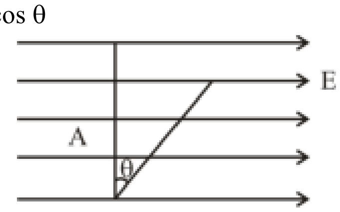
■ Electric flux -

- Electric flux linked to surface area is a quantity (Scalar) formed by intensity of field E and area normal to it A, such that flux linked to surface area is–

  $\phi_E = \vec{E}\cdot\vec{A}$

- Flux = Intensity of electric field × Area linked

  $\phi_E = E\,A\cos\theta$



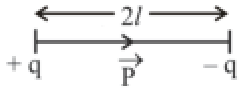
If $\vec{E}$ is not uniform throughout the area 'A' then $\phi = \int \vec{E}\bullet d\vec{A}$

- SI unit is $N\text{-}m^2C^{-1}$. It is scalar Quantity.
- 'ϕ' does not depend on the -

  (i) Shape and size of the closed surface.

  (ii) The charges located outside the closed surface.

■ Electric Dipoles –

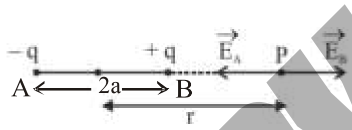


- Pair of two equal and opposite point charges attached at a fixed distance is called electric dipole.
- $Na^{\oplus}Cl^{\ominus}$ molecule represents an electric dipole.
- Dipole Moment ($\vec{P}$) = $q \times 2l$

  Where $\vec{P}$ is vector quantity its direction is along the line from negative to positive charge.
- SI unit - Coulomb meter (Cm)
- Dimension - [LTA]

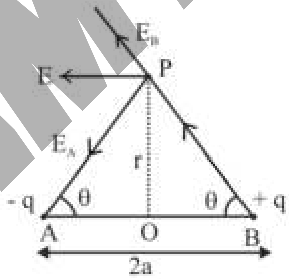
Electric field Intensity due to an Electric dipole –

- **On Axial line –** Electric field intensity at point P on a given axis, $E = \frac{1}{4\pi\varepsilon_0}\frac{2Pr}{\left(r^2 - a^2\right)^2}$



If r >> 2a then $E = \frac{1}{4\pi\varepsilon_0} \times \frac{2P}{r^3}$ or $\frac{2KP}{r^3}$

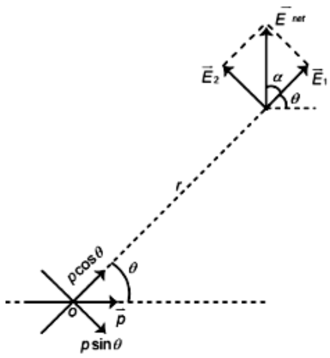
- **On equatorial line -** Electric field Intensity at point P on a given equatorial line is, $E = \frac{1}{4\pi\varepsilon_0}\frac{P}{\left(r^2 + a^2\right)^{3/2}}$



If r >> 2a then, $E = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}$.

- **At any point along a line making θ angle with dipole Axis -**

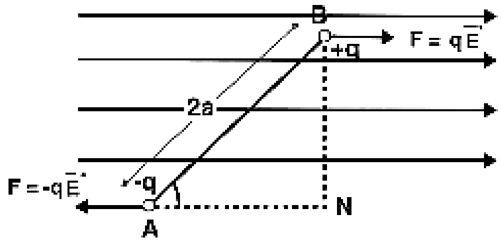
  Electric field intensity 'E' $= \frac{1}{4\pi\varepsilon_0}\frac{P\sqrt{1 + 3\cos^2\theta}}{r^3}$.



- Electric field due to a dipole deplete faster as it varies inversely to cube of r as compared to point charge which varies as square of r,

  $E_{point} \propto \frac{1}{r^2}$, $E_{Dipole} \propto \frac{1}{r^3}$

Dipole kept in Electric field -



- Torque(τ) $= \vec{P} \times \vec{E} = PE \sin\theta$

  Where θ is angle between $\vec{P}$ and $\vec{E}$

  If $\vec{P}$ is along E

  $\theta = 0°$, $\tau = 0$ (Stable equilibrium)

  If $\vec{P}$ is opposite to $\vec{E}$

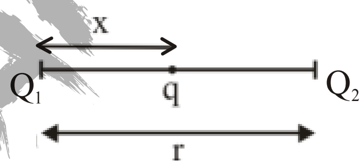
  $\theta = 180°$, $\tau = 0$ (Unstable Equilibrium)

  Work done in Rotating a dipole –

  $$W = \int dW = \int \tau d\theta = \int_{\theta_1}^{\theta_2} PE\sin\theta d\theta = PE\,[\cos\theta_1 - \cos\theta_2]$$

■ Equilibrium of charges: -

- **Equilibrium of three point charges -**

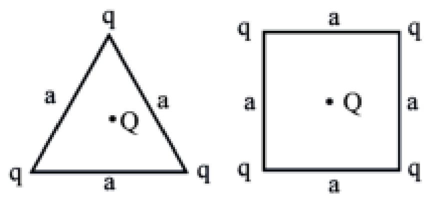


  (i) Two charges must be of like nature.

  (ii) Third charge should be of unlike nature.

  $$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \text{ and } q = \frac{-Q_1 Q_2}{\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2}$$

- **Equilibrium of Symmetric geometrical point charged system -**

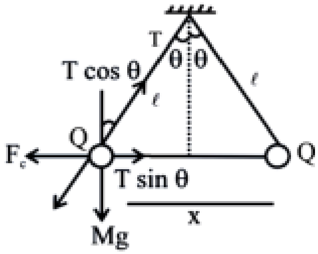


  Value of Q at centre for which system to be in state of equilibrium.

  For Equilateral triangle $Q = \frac{-q}{\sqrt{3}}$

  For Square $Q = \frac{-q\left(2\sqrt{2} + 1\right)}{4}$

- **Equilibrium of Suspended Point Charge System -**
  For equilibrium position,



$T \cos\theta = Mg$ (vertical equilibrium)

$T \sin\theta = F_e = \dfrac{KQ^2}{x^2}$ (Horizontal equilibrium)

$\tan\theta = \dfrac{F_e}{Mg} = \dfrac{KQ^2}{x^2 mg}$

If θ is small, $\tan\theta \approx \theta = \dfrac{x}{2\ell}$

$$\frac{x}{2\ell} = \frac{KQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2KQ^2\ell}{mg} \Rightarrow x = \left[\frac{Q^2\ell}{2\pi\varepsilon_0 mg}\right]^{\frac{1}{3}}$$

■ Electric Potential -

- The electric potential at a point in an electric field is the ratio of work done in carrying a test-charge from infinity to that point to the magnitude of the test charge. It is given by –

$$V = \frac{W}{q}$$

- Its S.I. unit is J/c or volt and dimension $[ML^2T^{-3}A^{-1}]$.
- For point charge, $V = \dfrac{Kq}{r}$.
- Electric potential due to an electric dipoles

  at axis of dipole $V = K\dfrac{P}{r^2}$.

  on a Equatorial line $V = 0$

  at any point a line making angle θ $V = K\dfrac{P\cos\theta}{r^2} = \dfrac{K\vec{P}.\vec{r}}{r^3}$

- For Several point charges, $V = K\sum_{i=1}^{n} \dfrac{q_i}{r_i}$

Potential gradient :-

- The rate of change of potential with distance is electric field is called Potential gradient.
- Potential gradient $= \dfrac{dV}{dr}$, its unit is V/m.
- Relation between potential gradient and electric field intensity is given by $E = -\left(\dfrac{dV}{dr}\right)$.

Equipotential Surface –

- Every point on a surface is at same potential called equipotential surface.

(i) Electric lines always join the equipotential surface perpendicularly.

(ii) Work done in moving a charge from one point to other on an equipotential surface is zero.

(iii) It is not affected by the path.

(iv) $\dfrac{W_{eq}}{q} = V_A - V_B \Rightarrow \quad W = 0$

(v) Equipotentials are parallel to each other (means does not intersect each other).

(vi) Equipotential potential surface are planer in an uniform electric field.

(vii) In a family of equipotential surface the surfaces are closer together where the electric field is stronger and farther apart where field weaker.

(viii) Equipotential surface due to line charge is cylindrical.

(ix) Equipotential surface due to an isolated point charge is spherical.

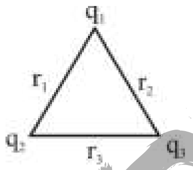
■ Electric potential energy -

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge from infinity.

- **Potential Energy of two Charge System -**

  Potential energy of two point charges system that contains charges q_1 and q_2 separated by a distance 'r' is given by $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

- **Potential Energy of three point charge system-**



$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_1} + \frac{q_3 q_1}{r_2} + \frac{q_2 q_3}{r_3}\right]$$

- **Potential Energy for a collection of More than two Charges -**

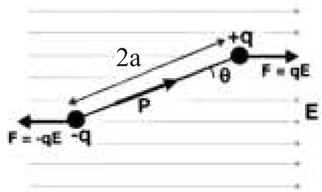
  The potential energy of a system of charges is given by

$$U = \frac{K}{2}\sum_{i \neq j}^{n}\frac{q_i q_j}{r_{ij}}$$

$$K = \frac{1}{4\pi\varepsilon_0}$$

The factor $\frac{1}{2}$ is applied only with the summation sign because on expanding the summation sign each pair is counted twice. It is represented by U.

- **Potential energy in a dipole –**



Potential energy of dipole is given by-

$U = -P.E\cos\theta$

If dipole is kept along E then U is given by -

$U = -P.E$

Where, P = dipole moment

E = Electric field

■ Motion of Charged Particle in electric field –

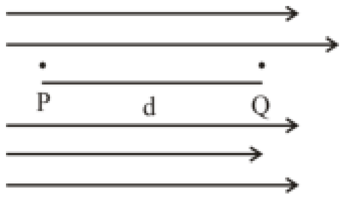
- Consider a charge particle having charge 'q' and mass m is initially at rest in an electric field of strength E. The particle will experience an electric force which causes its motion.

  The force experienced by the charge particle is F, where

  $F = qE$

  Acceleration produced by this force is

$$a = \frac{F}{m} = \frac{qE}{m} \quad \text{..............} \quad (i)$$



At point P particle is at rest and after some time 't' , it reaches the point Q and attains velocity v which is given by, v = at

If potential difference between P and Q be ΔV and the distance between them is d, then

$$v = \frac{qEt}{m} = \sqrt{\frac{2q\Delta V}{m}}$$

- **Momentum of charge particle-**

  As momentum,

  $$P = mv = m\left(\frac{qEt}{m}\right) \Rightarrow P = qEt = m\sqrt{\frac{2q\Delta V}{m}} = \sqrt{2mq\Delta V}$$

- **Kinetic Energy of Charged Particle -**

  Kinetic Energy gained by the particle in time t is-

  (a) $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qEt}{m}\right)^2 = \frac{q^2E^2t^2}{2m}$

  (b) $K.E. = \frac{1}{2}m \times \frac{2q\Delta V}{m} = q\,\Delta V$

■ Gauss's Law -

- According to Gauss theorem the electric flux linked with a closed surface is $\frac{1}{\varepsilon_0}$ times the charge enclosed by the surface.

  Flux linked through surface element ΔS is

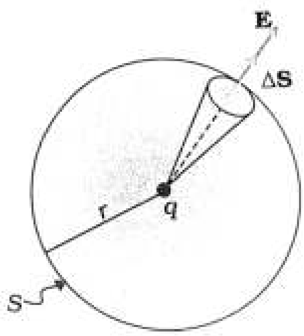
  $$\Delta\phi = E.\Delta S = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S$$

  Total flux linked through the entire surface S

  $$\phi = \int \Delta\phi = \int E.\Delta S = \frac{q}{4\pi\varepsilon_0 r^2}\int \Delta S = \frac{q}{4\pi\varepsilon_0 r^2}S$$

  For sphere $S = 4\pi r^2$

  $$\phi = \frac{q}{4\pi\varepsilon_0 r^2}.4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow \phi = \frac{q}{\varepsilon_0}$$



  Where, q = net charge enclosed by given surface.

  Similarly for a cube

  - Total electric flux linked with whole cube = $\frac{q}{\varepsilon_0}$
  - Electric flux linked with one face of cube = $\frac{q}{6\varepsilon_0}$

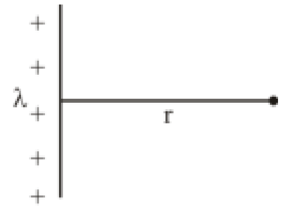
- This law is especially useful in determining electric field when source distribution has simple symmetry.
- This law is true for closed surface only.
- Charges outside makes the flux but net flux is only due to charge enclosed.
- Use the gauss theorem with symmetrical surface to calculate E at a point.
- Gauss theorem depends on inverse square law.

Gaussian Surface (Imaginary Surface)-

- The surface that we choose for the application of Gauss's law is called Gaussian surface.
- Gaussian surface does not applicable for discrete charge because electric field due to a system of discrete charges is not well defined at the location of any charge.
- Gaussian surface can pass through a continuous charge distribution.
- It is an imaginary surface; homogenous in nature, symmetrical surface.

Applications of Gauss' Theorem -

- **Electric field intensity due to an infinite line charge -**



Electric field, $E = \frac{1}{2\pi\varepsilon_0} \times \frac{\lambda}{r}$

Also, Potential difference $= \frac{\lambda}{2\pi\varepsilon_0} \ell n\left(\frac{r_2}{r_1}\right)$

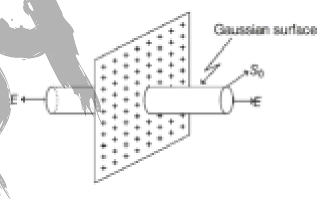
Where λ is linear charge density (q/L) and r is distance from the line charge.

Electric potential for infinite line charge is not defined.

- **Electric field near an infinite plane sheet of charge -**

Electric field, $E = \frac{\sigma}{2\varepsilon_0}$

Where, σ = surface charge density.

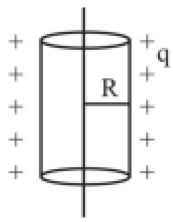


- **Electric field for a long charged conducting cylinder-**

For $r \geq R$

Electric field, $E = \frac{q}{2\pi\varepsilon_0 r}$

for, $r < R$, $E = 0$



- **Electric field just outside charge conductor-**

$E = \frac{\sigma}{\varepsilon_0}$, $(E_{inside} = 0)$

- **Electric field due to two infinite parallel sheets of Charge-**

$E = \frac{\sigma}{\varepsilon_0}$ {for both sheets having same charge density}

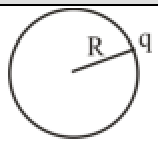
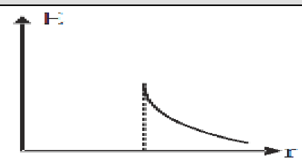
- **For non-conducting long sheet of surface charge density 'σ'-**

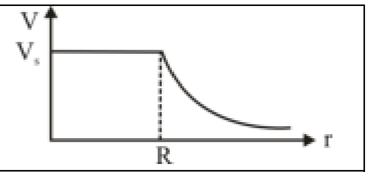
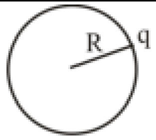
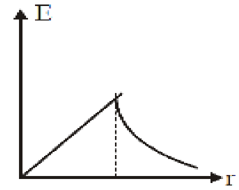
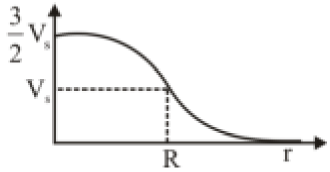
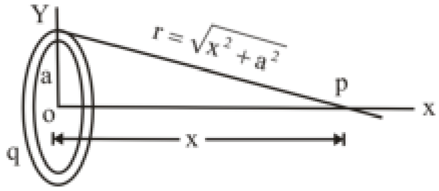
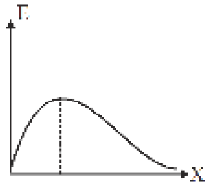
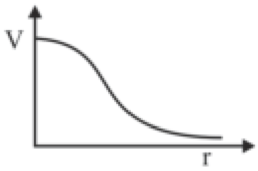
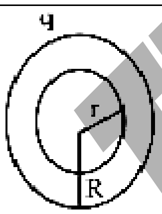
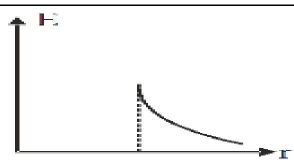
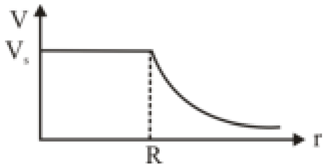
$E = \frac{\sigma}{2\varepsilon_0}$

- **For conducting long sheet of surface charge density 'σ' -**

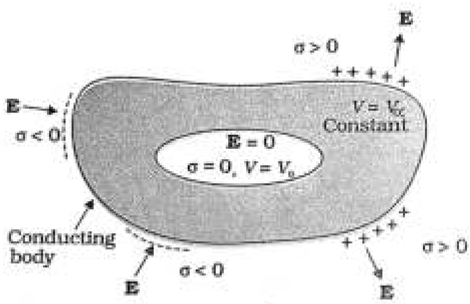
$E = \frac{\sigma}{\varepsilon_0}$

- **For uniformly Charged Spheres –**

Figure	Electric field (E) and electric potential (V)	Graphs
 Charged conducting solid sphere	For $r \geq R$ $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ for , $r < R$	

	$E = 0\ ,\ V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$	
 Charged Non conducting solid sphere	For $r \geq R$ $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\ ,\ V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ for, $r < R$ $E = \frac{1}{4\pi\varepsilon_0}\frac{q\,r}{R^3}$ $V = \frac{1}{4\pi\varepsilon_0}\frac{q\left(3R^2 - r^2\right)}{2R^3}$	
 Charged Ring	$E_x = \frac{1}{4\pi\varepsilon_0}\frac{x\,q}{\left(x^2 + a^2\right)^{3/2}}$ $E_{max} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{3\sqrt{3}a^2}\right)$ On the axis of uniformly charged ring $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x^2 + a^2\right)^{1/2}}$ At centre x = 0 $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$	
 For conducting/ non-conducting spherical shell	for $r \geq R$ $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right), V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$ For $r < R$ $E = 0, V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)$	

■ **Electrostatic Shielding -**



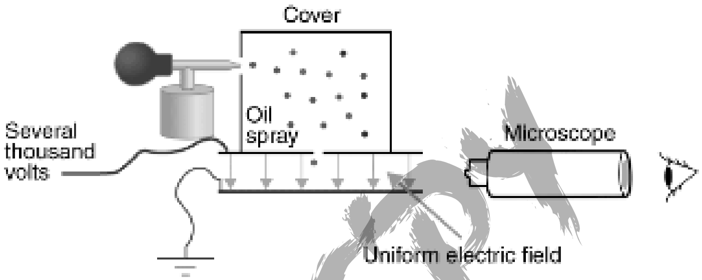
- Any cavity in conductor remains shielded from outside electric influence (i.e. field inside the cavity is always zero). This is known as Electrostatic Shielding.

■ Electrostatic of Conductor :-

- Inside a conductor, electrostatic field is zero.
- At the surface of charged conductor electrostatic field must be normal the surface at every point.
- The interior of a conductor can have no excess charge in the static situation.
- Electrostatic potential is constant throughout the volume of the conductor and has the same value on its surface.
- Electric field at the surface of charged conductor is $\frac{\sigma}{\varepsilon_0}$.

■ Millikan's Oil Drop Experiment :-

- Millikan allowed charged tiny oil droplets to pass through a hole an electric field. By varying the strength of the electric field the charge over an oil droplet was calculated, which always came as an integral value of 'e'.



Calculation:

$F_{up} = F_{down}$

$F_{up} = Q.E$

$F_{down} = m.g$

Q is an electron's charge, E is the electric field, m is the droplet's Q.E = m.g

$Q = \frac{m.g}{E}$

Conclusion:

The charge over any oil droplet is always an integral value of e (1.6×10^{-19}). Hence, the conclusion of Millikans Oil Drop Experiment is that the charge is said to be quantitzed, i.e. the charge on any particle will always e or an integral multiple of e.

(2) Capacitance and Capacitors

■ **Capacitor :-** A capacitor is a device that stores electrical energy. It consists of a pair of conductors of any shape and size carrying charges of equal magnitudes and opposite signs and separated by an insulating medium.

Capacitance of a Capacitor -

- The ability to store charge by a capacitor is called capacitance of the capacitor.
- It is also defined as the amount of charge required to raise the potential of the capacitor by one unit (In SI one volt).

  If 'Q' is the amount of charge given and 'V' is potential of capacitor, then capacitance of the capacitor is given by

$$C = \frac{Q}{V}.$$

- SI unit of capacitance is Farad or F.

  1 Farad = 1 Coulomb /Volt.
- Dimension of capacitance = $[M^{-1}L^{-2}T^4A^2]$.
- Capacitance is a scalar Quantity.
- The capacitance depends only on the geometry of the conductors and not on an external source of charge or potential difference.

Dielectric and Polarization :-

- A dielectric is an insulating material in which all electrons are tightly bonded to the nuclei of the atoms. Hence free movement of charges in not possible. It is given by

$$k = \frac{C}{C_0} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

where, C = capacitance when space between pair of conductor is filed with dielectric.
C_0 = capacitance when space is vacuum.
ε_0 = permittivity of free space, ε = permittivity of medium.
ε_r = relative permittivity

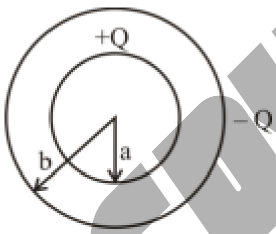
- For a polar and non-polar molecule the dipole moment per unit volume is called polarization. It is given by

$$P = \chi_e E$$

Where, χ_e is a constant characteristics of the dielectric and and is known electric susceptibility of the dielectric medium.

Spherical Capacitor:-

- A spherical capacitor consists of a solid or hollow spherical conductor of radius a, surrounded by another hollow concentric spherical of radius b



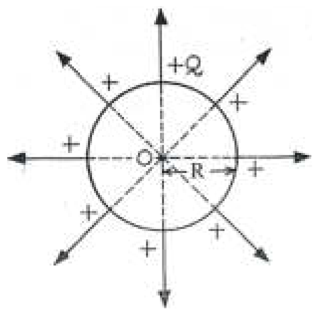
- Its capacitance C is given by -

$$C = \frac{4\pi\varepsilon_0 ba}{b - a} \text{ Farad}$$

- If radius of outer conductor approaches to infinity then,
$C = 4\pi\varepsilon_0 a$ Farad.
- When inner sphere is earthed then, it serves as two capacitors in parallel where,
$C_1 = 4\pi\varepsilon_0 b$ Farad. (Outer surface and earth)
$C_2 = \dfrac{4\pi\varepsilon_0 ba}{b-a}$ Farad. (Region between two spheres)

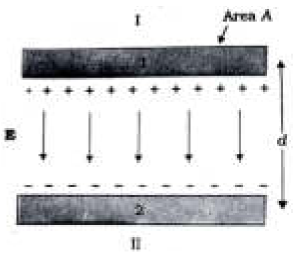
Capacitance of an Isolated Spherical Capacitor –

- $C = 4\pi\varepsilon_0 kR$ in a Medium
$C = 4\pi\varepsilon_0 R$ in a Air
Where, ε_0 = permittivity in free space
R = Radius of sphere.
k = dielectric constant



Parallel Plate capacitor :-

- Most commonly used capacitor is parallel plate capacitor, which consists of two charged parallel plates separated by a small distance. Capacitance of a parallel plate capacitor with vacuum between the plates is given by



Without dielectric– $C = \frac{\varepsilon_0 A}{d}$ Farad

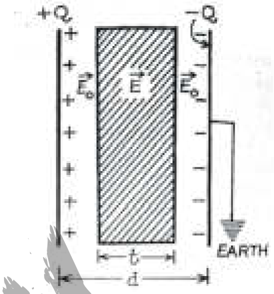
If the space between capacitor filled with dielectric then–

$$C = \frac{\varepsilon_0 k A}{d} \text{ Farad}$$

where, k = dielectric constant of the medium between the plates
A = Area of plates
d = distance between the plates

If medium is partly filled with air then-

$$C = \frac{\varepsilon_0 A}{d - \left(t - \frac{t}{k}\right)}$$



When a dielectric slab of thickness 't' and relative permittivity ε_r is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t - \frac{t}{\varepsilon_r}\right)$ irrespective of the position of the dielectric slab.

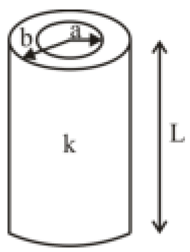
If the space between plates is completely filled with multiple dielectrics-

$$C = \frac{\varepsilon_0 A}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}}$$

Cylindrical Capacitor :-

- It consists of two co-axial cylinders of radii a and b the outer conductor is earthed. The dielectric constant of the medium filled in the space between the cylinders is k.

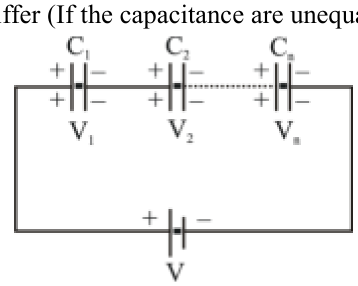
The capacitance per unit length is $C = \frac{2\pi\varepsilon_0 k}{ln\left(\frac{b}{a}\right)}$



■ Combination of Capacitors -

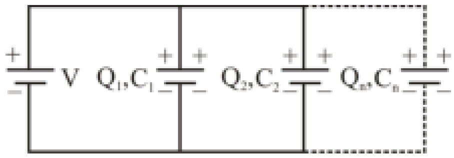
- **Capacitors in series**:- In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (If the capacitance are unequal).

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots \frac{1}{C_n}$$



- **Capacitors in Parallel :-** When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be in parallel connection. The capacitor have the same potential difference, 'V' but the charge on each one is different (If capacitance are unequal).

$C_{eq} = C_1 + C_2 + C_3 + \text{.........} + C_n$



■ Energy stored in a charged capacitor:-

- Energy stored in a charged capacitor is defined as the total amount of work in charging the capacitor stored up in the capacitor in the form of electrical potential energy.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

- This energy is stored in the electrostatic field setup in the dielectric medium between the conducting plates of the capacitor.

Energy Density :-

- The energy stored in a capacitor is not localized on the charges or the plates but is distributed in the field. In case of a parallel plate capacitor, electric field is only between the plates in a volume (A×d) which is so called energy density. It is given by-

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{A \times d}$$

$$= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)\frac{V^2}{Ad} \qquad \left(C = \frac{\varepsilon_0 A}{d}\right)$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

$$= \frac{1}{2}\varepsilon_0 E^2 \qquad \left(\frac{V}{d} = E\right)$$

- Energy density is the property of electric field and above formula is valid in all cases. The unit of energy density is J/m^3.

Heat Produced in Switching in Capacitive Circuit -

- Due to charge flow some amount of heat is produced when a switch is closed in a circuit which can be obtained by energy conservation as -

  Heat = Work done by battery – Energy absorbed by Capacitor.

Sharing of Charges :-

- When two charged capacitors of capacitance C_1 and C_2 at potential V_1 and V_2 respectively are connected in parallel by a conducting wire, the charge flows form higher potential conductor to lower potential, until the potential of the two capacitors becomes equal. The common potential (V) after sharing of Charges;

$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_2V_1 + C_2V_2}{C_1 + C_2}$$

After redistribution the charge in C_1 is Q_1' and on C_2 is Q_2'. Then

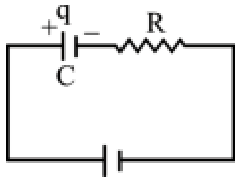
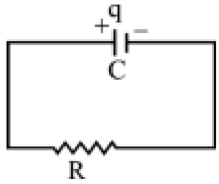
$Q_1' = C_1V$ and $Q_2' = C_2V$

- **Quantity of charge transferred** = $Q_1 - Q_1' = \dfrac{C_1C_2(V_1 - V_2)}{C_1 + C_2}$

- **Loss of energy in redistribution of charge-**

$$U_{initial} - U_{final} = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$$

■ Charging and discharging of Capacitors:-

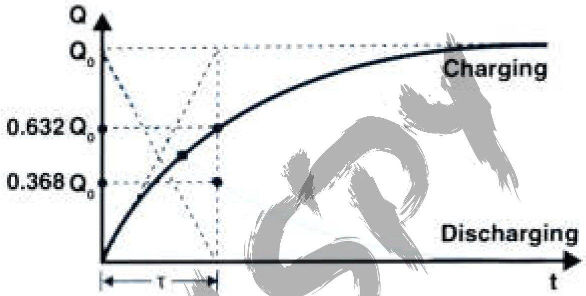
Charging	Discharging
 $q = q_0\left(1 - e^{-\frac{t}{RC}}\right)$, Where, $q_0 = CV_0$	 $q = q_0 e^{-t/RC}$

Where, $RC = \tau$ known as time constant of CR circuit.

q = charge on capacitors

q_0 = maximum value of charge at current $I = 0$.

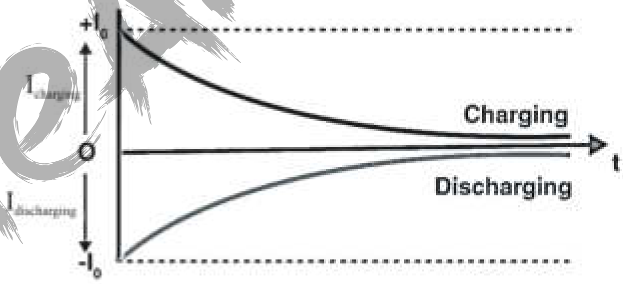
- **Time Constant (τ) -**



Time constant of a CR circuit is the time during which the charge on the capacitor becomes 0.632 (approx, 2/3) its maximum value.

Time constant of a CR circuit is also the time during which the charge on the capacitor falls from its maximum value to 0.368 (approx ... 1/3) of its maximum value.

- **Current during charging and discharging of a capacitor -**



$$I_{charging} = \frac{q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}$$

$$I_{discharging} = -\frac{q_0}{RC} e^{-t/RC} = -I_0 e^{-t/RC}$$

Where, $I_0 = \dfrac{q_0}{RC}$ = maximum value of current flow.

■ Force between Plates of Capacitor :-

- The two plates of capacitor attract each other because they are oppositely charged. Electric field due to any one plate (say positive plate),

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{q}{2\varepsilon_0 A}$$

Where σ = area charge density = $\dfrac{q}{A}$.

Force on other plate (negatively charged plate),

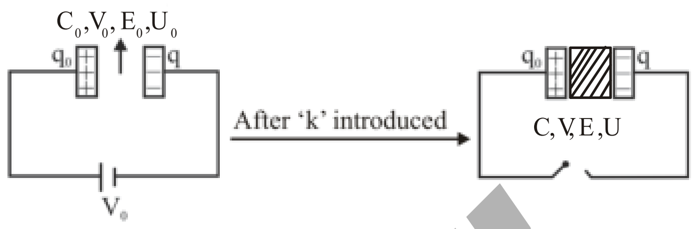
$$F = -qE = -q \times \frac{q}{2\varepsilon_0 A} = \frac{-q^2}{2\varepsilon_0 A}$$

Magnitude of force (F) = $\dfrac{q^2}{2\varepsilon_0 A}$.

■ Effect of Dielectric on Charge, Potential Difference, Field and Energy of Capacitor -

Effect of dielectric on various parameters of capacitors can be seen in two ways-

- **If Charge is held constant, i.e. Battery is disconnected after charging -**



(i) Charge remains constant, $q = q_0$.
(ii) Capacity increases as $C = kC_0$.
(iii) Potential difference between the plates decreases as $V = \dfrac{q}{C} = \dfrac{q_0}{kC_0} = \dfrac{V_0}{k}$.
(iv) Electric field between the plates decrease as $E = \dfrac{E_0}{k}$
(v) The energy stored decreases as $U = \dfrac{q^2}{2C} = \dfrac{q_0^2}{2kC_0} = \dfrac{U_0}{k}$.

- **If Potential is held constant, i.e. battery remains attached**

(i) Potential difference remains constant i.e. $V = V_0$ as battery is a source of constant potential difference.
(ii) Capacity increases as $C = kC_0$.
(iii) Charge on capacitor increases as $q = CV = (kC_0)V = kq_0$.
(iv) Electric field remains unchanged as $E = \left[\dfrac{V}{d}\right] = \dfrac{V_0}{d} = E_0$.
(v) Energy stored in the capacitor increases as - $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}kC_0V_0^2 = \dfrac{1}{2}kU_0$

	Dielectric is inserted	Charge (Q)	Voltage (V)	Electric Field (E)	Capacitance (C)	Energy (U)
1.	When the battery is disconnected	Constant	Decreases	Decreases	Increases	Decreases
2.	When the battery is connected	Increases	Constant	Constant	Increases	Increases

■ Van De Graff Generator :-

- This is a machine that can build up high voltages of the order of a few million volts.
- A Van De Graff generator consists of a large spherical conducting shell (a few meter in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to be shell and potential difference of the order of several millions volts is built up, which can be used for accelerating charged particles.

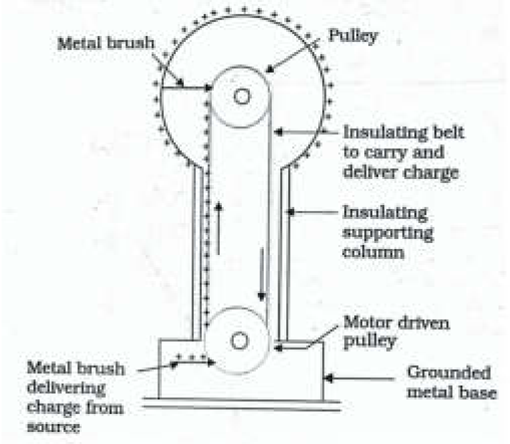


Figure (i)

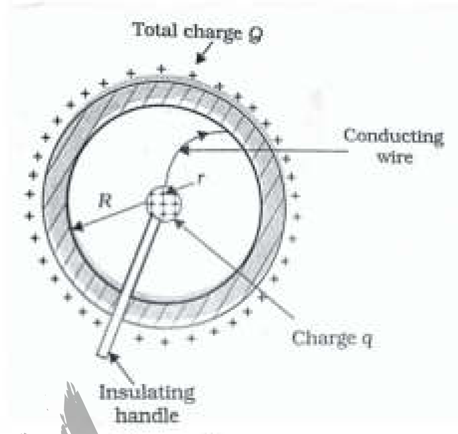


Figure (ii)

Let a small sphere be placed at the center of the large one such that the radius of the smaller sphere is r and the charge over its surface is q. The potential energy thus generated due to the smaller surface at different points in the system can be given as the following values,

At the surface of the small sphere:

$$V_{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

At the large spherical shell of radius R:

$$V_{r_2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

- If we consider the total charges in the system, that is q and Q, then the total potential energy due to the system of charges can be given as,

$$V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}+\frac{Q}{R}\right)$$

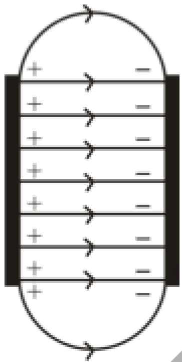
$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}+\frac{Q}{R}\right)$$

$$V_R - V_r = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R}-\frac{1}{r}\right)$$

■ Key Points –

- The energy of a charged conductor resides outside the conductor in its electric field, where as in a capacitor it is stored within the capacitor in its electric field.
- The energy of an uncharged capacitor = 0.
- The capacitance of capacitor depends only on its size & geometry the dielectric between the conducting surface and is independent of the conductor material, like whether it is Copper, Silver, Gold etc.
- The two adjacent conductors carrying same charge can be at different potential because the conductors may have different sizes and hence different capacitance.
- When a capacitor is charged by a battery both the plates receive charges equal in magnitude, no matter sizes of plates are identical or not because the charge distribution on the plates of a capacitor is in accordance with charge conservation principle.
- On filling the space between the plates of a parallel plate air capacitor with a dielectric, Capacity of the capacitor is increased because the same amount of charge can be stored at a reduced potential.
- The potential of grounded object is taken to be zero because capacitance of the earth is very large.
- Work done by battery to charge a capacitor (W) = $CV^2 = qV = \frac{q^2}{C}$.

- It is a very common misconception that a capacitor stores charge but actually a capacitor stores electric energy in the electrostatic field between the plates.
- Two plates of unequal area can also form a capacitor, but effective overlapping area is considered.
- Capacitance of a parallel plate capacitor doesn't depends upon the charge given. Potential raised or nature of metals and thickness of plates.
- The distance between the plates is kept small to avoid fringing or edge effect (non-uniformly of the field) at the boundaries of the plates.



- Radial and non-uniform electric field exists between the spherical surfaces of spherical capacitor.
- Two capacitors of capacitances C_1 and C_2 are charged to potential of V_1 and V_2 respectively. After disconnecting from batteries they are again connected to each other with reverse polarity i.e., Positive plate of a capacitor connected to negative plate of other. The common potential is given by-

$$V = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

(3) Current Electricity and Heating effect of current

■ Electric Current

- When a charge flows in a conductor from one point to the other then this flow of charge is called electric current (I).
- The charge flowing per unit time in an electric circuit is the measure of electric current i.e.

  $(I) = \frac{q}{t}$

- Its SI unit is Ampere (A).
  1Ampere = 1 coulomb/second
- The convectional direction of electric current is the direction of motion of positive charge.
- Dimension of current $= \frac{AT}{T} = [A]$.
- Current is a scalar quantity because it follows addition laws of scalar quantities and not of vector quantities.
- According to its magnitude and direction electric current is of two types-
  (i) Direct Current (DC)- In DC the current flows in a single direction steadily. Example: an electric cell.
  (ii) Alternating Current (AC)- In AC the flow of current changes its direction forward and backward periodically. Example: AC Dynamo.

Mechanism of flow of electric charge in metallic conductors:

- In metals the electric conduction occurs by movement of negative charge carriers (electrons) only.
- In liquids and gases it occurs by the movement of both positive and negative charge carriers (ions)

Mean free path and Relaxation time:

- The average distance moved by a free electron between two successive collisions is called 'mean free path' of the electrons and the average time-interval between two successive collisions is called the 'relaxation time of the electrons-

$$\text{Relaxation time } (\tau) = \frac{\text{mean free path}}{\text{rms velcity of electrons}}$$

Drift Velocity (V_d) :-

- Drift velocity of electrons is the average uniform velocity acquired by free electrons inside a metal conductor by the application of an electric field which is responsible for current through it. Drift velocity is very small, it is of the order of $10^{-4}\ ms^{-1}$ as compared to the thermal speed ($\approx 10^5 ms^{-1}$) of electrons at room temperature.
- The current related with drift velocity is

  $I = neAV_d \Rightarrow V_d = \frac{I}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{V}{\rho \ell ne}$

  Where, n = number of free electrons per unit volume of the conductor .
  A = Area of cross section, V = Potential difference across the conductor.
  E = Electric field inside the conductor (E) = $\frac{V}{\ell}$
  I = Current, J = Current density, ρ = Specific resistance
  σ = Conductivity $\left(\sigma = \frac{1}{\rho}\right)$
  e = Charge on charge carrier (Electron), ℓ = length of metallic conductor
- In terms of material parameter- $V_d = \frac{e\tau E}{m}$

  Where, m = mass of electron, τ = Relaxation time

Electric Current density :-

- The amount of electric current travelling per unit cross-section area is called current density. It is given by-

$$J = \frac{I}{A}$$

- It SI unit is $A\text{-}m^{-2}$ and dimension is $[L^{-2}A]$.
- Current density (J) is a vector quantity.
- ' J' depends upon area (A).
- Current density related with electric field is $J = \sigma E = \frac{E}{\rho} = neV_d$

  Where, σ = conductivity and ρ = resistivity or specific resistance of substance, n = number of free electrons per unit volume of the conductor, e = Charge on charge carrier (Electron), V_d = Drift velocity of electrons
- The direction of current density is same as that of electric field 'E'.
- In terms of material parameter-

$$J = \frac{ne^2}{m}\tau E$$

Where, m = mass of charge carrier (Electron), τ = Relaxation time

Mobility -

- Mobility of charge carrier is defined as the drift velocity of the charge carrier per unit of the electric field.
- It is also defined as mobility of a charge carrier in a particular medium is drift velocity developed when electric field applied is unity.

  Mobility (μ)= $\frac{V_d}{E}$
- Its SI Unit is $[m^2 Volt^{-1} s^{-1}]$ and its Dimension is $[M^{-1} L^0 T^2 A^1]$.
- In terms of material parameter $\mu = \frac{e\tau}{m}$.

■ Ohm's Law -

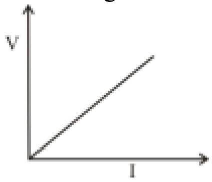
- According to ohm's law when physical conditions (temperature, mechanical strain etc) remains unchanged then the current flowing through a conductor is directly proportional to the potential difference across its ends.

  $I \propto V \quad \Rightarrow \quad V = RI$

  Where, R is a constant known as the electrical resistance of given conductor.
- Miniature form of ohm's law is $J = \sigma E$.

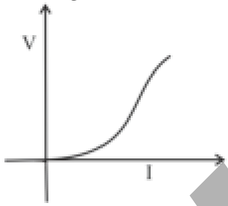
Ohmic conductors -

- Those conductors which obey ohm's law are called ohmic conductor, e.g. all metallic conductors are ohmic conductors. For ohmic conductor V-I graph is a straight line



Non-Ohmic conductor -

- Those conductors which do not obey ohmic law, are called non-ohmic conductors e.g. diode, triode valve transistor vacuum tubes etc.
  For non-ohmic conductors V-I graph is not a straight line.



■ Electrical Resistance :-

- Resistance is opposition faced by charges while moving in a conductor. It is given by -

$$R = \frac{V}{I}$$

  Where V = potential difference across the ends of conductor.
  I = Current flowing through the conductor.
- Its SI unit is ohm (Ω) and its dimensional formula is $[ML^2T^{-3}A^{-2}]$.
  $1\Omega = 1$ Volt-Ampere^{-1}
- It is also given by $R = \frac{\rho\ell}{A}$.

  Where, ℓ = length of the conductor
  A = cross-section area of conductor
  ρ = resistivity of the material of the conductor
- If a wire is stretched to a greater length, keeping volume constant, then

  $R \propto \ell \Rightarrow \frac{R_1}{R_2} = \left(\frac{\ell_1}{\ell_2}\right)$.

  and $R \propto \frac{1}{r^2} \Rightarrow \frac{R_1}{R_2} = \left(\frac{r_2}{r_1}\right)^2$

  Where, ℓ = length of the wire
  r = radius of cross-section area of wire.
- In terms of material parameter it is given by -

$$R = \frac{m\ell}{ne^2\tau A}$$

■ Specific Resistance or Electrical resistivity -

- The resistance of a conductor is directly proportional to its length 'l' and inversely proportional to the area of cross section 'A' i.e.

  $R \propto \ell$ and $R \propto \frac{\ell}{A}$

  $\Rightarrow R = \frac{\rho\ell}{A}$

  If $\ell = 1$ unit and A = 1 unit then $R = \rho$.

Therefore, **specific resistance** of the material of a conductor is equal to the resistance offered by wire of unit length and unit area of cross section of the material of wire. Its unit is ohm-meter. It remains constant for a material.

- Dimension of resistivity is $ML^3T^{-3}A^{-2}$
- The reciprocal of resistivity of the material of a conductor is called as conductivity(σ)

$$\sigma = \frac{1}{\rho} = \frac{J}{E}$$

The unit of conductivity is ohm^{-1} $metre^{-1}(\Omega\ m)^{-1}$.

- In terms of material parameter

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}\ \therefore$$

Temperature Dependence of Resistivity

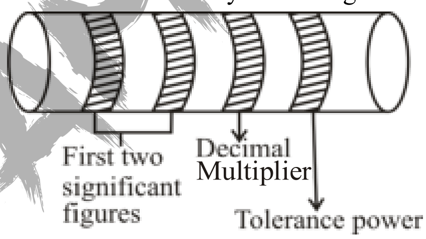
- When the temperature of a metallic wire is raised, its electric resistance increases.
- The resistance of a pure metallic wire or the resistivity ρ of pure metal is directly proportional to its absolute temperature, provided the temperature coefficient of the metal is $(1/273)^\circ C^{-1}$
- The resistivity of alloys also increases with rise in temperature, but this increase is much smaller compared to pure metals.
- The resistivity of semiconductor decreases rapidly with rise in temperature.

■ Superconductor -

- Super conductors are those materials whose resistance is zero. Superconductors are made at very low temperature.
- The lowers temperature at which a material becomes super conductor is called critical temperature.
  Example: Lead Pb = 7.2 K, Mercury Hg = 4.2 K.

■ Colour Coding of Carbon Resistors -

- In electronic circuits, carbon resistors are frequently used and their values vary over a wide range. A colour code is used to print the value of the resistance.
- The resistance of a carbon resistor can be calculated by the code given on it in the form of coloured strips



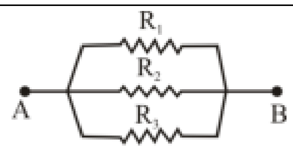
Colour	Figure	Multiplies
Black (B)	0	$10^0 = 1$
Brown (B)	1	10^1
Red (R)	2	10^2
Orange (O)	3	10^3
Yellow (Y)	4	10^4
Green (G)	5	10^5
Blue (B)	6	10^6
Violet (V)	7	10^7
Grey (G)	8	10^8
White (W)	9	10^9

Tolerance Power -	
Colour	Tolerance
Gold	5%
Silver	10%
No-colour	20%

- To read the value of a carbon resistance the following sentence may be memorized:

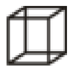
B	B	R	O	Y	Great	Boy	Very	Good	Wife
0	1	2	3	4	5	6	7	8	9

■ Combination of Resistors -

In Series	In Parallel
	
• Equivalent resistance, $R_s = R_1 + R_2 + R_3$.	• Equivalent resistance, $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
• Current through each resistor is same.	• Potential difference across each resistor is same.
• Sum of potential difference across individual resistors is equal to the potential difference by the source.	• Sum of electric currents flowing through individual resistors is equal to the electric current drawn from the source.

- If n-identical resistances are first connected in series and then in parallel, the ratio of the equivalent or maximum to minimum resistance is-

$$\frac{R_s}{R_p} = n^2,$$

- If a Skelton cube is made with 12 equal resistors, each having a resistance across  (each side has resistor R).

(a)The diagonal of a cube = $\frac{5}{6}R$

(b)The diagonal of a face = $\frac{3}{4}R$

(c)Along a side = $\frac{7}{12}R$

■ Thevenins Theorem (Network reduction theorem)

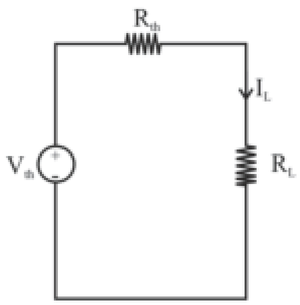
According to the thevenin's, 'Any linear bilateral network ir-respective of its complexities can be reduced into a Thevenin's equivalent circuit having the Thevenin's open circuit Voltage 'V_{th}' in series with the Thevenin's equivalent resistance R^{th} along with load resistor R_L.

OR

Any two terminal network containing a no. of e.m.f sources and resistance can be replaced by an equivalent series circuit having a resistance R. Where-

V_{th} or E_o = Open circuited voltage between the two terminals.

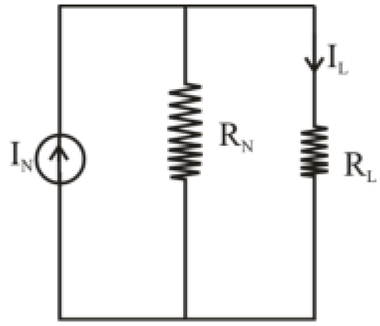
R_{th} or R_o = the resistance between two terminals of the circuit obtained by looking "in" the terminals with load removed and voltage sources replaced by their internal resistance, if any.



$$\boxed{I_L = \frac{V_{th}}{R_{th} + R_L}}$$

■ Norton's Theorem (Network reduction theorem)

According to Norton's theorem. 'Any linear bilateral network ir-respective of its complexity (Can be reduced into a Norton's equivalent circuit having a Norton's Short circuit 'I_N' Parallel with Norton's equivalent resistance R_N in Parallel with load resistor R_L.



$$I_L = \frac{I_N R_N}{R_N + R_L}$$

■ Electric cell

- An electric cell is a device which converts chemical energy into electrical energy. When connected in a circuit it acts as a source of D.C. current.

  It is of two types-

S.N.	Primary Cell	Secondary cell
1.	These cannot be recharged again after getting discharged once.	These can be recharged easily.
2.	These are cheap of low cost.	These are expensive as compare to Primary cell.
3.	These are easy to use.	These are difficult to use in comparison to Primary cell.
4.	These can be used only once.	These can be used more than once.
5.	In these cells irreversible reactions occur.	In these cells reversible reaction occurs.
6.	These have a lower self discharge rate.	These have a higher self discharge rate.
7.	These are used in torch and other portable devices as they produce electric current immediately.	These are used in inverters and automobiles.
8.	These cells don't require regular maintenance and can be disposed easily after use.	These cells require regular maintenance.
9.	They have a low or small lifetime.	They have high lifetime.
10.	Examples of these cells are Dry cells, Daniel cells etc.	Examples of these cells lead-acid cell, nickel-iron cell etc.

■ E.M.F. and Potential difference

S.N.	E.M.F.	Potential difference
1.	Emf or electromotive force is the maximum potential difference between two electrodes of this cell when no current is drawn from the cell i.e. When circuit is open	It is the difference of potentials between any two points in a closed circuit
2.	The unit of EMF is volt.	The unit of potential difference is volt.
3.	It is independent of resistance.	It depends upon resistance between two points.
4.	It is measured using an Emf meter	It is measured using a voltmeter
5.	The electric, gravitational and magnetic fields are responsible for this.	The electric field is the sole source of potential difference.

■ Internal resistance

Internal resistance refers to the opposition to the flow of current offered by the cells and batteries themselves resulting in the generation of heat. Internal resistance is measured in ohms. The relationship between internal resistance (r) and emf (E) of cell is given by.

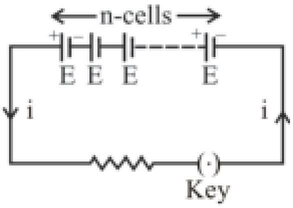
$$E = I(r + R)$$

Where, I = Current, r = Internal resistance, R = Resistance in circuit

■ Grouping of cells -

Series combination

- Series combination of n -cells each of emf E and internal resistance r.



$E_{eq} = E_1 + E_2 + E_3 + \ldots\ldots\ldots\ldots\ldots\ldots + E_n = nE$

$r_{eq} = r_1 + r_2 + r_3 + \ldots\ldots\ldots\ldots\ldots\ldots + r_n = nr$

Current through the circuit = $\dfrac{\text{Total emf}}{\text{Total resistance}}$.

$I = \dfrac{nE}{(R + nr)}$ .... (1)

(i) If nr << R then I = nE/R

If the value of the internal resistance is much lesser than the external resistance then the current in the circuit will be n times the circuit current due to the single cell.

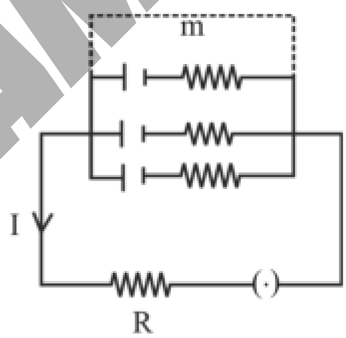
(ii) If nr >>R then I = E/r

If the value of the internal resistance is much greater than the external resistance then the current in the circuit will be equal to the short-circuited current obtained from a single cell.

Parallel combination-

- If *m* cells each of e.m.f. *E* and internal resistance *r* be connected in parallel and if this combination be connected to an external resistance then the e.m.f. of the circuit = *E*.

  Internal resistance of the circuit = $\dfrac{r}{m}$



$I = \dfrac{E}{R + \dfrac{r}{m}} = \dfrac{mE}{mR + r}$

If R << r then $I = \dfrac{mE}{r} \rightarrow$ parallel combination should be used.

If R >> r then $I = \dfrac{E}{R} \rightarrow$ No advantage of using parallel combination

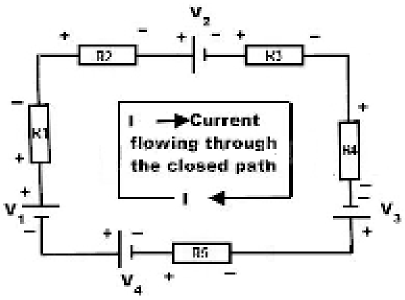
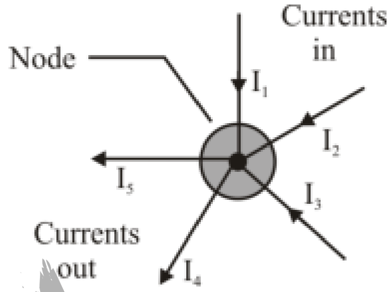
■ Kirchhoff's Law -

- Ohm's law is unable to give current in complicated circuit, Kirchhoff's in 1842 gave two law which are extremely useful to solve problem of complicated electrical circuits.

  Kirchohoff's two laws –

  (i) Kirchhoff's voltage law. (KVL)

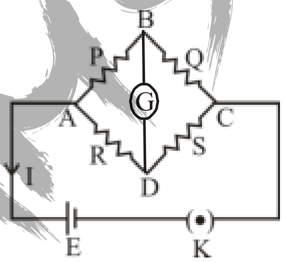
  (ii) Kirchhoff's current law. (KCL)

Kirchhoff's Voltage Law	Kirchhoff's Current Law
• In any closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total e.m.f. in the circuit i.e.- $\sum IR = \sum V$	• The algebraic sum of all currents meeting at junction in a closed circuit is zero i.e. $\sum I = 0$.
• This law follows law of conservation of energy. It is also called as loop rule.	• This law follows law of conservation of charge. It is a junction rule
 $\sum IR = \sum V$	 $I_1+I_2+I_3-I_4-I_5=0$

■ Applications of Kirchhoff's Law -

■ Wheatstone Bridge -

- Wheatstone bridge is also known as resistance bridge. It is an arrangement of four resistance connected to form the arms of quadrilateral ABCD. A battery with key is connected along two vertices and Galvanometer is connected across other two vertices.



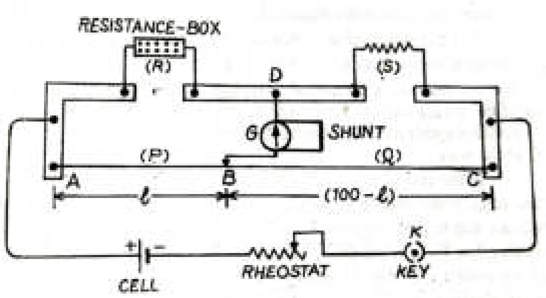
Where, PQ = Ratio arms
R = Known resistance
S = Unknown resistance

The bridge is said to be balanced, when
(i) $V_B = V_D$
(ii) There is no flow of current through galvanometer.
The Wheatstone Bridge is said to be sensitive if it gives ample deflection in the galvanometer even on slight change of resistance.

- For sensitivity of galvanometer the magnitude of four resistances P, Q, R, S should be of same order.
- For balanced condition $\frac{P}{Q} = \frac{R}{S}$, When deflection in galvanometer is zero $I_G = 0$.
- Wheatstone bridge is most sensitive if all the arms of bridge have equal resistance i.e. = P = Q = R = S.

■ Meter Bridge:-

- It is an electrical device used to determine the resistance of material of given wire/conductor.
- It is based on the principle of balanced Wheatstone bridge.
- For uniform wire AC of length 1 meter the balanced condition of meter bridge using Wheatstone bridge principle is given as-
$\frac{P}{Q} = \frac{R}{S}$ or $\frac{\ell}{100-\ell} = \frac{R}{S} \Rightarrow S = \frac{100-\ell}{\ell} \times R$.



Where S is the unknown resistance to be determined, R is variable Resistance and l is the balancing length.

- Meter Bridge is also known as slide wire bridge.

■ **Potentiometer:-**

It is an electrical device which can -

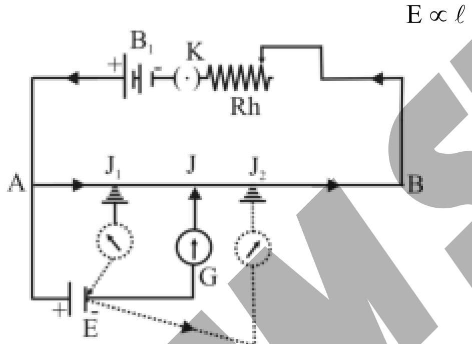
(i) Measure the potential difference with greater accuracy.

(ii) Compare the e.m.f. of two cells.

(iii) Used to determine the internal resistance of a primary cell.

It works on the principle when a constant current is passed through a wire of uniform cross-section, the potential difference across any portion of the wire is directly proportional to the length of that portion. In the null-point position the potential difference V is equal to the emf E of the cell-

$$E \propto \ell$$



Uses of potentiometer-

For various purposes the portion between A and B is modified-

(i) Comparison of emf of two cells

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$$

Where, E_1 = emf of known cell, E_2 = emf of unknown cell

ℓ_1 = distance of null point of cell 1 from A

ℓ_2 = distance of null point of cell 2 from A.

(ii) Comparison of Resistance

$$\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2}$$

where symbols have their usual meaning

(iii) Determination of internal Resistance

$$r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$$

$$r = R\left(\frac{E}{V} - 1\right)$$

Where E = E.m.f of given cell, V = potential difference between ends of resistor R.

(iv) Measurement of low resistance

$$x = R\left(\frac{\ell_2}{\ell_1}\right)$$

Where x is low resistance which is to be measured connected in series with R

■ Heating Effect of Electric Current -

- The generation of heat due to flow of electric energy in a conductor is known as heating effect of current.

■ Electrical Power -

- The energy liberated per second in device is called its power. The electrical power P delivered by an electrical device is given by P = VI,
  Where V = Potential difference across device then power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).
- Power consumed by a resistor $(P) = I^2R = VI = \dfrac{V^2}{R}$.
- SI unit of power is watt.
  1 watt = 1J/s
- Unit in the commercial unit of electric energy, 1 unit = 1 kWh = 3.6×10^6J.

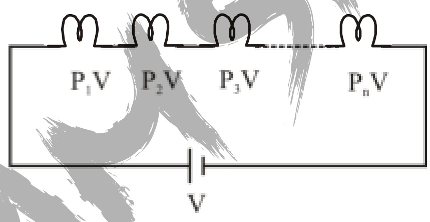
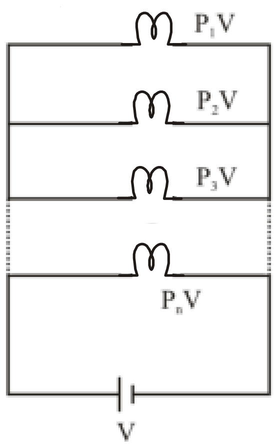
■ Joules Law of Electrical Heating -

- The heat generated (in Joules) when a current of I ampere flows through a resistance of R ohm for t second is given by

$$H = I^2Rt \text{ Joule} = \frac{I^2Rt}{4.2} \text{ Colories} = \frac{V^2}{R}t \quad \text{Joule}$$

- If current is variable passing through the conductor then we use for heat produced in resistance in time 0 to T is $H = \int_0^T I^2R\,\theta\,dt$.

■ Illumination of electric bulbs in parallel and in series-

Series combination of Bulbs		$\dfrac{1}{P_{total}} = \dfrac{1}{P_1} + \dfrac{1}{P_2} + \dfrac{1}{P_3} + ...$
Parallel combination of Bulbs		$P_{total} = P_1 + P_2 + P_3 + ...$

■ Chemical Effect of electric current-

- When an electric current flows through a conducing solution, chemical reactions take place in the solution. This is called the chemical effect of electric current. Some of the chemical effects of electric current are as follows-
  (i) Gas bubbles are formed at electrodes
  (ii) Metals are deposited on electrodes
  (iii) Change of colour of the solution.

■ Application of chemical effect of electric current

- Extracting Metals from their Ores
- Electroplating
- Purification of metals
- Production of compounds
- Decomposition of Compounds

■ Key points -

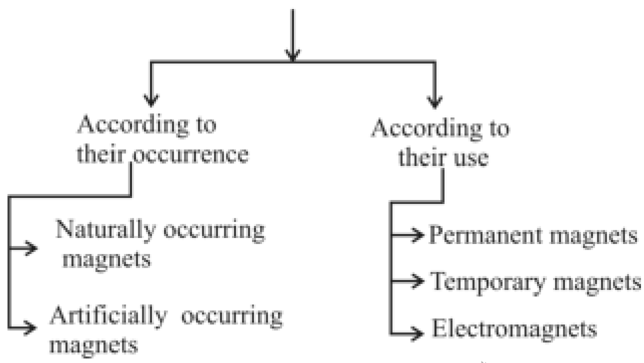
- A current flows through a conductor only when there is an electric field within conductor because the drift velocity of electrons is directly proportional to the applied field.
- Electric field outside the conducting wire which carries constant current is zero because net charge on a current carrying conductor is zero.
- A metal has a resistance and gets often heated by low current because when free electrons drift through a metal They make occasional collisions with lattice. There collisions are inelastic and transfer energy to the lattice as internal energy.
- Ohm's law holds only for small current in metallic wire, not for high currents because for small currents change in temperature is small and hence change in resistance is small.
- Potentiometer is an ideal instrument to measure the potential difference because potential gradient along the potentiometer wire can be made very small.
- An Ammeter is always connected in series where as voltmeter is connected in parallel because an ammeter is low resistance galvanometer while a voltmeter is high resistance galvanometer.
- The drift velocity of electrons is small because of the frequent collisions suffered by electrons.
- The small value of drift velocity produces a large amount of electric current, due to the presence of extremely large number of free electrons in a conductor.
- Free electron density in a metal is given by $n = \frac{N_A \times d}{A}$ where N_A = Avogadro's number, d = density of metal and A = Atomic weight of metal.
- In the absence of radiation loss, the time in which a fuse will melt does not depends on it's length but varies with radius as $t \propto r^4$
- If length (l) and mass (m) of a conducting wire is given then $R \propto \frac{\ell^2}{m}$
- Decoration of lights in festivals is an example of series grouping whereas all household appliances are connected in parallel grouping.
- Using n conductors of equal resistance, the number of possible combinations is 2^{n-1}.
- If the resistance of n conductors are totally different, then the number of possible combinations will be 2^n.
- E.m.f is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.
- If the temperature of the conductor placed in the right gap of metre bridge is increased, then the balancing length decreases and the jockey moves towards left.
- In Wheatstone bridge to avoid inductive effects the battery key should be pressed first and the galvanometer afterwards.
- In the measurement of resistance, Wheatstone bridge is not affected by the internal resistance of the cell.
- In case of zero deflection, the galvanometer current flows in the primary circuit of the potentiometer, not in the galvanometer circuit.
- A potentiometer can act as an ideal voltmeter.

(4) Magnetism and Matter

■ Magnet–

- A magnet is an object that produces a magnetic field around itself

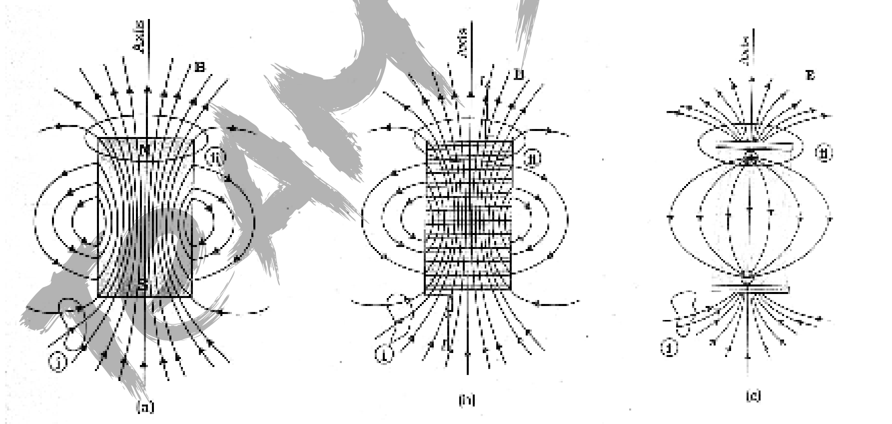
Classification of magnets



Properties of magnets –

- Magnet attracts iron and has two poles.
- A freely-suspended magnet always stays north-south.
- Unlike poles attract and like poles repel each other.
- A magnet induces magnetism in magnetic materials.
- An isolated magnetic pole does not exist.

Magnetic field lines



field lines of a bar magnet | field lines of a current carrying finite solenoid | field lines of an electric dipole

- The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.
- The tangent to the field lines at a given point represents the direction of the net magnetic field B at that point.

■ The bar Magnet:-

A bar magnet is a rectangular piece of an object, made up of iron, steel or any other ferromagnetic substance that shows permanent magnetic properties. It has two poles, a north and a south pole.

■ Magnetic Dipole -

- A magnetic dipole consists of two unlike poles of equal strength separated by small distance . It is another name of bar magnet. It is given by-

$$M = m \times 2\ell$$

Where m = pole strength

- Its SI unit is Am^2.

Magnetic dipole moment of a revolving electron

$$\vec{M} = \frac{evr}{2} = \left(-\frac{e}{2m_e}\right)\vec{L} \quad Am^2$$

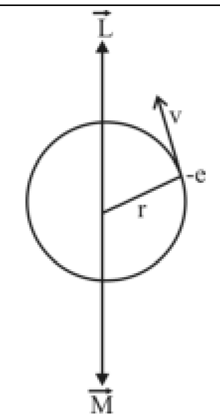
where L = angular momentum

m_e = Mass of electron

Interaction between two magnetic dipole is,

$$F = \frac{\mu_0}{4\pi}\frac{6m_1m_2}{r^4}.$$

where m_1, m_2 = Pole strength of north and south pole



Magnetic dipole Moment of current Loop

$\vec{M} = NiA$

where A = area of current loop

i = current carried by loop

N = number of turns of current carrying loop

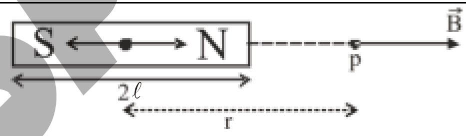
■ Magnetic field due to a bar magnet -

- **An axial point (end on position)**

$$B = \frac{\mu_0}{4\pi}\frac{2Mr}{\left(r^2 - \ell^2\right)^2}$$

when $\ell << r$

$$B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$$



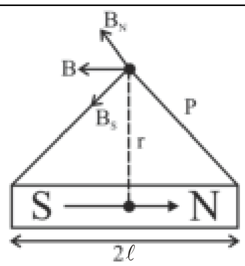
- **On equatorial point (broad side on-position)**

$$B = \frac{\mu_0}{4\pi}\frac{M}{\left(r^2 + \ell^2\right)^{3/2}}$$

when, $\ell << r$

$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}.$$

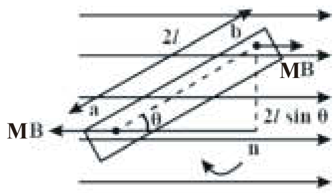
The direction of magnetic field is parallel to the magnetic dipole and opposite to the direction of dipole moment.



Torque on a bar magnet in a uniform magnetic field

$\vec{\tau} = MB\sin\theta = \vec{M}\times\vec{B}$.

Where, θ is the angle between M and B. Its SI unit is Joule per Tesla (JT^{-1}).



■ Potential Energy -

- Potential energy of a magnetic dipole in a magnetic field is given by

  $U = -MB\cos\theta = -\vec{M}.\vec{B}$

  where θ is the angle between 'M' and 'B'.
- Work done in rotating the dipole in a uniform magnetic field from θ_1 to θ_2 is given by-

  $W = MB(\cos\theta_1 - \cos\theta_2)$.

■ Oscillation of a freely Suspended magnet -

- The oscillations of a free suspended magnet (magnetic dipole) in a uniform magnetic field are SHM.
- The time period of oscillation $T = 2\pi\sqrt{\frac{I}{MB}}$.

  Where, I = moment of inertia of the magnet,
  M = magnetic moment
  B = magnetic field intensity

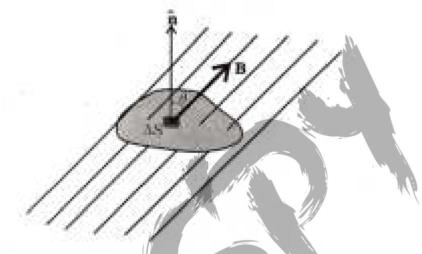
■ Bar Magnet as an Equivalent Solenoid -

- The expression of magnetic field at distance 'r' from centre is given by = $B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$.

  This expression is equivalent to that of bar magnet.

■ Gauss law of Magnetism:- It states that The net magnetic flux through any closed surface is zero.

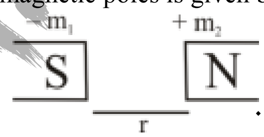
- The net magnetic flux $\phi_B = \sum_{all} \Delta\phi_B = \sum_{all} B \cdot \Delta S = 0$



- Comparing this with Gauss law of electrostatics. The flux through closed surface is given by $\sum E \cdot \Delta S = \frac{q}{\varepsilon_0}$
- The difference between these two laws is that electrostatics is a reflection of the facts that isolated magnetic poles(monopoles) don't exist.

■ Coulomb's Law in Magnetism -

- Magnitude of force acting between two magnetic poles is given by

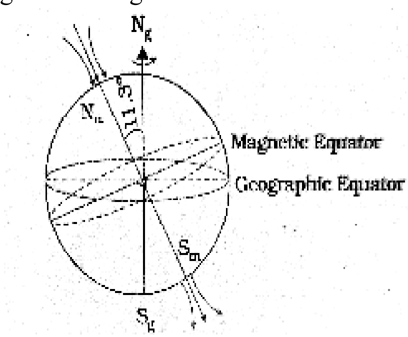


$F = k\frac{m_1 m_2}{r^2}$. Where, m_1 and m_2 are magnetic strength of poles and k is magnetic force constant

- SI unit of k is $N\text{-}A^{-2}$. and $k = \frac{\mu_0}{4\pi} = 10^{-7}$

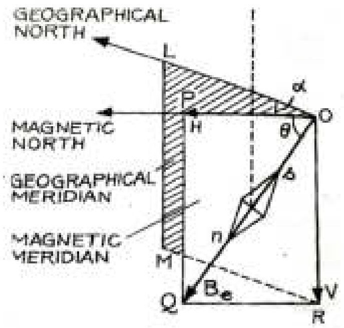
■ Earth's Magnetism -

- Earth's magnetism is generated by convection currents of molten iron and nickel in the earth's core which has streams of charged particles and generate magnetic fields.



- Earth behaves like a magnet whose North Pole is somewhere close to geographical South Pole and magnetic South pole is closed to geographical North Pole.

Magnetic meridian and geographical meridian -



- The vertical plane passing through the axis of a magnetic needle suspended freely through its center of gravity and at rest under earth's field is called the magnetic meridian.
- Similarly, the vertical plane passing through line joining the geographical north and south poles is called the geographical meridian.

Elements of Earth's Magnetism -

- **Magnetic Declination (α) -** The smaller angle subtended between the magnetic meridian and geographic meridian is called magnetic declination.
- **Angle of dip (θ) or Magnetic Inclination -** In Magnetic meridian, the angle made by resultant of the earth's magnetic field (B_e) with the horizontal in magnetic meridian is known as angle of dip (θ).
- **Horizontal and Vertical Component of Earth's Magnetic Field -** If B is the intensity of earth's magnetic field, then horizontal component of earth's magnetic field $H = B_e \cos\theta$

  Vertical component of earth's magnetic field $V = B_e \sin\theta$.

  $$B = \sqrt{H^2 + V^2}\ ,\ \tan\delta = \frac{V}{H}.$$

Neutral Points -

- These points are where net magnetic field due to the magnet and magnetic field of the earth is zero are called neutral points.
- When north pole of a bar magnet is placed towards South pole of the earth, then neutral point is obtained on axial line.

  $$B = \frac{\mu_0}{4\pi}\frac{2Mr}{\left(r^2 - l^2\right)^2} = H$$

  If $r >> l$, then

  $$B = \frac{\mu_0}{4\pi}\frac{2M}{r^3} = H$$
- When north pole of a bar magnet in placed towards north pole of the earth, then neutral point is obtained on equatorial line.

  $$B = \frac{\mu_0}{4\pi}\frac{M}{\left(r^2 - l^2\right)^{3/2}} = H$$

  If $r >> l$, then

  $$B = \frac{\mu_0}{4\pi}\frac{M}{r^3} = H$$

  where, H = horizontal component of earth of magnetic field

■ Tangent law -

The tangent law of magnetism is a way of measuring the strengths of two perpendicular magnetic fields. When a magnet is exposed to a magnetic fields B that is perpendicular to the Earth's horizontal magnetic field (H), the magnetic field will rest at an angle θ. The relation between the magnetic fields is as follows:

$B = H \tan\theta$

$\tan\theta = B/H$

■ Important Terms Used to Describe the Properties of Magnetic Materials -

To describe the magnetic properties of materials following terms are required.

- **Magnetic induction**

  When a piece of any substance is placed in an external magnetic field the substance becomes magnetised. The magnetism so produced in the substance is called induced magnetism and the phenomenon is called magnetic induction. It is denoted by $\vec{B}$

  Magnetic induction, $B = \mu_0 (H + I)$

  Unit is tesla or Wb/m^2

- **Magnetic Permeability -**

  It is the ability of a material to permit the passage of magnetic lines of force through it.

  Magnetic permeability $(\mu) = \dfrac{B}{H}$.

  Where, B is magnetic induction and H is magnetising force or magnetic intensity.

  Unit is Wb/A-m or NA^{-2}

- **Intensity of Magnetisation -**

  The magnetic dipole moment developed per unit volume of the magnetised substance is called intensity of magnetisation. Intensity of Magnetisation $(I) = \dfrac{M}{V}$.

  Where, V = volume and A = area of cross section of the specimen

  Unit is Am^{-1}

- **Magnetic Force or Magnetic Intensity -**

  The degree up to which a magnetic field can magnetise a material is defined in terms of magnetic intensity.

  Magnetic intensity $(H) = \dfrac{B}{\mu} - I$.

  Unit is same as that of Intensity of magnetisation

- **Magnetic Susceptibility (χ_m) -** It is equal to the ratio of intensity of magnetisation and magnetising field. It has no unit

$$\chi_m = \frac{I}{H}.$$

- **Relation between Magnetic permeability and Susceptibility is given by -**

$$\mu = \mu_0 (1 + \chi_m)$$

■ Types of magnetic substance

The magnetic materials placed in a uniform magnetic field are classified into three parts.

(i) Paramagnetic Substance

(ii) Diamagnetic Substance

(iii) Ferromagnetic Substance

Property	Paramagnetic Substance	Diamagnetic Substance	Ferromagnetic Substance
When placed in a uniform M.F.	Freely magnetise along applied field	Freely magnetise and opposite to magnetic field	Strongly magnetise along the magnetic field
Susceptibility (χ_m)	Small and positive $0 < \chi_m < \varepsilon$, ε = small numbers	Small and negative $-1 < \chi_m < 0$	Very large $\chi_m > 1$
Permeability	$\mu > \mu_0$	$\mu < \mu_0$	$\mu >> \mu_0$
Relative Permeability	$1 < \mu_r < (1 + \varepsilon)$ ε = small-number	Positive and less than 1 $0 < \mu_r < 1$	Large value $\mu_r > 1$
Effect of temperature	$\chi_m \propto \dfrac{1}{T}$	Independent with temperature	$\chi_m \propto \dfrac{1}{T - T_c} (T > T_c)$
Variation of I with H	Linear change	Linear change and saturable low temperature	Non-linear change

In a non- uniform magnetic field	Tends to move from weaker to stronger magnetic field	Tends to move from stronger to weaker magnetic field	Tends to move quickly from weaker to stronger magnetic field
Examples	Na, Ca, O_2, $CuCl_2$, Al	Pb, H_2O, NaCl, Bi, Cu, Si, Sb	Ni, Co, Fe, Fe_2, Gd

■ Curie's law

- According to this law the intensity of magnetisation I of a paramagnetic substance is directly proportional to magnetic intensity H of magnetising field and inversely proportional to kelvin temperature T.

$$I = C(H/T)$$

Where C is curie constant.

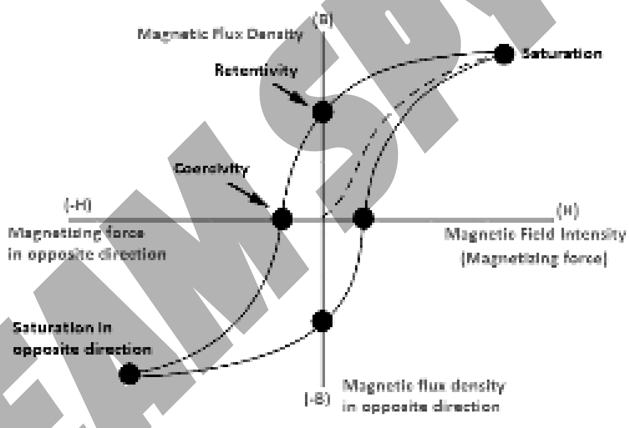
it can also the represented as $\chi = C/T$

where χ = magnetic susceptibility

- The temperature of transition from ferromagnetic substance to paramagnetism is called Curie temperature T_C.

$$\chi = \frac{C}{T - T_c} \qquad (T > T_C)$$

■ Hysteresis -



Hysteresis loop or B-H curve

- The lagging of intensity of magnetisation (I) or magnetic induction (B) behind magnetising field (H), when a specimen of a magnetic substance is taken through a complete of magnetisation is called hysteresis.

Retentivity or Residual Magnetism

- The value of the intensity of magnetisation of a material, then the magnetising field is reduced to zero is called rententivity or residual magnetism of the material.

Coercivity

- The coercivity of a substance is a measure of the reverse magnetising field required to distroy the residual magnetism of the substance.

Hysteresis Loss

- The energy supplied to the substance during magnetisation is not fully recovered. The balance of energy left in the substance is lost as heat.This is called Hysteresis loss.
- The energy lost per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop.

Differences in magnetic properties of soft Iron and steel

- The retentivity of soft iron is greater than the retentivity of steel.
- The coercivity of soft iron is less than the coercivity of steel.
- The hysteresis loss in soft iron is smaller than that in steel.
- Permeability of soft iron is greater than of steel

(5) Alternating Current

■ Alternating Current :-

- An alternating current that current which changes its direction with time.
- The voltage with which AC current is associated is called AC voltage and varies like a sine function with time.
- It is represented by-

  $E = E_0 \sin(\omega t + \phi)$

  $I = I_0 \sin(\omega t + \phi)$

  Where, I = Instantaneous current

  I_0 = Peak value or maximum value of AC

  $(\omega t + \phi)$ = Phase at any time t

  ϕ = Initial phase or phase constant

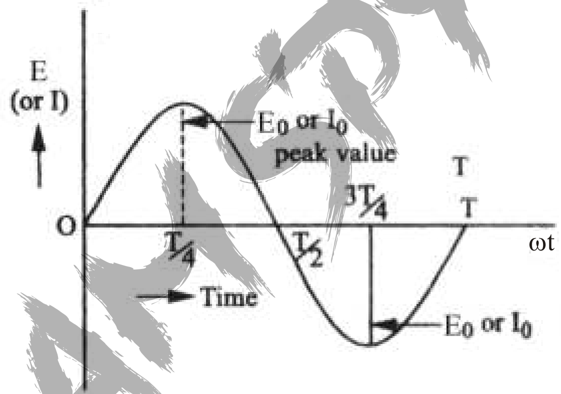
  ω = Angular frequency

  T = Time period = $\frac{2\pi}{\omega}$

  f = Frequency = $\frac{1}{T}$

  E_0 = amplitude or peak value of voltage

  E = Voltage at any time t



■ Key Points

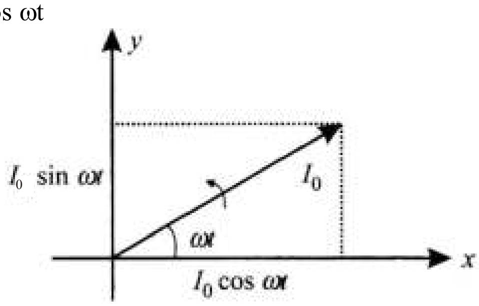
- The equation of current can also be written in cosine form as $I = I_0 \cos(\omega t + \phi)$.
- To produce alternating current, emf / voltage should be alternating. An alternating emf / voltage can be represented by $E = E_0 \sin(\omega t + \phi)$.
- An alternating emf is produced by a AC dynamo or an electronic oscillator.
- The frequency of an AC in India is 50 Hz, i.e. f = 50 Hz, so $\omega = 2\pi f = 100\ \pi$ rad/s^{-1}
- The AC can be converted into DC with the help of a rectifier while DC into AC with the help of an inverter.
- It cannot produce chemical effects of current such as electroplating.
- It can be stepped up or stepped down with the help of transformer.

■ Phasor Diagram -

- The length of arrow represents the peak value of the quantity and its projection on x or y-axis gives its instantaneous value.

  For Example - $I_y = I_0 \sin \omega t$

  $I_x = I_0 \cos \omega t$



■ Average Or Mean Value of Alternating Current Or Voltage Over One Complete Cycle

$$I_{av} = \int_{t_1}^{t_2} I(t)\,dt = I_{av} = \frac{\int_{t_1}^{t_2} I(t)\,dt}{t_2 - t_1}$$

Where, $I = I_0 \sin \omega t$

(i) Over first half cycle-

$$I_{av} = \frac{2L_0}{\pi} = 0.637\, I_0$$

(ii) Over full cycle -

$I_{av} = 0$

So, average value of current for the full cycle is zero.

- Average value of alternating emf over complete cycle is zero.
- Average value of voltage over one half cycle is

$$E_{av} = \frac{2E_0}{\pi} = 0.637\, E_0\,.$$

- Average value during + ve half cycle :

  $I_{av} = 0.637\, I_0$

  $E_{av} = 0.637\, E_0$

- Average value during – ve half cycle -

  $I_{av} = -\,0.637\, I_0$

  $E_{av} = -\,0.637\, E_0$

■ Root Mean Square Or Virtual Or Effective Value Of Sinusoidal Voltage Or Current –

- The effective or virtual value of an alternating current is that steady current which, when passed through a given resistance for a certain time will develop the same amount of heat as the actual alternating current shall develop when passed for the same time.

$$I_{eff} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

- The effective or RMS value of an alternating current is $\frac{1}{\sqrt{2}}$ times its peak value or maximum value. Similarly the effective or virtual value of alternating emf is given by

$$E_{eff} = \frac{E_0}{\sqrt{2}} = 0.707\, E_0$$

■ **Form Factor** - It is define as the ratio of the virtual or RMS value to the average value of an alternating current or voltage. Thus

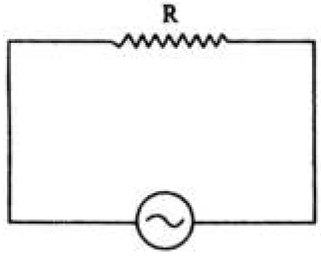
$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = \frac{E_{rms}}{E_{av}}\,.$$

■ A.C. Circuits -

- Basic ac circuit elements are resistors, inductors and capacitors.
- **Impedance -** The effective resistance offered to an LCR circuit or arising from combined effect of ohmic resistance and reactance is called Impedance .

■ Circuit containing pure resistance only -

- Consider resistor R is connected across emf $E = E_0 \sin \omega t$.
  As show in fig.

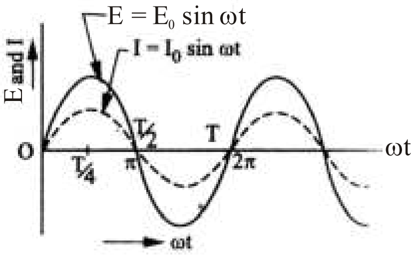


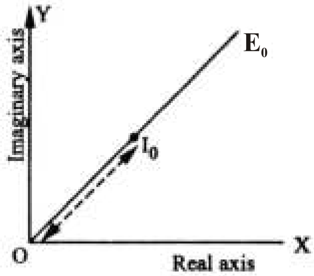
$E = E_0 \sin \omega t$

- If I is the instantaneous current in the circuit, at any time t, then current $I = \frac{E}{R} = \frac{E_0}{R}\sin \omega t$ Where,

$I_0 = \frac{E_0}{R}$ = maximum value of current.

- The current is in phase with applied voltage as shown in fig. (i) and fig. (ii) shows the complex number representation of voltage and current.





■ Circuit containing pure inductance only-

- Fig. (i) shows a circuit containing a inductor of self inductance L with an alternating voltage $E = E_0 \sin \omega t$ applied across it.

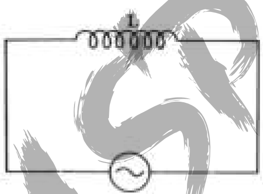


Fig. (i)

- $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) = \frac{E_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$

Where, $X_L = \omega L$ is the resistance offered by inductor. It is also known as inductive reactance. Its units is Ω. $I_0 = \frac{E_0}{X_L}$ = current amplitude

- The current lags behind the voltage by π/2 (or quarter of cycle) or voltage leads the current by π/2 as shown in fig (ii) and fig. (iii) shows the complex number representation.

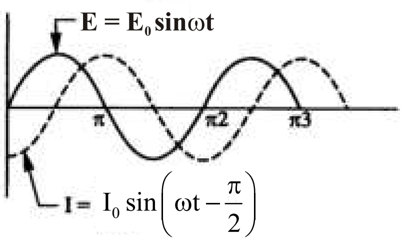


Fig. - (ii)

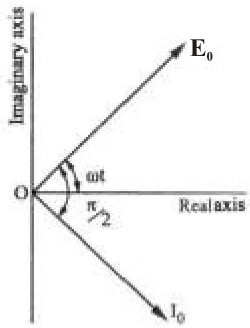


Fig. - (iii)

- **Key Points:–**

(i) A phase difference of π/2 is equivalent to time difference of T/4.

(ii) For dc circuit $\omega = 0 \Rightarrow X_L = 0$. So inductor offers zero resistance to a dc circuit.

(iii) X_L depends upon the frequency of source

■ A.C. circuit containing capacitor only

- Fig (i) shows a circuit containing a capacitor of capacitance c with an alternating voltage $E = E_0 \sin\omega t$ applied across it

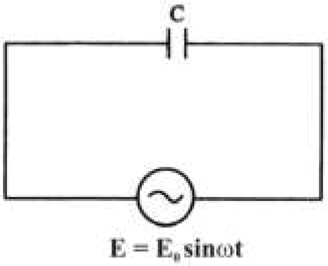


Fig (i)

- $I = I_0 \sin(\omega t + \pi/2) = \dfrac{E_0}{X_C}\sin\left(\omega t + \dfrac{\pi}{2}\right)$

  Where, $X_c = \dfrac{1}{\omega c}$ is known as capacitive reactance and $I_0 = \dfrac{E_0}{X_c}$ is the peak value of current

- In this case current leads the voltage by $\pi/2$

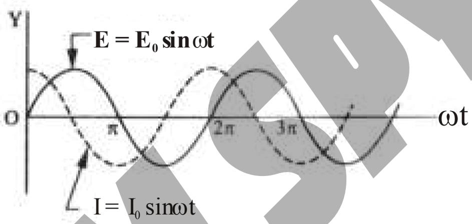


fig. (ii)

- Since the capacitive reactance $X_c = \dfrac{1}{\omega c}$ is inversely proportional to the frequency of applied voltage. Hence the condenser offers negligible reactance at high frequency and works as "by pass capacitor" and very high reactance at low frequency works as blocking condenser.
- The complex no representation is shown as-

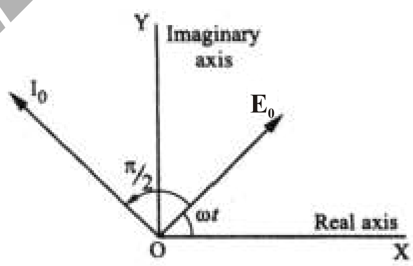


fig. (iii)

- In case of resistor, resistance is offered due to the obstruction to the passage of current. In inductor it is due to induced e.m.f. In capacitor, it is potential difference developed across the capacitor.
- Values of ϕ and Z for different types of circuits is given the table below-

Type of circuit	Phase factor	Impedance
Purely resistive circuit	$\phi = 0^\circ$	$Z = R$
Purely inductive circuit	$\phi = -\pi/2$	$Z = X_L = \omega L$
purely capacitive circuit	$\phi = \pi/2$	$Z = X_c = \dfrac{1}{\omega c}$

- The reciprocal of reactance is called susceptance and that of impedance is called admittance

■ A.C. circuit containing Inductance and Resistance In series:-

Let us consider a circuit as shown in fig (i) containing a series combination of an inductance L and resistance R with an alternating emf E = E_0 sinωt. applied across the combination. The instantaneous current in the circuit is given by –

$$I = \frac{\text{Applied emf}}{\text{vector impedence of the circuit}}$$

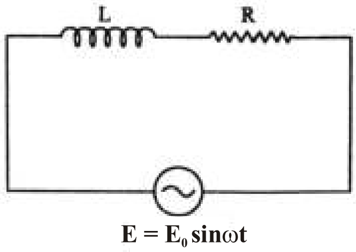


fig. (i)

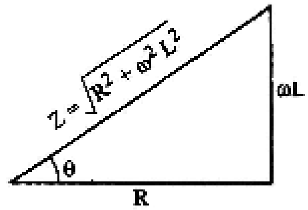


fig. (ii)

- $I = I_0 \sin(\omega t - \theta) = \frac{E_0}{Z}\sin(\omega t - \theta)$ ....(A)

  Where, $Z = \sqrt{R^2 + \omega^2 L^2}$, $I_0 = \frac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$

- Equation (A) represent the variation of current in the circuit with time as shown in fig (iii). It also indicates that the current lags behind the applied phase by an angle θ or emf in current by angle θ where, $\theta = \tan^{-1}\frac{\omega L}{R}$ as shown in fig. (ii)

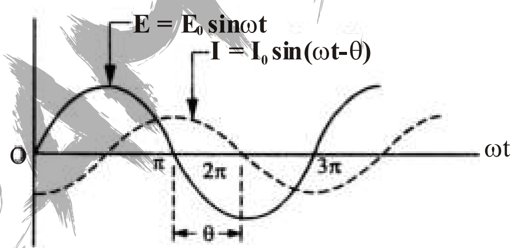


fig (iii)

- Fig (iv) represents the complex no representation–

The voltage E_R across the resistance while voltage E_L across the inductance leading the current I by an angle π/2 and the total voltage, $E_0 = I_0\sqrt{R^2 + \omega^2 L^2}$ leading the current by an angle $\theta = \tan^{-1}\frac{\omega L}{R}$

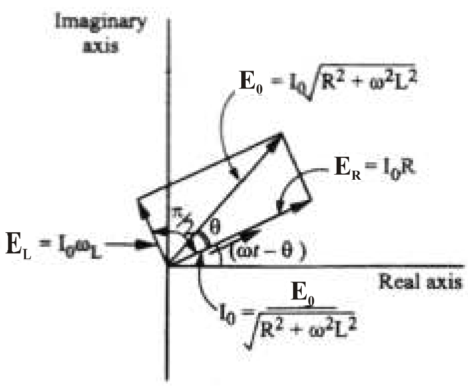


fig (iv)

• The impedance is defined as the ratio of r.m.s (or virtual) value of the e.m.f. applied across the L – R circuit to the r.m.s. (or virtual) value of current flowing through the circuit.

$$Z = \frac{E_{rms}}{I_{rms}} = \frac{\frac{E_0}{\sqrt{2}}}{\frac{I_0}{\sqrt{2}}} = \frac{E_{eff}}{I_{eff}}$$

Special Cases–

Case 1:

- If $R \to 0$ i.e. when inductive reactance ωL is very large than to ohmic resistance R which can be neglected. Then $Z = \omega L$ and $\theta = \pi/2$
- In practice, it occurs when the frequency is very high. Then current is given by–

  $I = I_0 \sin(\omega t - \pi/2)$ and $I_0 = = \frac{E_0}{\omega L}$

  I_0 is inversely proportional to both self inductance and frequency. Therefore, when the frequency is high, a high voltage can produce only small current.
- Phase lag is $\theta = \tan^{-1}\frac{\omega L}{R}$

  Hence greater the reactance compared with the ohmic resistance the phase lag comes into existence. Practically this is adopted in A.C voltmeters and wattmeter.

Case 2:-

- If $\omega_L \to 0$ i.e. When inductance or the frequency ($f = \omega/2\pi$) is small as to be negligible, then ωL will be very small compared to R and we get,

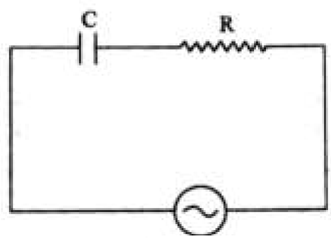
  $\theta = 0$ and $Z = R$

  and $I = I_0 \sin\omega t = \frac{E_0}{R}\sin\omega t = \frac{E}{R}$

■ A.C. Circuit Containing a Resistance And A Capacitance In Series:

Fig. (i) shows a circuit consisting of a series combination of a resistance R and a capacitor C with an alternating emf $E = E_0 \sin\omega t$ applied across the combination. The instantaneous current I is given by,

$$I = \frac{\text{Applied emf}}{\text{Vector impdeance of the circuit}}$$



$E = E_0 \sin\omega t$

fig. (i)

- $$I = I_0 \sin(\omega t + \theta) = \frac{E_0 \sin(\omega t + \theta)}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \text{ .... (A)}$$

Where, $I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$ Indicates the peak value of the current. Equation (A) represents the variation of current with time and shows that current leads the applied voltage in phase by angle θ as shown in Fig. (ii) given by $\theta = \tan^{-1}\frac{1/\omega C}{R}$

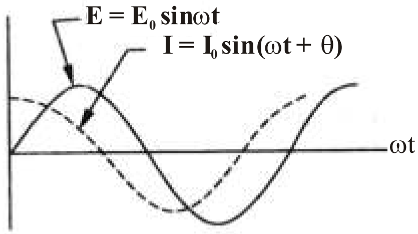


fig. (ii)

- The impedance, $Z=\sqrt{R^2+\frac{1}{\omega^2C^2}}=\sqrt{(\text{Resistance})^2+(\text{Capacitive reactance})^2}$
- **Fig. (iii)** Shows the complex number representation of current and voltage.

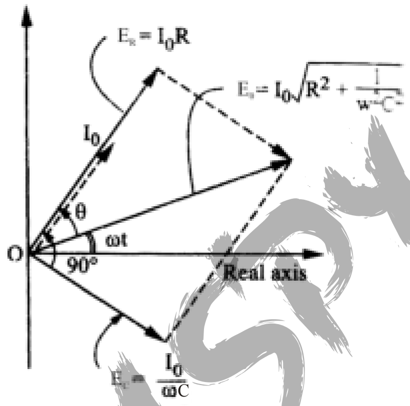


Fig. (iii)

- The current leads the applied voltage by phase is the same as the voltage lags behind the current in phase.
- The voltage E_R across the resistance is in phase with current, while the voltage E_C across the condenser lagging the current I by an angle $\pi/2$. Finally the total voltage, $E_0=I_0\sqrt{R^2+\frac{1}{\omega^2C^2}}$ lagging the current I by an angle $\theta=\tan^{-1}\frac{\frac{1}{\omega C}}{R}$.

Special Cases

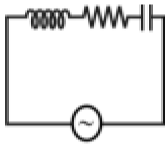
(i) If $C\to 0$, then $I_0\to 0$, this means no current will flow in the circuit with zero capacity and this causes a break in the circuit. The same result will be obtained with very low frequencies.

(ii) If $C\to\infty$, then $I_0\to\frac{E_0}{R}$. Thus a condenser of infinite capacity does not offer any resistance to A.C. and $\tan\theta=0$ or $\theta\to 0$. This means the introduction of such condenser causes no phase change. The same result follows if the frequency is very high.

(iii) If $R\to 0$, then $I=E_0/\omega C$ and $\tan\theta\to\infty$ or $\frac{\pi}{2}$. Thus the current is leading by $\frac{\pi}{2}$ in phase with respect to emf.

■ A.C. Circuit with resistance, capacitance and inductance in series

- Let a sinusoidal alternating emf $E=E_0\sin\omega t$ be applied to the series combination of L.C.R. as shown in Fig. (i)



$E=E_0\sin\omega t$

Fig. (i)

- The expression for current is given by

$$I = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \theta) \text{ ....(A)}$$

Where, $I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ I_0 is the peak value of the current and, $\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = Z$

Where Z is called the impedance of the circuit (in magnitude), which is the ratio of peak emf to the peak current. Again impedance, is given by

$$z = \frac{E_{rms}}{I_{rms}} = \frac{E_0/\sqrt{2}}{I_0/\sqrt{2}} = \frac{E_{eff}}{I_{eff}}$$

Thus the impedance of a LC.R. Circuit can be defined as the ratio of the r.m.s. value of emf applied to the r.m.s. current flowing through it. Eq. (A) shows that the current lags the applied voltage in phase by an angle θ given by

$$\theta = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R}$$

- The variation of current and voltage with time is shown in fig. (ii). In other words, the emf and current have a phase difference of θ. This phase difference is shown in Fig. (iii) in the complex plane. The voltage across the resistance E_R is shown in phase with the current.

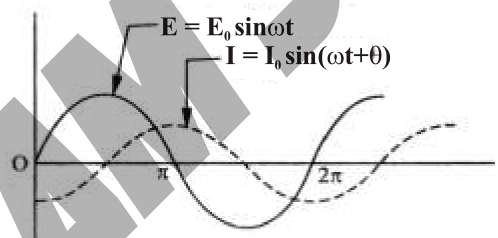


Fig. (ii)

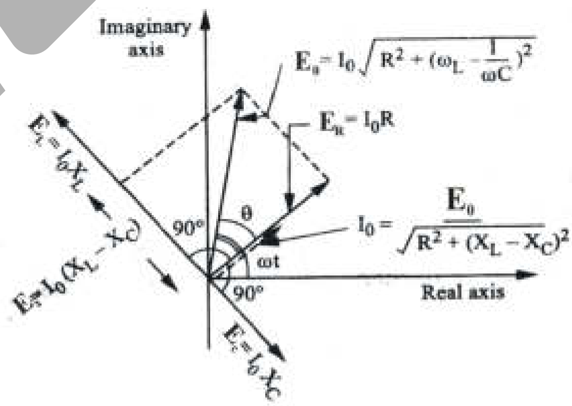


Fig. (iii)

- $E_X = E_L - E_C$ Which is the net voltage across the combination of L and C and leads the current by an angle $\pi/2$. The total voltage E_0 shown leading the current by an angle.

$$\theta = \tan^{-1} \frac{X_L - X_C}{R}$$

Special Cases

Depending on the values of X_L and X_C there are three important cases.

Case 1 :

If $\omega L > \frac{1}{\omega C}$ i.e. when impressed frequency $\left(f = \frac{\omega}{2\pi}\right)$ is very large, the phase angle θ will be positive and the current will lag behind the applied emf. The potential difference ($I_0 X_L = I_0\, \omega L$) across the inductance is greater than potential difference $I_0/\omega C$ across the capacitance and the circuit behaves as an inductive circuit. The vector voltage in complex plane is shown in Fig. (iii)

Case 2 :

If $\omega L < \frac{1}{\omega C}$ the phase angle θ will be negative and the current will lead the applied emf. The circuit behaves as capacitive circuit. The vector diagram for the case is shown in Fig (iv) in complex plane.

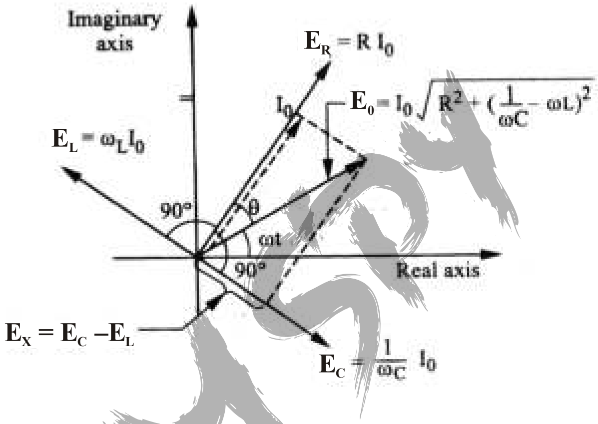


Fig. (iv)

Case 3 :

If $\omega L = \frac{1}{\omega C}$ a very important and interesting case arises. In this case,

$$\theta = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = 0$$

i.e. the phase angle θ becomes zero and the emf and current will be in phase. The potential difference across the inductance and capacitance are equal in magnitude but opposite in phase and therefore cancel out and the whole voltage is dropped across the resistance. The vector diagram in complex plane is shown in fig. (v)

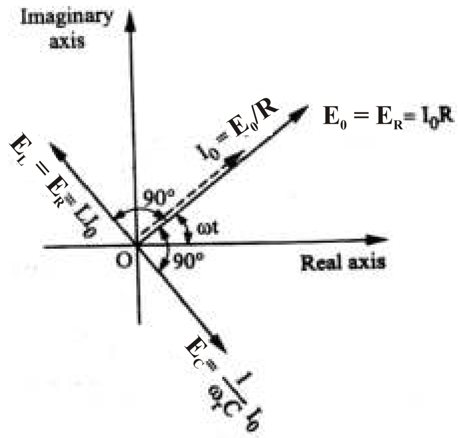


Fig. (v)

Under this condition, the peak current will be maximum given by

$$I_{0\max} = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{E_0}{R}$$

- Thus, in this particular case when $\omega L = \frac{1}{\omega C}$ the emf and current will be in phase and current is maximum. The circuit is said to be a series resonant circuit and the phenomenon of maximum current is called resonance. Thus at resonance,

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{LC} \quad \text{or} \quad f_r = \frac{1}{2\pi\sqrt{LC}} \text{ .... (B)}$$

Here subscript r corresponds to resonance.

- The frequency at which resonance occurs is known as resonance frequency.
- Evidently from Eq. (B) the resonant frequency is the same as the frequency of the oscillatory discharge of a series LCR circuit when the resistance is low.
- Thus the series LCR circuit is in resonance with applied voltage. The frequency of the applied voltage coincides with the natural frequency of the circuit. Fig. (v) gives the vector diagram of voltage and current for this circuit on the complex plane under this condition. Fig. (vi) shows the variation of peak current with frequency for resonant circuit.

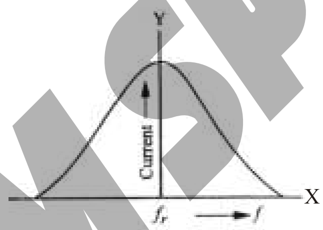


Fig. (vi)

■ Sharpness Of Resonance Of A Series LCR Cicuit

We have discussed in the previous article that the current amplitude is,

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \text{ for an LCR circuit}$$

At resonance $\omega L = \frac{1}{\omega C}$, therefore, $I_{0(\max)} = \frac{E_0}{R}$

- The curves obtained by plotting amplitude of current (or voltage drop across a fixed resistance) against ω (or f) are known as resonance curves.
- Fig. (i) shows the variation of current amplitude as function of frequency for different values of resistance R

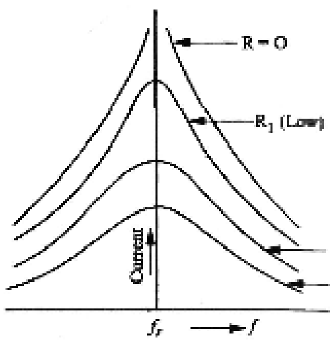


fig (i)

- Clearly, as the resistance in the circuit is reduced the resonance curve becomes sharper. The peak value of I_0 shows that the circuit responds only to the frequency exactly equal to its natural frequency of the circuit $\omega_r = \frac{1}{\sqrt{LC}}$ for low resistance in LCR circuit and to none others. The resonance here is, therefore, said to be sharp
- Thus, sharpness of resonance is, in a way, a measure of the rate of fall of amplitude from its maximum value at resonant frequency on either side of it. The more quickly the current amplitude falls for changes of frequency on both sides of the resonant frequency f_r the sharper is said to be resonance.
- On the other hand, if the amplitude remains more or less at its peak value an appreciable range of frequency on either side $\omega_r = \frac{1}{\sqrt{LC}}$ resonant frequency i.e. even when $\omega_r \neq \frac{1}{\sqrt{LC}}$. Thus, the circuit responds to a number of frequencies near about f_r on either side of it. The resonance in this case, is therefore, said to be flat.
- Fig. (ii) shows the variation of E_L and E_C with ω. Both of these curves are similar in shape in the vicinity of the resonance to the curves of Fig. (i). This is because the resonance phenomenon takes place in a very limited range. The reactance across which the voltage is developed remains substantially the same in the region about the resonance.

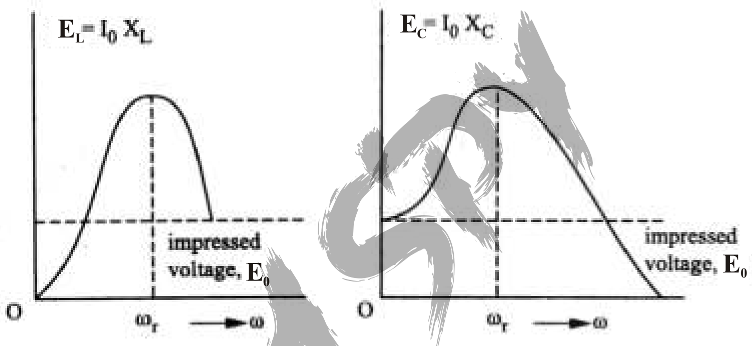


Fig. (ii)

■ Quality factor (Q) of a circuit

- Qualitatively the sharpness of resonance curve is determined by a quality factor called "Q" of the circuit. It is defined as the ratio of reactance of either the inductance or capacitance at the resonant frequency to the total resistance of the circuit. Thus, mathematically,

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R} \text{ or } Q = \frac{X_C}{R} = \frac{1}{\omega_r CR}$$

- If Q is large the resonance curve is sharp.

■ Voltage Magnification

- The Quality factor (Q) is also a measure of voltage magnification in the series L.C.R circuit which can be understood as follows.
- At resonance the potential difference across inductance and capacitance are equal and 180º out of phase and hence cancel. Therefore, the only potential difference at resonance is across the resistance.
- As resonant current is maximum and given by,

$$I_{0\,max} = \frac{E_0}{R}$$

Potential difference across resistance = $I_{0max} R = E_0$.

- At resonance the potential difference across resistance is equal to the applied emf. The voltage magnification in L.C.R. circuit is defined as,

$$\text{Voltage magnification} = \frac{\text{p.d. across the inductance or capacitance}}{\text{Applied voltage}}$$

For inductance-

$$m = \frac{I_{0\,max}\,\omega_r L}{E_0} = \frac{E_0}{R}\frac{\omega_r L}{E_0} = \frac{\omega_r L}{R} = Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

For capacitance-

$$m = \frac{I_{0\max}}{E_0}\frac{1}{\omega_r C} = \frac{E_0}{RE_0}\frac{1}{\omega_r C} = \frac{1}{\omega_r CR} = Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Thus, the voltage magnification of L.C.R. circuit is equal to its quality factor Q at resonance.

■ Comparative Study Of A Series Resonant And Parallel

	Series resonant circuit	Parallel resonant circuit
•	Series resonant frequency is $f_r = \frac{1}{2\pi\sqrt{LC}}$	Here, $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
•	At resonance the power factor is unity and the impedance is purely resistive $Z_r = R$	In this case the power factor is also unity at resonance frequency but the impedance is given by, $Z_r = \frac{L}{CR}$
•	At resonance, the impedance of the circuit is minimum while the admittance is maximum.	Here at resonance the impedance of the circuit is maximum while the admittance is minimum.
•	Series circuit admits maximum current at resonant frequency.	On the other hand this circuit allows a minimum current at the resonant frequency.
•	Series circuit is called acceptor circuit because it accepts a particular frequency and rejects others.	This circuit is called a rejecter because it rejects only one frequency and accepts others.
•	At resonance the circuit exhibits a voltage magnification equal to the quality factor.	This circuit shows a similar current magnification equal to the quality factor.

■ Tabular Form Of Power In A.C circuit–

	A.C. Circuit	Power
•	Purely Resistance circuit	$P = E_{rms} \times I_{rsm}$
•	Power in inductive circuit	$P = 0$
•	Purely capacitive circuit	$P = \frac{E_0 I_0}{2} \sin 2\omega t$
•	Circuit containing L, C and R	$P_{avg} = E_{rms} I_{rms} \cos\theta$

■ Power Factor-

The term E_{rms}. I_{rms} is called apparent power and $\cos\theta$ is called power factor

$$P.F = \frac{\text{True Power}}{\text{Apparent Power}} = \frac{P_{avg}}{I_{rms.} E_{rms}} = \cos\theta$$

■ Tabular Form Of Power Factor In A.C. Circuit:

	A.C. Circuit	Power Factor
•	L – C – R	$\cos\theta = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$
•	Purely inductive circuit	$\cos\theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$
•	Purely capacitive circuit	$\cos\theta = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

■ Key Points:

(i) Out of the number of available frequencies it selects one frequency for which the current is maximum and for other frequencies the current is comparatively very small, in other words it shows selectivity.

(ii) When emf is expressed in volt and current in ampere then unit of power is watt.

(iii) Power dissipation in a pure inductance is zero and the current in such circuit is known as idle or wattless current. The choke coil works on this principle.

(iv) As the average of sin $2\omega t$ over one cycle is zero. Hence the average power is zero. Thus the average power dissipation in a pure capacitive circuit is zero, hence the current through it is also wattless or idle.

■ Choke Coil:

- The inductor which is used for reducing the current in the A.C. circuit is called a choke or choking coil. The average power dissipated in the choke coil is given by.

$P = \frac{1}{2}E_0 I_0 \cos\theta$ Where the power factor, $\cos\theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

- In choke coil L is large while R is very small. Hence $\cos\theta$ is nearly zero. Therefore the power consumed by the coil is extremely small.
- Resistance is used in place of inductance the power loss is I^2R. Hence to avoid this loss, we prefer choke coil.

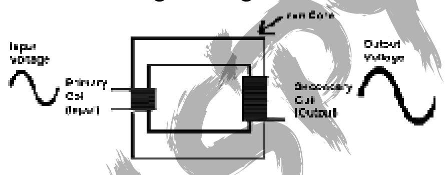
■ Transformer –

- It is a device which can change a low voltage of high current into a high voltage of low current and vice versa.
- Its working is based on mutual induction in which its converts magnetic energy into electrical energy.

There are two types of transformers -

(i) Step up Transformers-

- It converts a low voltage of high current into a high voltage of low current.



Step Up Transformer

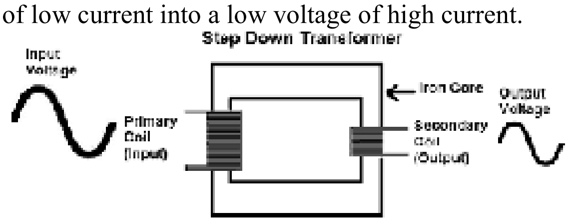
In this transformer-

(i) $N_S > N_P$
(ii) $E_S > E_P$
(iii) $I_P > I_S$,

Where symbols, N_S = number of coils in secondary
N_P = number of coils in primary
E_S = voltage of secondary
E_P = voltage of primary
I_P = current in primary coil
I_S = current in secondary coil

(ii) Step down transformer -

- It converts a high voltage of low current into a low voltage of high current.



In this transformer

(i) $N_P > N_S$
(ii) $E_P > E_S$
(iii) $I_P < I_S$

■ Transformation Ratio -

It is represented by k

$$k = \frac{N_S}{N_P} = \frac{E_S}{E_P} = \frac{I_P}{I_S}$$

- For step up transformer, k > 1.
- For step down transformer, k < 1.

■ **Key Points:**

(i) Transformer does not operate on direct current. It operates only on alternating voltage as input as well as at output.

(ii) Efficiency of transformer is given by $\eta = \frac{\text{Output power}}{\text{Input power}}$.

(iii) Generally efficiency ranges from 70% to 90%.

(iv) Transformer does not amplify power as vacuum tube.

(6) Magnetic effect of current and magnetism-

■ Oersted's Experiment-

In 1820 orested discovered that a current in a wire can also produce magnetic effect. It was demonstrated that when a compass needle is brought near a wire carrying current then the compass needle shows a deflection.

■ Magnetic Field $(\vec{B})$-

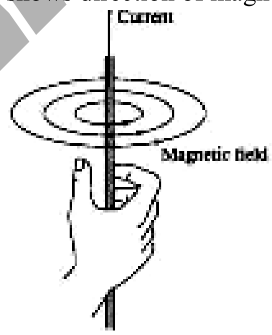
- The space in the surrounding of a magnet or any current carrying conductor in which its magnetic influence can be experienced is called magnetic field.
- S.I. unit of magnetic field is Wb/m^2 or T (tesla).
- The strength of magnetic field is called one tesla, if a charge of one coulomb, when moving with a velocity $1 ms^{-1}$ along a direction perpendicular to the direction of the magnetic field experiences a force of 1 Newton.
  1 tesla (T) = 1 weber /meter^2 (wbm^{-2}) = 1 Newton/ ampere–metre ($NA^{-1}m^{-1}$)
- C.G.S units of magnetic field are called gauss or oersted 1 gauss = 10^{-4} tesla.

■ Key Points:

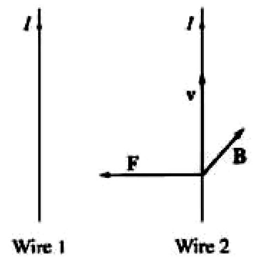
- A moving charge is a source of magnetic field.
- A current or moving charge creates a magnetic field in the surrounding space (in addition to its electric field)
- The magnetic field exerts a force or any other moving charge or current that is present in the field.
- At any position the direction of $\vec{B}$ is defined as that in which north pole of a compass needle tends to point.

■ Right Hand Thumb Rule –

If we hold a current carrying conductor in the grip of the right hand in such a way that thumb points in the direction of current then curling finger shows direction of magnetic field.



■ Magnetic Force: Lorentz Law



- The magnetic force on a charge Q, moving with velocity v in a magnetic field B is

$$\vec{F}_{mag} = Q(\vec{v} \times \vec{B})$$

This is known as the Lorentz force law. In the presence of both electric field and magnetic fields, the net force on Q would be –

$$\vec{F} = Q\left[\vec{E} + \left(\vec{V}\times\vec{B}\right)\right]$$

- Since, F and v are vectors, B is actually a pseudo vectors.
- In the fig at the second wire the magnetic field points into the page. The velocity of the charges is upward and the resulting force is to the left.

■ Key Points–

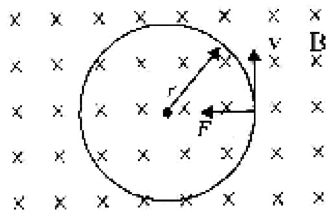
- The magnitude F_{mag} of the magnetic force exerted on the particle is proportional to the charge q and to the speed.
- When a charge particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle is at an angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in the direction between both $\vec{v}$ and $\vec{B}$.
- The magnetic forces on oppositely charged particles moving at the same velocity in a magnetic field are in opposite direction.
- The magnitude of the magnetic force exerted on the moving particle is proportional to sinθ, where θ is the angle of the velocity vector of particle makes with the direction of $\vec{B}$
- Magnetic force do not work because $\left(\vec{V}\times\vec{B}\right)$ is perpendicular $\vec{V}$, so $\left(\vec{V}\times\vec{B}\right) = 0$. Magnetic force may alter the direction in which a particle moves, but they cannot speed it up or slow it down.

■ Difference Between Electric and Magnetic Force on charged Particles

Electric Force	Magnetic Force
The electric force is always parallel or antiparallel to the direction of the electric field.	The magnetic force is perpendicular to the magnetic field.
The electric force acts on a charged particle independent of the particles velocity	The magnetic force acts on a charged particles only when the particle is in motion and the force is proportional to the velocity
The electric force does work in displacing a charged particle.	Magnetic force associated with a constant magnetic field.

■ Motion of A charge Particle In A Magnetic Field –

- Consider a charged particle of mass m projected in a uniform magnetic field $\vec{B}$ with an initial velocity vector $\vec{V}$ perpendicular to the field.



- A force will act on the particle as shown which is perpendicular to the velocity at any instant and also magnitude of the velocity will remain same, only direction will change.
- Finally, the particle will move in a circular path. The magnetic force provides the necessary centripetal force. So, we can write.

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB}$$

Where, r = radius of circular path

- The angular speed ω of the particle is given by –

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

- The time period T of the motion is given by–

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- Frequency is given by–

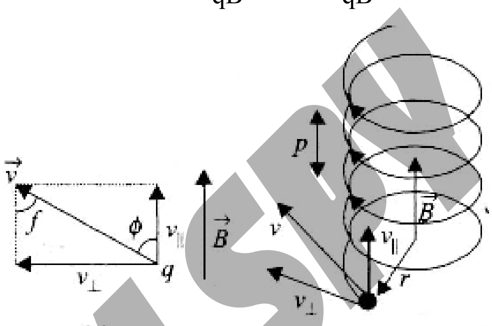
$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

- The time period is independent of velocity given to charge particle. If two identical charge particle are given different speeds from a point in the same direction, then they will return to the initial point simultaneously.
- If particle is projected at some angle ϕ with magnetic field.
  (a) With perpendicular component of velocity it moves in a circular path of radius

$$r = \frac{mv_{\perp}}{qB} = \frac{mv\sin\phi}{qB}$$

  (b) with parallel component of velocity it moves along the field lines. The resulting force is helical. The linear distance travelled in one revolution (time period) is called pitch.

$$p = v_{11}T = \frac{2\pi m v_{11}}{qB} = \frac{2\pi mv\cos\phi}{qB}$$



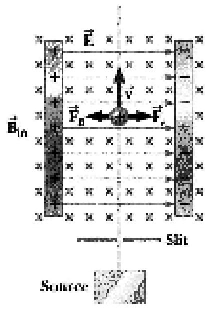
(a) (b)

■ Application Involving Charged Particles Moving In A Magnetic Field-

(a) Velocity Selector:

- In many experiments involving moving charged particles, it is important that all particles move with essentially the same velocity, which can be achieved by applying a combination of electric field and a magnetic field oriented. From the expression

  $qE = qvB$, we find that $v = \frac{E}{B}$

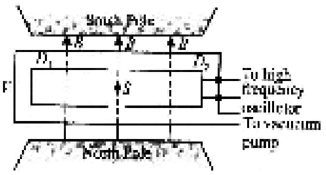


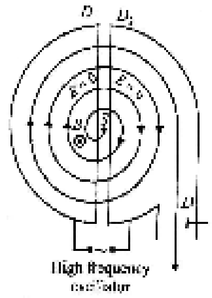
Only those particles having this speed pass undeflected through the mutually perpendicular electric and magnetic fields.

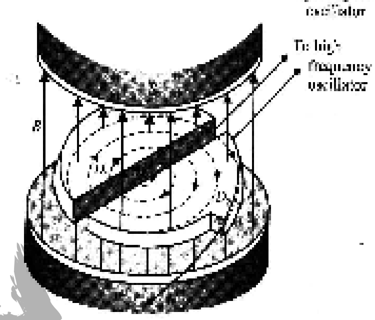
(b) Cyclotron:

- This machine was discovered by Lorentz in 1934 to increase the energy of the charge particle.
- It can impart the energy to the charge particle like α-particle, β-particle, dutron, ions, proton but not an electron
- The maximum energy that can be supplied by the cyclotron is a 25 MeV

- Radius of the circular path, $r = \dfrac{mv}{qB}$
- Cyclotron frequency, $\nu = \dfrac{Bq}{2\pi m}$
- Cyclotron velocity $v = \dfrac{qBr}{m}$
- Time period of cyclotron, $T = \dfrac{2\pi m}{qB}$
- Angular frequency of cyclotron, $\omega = \dfrac{qB}{m}$
- The energy gained by the charge particle is
  $E = 2m\pi^2\nu^2R^2$
  Where, m = mass of charge particle
  q = charge of the particle
  B = Magnitude of the applied field
  ν = velocity of revolving charge particle
  r = radius of the path







- The maximum energy that can be achieved by cyclotron is 25 MeV. Sometimes the modified cyclotron is impart the maximum energy to the charge particle and increased limits is 60 MeV. Such a cyclotron which produces 60 MeV energy is called 60 Cyclotron

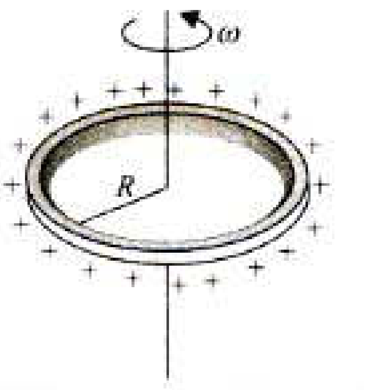
■ Limitation of cyclotron

- Cyclotron cannot accelerate uncharged particle like neutron
- The +vely charged particles having large mass i.e. ions cannot move at limitless speed in a cyclotron.

■ Magnetic Dipole And Dipole Moment–

- Magnetic dipole is the magnetic equivalent of electric dipole.
- The magnetic field pattern produced by a small current loop is similar to a bar magnet. Therefore, it also acts like a magnetic dipole.
- The magnetic moment of a flat current loop is defined as the product of current I and area A enclosed by it.
  $\vec{M} = I\vec{A}$
- The direction of the magnetic moment coincides with the direction of area vector (which is the direction of the magnetic field)
- If the loop contains N number of turns, the magnetic moment is given by M = NIA
- Sometimes a current carrying loop does not lie in a single plane.
- Sometimes a non-conducting body is related with some angular speed. In this case the ratio of magnetic moment and angular momentum is constant which is equal to q/2m, where q is the charge and m is the mass of the body. For example, in case of ring of mass m, radius R and charge q distributed on its circumference.

$$\frac{M}{L} = \frac{q}{2m}$$



■ Torque Acting on A current carrying coil placed Inside a uniform Magnetic field

Torque acting on a current carrying coil placed inside a uniform magnetic field is given by–

τ = NBIA Sinϕ

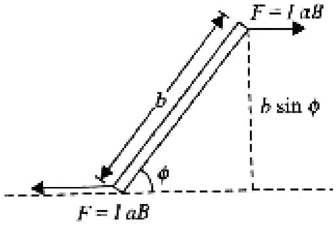
Where, N = number of turns in the coil

B = magnetic field intensity

I = current in the coil

A = area of cross-section of the coil

ϕ = angle between magnetic field and normal to the plane of the coil.



■ Energy of Dipole

- When a magnetic dipole changes orientation in a magnetic field, the field does work on it
- Energy needed to rotate the loop through on angle θ is

$$\Delta U = MB(\cos\theta_1 - \cos\theta_2)$$

If we choose θ_1 such that at $\theta_1 = 90^\circ$ $\theta_2 = 0$

$$U = -\vec{M}.\vec{B}$$

This is the energy stored in the loop.

■ Galvanometer–

It is a device used for the detection and measurement of the small electric currents.

■ Type of Galvanometer-

- Moving coil galvanometer
- Suspended type galvanometer
- Mirror type galvanometer
- Torsion head type galvanometer

■ Moving coil Galvanometer–

- The principle of moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.
- In equilibrium, deflecting torque = Restoring torque

$$NBIA = C\theta \Rightarrow I = \frac{C\theta}{NBA} = K\theta$$

Where, C = restoring torque per unit twist.

N = number of turns in the coil.

B = magnetic field intensity.

A = area of cross section of the coil.

θ = twist angle

K = galvanometer constant = $\frac{C}{NBA}$

- Therefore the deflection produced in the galvanometer is directly proportional to the current flowing through it.

■ Current Sensitivity -

- The deflection produced per unit current in galvanometer is called its current sensitivity.

Current sensitivity $I_s = \frac{\theta}{I} = \frac{NBA}{C}$

Where, $\theta = \frac{NBIA}{C}$.

- The unit of current sensitivity is radian per ampere or deflection per ampere.

■ Voltage Sensitivity -

- The deflection produced per unit voltage applied across the ends of the galvanometer is called its voltage sensitivity

$$\text{Voltage sensitivity, } V_S = \frac{\theta}{V} = \frac{NBIA}{kV}.$$

$$V_S = \frac{NBA}{CR}.$$

- $\frac{I}{V} = \frac{1}{R}$; Where, R is the resistance of the galvanometer.
- The unit of the voltage sensitivity is radian per volt or deflection per volt.

For the Sensitivity of the galvanometer -

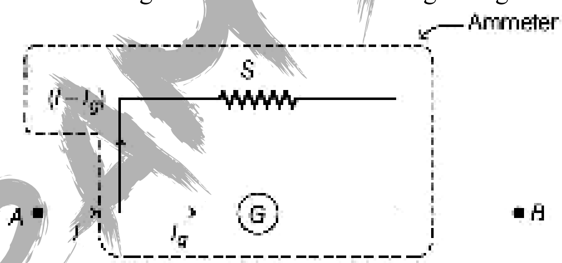
- A galvanometer is said to be more sensitive if it shows a large deflection for a small value of current. For a given value of I, θ will be large.
- Be can be increased using strong magnets.
- The value of C for a quartz and phosphor - bronze is very small. So the suspension wire of quartz or phosphor is used.
- 'N' should be large.
- 'B' should be large.
- 'A' should be large.
- 'k' should be small.

■ Ammeter

An ammeter is a low resistance galvanometer used for measuring the current in a circuit. It is always connected in series.

Conversion of a Galvanometer into a Ammeter

A galvanometer can be converted into an ammeter by connecting a low resistance into its parallel. If G is the resistance of a galvanometer and it give full scale deflection for current I_g, then required low resistance S, connected in its parallel for converting it into an ammeter of range I is given by



$$I_g \times G = (I - I_g) \times S \Rightarrow S = \left(\frac{I_g}{I - I_g}\right)G$$

The resistance of an ideal ammeter is zero.

■ Voltmeter

A voltmeter is a high resistance galvanometer used for measuring the potential difference between two points. It is always connected in parallel.

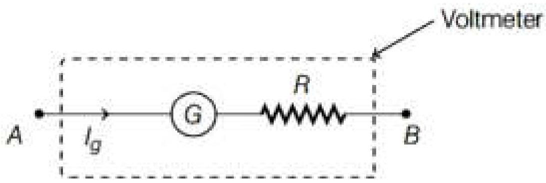
The resistance of an ideal voltmeter is infinity.

Conversion of Galvanometer into voltmeters

A galvanometer can be converted into a voltmeter by connecting a high resistance into its series.

If a galvanometer of resistance G shows full scale deflection for current I_g, then required high resistance R, connected in series for converting it into a voltmeter of range V is given by

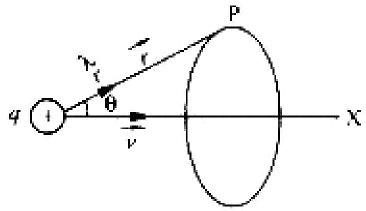
$$V = I_g (G + R) \Rightarrow R = \frac{V}{I_g} - G$$



■ Magnetic field of A Moving Point Charge (Laplace Rule)

- It was found experimentally that the magnetic induction resulting from a charge q moving with velocity $\vec{v}$ at a distance r away from the charge where r is a vector pointing from the charge to the point where the field is being found is related by–

$$\vec{B} = \frac{\mu_0}{4\pi} q\left(\frac{\vec{v}\times\vec{r}}{r^3}\right)$$



Where, $\vec{r}$ is unit vector along r,
μ_0 is called permeability of free space.

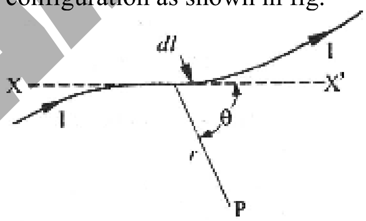
- In S.I. units. the numerical value of μ_0 is exactly $4\pi \times 10^{-7}$.
  $\mu_0 = 4\pi\times10^{-7}\ NS^2C^{-2} = 4\pi \times 10^{-7}\ \omega b\ (Am^{-1}) = 4\pi \times 10^{-7} \tau\ mA^{-1}$

■ Key Points–

- For Points lying very far from the particle the magnetic field is zero.
- The magnetic field due to a stationary particle is zero, As the velocity increases, the magnetic field increases.
- For points lying on a line parallel to $\vec{v}$ and passing through the charge, θ is zero and hence the magnetic field is zero.
- For points lying on a plane perpendicular to $\vec{v}$, the magnetic field is greatest since θ is 90º.
- Magnetic field of lines are concentric circles centered on the line of the velocity $\vec{v}$ and lying in planes perpendicular to this line.
- The direction of the magnetic field lines is given by the right hand rule.

■ The Biot-Savart Law–

- In 1820, Biot and Savart formulated the equation for the field due to current in a long straight wire. They produced mathematical formula for the field due to a single current element rather than a finite length of wire. The equation can be used to calculate configuration as shown in fig.



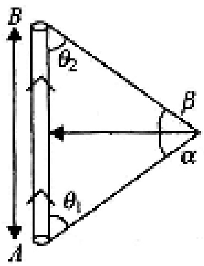
$$d\vec{B} = \frac{\mu_0}{4\pi} I\frac{(d\vec{\ell}\times\hat{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

■ Application of Biot-Savart Law–

(i) Magnetic field Due to current In A straight wire

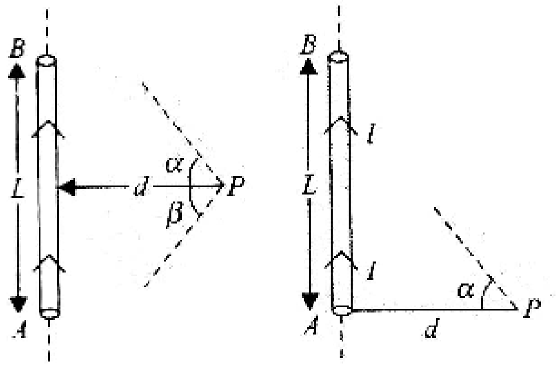
- Magnetic field due to a wire segment carrying current I at P, when the wire segments subtends angle α and β as shown is given by–

$$\vec{B} = \frac{\mu_0 I}{4\pi R}[\sin\alpha + \sin\beta]$$



■ **Key Points-**

- For points along the length of the wire (but not on it), the field is always zero.
- The field is always perpendicular to the plane containing the wire and the point. So, in a plane perpendicular to the wire and containing the point, the lines of force are concentric circles encircling the wire as shown in fig.



(a) (b)

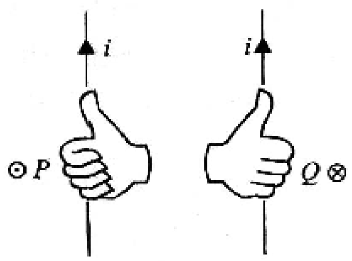
- If the wire is of infinite length and the point P is not near its ends as shown in fig above.
  $\alpha = \beta = (\pi/2)$, then –

$$B = \frac{\mu_0}{4\pi}\frac{2I}{d}$$

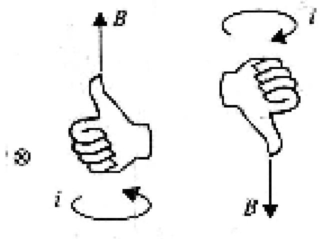
- If the point is near one end of an infinitely long wire as shown in fig $\alpha = \frac{\pi}{2}$ and $\beta = 0$

$$B = \frac{\mu_0}{4\pi}\frac{I}{d}$$

- Direction of magnetic field for straight current carrying wire:
  (a) When the current I is coming outward ⊙ (out of the page), the magnetic field B is a circle in the anticlockwise.
  (b) When the current I is pointing inward (x) (into the page) the magnetic field B is a circle in the clockwise sense.



- Direction of magnetic field in closed current carrying loop
  (a) When the current I is flowing in the anticlockwise sense, the magnetic field B is outwards ⊙ (out of page)
  (b) When the current I is flowing in the clockwise sense, the magnetic field B is inward (×) (into the page)

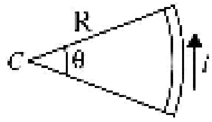


(b)

(ii) Magnetic field at the centre of a current. Carrying circular Arc–

- Magnetic field B at the centre of a circular arc of radius r is given by –

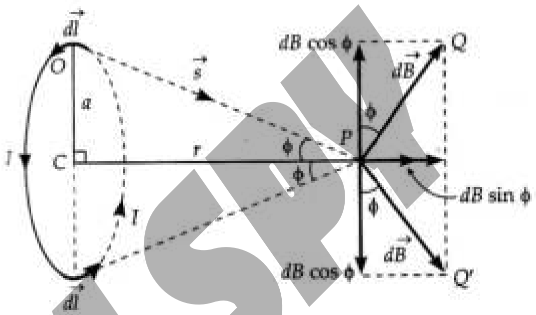
$$B_{arc} = \frac{\mu_0 I}{4\pi r}\theta$$



- For circular loop $\theta = 2\pi$, $B_{arc} = \frac{\mu_0 I}{2r}$

(iii) Magnetic field on the axis of a circular current loop-

- Consider a circular loop of wire of radius a and carrying current I, as shown in fig. Let the plane of the loop be perpendicular to the plane of paper and point P at a distance r from the centre C.



- As the direction of the field is along +ve x-direction, so we can write-

$$\vec{B} = \frac{\mu_0 I a^2}{2\left(r^2 + a^2\right)^{3/2}}\hat{i}$$

If the coil consists of N turns, then–

$$\vec{B} = \frac{\mu_0 N I a^2}{2\left(r^2 + a^2\right)^{3/2}}$$

- Variation of magnetic field along the axis of a circular current loop.

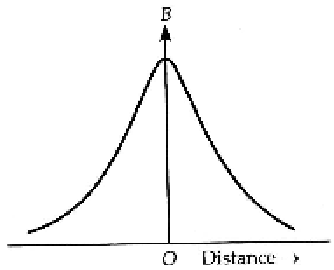


fig. shows the magnetic field along the axis of a circular loop with distance from its centre. The value of B is maximum at the centre and decrease as we go away the centre, on either side of loop.

Special Cases:

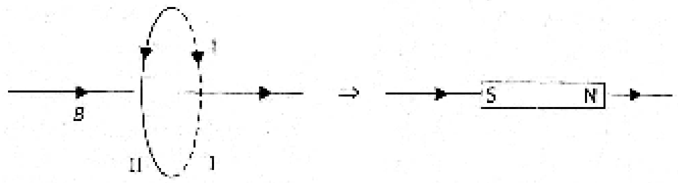
(a) At the centre of the current loop, r = 0 therefore

$$B = \frac{\mu_0 N I a^2}{2a^3} = \frac{\mu_0 N I}{2a}$$

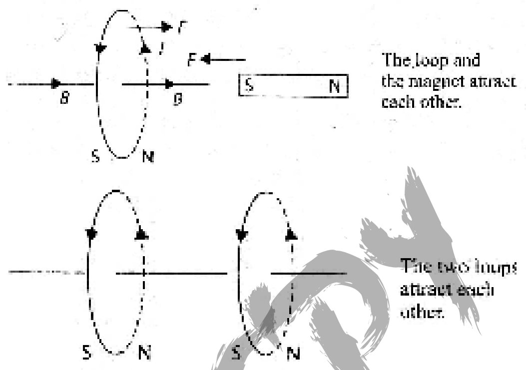
or $B = \frac{\mu_0 N I A}{2\pi a^3}$

■ **Key Points-**

- A loop as a magnet: The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet



- The side I (the side from which $\vec{B}$ emerges out) of the loop acts as "north pole and side" (the side in which $\vec{B}$ enter) acts as the "south pole". It can be verified by studying force on one loop due to a magnet or a loop.

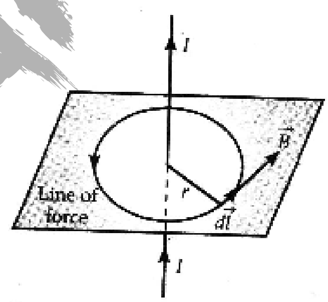


■ **Ampere's circuital Law** –

- Ampere's circuital law states that the line integral of the magnetic field $\vec{B}$ around any closed circuit is equal to μ_0 times the total current I threading or passing through this closed circuit. Mathematically.

$$\oint \vec{B}.\vec{d\ell} = \mu_0 I$$

In a simplified form $BL = \mu_0 I$



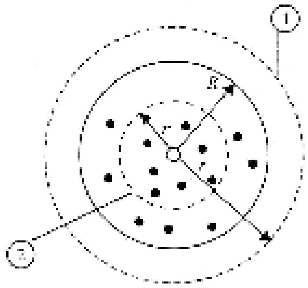
■ **Key points-**

- While applying the ampere's law, the following right hand convention is used-
  (a) Current into the plane of the paper is negative
  (b) Current out of plane of the paper is + ve
  (c) Circulation is taken in the counter clockwise direction is +ve.
  (d) Circulation taken in clockwise direction is –ve.
- If B is everywhere tangent to the integration path and has the same magnitude B at every point on the path, then its line integral is equal to B multiplied by the circumference of the path.
- If B is everywhere perpendicular to the path, for all or some portion of the path, that portion of the path makes an contribution to the line integral.
- In the integral $\oint \vec{B}.d\vec{\ell}, \vec{B}$ is always the total magnetic field at each point on the path. In general, this field is caused partly by current enclosed by the path and partly by current outside.
- Even when no current is enclosed by the path, the field at points on the path need not be zero. In that case, however, $\oint \vec{B}.d\vec{\ell}$ is always zero

- Some judgment is required in choosing an integration path. Two useful guiding principles are that the point or points at which the field is to be determined must lie on the path, and that the path must have enough symmetry so that he integral can be evaluated.

■ Application of Ampere's circuital law

- Magnetic field B outside and inside a cylindrical wire



(i) B outside the wire (r ≤ R)

Consider a circular loop of radius r (≥ R) as shown by loop 1 in fig.

Here, the integral $\oint \vec{B}.d\vec{\ell} = B\oint dl = B(2\pi r)$

Applying Ampere's Law,

$B\ (2\pi r) = \mu_0 I\ \ B = \dfrac{\mu_0 I}{2\pi r}$

(ii) B inside the wire (r ≤ R)

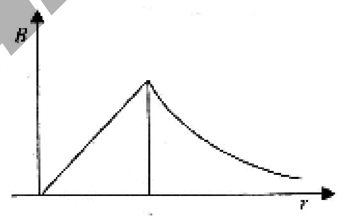
Consider the loop 2 as shown in fig.

The current enclosed is $I_e = I\dfrac{\pi r^2}{\pi R^2}$

Using Ampere's law,

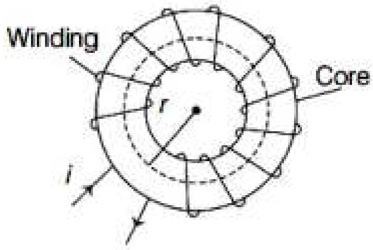
$B.(2\pi r) = \mu_0\left[\dfrac{I}{\pi R^2}(\pi r^2)\right]$ or $B = \dfrac{\mu_0 Ir}{2\pi R^2}$

The variation of B with r is shown in the figure.



■ Toroid

A toroidal solenoid is an anchor ring around which is large number of turns of a copper wire are wrapped.



A toroid is an endless solenoid in the form of a ring. Magnetic field inside the turns of toroid is given by $B = \mu_0 nI$

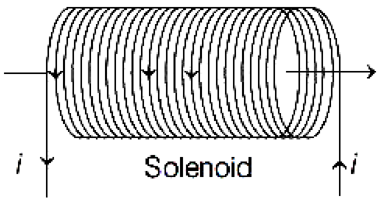
Where n = number of turns/length

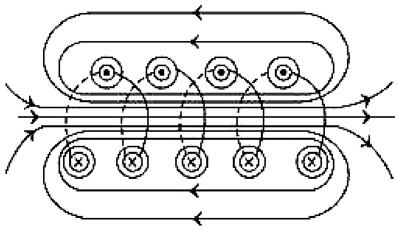
Magnetic field inside a toroid is constant and is always tangential to the circular closed path.

Magnetic field at any point inside the empty space surrounded by the toroid and outside the toroid is zero, because net current enclosed by these space is zero.

■ Solenoid

A solenoid is a closely wound helix of insulated copper wire.





Magnetic field at a point well inside a long solenoid is given by $B = \mu_0 nI$

where, n = number of turns per unit length and I = current flowing through the solenoid.

Magnetic field at a point on one end of a long solenoid is given by

$B = B = \frac{\mu_0 nI}{2}$

■ Force on a current carrying conductor in a Magnetic Field

The magnetic force on a segment of current carrying wire is evidently

$F_{mag} = \int(\vec{v}\times\vec{B})dq = \int(\vec{v}\times\vec{B})\lambda d\ell = \int(I\times\vec{B})d\vec{\ell}$

I and $d\ell$ both point in the same direction-

$$\vec{F}_{mag} = \int I(d\vec{\ell}\times\vec{B})$$

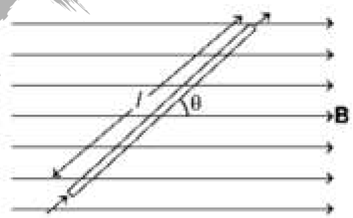
Typically, the current is constant (in magnitude) along the wire, and in that case I comes outside the integral.

$\vec{F}_{mag} = I\int(d\vec{l}\times\vec{B}) = BIl\sin\theta$

Where, $\lambda = \frac{q}{l}$ = linear charge density

$I = \lambda v$ (when a line charge λ travelling down a wire at speed v)

Idl = current element

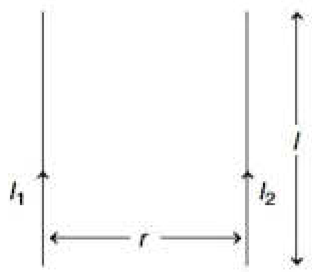


Direction of force on a current carrying conductor can be found out by Fleming's left had rule.

■ Force between Two infinitely Long Parallel Current Carrying Conductors

Force between two long parallel current carrying conductors is given by

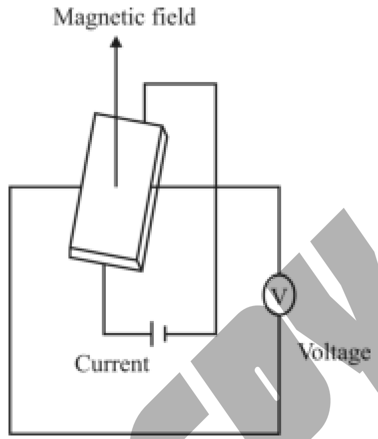
$F = F = \frac{\mu_0}{2\pi}.\frac{I_1 I_2}{r}\ell$



- The force is attractive if current in both conductors is in same direction and repulsive if current in both conductors is in opposite direction.
- If the current is both parallel wires are equal and in same direction, then magnetic field at a point exactly half way between the wire is zero

■ The Hall Effect:

- If a conductor is placed in a magnetic field such that the direction of flow of current is at right angle to the magnetic field, a voltage is developed across the conductor in the direction perpendicular to both current and field. The phenomena by which voltage developed is called Hall effect and voltage itself is Hall voltage



$$\boxed{K_H = \frac{V_H}{I.B}.t}$$

Where K_H = Hall coefficient

I = Applied current

B = Magnetic field

t = Thickness of crystal

■ Important Comments:

If the analogy of $\vec{H}$ to $\vec{E}$ and $\vec{B}$ to $\vec{D}$ is pursued, it is possible to achieve a formal or mathematical symmetry between many of the electric and magnetic field equations. However, electric and magnetic fields are fundamentally different, and the analogy of $\vec{B}$ to $\vec{E}$ and $\vec{H}$ to $\vec{D}$ has more meaningful physical significance. Static electric fields are due to electric charge, a scalar quantity while static magnetic fields are due to electric current, a vector quantity.

■ A Comparison of static Electrical and Magnetic Field Equations:

Description of Equation	*Electric Fields*	*Magnetic Fields*
Force	$\vec{F} = q\vec{E}$	$d\vec{F} = I\,d\vec{l} \times \vec{B}$ $\vec{F} = Q_m \vec{B}$
Basic relations for lamellar or solenoid field	$\nabla \times \vec{E}_c = 0^*$	$\nabla \times \vec{B} = 0$
Derivation from scalar or vector potential	$E_c = -\nabla V$ $V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r} dv'$	$\vec{B} = \nabla \times \vec{A}$ $\vec{A} = \frac{\mu_0}{4\pi}\int_v \frac{\vec{j}}{r} dv'$

Relations for $\vec{D}$ and $\vec{H}$	$\vec{D} = \varepsilon\vec{E}$ $\nabla \cdot \vec{D} = \rho f$	$\vec{H} = \frac{\vec{B}}{\mu}$ $\nabla \times \vec{H} = \vec{j}$
Energy density	$W_e = \frac{1}{2}\varepsilon E^2$	$W_m = \frac{1}{2}\frac{B^2}{\mu}$
Capacitance and inductance	$C = \frac{\text{Charge}}{\text{Potential diff.}}$	$L = \frac{\text{Flux linkage}}{\text{Current}}$
Closed path of integration	$\oint \vec{E} \cdot d\vec{l} = \text{emf}$ $\oint \vec{E} \cdot d\vec{l} = 0$	$\oint \vec{H} \cdot d\vec{l} = NI = \text{mmf}$ $\oint \vec{H} \cdot d\vec{l} = 0$ (no current enclosed)
Derivation from scalar potentials	$\vec{E}_c = -\nabla V$	$H = -\nabla V_m$ (In current free region)

* $\vec{E}_c$ is the static electric field intensity (due to charge)

$\vec{E}$ (without subscript) implies that emf producing field (not due to charges) may also be present.

■ Comparison on Equations involving Polarisation $\vec{P}$ Magnetisation $\vec{M}$

Description of Equation	*Electric Fields*	*Magnetic Fields*
Dipole moment relation	$\vec{P} = \frac{\vec{p}}{v} = \frac{Q\vec{d}}{v}$	$\overline{M} = \frac{\vec{m}}{v} = \frac{Q_m\vec{l}}{v}$
Flux density	$\vec{D} = \left(\varepsilon_0 + \frac{\vec{P}}{\vec{E}}\right)\vec{E}$	$\vec{B} = \mu_0\left(1 - \frac{\vec{M}}{\vec{H}}\right)\vec{H}$
Permittivity and permeability	$\varepsilon = \varepsilon_0 + \frac{\vec{P}}{\vec{E}}$	$\mu = \mu_0\left(1 + \frac{\vec{M}}{\vec{H}}\right)$
Relation to polarisation charge density and to equivalent current density	$\nabla \cdot \vec{P} = -\rho_p$	$\nabla \times \overline{M} = \vec{I}_e$
Poisson's eqn.	$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ in vacuum $\nabla^2 V = -\frac{\rho}{\varepsilon}$ in medium	$\nabla^2 V_m = \nabla \cdot \overline{M} = -\nabla \cdot \vec{H}$
Scalar and vector potentials	$V = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho - \nabla' \cdot P}{r} dV$	$A = \frac{\mu_0}{4\pi}\int_v \frac{\vec{J} + \nabla \times \vec{M}}{r} dV$
Magnetisation curve	$\vec{P} - \vec{E}$ curve	$\vec{M} - \vec{H}$ curve identical to $\vec{P} - \vec{E}$ curve
Susceptibility	$\varepsilon = \varepsilon_0(1 + \chi_{e})$ (χ_{e} is dimensionless)	$\mu = \mu_0(1 + \chi_m)$ (χ_m is dimensionless)

(7) Electromagnetic Induction (EMI)

■ Electromagnetic Induction (EMI) :-

- A current carrying coil produces a magnetic field.
- The magnetic field lines form appropriate closed curves.
- Whenever the total number of magnetic field lines linked normally with a coil/circuit changes (i.e. the magnetic flux linked with coil/circuit changes), an induced emf is produced in its as per the phenomena of electromagnetic induction.

- The phenomena of electromagnetic induction (EMI) is the basis of the working of power generators, dynamos, transformers, etc.

■ Magnetic Flux:-

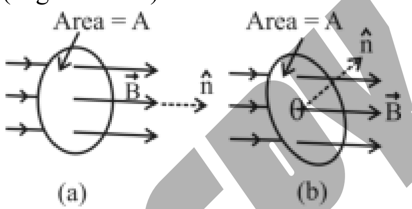
- Magnetic flux through any surface held in a magnetic field is measured as the total number of magnetic field lines crossing the surface. It is a scalar quantity.
- Now the magnetic flux passing through the area is defined as

  $\phi = \vec{B}.\vec{A}$

  or ϕ = BA cosθ = (B cos θ) A = $B_\perp$ A

  where $B_\perp$ = B cosθ is the component of the magnetic field B perpendicular to the face of the area. Direction of area vector is normal to the face of the area.
- **Special Cases**.

  1. if θ = 0º, then ϕ = BA cos0º ⇒ ϕ = BA (maximum flux)
  2. if θ = 90º, then ϕ = BA cos90º ⇒ ϕ = 0
  3. if θ < 90º, then cosθ > 0 ⇒ ϕ > 0 (positive flux)
  4. if θ > 90º, then cosθ < 0 ⇒ ϕ < 0 (negative flux)



- **SI unit of magnetic flux:**

  The SI unit of magnetic flux is weber (Wb). One weber is the flux produced when a uniform magnetic field of one tesla acts normally over an area of 1 m^2

  1 weber = 1 tesla × 1 $metre^2$

  or 1 Wb = 1 Tm^2
- **CGS unit of magnetic flux.**

  The CGS unit of magnetic flux is maxwell (**Mx**). One maxwell is the flux produced when a uniform magnetic field of one gauss acts normally over an area of 1 cm^2

  1 maxwell = 1 gauss × 1 cm^2

  or 1 Mx = 1 G cm^2
- **Relation between weber and maxwell**:

  1 Wb = 1 T × 1 m^2 = 10^4 G × 10^4 cm^2

  or 1 Wb = 10^8 maxwell.
- **Law of Electromagnetic Induction:-**

  There are two types of laws which govern the phenomena of electromagnetic induction.

  A. Faraday laws of electromagnetic Induction:

  These can be stated as follows-

  First Law : Whenever the magnetic flux linked with a closed circuit changes, an emf (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomena is called electromagnetic induction.

  Second Law : The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit mathematically.

  $$|E| = \frac{d\phi}{dt}$$

  If the flux changes from ϕ_1 to ϕ_2 in time t, then the average induced emf will be-

  $E = -N\frac{\phi_2 - \phi_1}{t}$ Where N= Number of turns in the coil

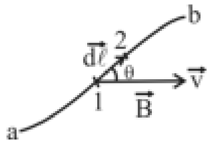
  B. Lenz's Law:

  This law states that the direction of induced current is such that it opposes the causes which produces it i.e. it opposes the change in magnetic flux.

■ Motional Electromotive Force-

$$E = \int_a^b (\vec{v} \times \vec{B}) \overrightarrow{d\ell}$$

As $\overrightarrow{d\ell} \times \vec{v}$ is the area swept per unit time by length $\overrightarrow{d\ell}$ and hence $\vec{B}.(\overrightarrow{d\ell} \times \vec{v})$ is the flux of induction through the area. Therefore, the motional emf is equal to the flux of induction cut by the conductor per unit time.



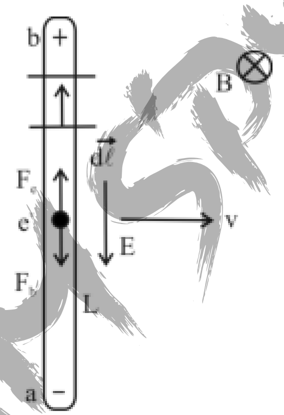
- We can write the expression for induced emf in various forms:

$$E = \int_a^b (\vec{v} \times \vec{B}) \overrightarrow{d\ell} = \int_a^b (\vec{B} \times \overrightarrow{d\ell}) \vec{v} = \int_a^b (\overrightarrow{d\ell} \times \vec{v}) \vec{B}$$

- If any two out of $\vec{v}$, $\vec{B}$ and $\overrightarrow{d\ell}$ becomes parallel or antiparallel, E will become zero.

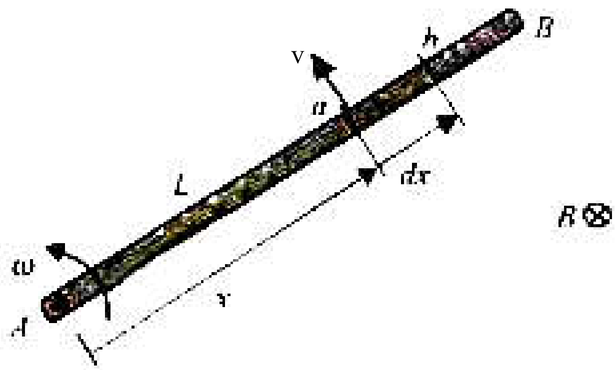
Special Cases:

- **A straight conductor moving in a magnetic field–**
  Induced emf in the rod is given by-



E = BVL
Where, L = length of conductor
B = magnetic field
v = velocity

- **Rotating straight conductor**:
  The magnitude of induced emf in the rod is–



$$E = \frac{1}{2} B \omega L^2$$

Where, ω = angular velocity
L = length of rod

■ Instaneous induced EMF -

$E = NAB\ \omega \sin \omega t = E_0 \sin \omega t$

Where, N = Number of turns in the coil.

A = Area of one turn.

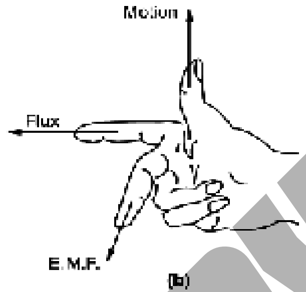
B = Magnetic Induction

ω = Uniform angular velocity of the coil.

E_0 = Maximum induced emf.

■ Fleming's Right Hand rule :-

- If the thumb forefinger and middle finger of right hand are stretched mutually perpendicular to each other such that the forefinger points the direction of magnetic field, middle finger points towards the direction of induced current in the conductor and thumb points towards the direction of magnetic force.



■ Eddy Currents :-

- Eddy Currents are the current induced in solid metallic masses, when magnetic flux threading through them changes .
- Direction of eddy currents can be given by Lenz's Law or by Fleming's Right Hand rule.

■ Application of Eddy currents :-

- Inspite of the undesirable effects, eddy currents are used in many ways. Some of them are given below.

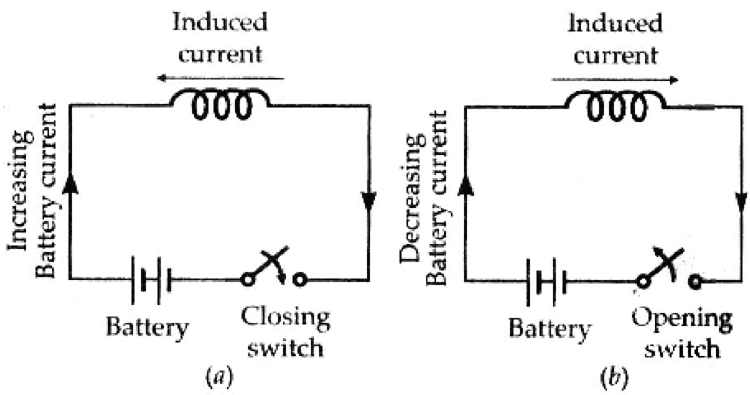
(i)	Speedometer	(ii)	Induction meter
(iii)	Induction furnance	(iv)	Electromagnetic shielding
(v)	Electromagnetic damping	(vi)	Energy meter

■ Key Points:

- Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes.
- Eddy currents tend to follow the path of least resistance inside a conductor. So they form irregularly shaped loops. However, their directions are not random, but guided by Lenz's law.
- Eddy currents have both undesirable effects and practically useful applications.
- Eddy currents can be induced in biological tissues. For example the cavity of the eye is filled with a conducting fluid. A large transient magnetic field of 1 T alternating at a frequency of 60 Hz then induces such a large current in the retina that it produces a sensation of intense brightness.

■ Self Induction :-

- The phenomenon of production of induced emf in a coil, when a changing current passes through it.



■ Coefficient of Self Induction:

- At any instant, the magnetic flux ϕ linked with a coil is proportional to the current I through it, i.e.,

$$\phi \propto I$$

or $$\phi = LI$$

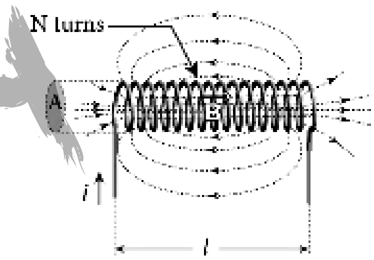
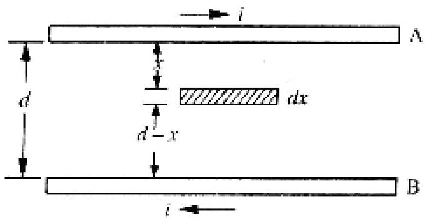
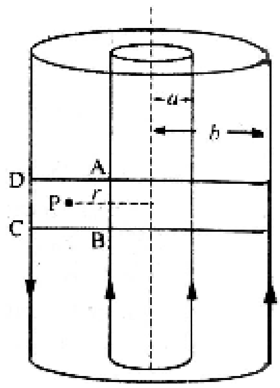
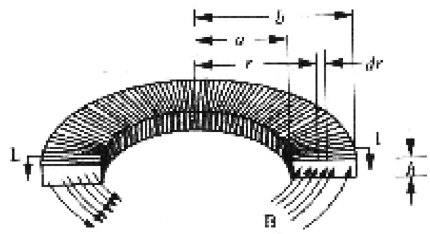
- Any change in current sets up an induced emf in the coil given by

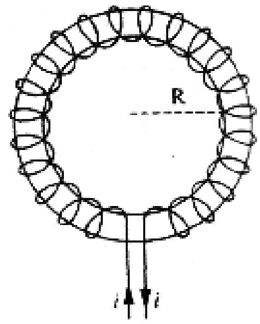
$$e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$

- SI unit of self induction is Henry (H)

  1 Henry (H) = 1 V-s/A or 1 Tm^2/A.

■ Self Induction of Different Cases–

Cases	Formula	Figure
• **Long Solenoid**	$L = \mu_0 n^2 l A$ Where n = no. of turns per unit length $= \frac{N}{l}$ • $\mu = \mu_0\mu_r$, μ_r = relative permeability • If solenoid is wound over a core of permeability μ, then $L = \mu n^2 l A$	
• **Two parallel wires**	$L = \frac{\mu_0}{\pi}\log_e\frac{d-r}{r}$ Where, d = distance between two parallel wires A and B r = radius of each wire	
• **Two coaxial Cylinder**	$L = \frac{\mu_0}{2\pi}\log_e\frac{b}{a}$ a = radius of inner cylinder b = radius of outer cylinder.	
• **Toroidal coil of rectangular cross-section**	$L = \frac{\mu_0 N^2 h}{2\pi}\log_e\frac{b}{a}$ If coil is wound over a core of permeability μ, then - $L = \frac{\mu N^2 h}{2\pi}\log_e\frac{b}{a}$	

• **A Toroidal coil of circular cross**	$L = \dfrac{\mu_0 N^2 A}{2\pi R}$ Where, A= cross-sectional area R = radius of circular path for a toroid wound on a core of constant permeability μ $L = \dfrac{\mu N^2 A}{2\pi R}$	

■ Energy stored in magnetic field of solenoid–

$$U_B = \frac{1}{2}Li^2$$

Where, $L = \dfrac{\mu_0 N^2 A}{\ell}$

$$i = \frac{B\ell}{\mu_0 N}$$

■ Energy Density-

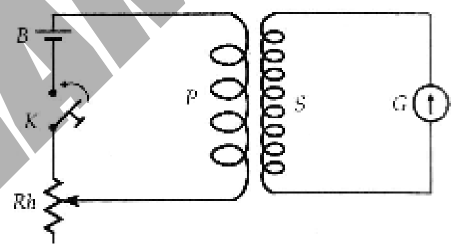
The energy is stored within the volume because B outside of the solenoid is zero. Moreover, the stored energy must be uniformly distributed throughout the volume of the solenoid because the magnetic field is uniform everywhere inside. The energy density is defined as the energy per unit volume. The energy stored per unit volume of the magnetic field.

$$\frac{U_B}{A\ell} = \frac{1}{2}\frac{B^2}{\mu_0} \text{ Joules/meter}^3$$

This equation gives the energy density stored at any point where the magnetic induction is $\vec{B}$

■ Mutual Inductance-

Mutual Induction is the phenomenon of production of induced emf in one coil due to a change of current in the neighboring coil.



■Coefficient of mutual induction

Magnetic flux linked with the secondary coil $\propto$ current in the primary coil

i.e. $\phi \propto I$

or $\phi = MI$

The proportionality constant M is called the mutual inductance or coefficient of mutual induction of the two coils. Any change in the current I sets up an induced emf in the secondary coil which is given by.

$$e = -\frac{d\phi}{dt} = -M.\frac{dI}{dt}$$

■ Unit of mutual inductance

$$\therefore \text{ SI unit of } M = \frac{1Vs}{1A} = 1VsA^{-1} = 1\text{henry}(H)$$

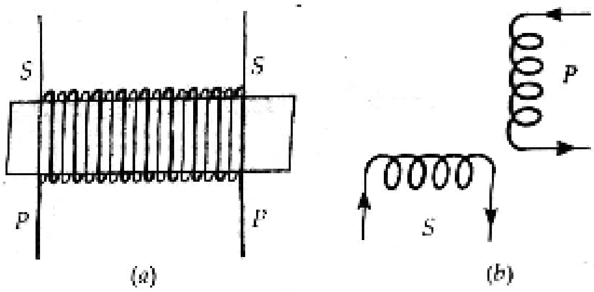
■ Factors on which mutual inductance depends.

a. **Number of turns** : Larger the number of turns in the two solenoids, larger will be their mutual inductance.
$M \propto N_1 N_2$

b. **Common cross-sectional area**: Larger the common cross-sectional area of two solenoids, larger will be their mutual inductance.

c. **Relative separation:** Larger the distance between two solenoids, smaller will be the magnetic flux linked with the secondary coil due to current in the primary coil. Hence smaller will be the value of M.

d. **Relative orientation of the two coil:** M is maximum when the entire flux of the primary is linked with the secondary, i.e., when the primary coil completely envelopes the secondary coil. M is minimum when the two coils are perpendicular to each other, as shown in figure.



e. Permeability of the core material. If the two coils are wonder over an iron core of relative permeability μ_r, their mutual inductance increases μ_r times.

Coefficient of coupling: The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together. If L_1 and L_2 are the self-inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

The value of K lies between 0 and 1.

■ Key Points-

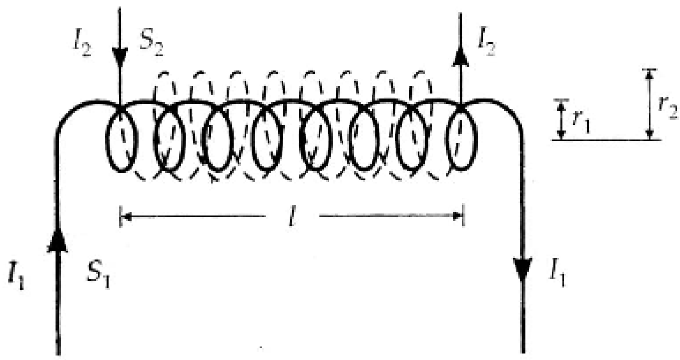
- When two coils are inductively coupled, in addition to the emf produced due to mutual indication, induced emf is set up in the two coils due to self-induction also.
- The mutual inductance of two coils is a property of their combination. The value of M remains unchanged irrespective of the fact that current is passed through one coil or the other.
- While calculating the mutual inductance of two long co-axial solenoids, the cross-sectional area of the inner solenoid is to be considered.
- While calculating the mutual inductance of two co-axial solenoids of different lengths, the length of the larger solenoid is to be considered.

■ Theorem of Reciprocity -

In given pair of coils, $M_{12} = M_{21}$, the two values of mutual inductance are equal in pairs. According to reciprocity theorem, for a given pair of coils, say 1 and 2, Irrespective of their shapes or size the magnetic flux linked with coil 2, due to a unit current in coil 1, is equal to the magnetic flux, linked with coil 1 due to a units current in coil 2.

■ Mutual Inductance of concentric solenoid –

$$M = \frac{\mu_0 N_1 N_2 A}{\ell} = \mu_0 n_1 n_2 A\ell$$

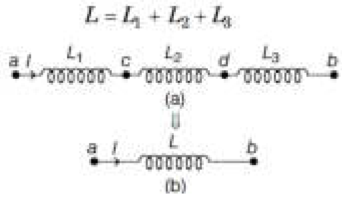


Where, N_1 and N_2 are total no. of turns in both coils, n_1 and n_2 are no of turn per unit length in coils, A is area of cross section of coils and ℓ is length of coil, S_1 and S_2 are inner and outer solenoid.

Grouping of Coils-

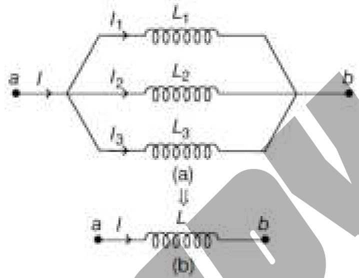
(a). When three coils of inductances L_1, L_2 and L3 are connected in series and the coefficient of coupling K = 0 as in series, then

$L = L_1 + L_2 + L_3$



(b). When three coils of inductance L_1, L_2 and L_3 are connected in parallel and the coefficient of coupling K = 0 as in parallel, then

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



If coefficient of coupling K = 1, then

(i) In series

(a) If current in two coils are in the same direction, then $L = L_1 + L_2 + 2M$

(b) If current in two coils are in opposite direction, then $L = L_1 + L_2 - 2M$

(ii) In parallel

(a) If current in two coils are in same direction, then $L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$

(b) If current in two coils are in opposite directions, then $L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$

■ Formula for self induction and Mutual Induction–

Self Inductance	Mutual Inductance
• $\phi = LI$ • Induced emf = $-L\frac{dI}{dt}$ • Self inductance in long solenoid $L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 A\ell$	• $\phi = MI$ • Induced emf = $-M\frac{dI}{dt}$ • M.I of two closely wound solenoid $M = \frac{\mu_0 N_1 N_2 A}{\ell} = \mu_0 n^2 A\ell$

(8) Electromagnetic Waves

■ Displacement Current:–

- Displacement current is that current which comes into existence, in addition to the conduction current whenever the electric field and hence the electric flux changes with time.

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

Where, ID = Displacement current

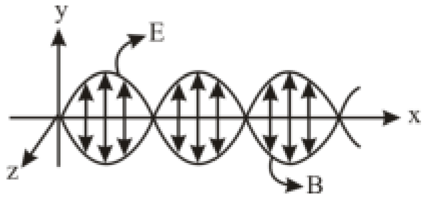
ϕ_E = Electric field × area = EA is the electric flux across the loop.

■ Need for Displacement current

- Ampere's circuit law for conduction current during charging of a capacitor was found inconsistent. Therefore Maxwell modified Ampere's circuit law.
- The displacement current produces in space due to change of electric flux linked with the surface. This reveals that, varying electric field is the source of magnetic field.

■ Electromagnetic waves:–

- An electromagnetic wave is a wave radiated by an accelerated or oscillatory charge in which varying magnetic field is the source of electric field and varying electric field is the source of magnetic field. Thus two fields becomes source of each other and the wave propagates in a direction perpendicular to both the fields.



$$E = E_0 \sin \omega \left(t - \frac{x}{c}\right)$$

$$B = B_0 \sin \omega \left(t - \frac{x}{c}\right)a$$

■ Properties of E M waves:–

1. The electric and Magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular to the direction in which the wave is travelling thus the electromagnetic wave is a transverse wave.
2. E M waves carry momentum and energy.
3. E M wave travel through vacuum with the speed of light 'c' where $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8\ ms^{-1}$.
4. The instanteous magnitude of E and B in an E M waves are related by the expression $\frac{E}{B} = c$
5. The cross product $\vec{E}\times\vec{B}$ always gives the direction in which the wave travels.
6. Energy associated with an electromagnetic wave is $U = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$
7. Linares Momentum delivered to the Surface $P = \frac{U}{c}$

U = Total energy transmitted by Electromagnetic waves
c = Speed of electromagnetic wave.

■ Poynting vector:–

- Poynting Vector can be defined as the rate at which the energy is carried out of the volume across the bounding surface. It is always in the direction of the propagation of wave, as it is perpendicular to both electric and magnetic field.
- The poynting vector is proportional to the cross product of electric and magnetic field $\vec{E}\times\vec{B}$. The poynting vector is given by-

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})$$

■ Key Points:–

1. An alternating current of frequency 50 Hz becomes zero, 100 times in one second because alternating current changes direction and becomes zero twice in a cycle.
2. An alternating current cannot be used to conduct electrolysis because the ions due to their inertia cannot follow the changing electric field.
3. Average value of AC is always defined over half cycle because average value of AC over a complete cycle is always zero.

■ Maxwell's Equations of Electromagnetic waves:–

- Maxwell's Equation are the basic laws of electricity and Magnetism. These equations give complete description of all electromagnetic interactions.

■ Maxwell's Equations in Differential and Integral Form-

	Differential form	Integral form
•	$\nabla \cdot \vec{E} = \frac{\rho}{E_0}$	$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$ (Gauss law in Electrostatics)
•	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E} \cdot d\vec{\ell} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$ (Faraday's law of Electomagnetic Induction)
•	$\nabla \cdot \vec{B} = 0$	$\oint \vec{B} \cdot d\vec{S} = 0$ (Gauss law in Magnetism)
	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{B}}{\partial t}$	$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}$ (Modified Ampere's circuital law)

■ Key Points:-

- Maxwell equations for different medium can be obtained by,

(i) For free space $\rho = 0$, $\vec{J} = 0$ and K_m(or μ_r) = relative permeability = 1

$\vec{D} = \varepsilon_0 \vec{E}$

$\vec{B} = \mu_0 \vec{H}$

(ii) For Isotropic non-conducting medium

$D = K\varepsilon_0 \vec{E} = \varepsilon \vec{E}$

$B = \mu_0 \mu_r \vec{H} = \mu \vec{H}$

$\vec{J} = 0$

$\rho = 0$

■ Electromagnetic Spectrum

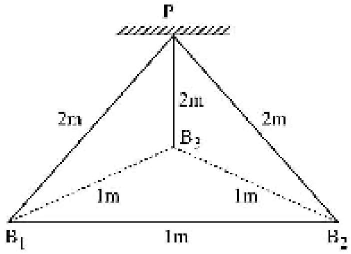
- The arranged array of electromagnetic radiation in the sequence of their wavelength or frequency is called electromagnetic spectrum.

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1 mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells, Photographic film
Ultraviolet	400 nm to 100 nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	10 nm to 10^{-2}nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	< 10^{-3} nm	Radioactive decay of the nucleus	-do-

Exemplar Problems
Electricity & Magnetism

Electrostatics

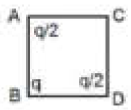
1. **Three identical charged balls each of charge 2C are suspended from a common point P by silk threads of 2m each (as shown in figure). They form an equilateral triangle of side 1m.**
The ratio of net force on a charged ball to the force between any two charged balls will be :



(a) 1 : 1 (b) 1 : 4
(c) $\sqrt{3}:2$ (d) $\sqrt{3}:1$

JEE MAIN 2022

2. **The three charges q/2, q and q/2 are placed at the corners A, B and C of a square of side 'a' as shown in figure. The magnitude of electric field (E) at the comer D of the square, is :**



(a) $\frac{q}{4\pi\epsilon_0 a^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{2}\right)$ (b) $\frac{q}{4\pi\epsilon_0 a^2}\left(1+\frac{1}{\sqrt{2}}\right)$

(c) $\frac{q}{4\pi\epsilon_0 a^2}\left(1-\frac{1}{\sqrt{2}}\right)$ (d) $\frac{q}{4\pi\epsilon_0 a^2}\left(\frac{1}{\sqrt{2}}-\frac{1}{2}\right)$

JEE MAIN 2022

3. **Two point charges A and B of magnitude $+8\times10^{-6}$C and -8×10^{-6}C respectively are placed at a distance d apart. The electric field at the middle point O between the charges is 6.4×10^{4}NC^{-1}. The distance 'd' between the point charges A and B is:**

(a) 2.0 m (b) 3.0 m
(c) 1.0 m (d) 4.0 m

JEE MAIN 2022

4. **An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field 3.55×10^{5} V m^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess? (consider g = 9.81 m/s^2)**

(a) 48.8×10^{11} (b) 1.73×10^{10}
(c) 17.3×10^{10} (d) 1.73×10^{12}

JEE MAIN 2021

5. **Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/ m lying along the z-axis in free space.**

(a) 0.424 nC m^{-2} (b) 47.88 C/m
(c) 0.07 nC m^{-2} (d) 4.0 nC m^{-2}

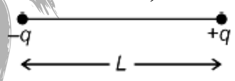
JEE MAIN 2021

6. **Two hollow conducting spheres of radii *R*1 and *R*2 (*R*1 >> *R*2) have equal charges. The potential would be**

(a) Dependent on the material property of the sphere
(b) More on bigger sphere
(c) More on smaller sphere
(d) Equal on both the spheres

NEET 2022

7. **Two point charges –q and +q are placed at a distance of L, as shown in the figure.**



The magnitude of electric field intensity at a distance r(R>>L) varies as :

(a) $\frac{1}{R^6}$ (b) $\frac{1}{R^2}$

(c) $\frac{1}{R^3}$ (d) $\frac{1}{R^4}$

NEET 2022

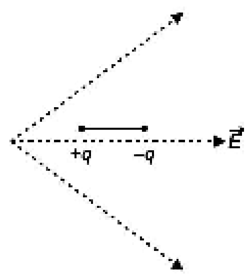
8. **Two charged spherical conductors of radius R_1 and R_2 are connected by a wire. Then the ratio of surface charge densities of the spheres (σ_1/σ_2) is**

(a) $\frac{R_1^2}{R_2^2}$ (b) $\frac{R_1}{R_2}$

(c) $\frac{R_2}{R_1}$ (d) $\sqrt{\left(\frac{R_1}{R_2}\right)}$

NEET 2021

9. **A dipole is placed in an electric field as shown. In which direction will it move?**



(a) Towards the right as its potential energy will increase.
(b) Towards the left as its potential energy will increase.
(c) Towards the right as its potential energy will decrease.
(d) Towards the left as its potential energy will decrease.

NEET 2021

10. The electrostatic force between the metal plates of an isolated parallel plate capacitor *C* having a charge *Q* and area *A*, is
(a) Independent of the distance between the plates
(b) Linearly proportional to the distance between the plates
(c) Proportional to the square root of the distance between the plates
(d) Inversely proportional to the distance between the plates.

NEET 2018

11. In going from the surface of a charged metallic sphere towards the centre of the sphere, the electric filed
(a) increases
(b) decreases
(c) remains same as at the surface
(d) is zero everywhere

UPPSC Ashram Paddhati PGT - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)

12. Electric flux in an electric field $\vec{E}$ through a small area $d\vec{s}$ is given by -
(a) $\vec{E}.d\vec{s}$
(b) $\oint_s \vec{E}.d\vec{s}$
(c) $\varepsilon_0 \vec{E}.d\vec{s}$
(d) $\vec{E}\times d\vec{s}$

UPPSC Poly. Lect. 2021 Paper - II

13. The flux of the electric field strength through each of the faces of a closed cube of edge length *l*, if a charge q is placed at its centre, is
(a) $\frac{q}{6\in_0 l^2}$
(b) $\frac{q}{6\in_0 l}$
(c) $\frac{q}{6\in_0}$
(d) $\frac{q}{6\in_0 l^3}$

UP PGT - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)

14. 4 equal point charges, +4μC, are placed at four corners of a square that is 50 cm on a side. The force on any of the charges, is
(a) 1.10 N inward along diagonal
(b) 0.72 N inward along diagonal
(c) 0.72 N outward along diagonal
(d) 1.10 N outward along diagonal

UP PGT - 2021

15. The electric potential due to dipole p, at a point distant r from it and in the direction θ from the dipoles is
(a) $\frac{1}{4\pi\in_0}\frac{p\cos\theta}{r^2}$
(b) $\frac{1}{4\pi\in_0}\frac{p}{r^3}$
(c) $\frac{1}{4\pi\in_0}\frac{p\sin\theta}{r^2}$
(d) $\frac{1}{4\pi\in_0}\frac{p\tan\theta}{r^2}$

UP PGT - 2021

16. Infinite equal charge q are placed along a line at distances 1 meter, 2 meter, 4 meter, 8 meter, 16 meter, 32 meter and so on. Potential due to all charges are
(a) Zero
(b) $\frac{q}{4\pi\varepsilon_0}$
(c) $\frac{q}{2\pi\varepsilon_0}$
(d) $\frac{q}{8\pi\varepsilon_0}$

UP PGT - 2021
UKPSC Lecturer (Mains) 2020

Capacitance & Capacitors

17. Two metallic plates form a parallel plate capacitor. The distance between the phase is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor?
(a) 2:1
(b) 1:2
(c) 1:4
(d) 4:1

JEE MAIN 2022

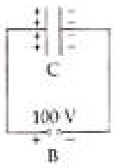
18. A force of 10N acts on a charged particle placed between two plates of a charged capacitor. If one plate of capacitor is removed, then the force acting on that particle will be :
(a) 5 N
(b) 10 N
(c) 20 N
(d) Zero

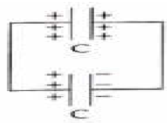
JEE MAIN 2022

19. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}$d, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation:
(a) $C' = \frac{3+K}{3}C_0$
(b) $C' = \frac{4+K}{3}C_0$
(c) $C' = \frac{4K}{K+3}C_0$
(d) $C' = \frac{4}{3+K}C_0$

JEE MAIN 2021

20. **A capacitor of capacitance C = 900 pF is charged fully by 100 V battery B as shown in figure (a). Then it is disconnected from the battery and connected to another uncharged capacitor of capacitance C = 900 pF as shown in figure (b). The electrostatic energy stored by the system (b) is**

(1) 

(2) 

(a) 1.5×10^{-6} J (b) 4.5×10^{-6} J
(c) 3.25×10^{-6} J (d) 2.25×10^{-6} J

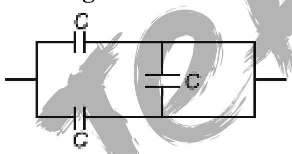
NEET 2022

21. **A parallel plate capacitor has a uniform electric field '$\vec{E}$' in the space between the plates. If the distance between the plates is 'd' and the area of each plate is 'A', the energy stored in the capacitor is**
(ε_0 = permittivity of free space)

(a) $\frac{E^2Ad}{\varepsilon_0}$ (b) $\frac{1}{2}\varepsilon_0E^2$
(c) ε_0EAd (d) $\frac{1}{2}\varepsilon_0E^2Ad$

NEET 2021

22. **The equivalent capacitance of the combination shown in the figure is**



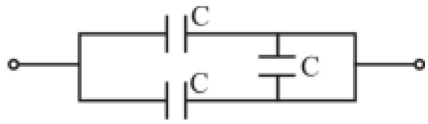
(a) $\frac{3C}{2}$ (b) 3C
(c) 2C (d) $\frac{C}{2}$

NEET 2021

23. **The capacitance of a parallel plate capacitor with air as medium is 6 μF. With the introduction of a dielectric medium, the capacitance becomes 30 μF. The permittivity of the medium is :**
($\epsilon_0 = 8.85\times10^{-12}\ C^2N^{-1}m^{-2}$)
(a) $1.77\times10^{-12}\ C^2N^{-1}m^{-2}$
(b) $0.44\times10^{-10}\ C^2N^{-1}m^{-2}$
(c) $5.00\ C^2N^{-1}m^{-2}$
(d) $0.44\times10^{-13}\ C^2N^{-1}m^{-2}$

NEET 2020

24. **The equivalent capacitance of the combination shown in figure below, is**



(a) 3C (b) 2C
(c) C (d) $\frac{C}{2}$

UP PGT - 2021

25. **A capacitor with air between its plates has a capacitance of 8 μF. Its capacitance on introducing a dielectric with dielectric constant 6 between its plates, is**
(a) 48 μF (b) 40 μF
(c) 32 μF (d) 16 μF

UP PGT - 2021

26. **Two capacitors C_1 and C_2 are charged to 120 V and 200 V, respectively. It is found that by connecting them together, the potential on each one can be made zero. Then -**
(a) $3C_1 = 5C_2$ (b) $9C_1 = 4C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $5C_1 = 3C_2$

UKPSC Lecturer (Mains) 2020

27. **Three capacitors each of 3 μF are provided. These cannot be combined to provide resultant capacitance of :**
(a) 1 μF (b) 2 μF
(c) 4.5 μF (d) 6 μF

UKPSC Lecturer (Mains) 2020

28. **A capacitor of 20 μF charged to 500 volts is connected in parallel with another capacitor of 10 μF charged to 200 volts. Common potential is:**
(a) 300 volts (b) 500 volts
(c) 200 volts (d) 400 volts

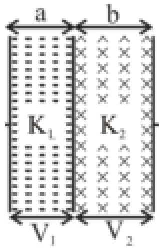
UKPSC Lecturer (Mains) 2020

29. **A parallel plate capacitor has a uniform electric field E in the space between the plates. If the distance between the plates is d and area of each plate is A, the energy stored in the capacitor is**

(a) $\frac{1}{2}\varepsilon_0E^2Ad$ (b) $\frac{1}{2}\varepsilon_0EAd$
(c) $\frac{1}{2}\frac{\varepsilon_0EA}{d}$ (d) $\frac{\varepsilon_0E^2A}{d}$

RPSC School Lect. 08.01.2020

30. **A parallel plate capacitor has two layers of different dielectrics as shown in figure. The ratio of potential difference across the dielectric layers when connected to the battery is**



(a) $\frac{K_1}{K_2}$ (b) $\frac{K_1 a}{K_2 b}$

(c) $\frac{K_2 a}{K_1 b}$ (d) 1 : 1

RPSC School Lect. 08.01.2020

Current electricity & Heating effect of current

31. If n represents the actual number of deflections in a converted galvanometer of resistance G and shunt resistance S. Then the total current I when its figure of merit is K will be :

(a) $\frac{KS}{(S+G)}$ (b) $\frac{(G+S)}{nKS}$

(c) $\frac{nKS}{(G+S)}$ (d) $\frac{nK(G+S)}{s}$

JEE MAIN 2022

32. Two cells of same EMF but different internal resistances r_1 and r_2 are connected in series with a resistance R. The value of resistance R, for which the potential difference across second cell is zero, is

(a) $r_2 - r_1$ (b) $r_1 - r_2$

(c) r_1 (d) r_2

JEE MAIN 2022

33. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is:-

(a) 3.9 (b) 8.4

(c) 7.5 (d) 3.0

JEE MAIN 2021

34. A conducting wire of length 'l', area of cross section A and electric resistivity r is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

(a) $\frac{1}{4}\frac{VA}{\rho l}$ (b) $\frac{3}{4}\frac{VA}{\rho l}$

(c) $\frac{1}{4}\frac{\rho l}{VA}$ (d) $4\frac{VA}{\rho l}$

JEE MAIN 2021

35. As the temperature increases, the electrical resistance

(a) Decreases for conductors but increases for semiconductors

(b) Increases for both conductors and semiconductors

(c) Decreases for both conductors and semiconductors

(d) Increases for conductors but decreases for semiconductors

NEET 2022

36. A copper wire of length 10 m and radius $\left(\frac{10^{-2}}{\sqrt{\pi}}\right)$ m has electrical resistance of 10Ω. The current density in the wire for an electric field strength of 10 (V/m) is

(a) $10^5 A/m^2$ (b) $10^4 A/m^2$

(c) $10^6 A/m^2$ (d) $10^{-6} A/m^2$

NEET 2022

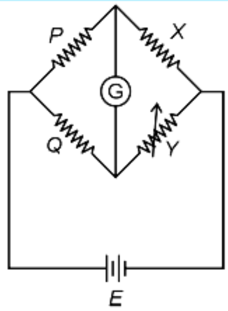
37. The energy that will be ideally radiated by a 100 kW transmitter in 1 hour is

(a) 1×10^6 J (b) 36×10^7 J

(c) 36×10^4 J (d) 36×10^6 J

NEET 2022

38. A Wheatstone bridge is used to determine the value of unknown resistance X by adjusting the variable resistance Y as shown in the figure. For the most precise measurement of X, the resistances P and Q



(a) Do not play any significant role

(b) Should be approximately equal to 2X

(c) Should be approximately equal and are small

(d) Should be very large and unequal

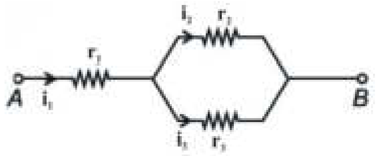
NEET 2022

39. The effective resistance of a parallel connection that consists of four wires of equal length, eaual area of cross-section and same material is 0.25 Ω. What will be the effective resistance if they are connected in series?

(a) $4\,\Omega$ (b) $0.25\,\Omega$
(c) $0.5\,\Omega$ (d) $1\,\Omega$

NEET 2021

40. **Three resistors having resistances r_1, r_2 and r_3 are connected as shown in the given circuit. The ratio $\frac{i_3}{i_1}$ of currents in terms of resistances used in the circuit is**



(a) $\frac{r_2}{r_1+r_3}$ (b) $\frac{r_1}{r_2+r_3}$
(c) $\frac{r_2}{r_2+r_3}$ (d) $\frac{r_1}{r_1+r_2}$

NEET 2021

41. **A resistance wire connected in the left gap of a metre bridge balances a 10 Ω resistance in the right gap at a point which divides the bridge wire in the ratio 3 : 2. If the length of the resistance wire is 1.5 m, then the length of 1 Ω of the resistance wire is:**

(a) 1.0×10^{-1} m (b) 1.5×10^{-1} m
(c) 1.5×10^{-2} m (d) 1.0×10^{-2} m

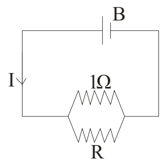
NEET 2020

42. **A charged particle having drift velocity of 7.5×10^{-4} m s^{-1} in an electric field of 3×10^{-10} Vm^{-1}, has a mobility in m^2 V^{-1} s^{-1} of :**

(a) 2.5×10^{6} (b) 2.5×10^{-6}
(c) 2.25×10^{-15} (d) 2.25×10^{15}

NEET 2020

43. **In the circuit below, battery B has electro motive force 2 V and internal resistance 0.5 Ω. What should be the value of Resistance R for current I to be maximum ?**



(a) Infinite (b) 1 Ω/1 Ω
(c) 0.5 Ω/0.5 Ω (d) Zero

UPTGT Science - 2021

44. **The internal resistance of an ideal constant voltage source is -**

(a) Zero
(b) Infinite
(c) equal to resistance of load
(d) None of the above

UPPSC Poly. Lect. 2021 Paper - II
(UPPGT 2013)

45. **$\vec{J} = \sigma\vec{E}$, where J is current density, σ is conductivity and E is electric field, is**

(a) Maxwell's equation
(b) Continuity equation
(c) Ampere's law
(d) A form of Ohm's law

UP PGT - 2021

46. **20 μA current is flowing in a wire for 30 seconds. The number of electrons passing through wire is**

(a) 375×10^{13} (b) 375×10^{15}
(c) 375×10^{16} (d) 375×10^{17}

UP PGT - 2021

47. **If the standard electrode potentials for zinc and copper electrodes in the cell $Zn|ZN^{2+}||Cu^{2+}|Cu$ are –0.76 V and 0.34 V respectively, the value of standard potential of the cell in volts will be**

(a) 0.11 (b) 11.0
(c) 1.10 (d) 0.011

UPTGT Science - 2021

48. **A current is flowing through a metallic wire. If the wire is heated, which quantities change?**

(a) Drift speed only (b) Resistivity only
(c) Resistance only (d) All of the above

UPTGT Science - 2021

49. **The resistances in a post office box are made of**

(a) Copper (b) Iron
(c) Manganin (d) Brass

UPTGT Science - 2021

50. **The potential difference of a cell in an open circuit is 6 volts but it falls to 4 volts, when a current of 2 Amp is drawn from it. The internal resistance of the cell is :**

(a) $0.5\,\Omega$ (b) $2\,\Omega$
(c) $1.5\,\Omega$ (d) $1\,\Omega$

UKPSC Lecturer (Mains) 2020

51. **A 220 V, 100 W bulb is connected to 110 V source. The power consumed by the bulb is :**

(a) 20 W (b) 25 W
(c) 15 W (d) 50 W

UKPSC Lecturer (Mains) 2020

52. **Daniel cell consists of**

(a) Zn and Cu as electrodes
(b) Zn and Fe as electrodes
(c) An and Si as electrodes
(d) Zn and Ag as electrodes

UP TGT Physics 2016

53. **The internal resistance of an ideal voltmeter should be**

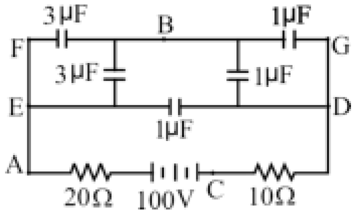
(a) Zero
(b) Infinite
(c) Equal to internal resistance of source
(d) Negative

UP PGT Physics 2016 (Exam : 02.02.2019)

54. **The cell reaction of a secondary battery is**
(a) Reversible (b) Irreversible
(c) Equilibrium (d) Endothermic
UP TGT Physics 2016

55. **Kirchhoff's Laws are applicable to**
(a) DC only (b) AC only
(c) AC and DC both (d) None of the above
UP PGT - 2021

56. **The potential difference between the points a and B in steady state in adjoining diagram is :**



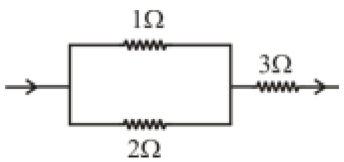
(a) 25 V (b) 20 V
(c) 15 V (d) 10 V
UKPSC Lecturer (Mains) 2020

57. **A piece of copper and the other of germanium are heated from room temperature to 400 K. Then choose the correct statement.**
(a) Resistance of each decreases.
(b) Resistance of each increases.
(c) Resistance of copper increases while that of germanium decreases.
(d) Resistance of copper decreases while that of germanium increases.
RPSC School Lect. 26.07.2016

58. **The value of current I in the following figure is:**

(a) I = 2.6 A (b) I = 4.6 A
(c) I = 3.6 A (d) I = 5.6 A
UKPSC Lecturer (Mains) 2020

59. **In the circuit show in figure, power developed across 1 Ω, 2 Ω and 3 Ω resistance are in the ratio of**



(a) 1 : 2 : 3 (b) 4 : 2 : 27
(c) 6 : 4 : 9 (d) 2 : 1 : 27
RPSC School Lect. 08.01.2020

Magnetism and Matter

60. **At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is :**
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$
(c) $\sqrt{3}$ (d) 1
JEE MAIN 2022

61. **A metallic conductor of length lm rotates in a vertical plane parallel to east-west direction about one of its end with angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is 0.2 × 10^{-4} T, then EMF induced between the two ends of the conductor is :**
(a) 5μV (b) 50μV
(c) 5mV (d) 50mV
JEE MAIN 2022

62. **The soft-iron is a suitable material for making an electromagnet. This is because soft-iron has:**
(a) low coercively and high retentively
(b) low coercively and low permeability
(c) high permeability and low retentively
(d) high permeability and high retentively
JEE MAIN 2022

63. **A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If BH = 0.4 G, the magnetic moment of the magnet is (1 G=10^{-4}T)**
(a) 2.880×10^3 J T^{-1} (b) 2.880×10^2 J T^{-1}
(c) 2.880 J T^{-1} (d) 28.80 J T^{-1}
JEE MAIN 2021

64. **A long solenoid of radius 1 mm has 100 turns per mm. If 1 A current flows in the solenoid, the magnetic field strength at the centre of the solenoid is**
(a) 6.28×10^{-4}T (b) 6.28×10^{-2}T
(c) 12.56×10^{-2}T (d) 12.56×10^{-4}T
NEET 2022

65. **A long solenoid of 50 cm length having 100 turns carries a current of 2.5 A. The magnetic field at the centre of the solenoid is :**
($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(a) 3.14×10^{-4} T (b) 6.28×10^{-5} T
(c) 3.14×10^{-5} T (d) 6.28×10^{-4} T
NEET 2020

66. **An iron rod of susceptibility 599 is subjected to a magnetising field of 1200 A m^{-1}. The permeability of the material of the rod is :**
($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})
(a) 8.0×10^{-5} T m A^{-1}
(b) $2.4\pi \times 10^{-5}$ T m A^{-1}
(c) $2.4\pi \times 10^{-7}$ T m A^{-1}
(d) $2.4\pi \times 10^{-4}$ T m A^{-1}
NEET 2020

67. A 250-turn rectangular coil of length 2.1 cm and width 1.25 cm carries a current of 85 mA and subjected to a magnetic field of strength 0.85 T. Work done for rotating the coil by 180° against the torque is

(a) 4.55 μJ (b) 2.3 mJ
(c) 1.15 mJ (d) 9.1 mJ

NEET 2017

68. An iron rod of susceptibility 599 is subjected to a magnetising field of 1200 A m^{-1}. The permeability of the material of the rod is (m_0 = 4π × 10^{-7} T m A^{-1})

(a) 2.4π ×5 10^{-4} T m A^{-1}
(b) 8.0 × 10^{-5} T m A^{-1}
(c) 2.4π × 10^{-5} T m A^{-1}
(d) 2.4 p × 10^{-7} T m A^{-1}

NEET 2020

69. A thin diamagnetic rod is placed vertically between the poles of an electromagnet. When the current in the electromagnet is switched on, then the diamagnetic rod is pushed up, out of the horizontal magnetic field. Hence the rod gains gravitational potential energy. The work required to do this comes from

(a) the current source
(b) the magnetic field
(c) the lattice structure of the material of the rod
(d) the induced electric field due to the changing magnetic field

NEET 2018

70. The magnetic susceptibility is negative for

(a) ferromagnetic material only
(b) paramagnetic and ferromagnetic materials
(c) diamagnetic material only
(d) paramagnetic material only

NEET 2016

71. The dipole moment of current loop does not depend upon :

(a) Current in the loop
(b) Area of the loop
(c) Number of turns in the loop
(d) Shape of the loop

UKPSC Lecturer (Mains) 2020

72. A magnetized steel wire 31.4 cm long has a pole strength of 5 CGS units. It is then bent in the form of a semi-circle. The magnetic moment of this magnet is

(a) 100 CGS units (b) 120 CGS units
(c) 150 CGS units (d) 175 CGS units

UKPSC Lecturer (Mains) 2020

73. The substances which are attracted by the magnetic field are–

(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Ferrimagnetic

UP TGT Physics 2016

74. For a 1m long solenoid, if n=500 and I=5 Amp, then calculate 'B' at its centre -

(a) 3.14×10^{-3} Weber / m^2
(b) 3.14 Weber / m^2
(c) 3.14×10^{-2} Weber / m^2
(d) 3.14×10^{-7} Weber / m^2

UPPSC Poly. Lect. 2021 Paper - II

75. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then

(a) $\vec{F_1}$ is radially inwards and $\vec{F_1}$ is radially outwards
(b) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$
(c) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$
(d) $\vec{F_1} = \vec{F_2} = 0$

UKPSC Lecturer (Mains) 2020

76. The dilute solution of all the alkali metals in anhydrous liquid ammonia is

(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Anti-ferromagnetic

UPTGT Science - 2021

77. The magnetic permeability is maximum for which of the following ?

(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Anti ferromagnetic

UKPSC Lecturer (Mains) 2020

78. When a superconductor is placed in an external magnetic field, its superconductivity

(a) decreases with increase in the intensity H of the external field.
(b) increases with increase in the intensity H of the external field.
(c) is independent of the intensity H of the external field.
(d) None of these

UKPSC Lecturer (Mains) 2020

79. A paramagnetic material is placed in a magnetic field. The magnetic field is increased till magnetization becomes constant. Now temperature is decreased, the magnetization will:

(a) Increase (b) Decrease
(c) Remain constant (d) None of these

UKPSC Lecturer (Mains) 2020

80. Choose the wrong statement for magnetic susceptibility of ferromagnetic material through a hysteresis loop.

(a) It has a fixed value (b) It may be zero
(c) It may be infinite (d) It may be negative

RPSC School Lect. 08.01.2020

Alternating Current

81. A sinusoidal voltage V(t) = 210 sin 3000t volt is applied to a series LCR circuit in which L = 10 mH, C = 25 μ F and R = 100Ω. The phase difference (ϕ) between the applied voltage and resultant current will be :
(a) $\tan^{-1}(0.17)$ (b) $\tan^{-1}(9.46)$
(c) $\tan^{-1}(0.30)$ (d) $\tan^{-1}(13.33)$
JEE MAIN 2022

82. The current flowing through an ac circuit is given by
I = 5 sin (120πt) A
How long will the current take to reach the peak value starting from zero?
(a) $\frac{1}{60}$s (b) 60s
(c) $\frac{1}{120}$s (d) $\frac{1}{240}$s
JEE MAIN 2022

83. In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value:
(a) The bandwidth of resonance circuit will increase.
(b) The resonance frequency will increase.
(c) The quality factor will increase.
(d) The quality factor and the resonance frequency will remain constant.
JEE MAIN 2021

84. An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is :
(a) 2.5 ms (b) 25 ms
(c) 2.5 s (d) 0.25 ms
JEE MAIN 2021

85. The peak voltage of the ac source is equal to
(a) $1/\sqrt{2}$ times the rms value of the ac source
(b) The value of voltage supplied to the circuit
(c) The rms value of the ac source
(d) $\sqrt{2}$ times the rms value of the ac source
NEET 2022

86. A series *LCR* circuit is connected to an ac voltage source. When *L* is removed from the circuit, the phase difference between current and voltage is p/3. If instead *C* is removed from the circuit, the phase difference is again p/3 between current and voltage. The power factor of the circuit is
(a) zero (b) 0.5
(c) 1.0 (d) –1.0
NEET 2020

87. A circuit when connected to an AC source of 12 V gives a current of 0.2 A. The same circuit when connected to a DC source of 12 V, gives a current of 0.4 A. The circuit is
(a) series LR (b) series RC
(c) series LC (d) series LCR
Odisha NEET 2019

88. An inductor 20 mH, a capacitor 100 mF and a resistor 50 W are connected in series across a source of emf, *V* = 10 sin 314*t*. The power loss in the circuit is
(a) 0.79 W (b) 0.43 W
(c) 2.74 W (d) 1.13 W
NEET 2018

89. The average value of I from t = 0 to t = T, for I $= I_0\sin^2\omega t$ where $\omega = \frac{2\pi}{T}$, is
(a) $2I_0$ (b) $3I_0$
(c) $I_0/2$ (d) $I_0/3$
UP PGT - 2021
UP PGT Physics 2016 (Exam : 02.02.2019)

90. What is Fourier transform of unity?
(a) 1 (b) δ(s)
(c) $\sqrt{2\pi}\,\delta(s)$ (d) $\frac{1}{\sqrt{2\pi}}\delta(s)$
UPPSC GDC - 2021

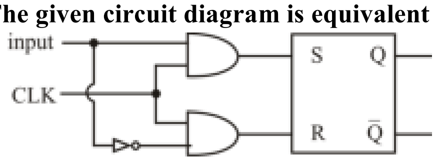
91. The current in an AC circuit is wattless when the phase difference between current and voltage is
(a) zero (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π
UP PGT - 2021

92. The peak voltage in a 240V A.C. source is
(a) 240 V (b) 300 V
(c) 340 V (d) 380 V
UP PGT - 2021

93. Fourier cosine transform of $f(x)=1$ in interval $(0,\pi)$ is -
(a) $\frac{1-\cos p\pi}{p}$ (b) $\frac{1+\cos p\pi}{p}$
(c) $\frac{2}{p}$ (d) Zero
UPPSC Poly. Lect. 2021 Paper - I

94. Instantaneous current in a circuit is given by i $= 2\sqrt{2}\cos(\omega t + \phi)$ Ampere. The value of r.m.s current is :
(a) $2\sqrt{2}$ Ampere (b) 4 Ampere
(c) $\sqrt{2}$ Ampere (d) 2Ampere
UKPSC Lecturer (Mains) 2020

95. **The given circuit diagram is equivalent to**



(a) T Flip-flop
(b) Clocked RS Flip-flop
(c) JK Flip-flop
(d) D Flip-flop

RPSC School Lect. 08.01.2020

96. **The alternating current cannot be used for**
(a) Heating
(b) Lighting
(c) Generate mechanical energy
(d) Eletroplating

UP TGT Physics 2016

97. **An AC source (V volts) sends current through a series L, C, R circuit. If voltages V_L, V_C, V_R are measured across L, C, R, then**
(a) $V = V_L + V_C + V_R$
(b) $V = V_R + (V_L - V_C)$
(c) $V = \left[V_R^2 + V_L^2 + V_C^2\right]^{1/2}$
(d) $V = \left[V_R^2 + (V_L - V_C)^2\right]^{1/2}$

UP PGT Physics 2016 (Exam : 02.02.2019)

98. **A 220 volt A.C. sources is connected across an inductance of One Henry. If the frequency of the source is 50Hz then what is the power loss?**
(a) 220W (b) 110W
(c) Zero (d) 440W

UP PGT Physics 2016 (Exam : 02.02.2019)
UP LT Grade Science 2018 (Exam : 29.07.2018)

99. **In an AC circuit resonance condition is**
(a) $L > \frac{1}{\omega^2 C}$ (b) $L < \frac{1}{\omega^2 C}$
(c) $L = \frac{1}{\omega^2 C}$ (d) L = C

UP PGT Physics 2016 (Exam : 02.02.2019)

100. **If the phase difference between an alternating current and voltage is $\pi/2$, which of the following can not be part of the circuit?**
(a) R, L both present (b) C alone present
(c) L alone present (d) L and C both present

UP PGT Physics 2016 (Exam : 02.02.2019)

101. **A transformer is used to–**
(a) Change the alternating potential
(b) Change the direct current
(c) Both (a) & (b)
(d) None of these

(UPPGT 2011)

102. **If in a transformer keeping the peak voltage in the primary coil the same, the frequency of voltage is changed from 50Hz to 60Hz, The voltage in the secondary coil–**
(a) Will increase in the ratio 60/50
(b) Will decrease in the ratio 50/60
(c) Will remain unchanged
(d) None of these

(UPPGT 2011)

103. **Power consumed in an a.c. circuit is zero if :**
(a) inductance and resistance both are high
(b) inductance and resistance both are low
(c) inductance is very low and resistance is zero
(d) inductance may be high or low and the resistance is zero

UPPCS Pre 2010

104. **In an a.c. circuit the voltage across L, C and R in series are 60, 20 and 30 volts respectively. The applied e.m.f. is :**
(a) 110 volts (b) 70 volts
(c) 50 volts (d) 10 volts

UPPCS Pre 2010

Magnetic effect of current & Magnetism

105. **The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s.The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:**
(a) 2.25 (b) 4.25
(c) 6.25 (d) 8.25

JEE MAIN 2022

106. **A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to**
(a) B (b) 2B
(c) 4 B (d) $\frac{B}{2}$

JEE MAIN 2022

107. **Two long current carrying conductors are placed parallel to each other at a distance of 8 cm between them. The magnitude of magnetic field produced at mid-point between the two conductors due to current flowing in them is 300 μT. The equal current flowing in the two conductors is :**
(a) 30A in the same direction.
(b) 30A in the opposite direction.
(c) 60A in the opposite direction.
(d) 300A in the opposite direction.

JEE MAIN 2022

108. **A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.**
(a) Loop assumes circular shape with its plane normal to the field.
(b) Loop assumes circular shape with its plane parallel to the field.
(c) Wire gets stretched to become straight.
(d) Shape of the loop remains unchanged.

JEE MAIN 2021

109. **A charge Q is moving $\vec{dl}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$**

(a) 1 (b) Infinite
(c) Zero (d) 1

JEE MAIN 2021

110. **A proton, an electron and a helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane.**
Let r_p r_e and r_{He} be their respective radii, then

(a) $r_e < r_p = r_{He}$ (b) $r_e < r_p = r_{He}$
(c) $r_e > r_p = r_{He}$ (d) $r_e > r_p > r_{He}$

JEE MAIN 2019

111. **A square loop of side 1 m and resistance 1 Ω is placed in a magnetic field of 0.5 T. If the plane of loop is perpendicular to the direction of magnetic field, the magnetic flux through the loop is**

(a) Zero weber (b) 2 weber
(c) 0.5 weber (d) 1 weber

NEET 2022

112. **A big circular coil of 1000 turns and average radius 10 m is rotating about its horizontal diameter at 2 rad s^{-1}. If the vertical component of earth's magnetic field at that place is 2×10^{-5}T and electrical resistance of the coil is 12 : 56 Ω, then the maximum induced current in the coil will be**

(a) 2A (b) 0.25 A
(c) 1.5 A (d) 1 A

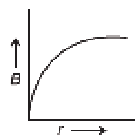
NEET 2022

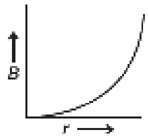
113. **From Ampere's circuital law for a long straight wire of circular cross-section carrying a steady current, the variation of magnetic field in the inside and outside region of the wire is**

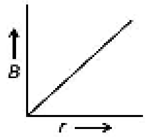
(a) A linearly decreasing function of distance upto the boundary of the wire and then a linearly increasing one for the outside region.
(b) Uniform and remains constant for both the regions.
(c) A linearly increasing function of distance upto the boundary of the wire and then linearly decreasing for the outside region.
(d) A linearly increasing function of distance r upto the boundary of the wire and then decreasing one with $\frac{1}{r}$ dependence for the outside region.

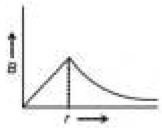
NEET 2022

114. **A thick current carrying cable of radius 'R' carries current 'I' uniformly distributed across its crosssection. The variation of magnetic field B(r) due to the cable with the distance 'r' from the axis of the cable is represented by**

(a) 

(b) 

(c) 

(d) 

NEET 2021

115. **Ionized hydrogen atoms and a-particles with same momenta enters perpendicular to a constant magnetic field, *B*. The ratio of their radii of their paths $r_H : r_\alpha$ will be**

(a) 1 : 4 (b) 2 : 1
(c) 1 : 2 (d) 4 : 1

NEET 2019

116. **The differential form of Ampere's law is**

(a) $\vec{\nabla}\times\vec{H} = \vec{J}$ (b) $\vec{\nabla}.\vec{H} = J$
(c) $\vec{\nabla}\times\vec{H} = I$ (d) $\vec{\nabla}\times\vec{H} = \mu_0\vec{J}$

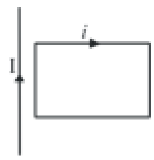
UPPSC Ashram Paddhati PGT - 2021

117. **According to modified Ampere's law -**

(a) $\text{curl}\vec{B} = \mu_0\left(\vec{j}_{free} + \hat{j}_d\right)$
(b) $\text{curl}\vec{B} = \mu_0\left(\vec{j}_{free} - \hat{j}_d\right)$
(c) $\text{curl}\vec{E} = \mu_0\left(\vec{j}_{free} + \hat{j}_d\right)$
(d) $\text{curl}\vec{E} = \mu_0\left(\vec{j}_{free} - \hat{j}_d\right)$

UPPSC Poly. Lect. 2021 Paper - II

118. **Current *i* is flowing in a rectangular loop. It is kept near a long straight wire such that the wire is parallel to one arm of the loop and it is also in the plane of the loop. If a stationary current I is flowing in the wire as shown in figure.**



then loop

(a) will rotate along an axis parallel to the wire
(b) will move away from the wire
(c) will remain stationary
(d) will move towards the wire

UPPCS Pre 2006

119. **Two long parallel wires are at a separation of r and carry current I. The force on 1 mt of wire will be**

(a) $\frac{\mu_0 I}{2\pi r}$ (b) $\frac{\mu_0 I^2}{r^2}$
(c) $\frac{\mu_0 I^2}{2\pi r}$ (d) $\frac{\mu_0 I}{r^2}$

(UPPGT 2013)
UPPCS Pre 2005

120. **The phenomenon of adiabatic demagnetization is used to**
(a) Demagnetize a magnet
(b) Produce very low temperature
(c) Generate electricity
(d) Purify a gas

UPPSC Ashram Paddhati PGT - 2021

121. **Faraday's law relates electric field $\vec{E}$ and magnetic field $\vec{B}$ as -**
(a) $\oint_c \vec{E}.d\vec{r} = -\frac{d}{dt}\int_s \vec{B}.d\vec{s}$ (b) $\oint_c \vec{E}.d\vec{r} = -\int_s \vec{B}.d\vec{s}$
(c) $\oint_c \vec{E}.d\vec{r} = \frac{d}{dt}\int_s \vec{B}.d\vec{s}$ (d) $\oint_c \vec{E}.d\vec{r} = \int_s \vec{B}.d\vec{s}$

UPPSC Poly. Lect. 2021 Paper - II

122. **An electron is injected into a region of magnetic flux density with components of velocity parallel to and normal to the flux. The path of the electron is -**
(a) Helix (b) Parabola
(c) Straight line (d) Circle

UPPSC Poly. Lect. 2021 Paper - II

123. **The maximum kinetic energy of the ion emerging from the cyclotron is -**
(a) $\frac{qBR}{m}$ (b) $\frac{qBR^2}{2m}$
(c) $\frac{q^2B^2R^2}{2m}$ (d) $\frac{q^2B^2R^2}{m}$

UPPSC Poly. Lect. 2021 Paper - II

124. **A circular coil of radius 'r' carries a current and the magnetic field at its centre is 'B'. At what distance from the centre on the axis of coil, the magnetic field will be B/8?**
(a) r (b) 2r
(c) $\sqrt{3}$r (d) 8r

UPPSC Poly. Lect. 2021 Paper - II

125. **Hall voltage is produced**
(a) At right angle to the direction of current
(b) At right angle to the direction of mangetic field
(c) At right angle to the direction of both current and magnetic field
(d) In the direction of magnetic field

UPPSC GDC - 2021

Electromagnetic Induction

126. **A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to :**
(a) $\sqrt{L_1L_2}$ (b) $\frac{L_1+L_2}{2}$
(c) $2L_1-L_2$ (d) $3L_1-L_2$

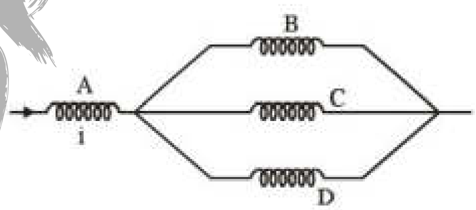
JEE MAIN 2022

127. **Two coils of self inductance L_1 and L_2 are connected in series combination having mutual inductance of the coils as M. The equivalent self inductance of the combination will be :**

(a) $\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{M}$ (b) L_1+L_2+M
(c) L_1+L_2+2M (d) L_1+L_2-2M

JEE MAIN 2022

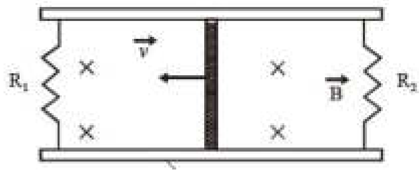
128. **Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3T, the field at the center of C would be : (Assume that the magnetic field is confined with in the volume of respective solenoid).**



(a) 12T (b) 6T
(c) 9T (d) 1T

JEE MAIN 2021

129. **A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure**



Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field B pointing into the page. An external agent pulls the bar to the left at a constant speed *v*. The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is :
(a) Both I_1 and I_2 are in anticlockwise direction
(b) Both I_1 and I_2 are in clockwise direction
(c) I1 is in clockwise direction and I_2 is in anticlockwise direction
(d) I_1 is in anticlockwise direction and I_2 is in clockwise direction

JEE MAIN 2021

130. **A 10 m long horizontal wire extends from North-East to South-West. It is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field of 0.3×10^{-4} Wb/m^2. The value of the induced emf in wire is**

(a) 1.5×10^{-3} V (b) 1.1×10^{-3} V
(c) 0.2×10^{-3} V (d) 2.5×10^{-3} V

JEE MAIN 2019

131. **A series LCR circuit with inductance 10H, capacitance 10μF, resistance 50Ω is connected to an ac source of voltage, V = 200sin (100t) volt. If the resonant frequency of the LCR circuit is v_0 and the frequency of the ac source is v, then**

(a) $v = 100Hz;\ v_0 = \frac{100}{\pi}Hz$

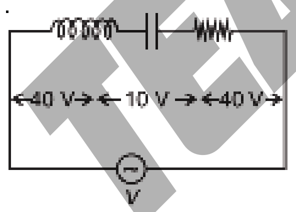
(b) $v_0 = v = 50Hz$

(c) $v_0 = v = \frac{50}{\pi}Hz$

(d) $v_0 = \frac{50}{\pi}Hz,\ v = 50Hz$

NEET 2022

132. **An inductor of inductance L, a capacitor of capacitance C and a resistor of resistance 'R' are connected in series to an ac source of potential difference 'V' volts as shown in figure.**
Potential difference across L, C and R is 40 V, 10 V and 40 V, respectively. The amplitude of current flowing through LCR series circuit is $10\sqrt{2}$ A. The impedance of the circuit is



(a) 5 Ω (b) $4\sqrt{2}\,\Omega$
(c) $5\sqrt{2}\,\Omega$ (d) 4 Ω

NEET 2021

133. **A series LCR circuit is connected to an ac voltage source. When L is removed from the circuit, the phase difference between current and voltage is $\frac{\pi}{3}$. If instead C is removed from the circuit, the phase difference is again $\frac{\pi}{3}$ between current and voltage. The power factor of the circuit is :**

(a) 0.5 (b) 1.0
(c) –1.0 (d) zero

NEET 2020

134. **A 800 turn coil of effective area 0.05 m^2 is kept perpendicular to a magnetic field 5×10^{-5} T. When the plane of the coil is rotated by 90° around any of its coplanar axis in 0.1 s, the emf induced in the coil will be**

(a) 0.02 V (b) 2V
(c) 0.2 V (d) 2×10^{-3} V

NEET 2019

135. **Which material is used in transformer core?**

(a) Nickel (b) Soft iron
(c) Steel (d) Copper

UPPSC Ashram Paddhati PGT - 2021
UPPCS Pre 1995

136. **With a decrease of current in primary coil from 2 Amp to 0 Amp in 0.01 sec, the e.m.f. generated in secondary coil is 100 Volt. The mutual inductance will be**

(a) 10 Henry (b) 5 Henry
(c) 1 Henry (d) 0.5 Henry

UPPSC Ashram Paddhati PGT - 2021

137. **The total induction of two coupled coils in positive coupling and negative coupling are 1.6 mH and 0.8 mH respectively. The value of mutual inductance will be -**

(a) 0.2 mH (b) 0.4 mH
(c) 4 mH (d) 2 mH

UPPSC Poly. Lect. 2021 Paper - II

138. **An electric transformer has 100 turns in primary and N turns in secondary. If the input is 220 V A.C. and we want 11 V output, what is the value of N?**

(a) 1 (b) 5
(c) 100 (d) 500

UPTGT Science - 2021

139. **A 200 km long telegraph wire has capacity of 0.014 μF/km. If it carries as A.C. of frequency 5 kHz, what is the value of inductance required to be connected in series such that impedance is minimum?**

(a) 0.36 mH (b) 0.45 mH
(c) 0.52 mH (d) 0.64 mH

UKPSC Lecturer (Mains) 2020

140. **A transformer is required to match the 400 Ω output impedance of a transistor power amplifier to a 4 Ω speaker coil. What should be the turn ratio ?**

(a) $\frac{10}{1}$ (b) $\frac{1}{10}$

(c) $\frac{16}{1}$ (d) None of these

UKPSC Lecturer (Mains) 2020

141. **Two coils of same length and same area of cross section are placed near each other. If L_1 and L_2 are their self-inductances, then the mutual inductance between two coils is proportional to :**

(a) $\sqrt{\frac{L_1}{L_2}}$ (b) $\sqrt{\frac{L_2}{L_1}}$

(c) $\sqrt{L_1L_2}$ (d) $\sqrt{\frac{1}{L_1L_2}}$

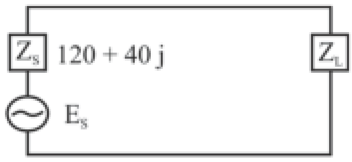
UKPSC Lecturer (Mains) 2020
UPPGT 2010, 2004
UPPCS (Pre) 1998

142. A 220 V input is supplied to a transformer. The output circuit draws a current of 2.0 A at 440 V. If the efficiency of the transformer is 80% the current drawn by the primary winding of transformer is

(a) 3.2 A (b) 4.0 A
(c) 4.4 A (d) 5.0 A

RPSC School Lect. 08.01.2020

143. In given circuit, if source deliver maximum power to load, then load impedance must be



(a) 120 + 40j (b) 120 – 40j
(c) 40 + 120j (d) 110Ω

RPSC School Lect. 08.01.2020

144. The current flowing in a coil of self induction 0.4 mH is changed by 250mA in 0.1 seconds. What is the induced emf in the coil.

(a) –1mV (b) –2mV
(c) –3mV (d) –5mV

UP PGT Physics 2016 (Exam : 02.02.2019)

145. The power factor of LCR circuit at resonance is

(a) 0.707 (b) 1
(c) zero (d) 0.5

UPPSC Ashram Paddhati PGT - 2021
(UPPGT 2005)

146. The best filter used in a power supply is

(a) L – section filter
(b) Capacitor input filter
(c) Choke input filter
(d) π – section filter

UP PGT - 2021

147. In a driven series LCR circuit, the resonant frequency f_r is given by

(a) $f_r = \frac{1}{2\pi\sqrt{LC}}$ (b) $f_r = 2\pi\sqrt{LC}$

(c) $f_r = 2\pi\sqrt{\frac{L}{C}}$ (d) $f_r = 2\pi\sqrt{\frac{C}{L}}$

UP PGT - 2021

148. A coil having inductance 0.15 H and resistance 15Ω is connected across a 220 V, 50 Hz line. Compute the current in the coil.

(a) 4.45 A (b) 4.25 A
(c) 4.00 A (d) 3.50 A

UP PGT - 2021

149. Anderson's Bridge is used for measurement of

(a) Capacitance
(b) Self-inductance
(c) Resistance
(d) Frequency of AC supply

UP PGT - 2021

150. AC, voltage 100 V, 50 Hz is applied to the ends of a choke of L = 2 Henry. What current will flow through it (in amperes)?

(a) $\frac{1}{2\pi}$ A.C. (b) $\frac{1}{2\pi}$ D.C.

(c) $\frac{2}{\pi}$ A.C. (d) $\frac{2}{\pi}$ D.C.

UPTGT Science - 2021

151. A sinusoidal voltage v(t) = 100 sin (500 t) is applied across a pure inductance L = 0.02 H. The current through the coil is.

(a) 10 cos (500 t) (b) –10 cos (500 t)
(c) 10 sin (500 t) (d) –10 sin (500 t)

UKPSC Lecturer (Mains) 2020

Electromagnetic waves

152. Match List-I with List-II

	List - I		List - II
A	Television signal	I	03 KHz
B	Radio signal	II	20 KHz
C	High Quality Music	III	02 MHz
D	Human speech	IV	06 MHz

Choose the correct answer from the options given below :

(a) A-I, B-II, C-III, D-IV
(b) A-IV, B-III, C-I, D-II
(c) A-IV, B-III, C-II, D-I
(d) A-I, B-II, C-IV, D-III

JEE MAIN 2022

153. A radar sends an electromagnetic signal of electric field (E_0) = 2.25 V/m and magnetic field (B_0) = 1.5 x 10^{-8} T which strikes a target on line of sight at a distance of 3 km in a medium. After that, a pail of signal (echo) reflects back towards the radar vit1i same velocity and by same path. If the signal was transmitted at time t_0 from radar. then after how much time echo will reach to the radar?

(a) 2.0×10^{-5}s (b) 4.0×10^{-5}s
(c) 1.0×10^{-5}s (d) 8.0×10^{-5}s

JEE MAIN 2022

154. A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B}$ 2.0×10^{-8} $\hat{k}$T (where, $\hat{k}$ is unit vector along z-direction) What is $\vec{E}$ at this point?

(a) $0.6\hat{j}\,V/m$ (b) $0.6\hat{k}\,V/m$

(c) $6.0\hat{j}\,V/m$ (d) $6.0\hat{k}\,V/m$

JEE MAIN 2021

155. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At particular point in space and time, $\vec{B} = 8.0\times10^{-1}\hat{z}T$.. The value of electric field at this point is:

(speed of light = 3 × 10^{8} ms^{-1})

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z direction.

(a) $-24\,\hat{x}$ V/m (b) $2.6\,\hat{x}$ V/m

(c) $24\,\hat{x}$ V/m (d) $-2.6\,\hat{x}$ V/m

JEE MAIN 2021

156. Match List-I with List-II

	List-I (Electromagnetic waves)		List-I (Electromagnetic waves)
(a)	AM radio waves	(i)	10^{-10}m
(b)	Microwaves	(ii)	10^{2} m
(c)	Infrared radiations	(iii)	10^{-2} m
(d)	X-rays	(iv)	10^{-4} m

Choose the correct answer from the options given below

(a) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

(b) (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)

(c) (a) - (iii), (b) - (ii), (c) - (i), (d) - (iv)

(d) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

NEET 2022

157. For a plane electromagnetic wave propagating in x-direction, which one of the following combination gives the correct possible directions for electric field (E) and magnetic field (B) respectively?

(a) $-\hat{j}+\hat{k}, -\hat{j}+\hat{k}$ (b) $\hat{j}+\hat{k}, \hat{j}+\hat{k}$

(c) $-\hat{j}+\hat{k}, -\hat{j}-\hat{k}$ (d) $\hat{j}+\hat{k}, -\hat{j}-\hat{k}$

NEET 2021

158. The ratio of contributions made by the electric intensity of an electromagnetic wave is :

(c = speed of electromagnetic waves)

(a) 1 : 1 (b) 1 : c

(c) 1 : c^2 (d) c : 1

NEET 2020

159. A parallel plate capacitor of capacitance 20 μF is being charged by a voltage source whose potential is changing at the rate of 3 V/s. The conduction current through the connecting wires, and the displacement current through the plates of the capacitor, would be, respectively

(a) zero, zero (b) zero, 60 μA

(c) 60 μA, 60 μA (d) 60 μA, zero

NEET 2019

160. The dominant TE mode in rectangular wave guide is -

(a) TE_{11} (b) TE_{20}

(c) TE_{00} (d) TE_{10}

UPPSC Poly. Lect. 2021 Paper - II

161. Hartley oscillator uses

(a) Positive feedback

(b) Negative feedback

(c) No feedback at all

(d) Both negative and positive feedback

UPPSC Ashram Paddhati PGT - 2021

162. Lorentz transformation changes to Galilean transformation, when -

(a) $v \approx c$ j (b) $v << c$

(c) $v = \frac{1}{c}$ (d) None of the above

UPPSC Poly. Lect. 2021 Paper - I

163. What is the speed of electromagnetic waves in free space? (Symbols have their usual meaning)

(a) $\sqrt{\frac{\mu_0}{\epsilon_0}}$ (b) $\sqrt{\frac{\epsilon_0}{\mu_0}}$

(c) $\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (d) $\sqrt{\mu_0 \epsilon_0}$

UPPSC GDC - 2021

UPPSC Poly. Lect. 2021 Paper - I

164. An electromagnetic wave going through vacuum is described by

$E = E_o \sin(kx - \omega t)$ and

$B = B_o \sin(kx - \omega t)$ then :

(a) $E_o k = B_o \omega$ (b) $E_o B_o = \omega k$

(c) $E_o \omega = B_o k$ (d) None of these

UKPSC Lecturer (Mains) 2020

165. Electromagnetic waves are produced by:

(a) a static charge

(b) a moving charge

(c) an accelerating charge

(d) chargeless particle

(UPPGT 2013)
UPPCS Pre 2007
UPPCS Pre 2005
UPPCS (Pre) 2002

166. In electromagnetic wave, electric and magnetic vectors oscillate perpendicular to each other. What is the phase difference of oscillation between the two vectors?

(a) Zero (b) $\pi/2$

(c) $\pi/4$ (d) π

UPPCS Pre 2007

167. The ray which can penetrate into 20 cm thick iron sheet, is

(a) γ-ray (b) α-ray

(c) β-ray (d) UV-ray

UPPCS Pre 2009

168. High frequency electromagnetic vibration can be converted into high frequency mechanical vibrations utilizing

(a) Piezocelectric effect only

(b) Magnetostricition effect only

(c) Magnetostricition effect and Piezoelectric effect both

(d) Kerr effect

UPPCS Pre 2009

UPPCS (Pre) 1997

169. Which of the following radiation is electromagnetic in nature?

(a) Alpha-rays (b) Beta-rays

(c) Gamma-rays (d) Ultrasonic rays

UPPCS Pre 2010

170. The angle between the equipotential surface and electric lines of force is :

(a) 0^0 (b) 180^0

(c) 90^0 (d) 45^0

UPPCS Pre 2010

171. The velocity of plane electromagnetic waves in a dielectric medium varies as: Where $\in_r$ is the relative permittivity of the medium.

(a) $\in_r$ (b) $\frac{1}{\in_r}$

(c) $\sqrt{\in_r}$ (d) $\frac{1}{\sqrt{\in_r}}$

UPPCS (Pre) 2003

172. Which of the following statements is not true of electromagnetic waves:

(a) They are transverse in nature

(b) They always travel with the speed of light

(c) Gamma rays are electromagnetic waves of very short wavelength

(d) Radio waves are electromagnetic waves of very high frequency

UPPCS (Pre) 2003

173. In an electromagnetic wave, the phase difference between the electric and the magnetic fields is:

(a) zero (b) $\pi/4$

(c) $\pi/2$ (d) π

(UPPGT 2013)

UPPCS (Pre) 2000

174. The correct order for different electromagnetic radiations in order of increasing energy is

(a) Radiowave, ultraviolet, X–rays, γ–rays

(b) γ–rays, X–rays, ultraviolet, radiowave

(c) radiowave, ultraviolet, γ–rays, X–rays

(d) ultraviolet, radiowave, γ–rays, X–rays

UPPCS Pre 2008

175. An electromagnetic wave has–

(a) electric vector only

(b) magnetic vector only

(c) electric and magnetic vectors perpendicular to each other

(d) neither the electric vector nor the magnetic vector

UPPCS (Pre) 1998

Answer Key

1. (d)	2. (a)	3. (b)	4. (b)	5. (a)	6. (c)	7. (c)	8. (c)	9. (c)	10. (a)
11. (d)	12. (b)	13. (c)	14. (d)	15. (a)	16. (c)	17. (a)	18. (a)	19. (c)	20. (d)
21. (d)	22. (c)	23. (b)	24. (b)	25. (a)	26. (a)	27. (d)	28. (d)	29. (a)	30. (c)
31. (d)	32. (a)	33. (a)	34. (a)	35. (d)	36. (a)	37. (b)	38. (c)	39. (a)	40. (c)
41. (a)	42. (a)	43. (d)	44. (a)	45. (d)	46. (a)	47. (c)	48. (d)	49. (c)	50. (d)
51. (b)	52. (a)	53. (b)	54. (a)	55. (c)	56. (a)	57. (c)	58. (d)	59. (b)	60. (a)
61. (b)	62. (c)	63. (c)	64. (c)	65. (d)	66. (d)	67. (d)	68. (a)	69. (a)	70. (c)
71. (d)	72. (a)	73. (c)	74. (a)	75. (d)	76. (b)	77. (c)	78. (a)	79. (c)	80. (a)
81. (a)	82. (d)	83. (a)	84. (a)	85. (d)	86. (c)	87. (a)	88. (a)	89. (c)	90. (c)
91. (c)	92. (c)	93. (d)	94. (d)	95. (d)	96. (d)	97. (d)	98. (c)	99. (c)	100. (a)
101. (a)	102. (c)	103. (d)	104. (c)	105. (a)	106. (a)	107. (b)	108. (a)	109. (c)	110. (c)
111. (c)	112. (d)	113. (d)	114. (d)	115. (b)	116. (a)	117. (a)	118. (d)	119. (c)	120.(b)
121. (a)	122. (a)	123. (c)	124. (c)	125. (c)	126. (c)	127. (d)	128. (d)	129. (c)	130. (a)
131. (c)	132. (a)	133. (b)	134. (a)	135. (b)	136. (d)	137. (a)	138. (b)	139. (a)	140.(b)
141. (c)	142. (d)	143. (b)	144. (a)	145. (b)	146. (d)	147. (a)	148. (a)	149. (b)	150. (a)
151. (b)	152. (c)	153. (b)	154. (c)	155. (a)	156. (a)	157. (c)	158. (a)	159. (c)	160.(d)
161. (a)	162. (b)	163. (c)	164. (a)	165. (c)	166. (a)	167. (a)	168. (c)	169. (c)	170. (c)
171. (d)	172. (d)	173. (a)	174. (a)	175. (c)					

07.

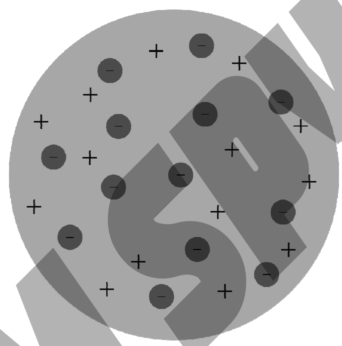
Modern Physics

(1) Atomic Physics

■ **Dalton's Atomic Theory:-** According to Dalton's Atomic Theory, all elements are consist of very small invisible particles, called atoms. Atoms of same elements are exactly same and atoms of different elements are different.

■ **Thomson's model of the atom:-**

The first model of atom was proposed by J.J. Thomson in 1898. According to this model, every atom is uniformly positive charged sphere of radius of order of 10^{-10}m. The positive charge of the atom is uniformly distributed throughout the volume of the atom and the negatively charged electrons are embedded in it like seeds in a watermelon. This model was picturesquely called plum pudding model of the atom. The atom is considered as a whole neutral.



Limitations of Thomson's atomic model:

- It could not explain the origin of spectral series of hydrogen and other atoms.
- It could not explain large angle scattering of α-particles.

■ **Rutherford's Atomic Model:**

Ernst Rutherford proposed an experiment of scattering of α-particles by atoms to investigate the atomic structure.

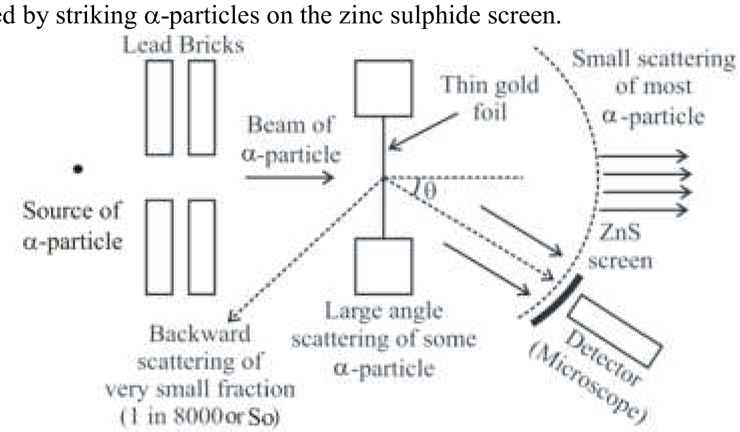
Rutherford's Experiment of Alpha – Particle:-

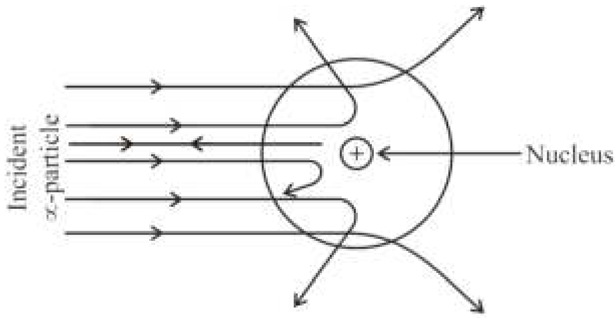
At the suggestions of Ernst Rutherford in 1911, Geiger and Marsden performed α-scattering experiment.

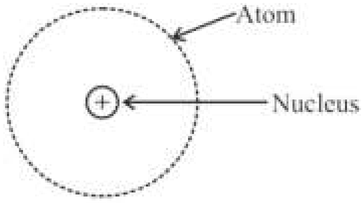
They directed a beam of 5.5 MeV α-Particles emitted from a $^{214}Bi_{83}$ radioactive source on a thin metal foil made of gold.

The beam was allowed to fall on a thin foil of gold of thickness 2.1×10^{-7}m. Alpha particles emitted by radioactive source were collimated into a narrow beam by passing through lead bricks. The scattered α-particles were received by a rotatable detector with zinc sulphide screen and a microscope.

Distribution of the number of scattered particles was studied as a function of angle of scattering by flashes or scintillations produced by striking α-particles on the zinc sulphide screen.







Size of the nucleus = 1 Fermi = 10^{-15}m,
Size of the atom= 1Å = 10^{-10}m

On the basis of this experiments, Rutherford made following observations :

- The entire positive charge and almost entire mass of the atom is concentrated at its centre in a very tiny region of the order of 10^{-15}m, called nucleus.
- The negatively charged electrons revolve around the nucleus in different orbits.
- The total positive charge on nucleus is equal to the total negative charge on electron. Therefore, atom is neutral.
- The centripetal force required by electron for revolution is provided by the electrostatic force of attraction between the electrons and the nucleus.

  For electron orbits:

  The electrostatic force of attraction, F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to a dynamically stable orbit in a hydrogen atom.

  $F_e = F_c$

  $\frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} = \frac{mv^2}{r}$ thus the relation between the orbit radius and the electron velocity is

  $$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

  The kinetic energy (E_K) and electrostatic potential energy (U) of the electron in hydrogen atom are,

  $E_K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}$ and $U = -\frac{e^2}{4\pi\varepsilon_0 r}$

  The negative sign in U signifies that the electrostatic force is in the r-direction. Thus the total energy E of as the electron in a hydrogen atom is.

  $$E = E_K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r}$$

  The total energy of electron is negative. This implies the fact that the electron is bound to the nucleus. If E were positive, an electron will not follow a closed orbit around the nucleus.
- Most of the α-particles passes through the gold foil without any deflection. This shows that most of the space in an atom is empty.
- Few α-particles got scattered, deflecting at various angles from 0 to π. This shows that atom has a small positively charged core called 'nucleus' at centre of atom, which deflects the positively charged α-particles at different angles depending on their distance from centre of nucleus.
- Very few α-particles (1 in 8000) suffers deflection of 180º.

Rutherford scattering formula:

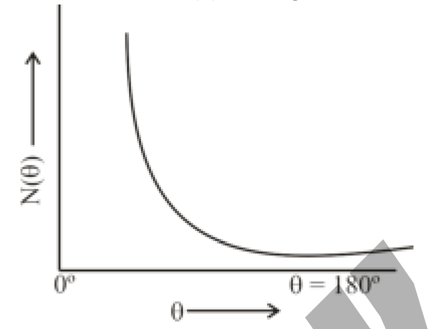
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 E_K{}^2 \sin^4\left(\frac{\theta}{2}\right)}$$

Where, $N(\theta)$= number of α-particles, N_i=Total number of α-particles reaching the screen, n=number of atoms per unit volume in the foil, Z= atomic number E_K = kinetic energy of the α-particles and t= foil thickness, r= target to detector distance, θ = scattering angle

Number of scattered α-particles,

$$N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

The graph between Number of particles scattered [$N(\theta)$] & angle of Scattering (θ) is shown in figure.



Impact parameter:

The perpendicular distance of the velocity vector of α-particles from the central line of the nucleus, when the particle is far away from the nucleus is called impact parameter. Impact parameter is denoted by b.

Rutherford had analytically calculated the relation between the impact parameter b and the scattering angle θ, given by,

$$\boxed{b = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2\cot\frac{\theta}{2}}{E_k} \Rightarrow b \propto \cot\left(\frac{\theta}{2}\right)}$$

Where, E_k is the kinetic energy of the incident alpha particle.

- If b = 0, $\Rightarrow$ Cot $\theta/2 = 0$ or $\theta/2 = 90^o$ or $\theta = 180^o$.
  i.e.In case of head on collision, the impact parameter is zero and the Alpha-particle rebounds back.
- If $b = \infty \Rightarrow \text{Cot}\ \frac{\theta}{2} = \infty$ or $\frac{\theta}{2} = 0^o$ or $\theta = 0^o$
  i.e.The alpha particle goes nearly undeviated for a large impact parameter.

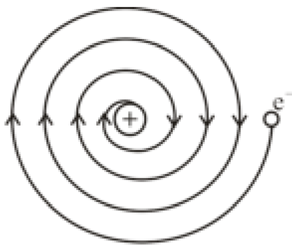
Distance of closest approach:

Distance of closest approach is given by,

$$\boxed{r_0 = \frac{2Ze^2}{4\pi\varepsilon_o E_k}}$$

Limitations of Rutherford's Model:

- **Stability of atom:** It could not explain stability of atom because according to classical electrodynamics theory an accelerated charged particle should continuously radiate energy. Thus an electron moving in a circular path around the nucleus should also radiate energy. As a result the electron should move into orbits of gradually decreasing radius and should ultimately fall into nucleus.



- **Line Spectrum:** Rutherford atomic model cannot explain atomic line spectrum.

■ Bohr's atomic model:

Bohr proposed a model for hydrogen atom which is also applicable for some lighter atom in which a single electron revolves around a stationary nucleus of positive charge Ze (called hydrogen like atom).

Bohr's model is based on the following postulates.

i. He postulated that an electron in an atom can move around the nucleus in certain circular stable orbits without emitting radiations.

ii. Bohr's found that the magnitude of the electron's angular momentum is quantized. i.e.

$$L = mv_n r_n = n\left(\frac{h}{2\pi}\right)$$

Where n = 1,2,3, ….. and each value of n corresponds to a permitted value of the orbit radius.
r_n = radius of n^{th} orbit and v_n = Corresponding Speed in n^{th} orbit.

iii. The radiation of energy occurs only when an electron jumps from one permitted orbit to another.
When electron jumps from higher energy orbit (E_2) to lower energy orbit (E_1) then difference of energies of these orbits, i.e. $E_2 - E_1$ emits in the form of photon.
But if electron goes from E_1 to E_2 it absorbs the same amount of energy.

Drawbacks of Bohr's atomic model:

- It is valid only for one electron atoms e.g. H, He^+, Li^{+2}, Na^{+1} etc.
- Orbits were taken as circular but according to Sommerfeld these are elliptical.
- Intensity of spectral lines could not be explained.
- Nucleus was taken as stationary but it also rotates on its own axis.
- It could not explain the fine structure in spectrum line.
- It does not explain the Zeeman Effect.
- It does not explain the doublets in the spectrum of some of the atoms like sodium (5890A° & 5896A°).

Bohr's orbits (for Hydrogen and H-like atoms) :

i. Bohr's orbits for H

(i). Electrons revolve around the nucleus in stationary circular orbits where centripetal acceleration is provided by the columbic attraction of protons on electrons as:-
Calculating radius (r_n) and speed (V_n) of n^{th} orbit we know that

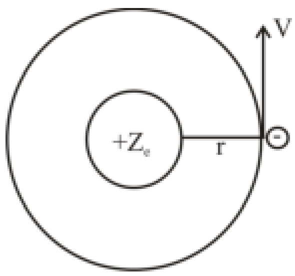
$$\frac{Ze^2}{4\pi\varepsilon_0 r} = mv^2 \text{ --------(i)} \qquad \text{and} \qquad L = mvr = \frac{nh}{2\pi} \text{------------(ii)}$$

From equation (i) & (ii) we have $r_n = \frac{n^2h^2\varepsilon_0}{Ze^2\pi m} = (0.53\text{Å})\frac{n^2}{Z}$

$$V_n = \frac{Ze^2}{2nh\varepsilon_0} = \left(2.18\times10^6\,m/sec\right)\frac{Z}{n}$$

For H- atoms, Z = 1
Radius of 1st orbit $r_1 = 0.53$Å
Speed of 1st orbit, $V_1 = 2.18\times10^6$m/sec

- Kinetic energy of electron in n^{th} orbit- $KE_n = \frac{1}{2}\left[\frac{mz^2e^4}{4\varepsilon_0^2n^2h^2}\right] \propto \frac{z^2}{n^2}$
- Potential energy of electron, in n^{th} orbit- $U_n = -\left[\frac{mz^2e^4}{4\varepsilon_0^2n^2h^2}\right] \propto \frac{z^2}{n^2}$
- Total Energy of Electron in n^{th} orbit- $E_n = KE_n + U_n = -\frac{1}{2}\left[\frac{mz^2e^4}{4\varepsilon_0^2n^2h^2}\right] \propto \frac{z^2}{n^2}$
- If we observe the relations carefully then- $E_n = -KE_n = \frac{U_n}{2}$



(ii). Speed of electron :

From the above relations, speed of electron in n^{th} orbit can be calculated as -

$$v_n = \frac{2\pi kZe^2}{nh} = \left(\frac{c}{137}\right)\frac{Z}{n} = 2.2\times10^6\frac{Z}{n}\,m/s$$

Where, c = Speed of light = $3\times10^8\,m/s$

(iii). Ionization energy and potential :

The energy required to ionize an atom is called ionization energy. It is the energy required to make the electron jumps from the present orbit to the infinite orbit.

Hence, $$E_{ionization} = E_\infty - E_n = 0 - \left(-13.6\frac{z^2}{n^2}\right) = +\frac{13.6z^2}{n^2}eV$$

For H-atom in the ground sate

$$E_{ionization} = \frac{+13.6(1)^2}{n^2} = 13.6eV \qquad (n=1)$$

The potential through which an electron needs to be accelerated so that it acquires energy equal to the ionization energy is called ionization potential.

$$V_{ionization} = \frac{E_{ionization}}{e}$$

(iv). Energy level for Hydrogen atom:

The description of the energy of the electron in different orbits around the nucleus is called energy level.

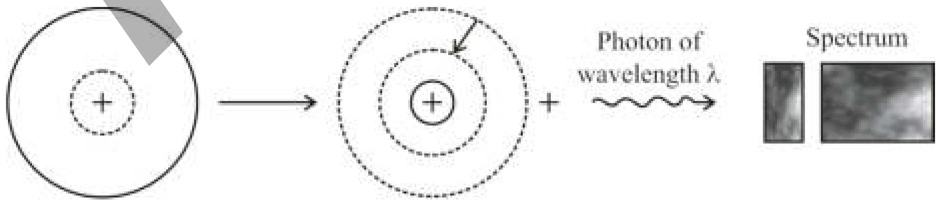
Energy level of hydrogen is given below-

Principle quantum number	**Orbit**	**Excited state**	**Energy for H-atom**
$n=\infty$	Infinite	Infinite	0 eV
$n=4$	Fourth	Third	–0.85eV
$n=3$	Third	Second	–1.51eV
$n=2$	Second	First	–3.4eV
$n=1$	First	Ground	–13.6eV

- **Hydrogen spectrum and spectral series :**

When hydrogen atom is excited, it returns to its normal unexcited state by emitting the energy it had absorbed earlier. This energy is given out by the atom in the form of radiations of different wavelengths as the electron jumps down from a higher to a lower orbit.

Transition from different orbits cause different wave lengths,



The spectral lines arising from the transition of electron forms a spectra series.

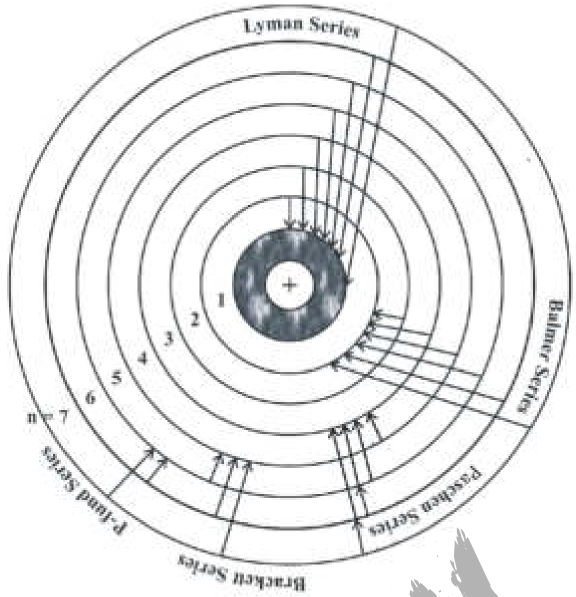
(i). Mainly there are five series and each series is named after it's discoverer as **Lyman series, Balmer series, Paschen series**, **Brackett series** and **p-fund series.**

(ii). According to the Bohr's theory the wavelength of the radiations emitted from hydrogen atom is given by.

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] \Rightarrow \lambda = \frac{n_1^2n_2^2}{(n_2^2-n_1^2)R} = \frac{n_1^2}{\left(1-\frac{n_1^2}{n_2^2}\right)R}$$

Where n_2 = outer orbit (electron jumps from this orbit). n_1 = inner orbit (electron falls in this orbit). The wavelength of spectral lines increases with the increases of order of the series i.e.,

$$\lambda_{\text{p-fund}} > \lambda_{brackett} > \lambda_{paschen} > \lambda_{Balmer} > \lambda_{Lyman}$$



(iii). First line of the series is called first member for this line wavelength is maximum (λ_{max})
For maximum wavelength if $n_1 = n$ then $n_2 = n + 1$

$$\lambda_{max} = \frac{n^2(n+1)^2}{(2n+1)R}$$

(iv). Last line of the series is called series limit for this line wavelength is minimum (λ_{min}).
For minimum wavelength $n_2 = \infty$ $n_1 = n$

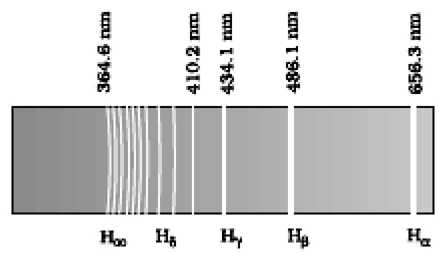
$$\lambda_{min} = \frac{n^2}{R}$$

(v). The ratio of first member and series limit can be calculated as :

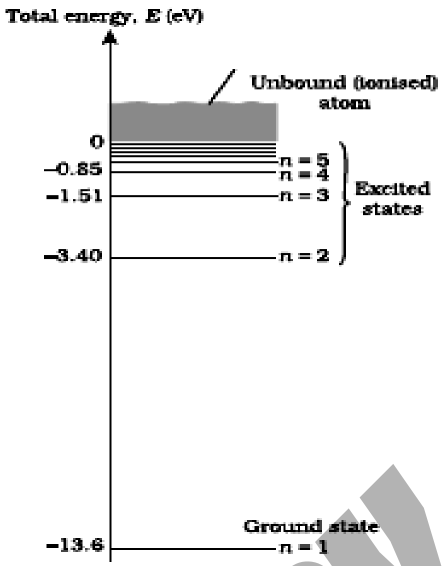
$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{(n+1)^2}{(2n+1)}$$

Spectral series	**Transition**	λ_{max}	λ_{min}	$\frac{\lambda_{max}}{\lambda_{min}}$	**Region**
1. Lyman series	$n_2 = 2,3,4,\ldots\infty$ $n_1 = 1$	$\frac{4}{3R}$	$\frac{1}{R}$	$\frac{4}{3}$	Ultraviolet
2. Balmer series	$n_2 = 3,4,5,\ldots\infty$ $n_1 = 2$	$\frac{36}{5R}$	$\frac{4}{R}$	$\frac{9}{5}$	Visible region
3.Paschen series	$n_2 = 4,5,6,\ldots\infty$ $n_1 = 3$	$\frac{144}{7R}$	$\frac{9}{R}$	$\frac{16}{7}$	Near Infrared region
4. Brackett series	$n_2 = 5,6,7,\ldots\infty$ $n_1 = 4$	$\frac{400}{9R}$	$\frac{16}{R}$	$\frac{25}{9}$	Far Infrared region
5. P-fund series	$n_2 = 6,7,8,\ldots\infty$ $n_1 = 5$	$\frac{900}{11R}$	$\frac{25}{R}$	$\frac{36}{11}$	Far Infrared region

■ **Balmer series in the emission spectrum of hydrogen:**



■ **The energy level diagram for the hydrogen atom.**



(2) Nuclear Physics

■ **Nucleus:**

- Rutherford's α-scattering experiment established that the mass of atom is concentrated with small positively charged region at the centre which is called 'nucleus'.
- The total number of protons (Z) termed as atomic number.
- Nuclei are made up of protons and neutrons. The total number of neutrons (N) and protons (Z) in a nucleus is called mass number A. So,

$$\boxed{A = Z + N}$$

- Neutrons and proton's when described collectively are called nucleons.
- A single nuclear species having specific value of both Z and N is called a nuclide. Nuclides are represented as $_{Z}X^{A}$; where X denotes the chemical symbols of the elements.

■ **Proton:**

Proton is a stable subatomic particle, symbol p, H^+ or ^1_1H with a positive charge '+1e' elementary charge.

Charge = 1.6×10^{-19}C

Mass = 1.6726×10^{-27} kg = 1 amu

■ **Neutron:**

Neutron is a fundamental particle which is essential constituent of all nuclei except that of hydrogen atom. It was discovered by Chadwick. A free neutron outside the nucleus is unstable and decays into proton and electron.

$$_0n^1 \rightarrow \underset{\text{Proton}}{_1H^1} + \underset{\text{Electron}}{_{-1}\beta^0} + \underset{\text{Antineutrino}}{\bar{V}}$$

- The charge of neutron : neutral (0) , symbol → ^1_0n
- The mass of neutron : 1.6750×10^{-27} kg
- It's spin angular momentum : $\frac{1}{2}\times\left(\frac{h}{2\pi}\right)J-S$
- It's magnetic moment : 9.57×10^{-27} J/tesla
- It's half life : 12 minutes
- Penetration power : High
- Neutrons are of two types slow neutron and fast neutron. Both are fully capable of penetrating a nucleus and causing artificial disintegration.

Classification of Nuclei:

The nuclei have been classified on the basis of the number of protons (atomic number) or the total number of nucleons (mass number) as follows:

	Condition	Examples
Isotopes	$Z \to$ equal $A \to$ different (same atomic number)	Isotopes of Hydrogen: $_1H^1$, $_1H^2$, $_1H^3$ Isotopes of Oxygen: $_8O^{16}$, $_8O^{17}$, $_8O^{18}$ Isotopes of Chlorine: $_{17}C\ell^{35}$, $_{17}C\ell^{37}$, Isotopes of Helium: $_2He^3$, $_2He^4$ Isotopes of Uranium: $_{92}U^{235}$, $_{92}U^{238}$
Isobars	$Z \to$ different $A \to$ equal (same mass number)	$_1H^3$ and $_2He^3$ $_6C^{14}$ and $_7N^{14}$ $_8O^{17}$ and $_9F^{17}$
Isotones	$(A - Z) \to$ same $A \to$ different $Z \to$ different (same neutrons)	$_4Be^9$ and $_5B^{10}$ $_7N^{14}$ and $_6C^{13}$ $_8O^{18}$ and $_9F^{19}$ $_3Li^7$ and $_4Be^8$ $_1H^3$ and $_2He^4$

Note: - Average Atomic = $M_1 \times n_1 + M_2 \times n_2$

Where,

M_1 = Mass of 1 isotope

M_2 = mass of 2 isotope

n_1 = % of 1 isotope

n_2 = % of 2 isotope

- **Nuclear radius:**
  Experimental results indicates that the nuclear radius is proportional to $A^{1/3}$, where A is the mass number of nucleus. i.e. $R \propto (A)^{1/3} \Rightarrow R = R_o (A)^{1/3}$, where $R_o = 1.2 \times 10^{-15} m = 1.2 fm$.
- **Nuclear volume:**
  The volume of nuclear is given by :
  $$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A \Rightarrow V \propto A$$
- **Nuclear density:**
  Mass per unit volume of a nucleus is called nuclear density.

$$\text{Nuclear density } (\rho) = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{mA}{\frac{4}{3}\pi\left(R_o A^{1/3}\right)^3}$$

$$= \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi R_0^3 A} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi \times \left(1.2 \times 10^{-15}\right)^3} = 2.38 \times 10^{17}\ kg/m^3$$

Where,

m = Average mass of a nucleon (mass of proton + mass of neutron) = 1 amu = 1.66×10^{-27} kg

and mA = Mass of nucleus

- **Number of nucleon per unit volume :**

$$= \frac{\text{Density of Nucleus}}{\text{Mass of one Nucleus}} = \frac{2.38 \times 10^{17}}{1.67 \times 10^{-27}} \cong 10^{44}\ \frac{\text{Nucleons}}{m^3}$$

Note:-

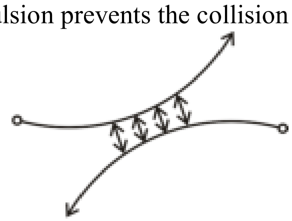
(i) Nuclear density is independent of mass number A. It is same for nuclei of all atoms.

(ii) Nuclear density is 10^{14} times more than that of water i.e. 10^3 kg/m^3

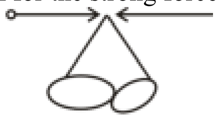
■ Nuclear force :

Forces that keep the nucleons bound in the nucleus are called nuclear forces.

(A) At low speeds, electromagnetic repulsion prevents the collision of nuclei



(b) At high speeds, nuclei came close enough for the strong force to bind them together.



- Nuclear forces are short range forces, these do not exist at large distances greater than 10^{-15}m.
- Nuclear forces are the strongest forces in nature.
- These are attractive force and cause stability of the nucleus.
- These forces are charge independent.
- Nuclear forces are non-central force.

■ Atomic Mass Unit (AMU) :

i. In nuclear physics, a convenient unit of mass is the unified atomic mass unit abbreviated amu or u.

ii. The amu is defined as $\frac{1}{12}$th mass of a ${}_6C^{12}$ atom.

iii. 1 amu (or 1u) = 1.6605402×10^{-27}kg

■ Masses of electrons, proton and neutrons :

Mass of electron (M_e) = 9.1×10^{-31} kg = 0.0005486 amu
Mass of proton (M_p) = 1.6750×10^{-27} kg = 1.007276 amu
Mass of neutron (M_n) = 1.6750×10^{-27}kg = 1.0078 amu
Mass of Hydrogen atom ($M_e + M_p$) = 1.6729×10^{-27}kg = 1.0078 amu
The energy associated with a nuclear process is usually large, of the order of Mev.
According to Einstein, mass and energy are inter convertible. The Einstein's mass energy relationship is given by:

$\boxed{E = mc^2}$

If m = 1amu, c = 3×10^8 m/s then E= 931 MeV
i.e. 1 amu (or 1u) = 931 MeV

$(1\ u)\ c^2 = 931\ \text{MeV} \Rightarrow 1\text{amu} = 931\ \frac{\text{MeV}}{c^2}$ or $c^2 = 931\frac{\text{MeV}}{u}$

■ Neutral atomic masses for some light nuclides–

Element and Isotope	Atomic mass (u)	For calculation (u)
Hydrogen $({}_1H^1)$	1.007825	1
Deuterium $({}_1H^2)$	2.014102	2
Tritium $({}_1H^3)$	3.016049	3
Helium $({}_2He^3)$	3.016029	3
Helium $({}_2He^4)$	4.002603	4
Lithium $({}_3Li^7)$	7.016004	7
Beryllium $({}_4Be^9)$	9.012182	9
Carbon $({}_6C^{12})$	12.000000	12
Nitrogen $({}_7N^{14})$	14.003074	14
Oxygen $({}_8O^{16})$	15.994915	16

■ Mass defect (Δm) :

It is found that the mass of a nucleus is always less than the sum of masses of it's constituent nucleons in free state.
This difference in masses is called mass defect.
Hence mass defect(Δm) = Sum of masses of nucleons – Mass of nucleus
$= \{Zm_p + (A - Z)m_n\} - M$
Where,
m_p = mass of proton
m_n = Mass of neutron
m_e = mass of electron
M = Mass of Nucleus
Z = Atomic number
A = Mass number

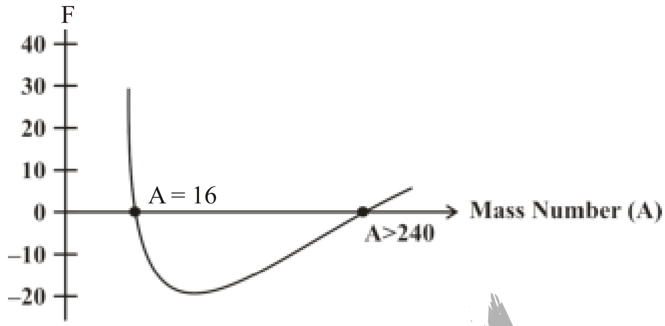
■ **Packing fraction**

Mass defect per nucleon is called packing fraction (f) : $\frac{\Delta m}{A}$

Where M = Mass of nucleus
A = Mass number

Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, longer is the stability of the nucleus. Packing fraction may be of positive, negative or zero.

At A = 16, f = 0



■ **Binding Energy:**

- The neutrons and Protons in a stable nucleus are held together by nuclear forces and energy is needed to pull them infinitely apart. This energy is called the binding energy of the nucleus.
  If Δm is mass defect then according to Einstein's mass energy relation.
- Binding Energy = $\Delta mc^2 = [\{m_pZ + m_n(A - Z)\} - M]c^2$
  The binding energy expressed in Joule, because Δm is measured in kg.
  If Δm is measured in amu then binding energy = Δm amu = $[\{M_pZ + M_n(A - Z)\} - M]$amu = $\Delta m \times 931$ MeV

■ **Binding Energy per nucleon :**

The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

Binding energy per nucleon. $= \frac{\text{Total binding energy}}{\text{Mass number (i.e. total number of nucleons)}} = \frac{\Delta m \times 931}{A} \frac{\text{MeV}}{\text{Nucleon}}$

Binding energy per nucleon $\propto$ stability of nucleus.

■ **Nuclear reactions:**

The process by which the identity of a nucleus is changed when it is bombarded by an energetic particle is called nuclear reaction the general expression for the nuclear reaction is as follows :

X	+	a	$\longrightarrow$	c	$\longrightarrow$	y	+	b	+	Q
(Parent nucleus)		(Incident Particle)		(Compound nucleus)		(Product nucleus)		(Daughter nucleus)		(Energy)

Here X and a are known as reactants and y and b are known as products. This reaction is known as (a, b) reaction and can be represented as X(a,b)y.

■ **Q Value or energy of nuclear reaction:**

The energy absorbed or released during nuclear reaction is known as Q value of nuclear reaction.

Q Value = (Mass of reactants – Mass of Products) C^2 Joules
= (Mass of reactants – Mass of Products) amu.

- If $Q < 0$, then nuclear reaction will be **endothermic.**
- If $Q > 0$, then nuclear reaction will be **exothermic.**

■ **Law of conservation in nuclear reactions:**

i. Conservation of mass number and charge number.

ii. Conservation of momentum: Linear momentum/angular momentum of particles before reaction is equal to the linear/angular momentum of the particles after the reaction. That is $\sum p = 0$

iii. Conservation of Energy: Total energy before the reaction is equal to total energy after the reaction. Term Q is added to balance the total energy of the reaction.

■ **Common nuclear reaction:**

The nuclear reactions lead to artificial transmutation of nuclei. Rutherford was the first to carry out artificial transmutation of nitrogen to oxygen in the year 1919. It is called (α, p) reaction which is give below,

$${}_2He^4 + {}_7N^{14} \rightarrow {}_9F^{18} \rightarrow {}_8O^{17} + {}_1H^1$$

Some other nuclear reactions are (p, n)

(p,n) reaction $\Rightarrow {}_1H^1 + {}_5B^{11} \rightarrow {}_6C^{12} \rightarrow {}_6C^{11} + {}_0N^1$

(P, α) reaction $\Rightarrow {}_1H^1 + {}_3Li^7 \rightarrow {}_4Be^8 \rightarrow {}_2He^4 + {}_2He^4$

(p, γ) reaction $\Rightarrow {}_1H^1 + {}_6C^{12} \rightarrow {}_7N^{13} \rightarrow {}_7N^{13} + \gamma$

(n, p) reaction $\Rightarrow {}_0n^1 + {}_7N^{14} \rightarrow {}_7N^{15} \rightarrow {}_6C^{14} + {}_1H^1$

(γ, n) reaction $\Rightarrow \gamma + {}_1H^2 \rightarrow {}_1H^1 + {}_0n^1$

■ **Nuclear Fission:**

- The Process of splitting of a heavy nucleus into two lighter nuclei of comparable masses with liberation of energy is called nuclear fission.
- The phenomenon of nuclear fission was discovered by Scientist **Ottohann** and **F. Strassman** and was explained by **N. Bohr** and **J.A. Wheeler** on the basis of liquid drop model of nucleus.
- **Fission reaction of U^{235}:**

$$ {}_{92}U^{235} + {}_0n^1 \rightarrow \underset{\text{(Unstable nucleus)}}{{}_{92}U^{236}} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_0n^1 + Q $$

- The energy released in U^{235} fission is about 200 MeV or 0.8 MeV per nucleon.
- **Nuclear Reactor:** The nuclear reactor is a device in which nuclear fission can be carried out through a sustained and a controlled chain reaction. It is also called an atomic pile. It is thus a source of controlled energy which is utilised for many useful purposes.

■ **Nuclear fusion:**

- In nuclear fusion two or more than two lighter nuclei combine to form a single heavy nucleus. The mass of single nucleus so formed is less than the sum of the masses of parent nuclei. This difference in mass results in the release of tremendous amount of energy.
- For fusion high pressure ($\approx 10^6$ atm) and high temperature (of the order of 10^7K to 10^8K) is required and so the reaction is called thermonuclear reaction.
- Here are three examples of energy – liberating fusion reactions, written in terms of the neutral atoms. Together the reactions make up the process called the Proton – Proton chain.

$$ {}_1H^1 + {}_1H^1 \rightarrow {}_1H^2 + \beta^+ + Ve $$
$$ {}_1H^2 + {}_1H^1 \rightarrow {}_2He^3 + \gamma $$
$$ \underline{{}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + {}_1H^1 + {}_1H^1} $$
$$ 4{}_1H^1 \rightarrow {}_2He^3 \rightarrow 2\beta^+ + 2\gamma + 26.73\text{MeV} $$

Note: The Proton chain takes place in the interior of the sun and other stars.

■ **Radioactivity:** The phenomenon of spontaneous emission of radiations by heavy elements is called radioactivity and the elements which show this phenomenon are called radioactive elements.

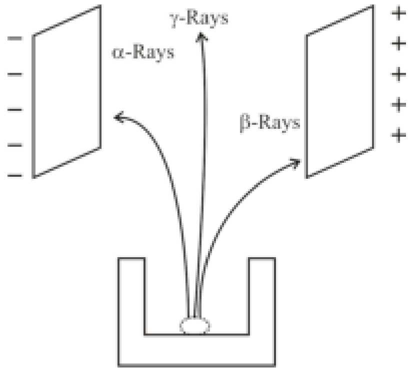
- Radioactivity was discovered by **Hennery Becquerel** in Uranium Salt in the year 1896.
- After the discovery of radioactivity in uranium, **Pierre curie** and **Madame curie** discovered a new radioactive element called radium,

Example of radioactive Elements: - Thorium, Polonium, Neptunium etc.

- All the elements with atomic number **(Z) > 82** are naturally radioactive.
- The conversion of lighter elements into radioactive elements by the bombardment of fast moving particles is called artificial or induced radioactivity.
- Radioactivity is a nuclear event and not Atomic.

■ **Nuclear radiations:**

According to Rutherford's experiment when a sample of radioactive substance is put in a lead box and allow the emission of radiation through a small hole only when the radiation enters into the external electric field, they split into three parts (α-rays, β-rays & γ-rays)



- **α- Decay-** Alpha particle is a He nucleus

  $${}^{A}_{Z}X \longrightarrow {}_{2}\alpha^{4} + {}^{A-4}_{Z-2}\gamma$$

  e.g.

  $${}^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}^{4}_{2}He \quad \left(\frac{n}{p}\text{ increases}\right)$$

  In any decay the sum of mass number & atomic number remains constant.

- **β Decay -** electron emission

  $$\left({}^{0}_{-1}e\right)\left({}^{0}_{-1}\beta\right)$$

  $${}^{A}_{Z}X \longrightarrow {}^{0}_{-1}\beta + {}^{A}_{Z+1}\gamma + \bar{\nu} \qquad \rightarrow (\beta \text{ minus decay})$$

  e.g.

  $${}^{14}_{6}C \longrightarrow {}^{0}_{-1}\beta + {}^{14}_{7}N + \bar{\nu}$$

  $${}^{A}_{Z}X \longrightarrow {}^{0}_{+1}\beta + {}^{A}_{Z-1}\gamma + \nu \qquad \rightarrow (\beta \text{ plus decay})$$

  e.g.

  $${}^{10}_{6}C \longrightarrow {}^{10}_{1}B + {}^{0}_{1}\beta + \nu$$

  Actually β decay is conversion of a neutron to proton inside the nucleus.

- **γ- Decay -** Electromagnetic wave.

  $${}^{A}_{Z}X \longrightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma$$

  After α or β emission the nucleus is left in an excited state, undergoes to a lower energy state by emitting a high energy photon called the γ - ray photon.

Property	Alpha	Beta	Gamma
Symbol	α, ${}_{2}He^{4}$, He^{2+}	β, ${}_{-1}e^{0}$, ${}_{-1}\beta^{0}$	γ, ${}_{0}\gamma^{0}$
Charge	+2 (Positive)	–1 (Negative)	0 (Neutral)
Speed	slow	fast	very fast (speed of light)
Ionizing ability	very high	medium	very slow
Penetrating power	low	medium	high
Stopped by	paper	Aluminum foil	Lead sheet

■ **Radioactive disintegration:**

According to Rutherford and Soddy, **Law for radioactive decay** is as follows –

"At any instant the rate of decay of radioactive atoms is proportional to the number of atoms present at that instant i.e.

$$\mathbf{\frac{-dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = -\lambda N \;\Rightarrow N = N_o\, e^{-\lambda t}}$$

It terms of mass $M = M_o\, e^{-\lambda t}$

Where, N = Number of atoms remained un-decayed after time t,

N_o = Number of atoms present initially (i.e. at t = 0) M = mass of radioactive nuclei at time t

$N_o - N$ = Number of disintegrated nucleus in time t, M_o = mass of radioactive nuclei at time t = 0

$$\frac{dN}{dt} = \text{Rate of decay,}$$

λ = Decay constant or disintegration, Constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

(3) Electrons, Photons, and X-rays

■ **Cathode rays:** Cathode ray was discovered by Sir William Crooke. They are streams of fast moving electrons. They can be produced by using a discharge tube containing gas at a low pressure of the order of 10^{-2} mm of Hg. The cathode rays in the discharge tube are the electrons produced due to ionisation of gas and are emitted by cathode due to collision of positive ions.

Properties of Cathode Rays:

- Cathode rays are emitted normally from the cathode surface. Their direction is independent of the position of the anode.
- Cathode rays exert mechanical force on the objects they strike.
- Cathode rays produce heat when they strike a metal surface.
- Cathode rays produce fluorescence
- Cathode rays are deflected by electric and magnetic field.
- Cathode rays can ionize the gases through which they are passed.
- Cathode rays can penetrate through thin foils of metal.
- Cathode rays are found to have velocity ranging $\left(\frac{1}{30}\right)^{th}$ to $\left(\frac{1}{10}\right)^{th}$ of velocity of light.

■ **J.J. Thomson's Experiment :**

- It's working is based on the fact that if a beam of electron is subjected to the crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, it experiences a force due to each field. In case the forces on the electrons in the electron beam due to these fields are equal and opposite, the beam remains un-deflected.
- When no field is applied the electron beam produces illumination at point P.
- In the presence of any field electron beam is deflected up or down (Illumination (at p' or p")) if both the fields are applied simultaneously and adjusted such that electron beam passes un-deflected and produces illumination at point p.

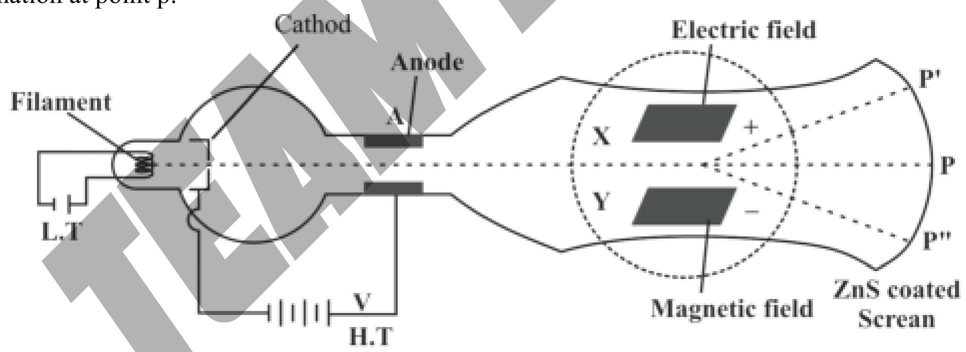


Fig : J.J. Thomson's cathode ray tube experiment

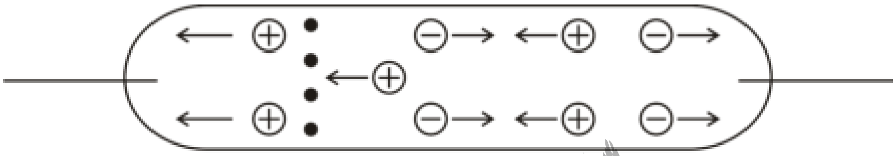
- In this case, electric force = magnetic force
  $\Rightarrow eE = evB$
  $\Rightarrow v = \frac{E}{B}$; v = velocity of electron
- As electron beam is accelerated from cathode to anode its loss in potential energy appears as gain in the K.E. at the anode. If suppose V is the potential difference between cathode and anode then, loss in potential energy eV. And gain in K.E. at anode will be K.E. = $\frac{1}{2}mv^2 \Rightarrow \frac{e}{m} = \frac{v^2}{2V} \Rightarrow \frac{e}{m} = \frac{E^2}{2VB^2}$

  Thomson found, $\frac{e}{m} = 1.77 \times 10^{11}\, C/kg$
- If one include the relativistic variation of mass with speed $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$, then specific charge of an electron decreases with increase in its velocity.

- The deflection of an electron in a electric field is given by $y = \frac{1}{2}\left(\frac{eE}{m}\right).\frac{l^2}{v^2}$; where l = Length of each plate, y = deflection of electron in the field region, v = speed of electron.

■ **Positive Rays or Canal Rays :**

Canal rays were discovered by Goldstein when potential difference is applied across the electrodes of a discharge tube (10^{-3}mm of Hg), electrons are emitted from the perforated cathode. As they move towards anode, they gain energy these energetic electrons when collide with the atoms of the gas in the discharge tube, they ionize the atoms. The positive ions when reach the cathode, some pass through the holes in the cathode and a faint luminous glow comes out from each holes on the backside of the cathode. It is called positive rays, which are coming out from the holes

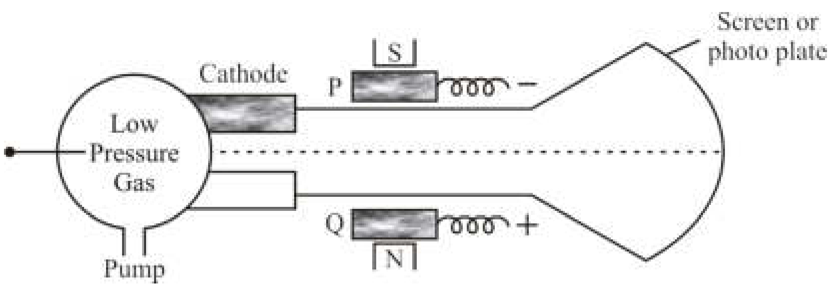


Positive Rays

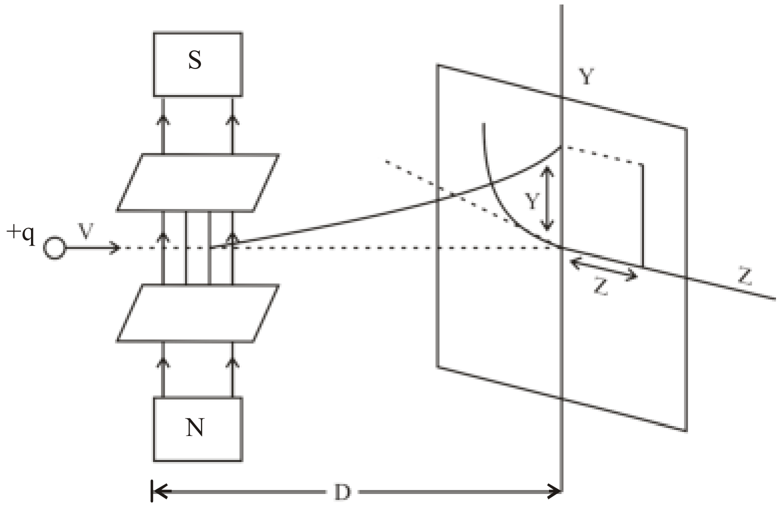
- Positive rays are positive ions having same mass if the experimental gas does not have isotopes. However if the gas has isotopes then positive rays are group of positive ions having different masses.
- They travel in straight lines and cast shadows of objects placed in their path but the speed of the positive rays is much smaller than that of cathode rays.
- They are deflected by electric and magnetic fields but the deflections are smaller than that of cathode rays.
- They show a spectrum of velocities. Different positive ions move with different velocities being heavy, their velocity is much less than that of cathode rays.
- q/m ratio of these rays depends on the nature of the gas in the tube. q/m for hydrogen is maximum.
- They carry energy and momentum the kinetic energy of positive rays is more than that of cathode rays.
- The value of charge on positive rays is an integral multiple of electronic charge.
- They cause ionization (which is much more than that produced by cathode rays.)
- $\left(\frac{q}{m}\right)$ of positive rays is much smaller than that of cathode rays and is not a universal constant.

■ **Thomson's Mass spectrograph:**

- It is used to measure atomic masses of various isotopes in gas this is done by measuring q/m of singly ionized positive ion of the gas.



- The positive ions are produced in the bulb at the left hand side. These ions are accelerated towards cathode. Some of the positive ions pass through the fine hole in the cathode. This fine ray of positive ions is subjected to electric field E and magnetic field B and then allowed to strike a fluorescent screen $(\vec{E} \parallel \vec{B}$ but $\vec{E}$ or $\vec{B} \perp \vec{V})$.
- If the initial motion of the ions is in +x direction and electric and magnetic fields are applied along +y axis then force due to electric field is in the direction of y-axis and due to magnetic field it is along z – direction.



- The deflection due to electric field alone.

$$y = \frac{qELD}{mv^2} \qquad ...(i)$$

The deflection due to magnetic field alone

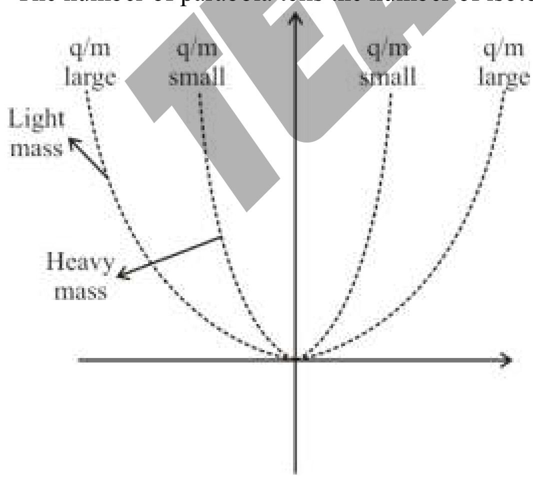
$$Z = \frac{qBLD}{mv} \qquad ...(ii)$$

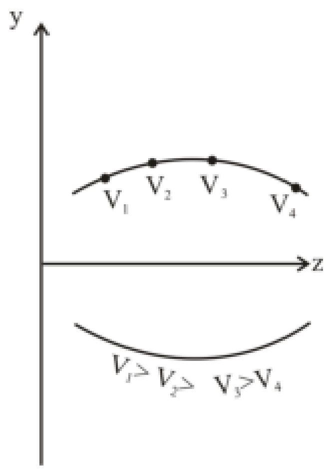
From eq^n (i) and (ii), $Z^2 = K\left(\frac{q}{m}\right)y$ i.e. equation of parabola.

Where, $K = \frac{B^2LD}{E}$;

Note:

- All the positive ions of same q/m moving with different velocity lie on the same parabola. Higher is the velocity lower is the value of y and z. The ions of different specific charge will lie on different parabola.
- The number of parabola tells the number of isotopes present in the given ionic beam.





■ Matter waves (De-Broglie waves)

According to De-Broglie a moving material particle sometimes acts as a wave and sometimes as a particle.
The wave associated with moving particle is called matter wave or De-Broglie wave and it propagates in the form of wave packets with group velocity.

i. De-Broglie wavelength : According to de-Broglie theory the wave length of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \frac{1}{p} \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}}$$

Where h = Planck's constant = 6.62×10^{-34} Jsec.
m = Mass of the particle
v = Speed of the particle
E = Kinetic energy of particle

- The smallest wavelength whose measurement is possible is that of γ- rays.
- The smallest wavelength of matter waves associates with the microscopic particles like electron, proton, neutron, α-particle etc. is of the order of 10^{-10}m.

ii. De-Broglie wavelength associated with the charged particles:

The energy of a charged particle accelerated through potential difference V is

$$E = \frac{1}{2}mv^2 = qV$$

Hence, De-Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$\lambda_{Electron} = \frac{12.27}{\sqrt{V}} A^o, \quad \lambda_{proton} = \frac{0.286}{\sqrt{V}} A^o, \quad \lambda_{Deuteron} = \frac{0.202}{\sqrt{V}} A^o, \quad \lambda_{\alpha\text{-particle}} = \frac{0.101}{\sqrt{V}} A^o$$

iii. De-Broglie wavelength associated with uncharged particle.

For Neutron De-Broglie wavelength is given as :

$$\lambda_{Neutron} = \frac{0.286 \times 10^{-10}\text{ m}}{\sqrt{E(\text{in eV})}} = \frac{0.286}{\sqrt{E(\text{in eV})}} A^o \qquad (1Å = 10^{-10}\text{ m})$$

- The De- Broglie wavelength of neutron in thermal equilibrium with heavy water at temperature T (Kelvin) and a mass m is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(KE)}}$$

$$= \frac{h}{\sqrt{2m \times \frac{3}{2}kT}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

- The De-Broglie wavelength of neutrons in thermal equilibrium at temperature T is -

$$\boxed{\lambda = \frac{h}{\sqrt{2mkT}} = \frac{30.8}{\sqrt{T}} Å}$$

iv. Ratio of wave length of photon and electron :

- The wavelength of a photon of energy E is given by $\lambda_{ph} = \frac{hc}{E}$
- The wavelength of an electron of kinetic energy K is given by $\lambda_e = \frac{h}{\sqrt{2mK}}$
- The ratio of wavelength of photon & electron $\frac{\lambda_{ph}}{\lambda_e} = \frac{c}{E}\sqrt{2mk} = \sqrt{\frac{2mc^2k}{E^2}}$
- For same energy (K =E) $\boxed{\frac{\lambda_{ph}}{\lambda_e} = c\sqrt{\frac{2m}{E}}}$

Characteristics of matter waves :

i. Matter wave represents the probability of finding a particle in space.
ii. Matter waves are not electromagnetic in nature.
iii. Particle observation of matter waves is possible only when the De-Broglie wavelength is of the order of the size of the particles.
iv. Electron microscope works on the phenomena of De-Broglie waves.
v. The phase velocity of the matter waves can be greater than the speed of the light.
vi. Matter waves can propagate in vacuum hence they are not mechanical waves.
vii. The number of De-Broglie waves associated with n^{th} orbital electron is n.

■ Phase Velocity (Vp) and Group Velocity (Vg)

- **Phase Velocity (Vp) :-** The phase velocity of a matter wave of De Broglie wave represents a velocity of individual wave of particle and is defined as the ratio of angular frequency to the wave vector (K)

$$\text{Phase velocity (Vp)} = \frac{\text{Angular Frequency}}{\text{Wave Vector}} = \frac{\omega}{K} = \mu$$

Where μ is a Wave velocity.

- **Group Velocity (Vg):-** The group velocity of a matter wave or De-Broglie wave gives the idea of the velocity of wave group and is defined as the ratio of change of angular frequency with wave vector.

$$\text{Group velocity } Vg = \frac{d\omega}{dK}$$

- Product of Phase velocity and Group velocity- $Vg \times Vp = c^2$
- Phase velocity & Group velocity in terms of Energy and Momentum-

$$Vp = E/p \text{ and } Vg = d\omega/dK$$

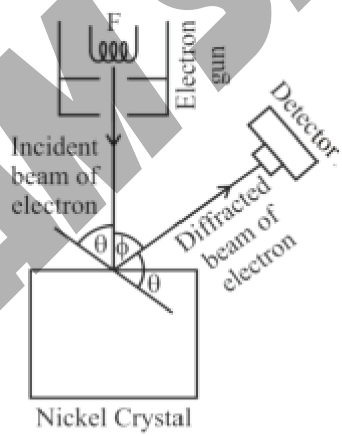
$$Vg = \left[\frac{E}{p} - \lambda\frac{d(E/p)}{d\lambda}\right]$$

- Relation between the Phase velocity and Group velocity

$$Vg = Vp - \lambda\frac{dVp}{d\lambda}$$

■ Davisson and Germer Experiment:

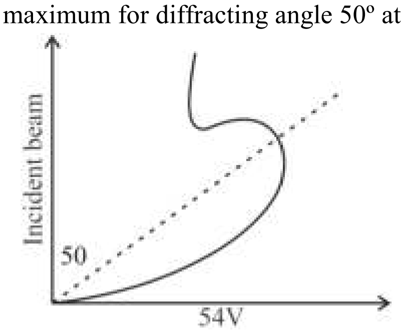
It is used to study the scattering of electron from a solid or to verify the wave nature of electron. A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. Ni crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.



The diffracted beam of electrons is received by the detector which can be positioned at any angle by rotating it about the point of incidence.

The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.

- According to classical physics, the intensity of scattered beam of electron at all scattering angle will be same but Davisson and Germer, found that the intensity of scattered beam of electrons was not the same but different at different angles of scattering it is maximum for diffracting angle 50º at 54 volt potential difference.



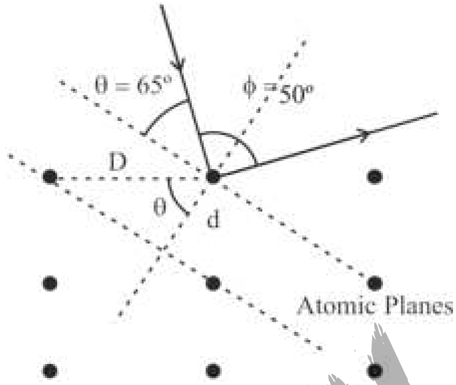
- If the De-Broglie waves exist for electrons then these should be diffracted as x-rays, so the **Bragg's formula**,

$$\mathbf{2d\ sin\theta = n\lambda.}$$

We can determine the wave length of these waves

Where, d = distance between diffracting planes

$\theta = \frac{(180^0 - \phi)}{2}$ = glancing angle for incident beam = Bragg's angle.



The distance between diffracting planes in Ni-Crystal for this experiment is d = 0.91 Å and the Braggs angle = 65°

This gives for n = 1, $\lambda = 2 \times 0.91 \times 10^{-10}$, sin 65° = 1.65 Å

Now the De-Broglie wavelength can also be determined by using the formula.

$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67Å$ thus the De-Broglie hypothesis is verified.

The Bragg's formula can be rewritten in the form containing inter-atomic distance D and angle ϕ

$\because \quad \theta = 90 - \frac{\phi}{2}$ and $d = D\cos\theta = D\sin\frac{\phi}{2}$

Using $\sin\theta = \cos\frac{\phi}{2}$

$2d\sin\theta = \lambda \Rightarrow 2\left(D\sin\frac{\phi}{2}\right).\cos\frac{\phi}{2} \Rightarrow D\sin\phi = \lambda$

■ Heisenberg's Uncertainty Principle:

According to Heisenberg's uncertainty principle, :"It is impossible to measure simultaneously both the position and the momentum of the microscopic particle."

Let Δx and Δp be the uncertainty in the simultaneous measurement of the position and momentum of the particle, then

$$\mathbf{\Delta x \Delta p = \hbar}$$

Where, $\hbar = \frac{h}{2\pi}$ and $h = 6.63 \times 10^{-34}$ J – s is the Planck's constant; $\left(\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34} J - sec\right)$. A more rigorous treatment gives $\Delta x \Delta p \geq \frac{\hbar}{2}\left(or \frac{h}{4\pi}\right)$

If $\Delta x = 0$, then $\Delta p = \infty$ and if $\Delta p = 0$, then $\Delta x = \infty$

The Heisenberg uncertainty principle is also applicable to energy and time, angular momentum and angular displacement. Hence

$\Delta E\ \Delta t \geq \frac{h}{2\pi}$ and $\Delta L\ \Delta\theta \geq \frac{h}{2\pi}$

If the radius of the nucleus is r then the probability of finding the electron inside of the nucleus is $\Delta x = 2r$ and un-certainty in momentum is $\Delta p = \frac{h}{4\pi r}$

■ Photo-electric effect :

The photo electric effect is the emission of electrons (called photo-electrons) when light strikes a surface. To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of positive ions in the material of the surface.

The photoelectric effect was first observed by Heinrich Hertz and it was investigated in details by Whilelm Hallwach and Philipp Lenard.

The photo electric effect is based on the principle of conservation of energy.

- **Work function (or threshold energy) (W_o) :**

  The minimum energy of incident radiation required to ejects the electrons from metallic surface is defined as work function of that surface.

  $W_o = h\nu_o = \dfrac{hc}{\lambda_o}$ Joules

  ν_o = Threshold frequency

  λ_o = Threshold wave length.

  Work function in electron volt $W_o\,(eV) = \dfrac{hc}{e\lambda_o} = \dfrac{12375}{\lambda_o(\overset{0}{A})}$

Element	Work function (eV)	Element	Work function (eV)
Platinum	6.4	Aluminum	4.3
Gold	5.1	Silver	4.3
Nickel	5.1	Sodium	2.7
Carbon	5.0	Lithium	2.5
Silicon	4.8	Potassium	2.2
Copper	4.7	Cesium	1.9

- **Threshold frequency (ν_o) :**

  The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency. If incident frequency $\nu < \nu_o$, No photoelectron emission.

  For most metals the threshold frequency is in the Ultraviolet region (corresponding to wavelength 200 and 300 nm) but for potassium and cesium oxides it is in the visible spectrum (λ between 400 and 700 nm)

- **Threshold wavelength (λ_o) :**

  The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength.

  If incident wavelength $\lambda > \lambda_o \Rightarrow$ No photoelectron emission.

- **Einstein's photoelectron equations :**

  According to Einstein, photoelectric effect is the result of one to one inelastic collision between photon and electron in which photon is completely absorbed.

  Einstein's photoelectric equation is –

  $E = W_o + K_{max}$

  Where, $K_{max} = \dfrac{1}{2}mv_{max}^2$ = maximum kinetic energy of emitted electrons.

Important formula for photoelectric effect :

- $h\nu = h\nu_o + K_{max}$ and $K_{max} = eV_o$
- $K_{max} = eV_o = h(\nu - \nu_o) \Rightarrow \dfrac{1}{2}mv_{max}^2 = h(\nu - \nu_o)$
- $V_{max} = \sqrt{\dfrac{2h(\nu - \nu_o)}{m}}$
- $K_{max} = \dfrac{1}{2}mv_{max}^2 = eV_o = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_o}\right) = hc\left(\dfrac{\lambda_o - \lambda}{\lambda.\lambda_o}\right)$
- $V_{max} = \sqrt{\dfrac{2hc}{m}\dfrac{(\lambda_o - \lambda)}{\lambda.\lambda_o}}$
- $V_o = \dfrac{h}{e}(\nu - \nu_o) = \dfrac{hc}{e}\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_o}\right) = 12375\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_o}\right)$

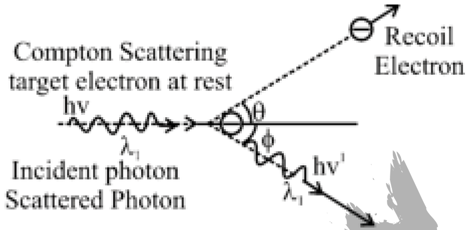
■ Compton Effect:

The scattering of a photon by an electron is called Compton effect. The energy and momentum is conserved scattered photon will have less energy (more wave length) as compared to incident photon (less wave length).
The energy lost by the photon is taken by electron as kinetic energy.
The change in wavelength due to Compton effect is called Compton shift.

$$\text{Compton shift } \boxed{\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_o c}(1-\cos\phi)}$$

If $\phi = 90^\circ$, $\Delta\lambda = \frac{h}{m_o c} = 0.24\text{nm}$

$\phi = 180^\circ$, $\Delta\lambda = \frac{2h}{m_o c} = 0.48\text{nm}$ (called Compton wavelength)



Note : Compton shift depends only on the scattering angle. It does not depend on the incident wavelength or on the scattering material.

■ X-Rays :

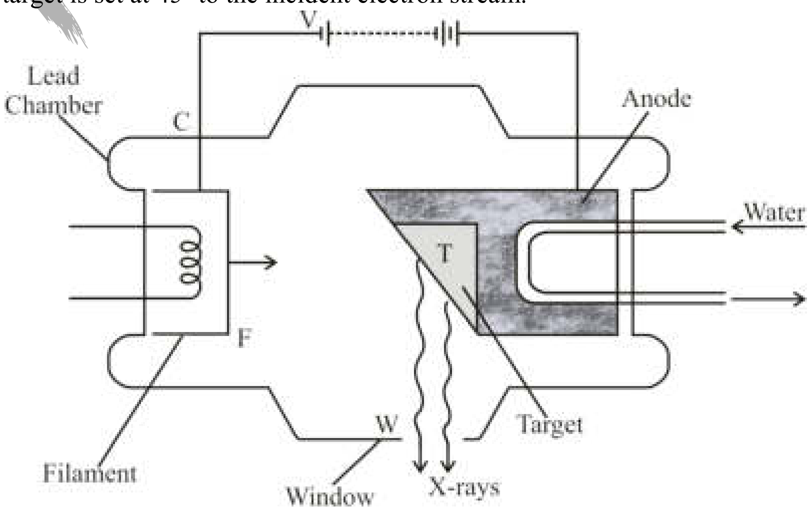
X-rays were discovered by scientist Roentgen. they are also called Roentgen rays.
Roentgen discovered that when pressure inside a discharge tube is kept 10^{-3} mm of Hg and potential difference is kept 25 kV. The some unknown radiations (x-rays) are emitted by anode.
There are three essential requirements for the production of X-rays.

i. A source of electrons.
ii. An arrangement to accelerate the electrons.
iii. A target of suitable material of high atomic weight and high melting point on which these speed electrons strike.

Coolidge X-Rays tube :

It consists of a highly evacuated glass tube containing cathode and target (also known as filament type X-ray tube). The Cathode consists of a tungsten filament. The filament is coated with oxides of Barium or Strontium to have an emission of electrons event at low temperature. The filament is surrounded by a Molybdenum cylinder kept at negative potential. w.r.t. the target.

- The target (it is a material of high atomic weight, high melting point and high thermal conductivity). Made of tungsten or molybdenum is embedded in a copper block.
- The face of the target is set at 45° to the incident electron stream.



The filament is heated by passing the current through it. A high potential difference (≈ 10kV to 80kV) is applied between the target and cathode to accelerate the electrons which are emitted by filament the stream of highly energetic electrons are focused on the target.

⇒ Most of the energy of the electrons is converted into heat (above 98%) and only a fraction of the energy of the electrons about 2% is used to produce X-rays.

Control of intensity of X-rays :

Intensity implies the number of X-ray photons produced from the target. The intensity of X-rays emitted is directly proportional to the electrons emitted per second from the filament and this can be increased by increasing the filament current. So, intensity of X-rays $\propto$ filament current.

Types of X-rays :

Hard X-rays	**Soft X-rays**
More Potential power	Less penetration power
More frequency of the order of $\approx 10^{19}$ Hz	Less frequency of the order of $\approx 10^{16}$ Hz
Lesser wavelength range (0.1 A° – 4A°)	More wavelength range (4A° – 100A°)

Properties of X-rays :

- X-rays are electromagnetic waves with wavelength range 0.1A°–100A°.
- The wavelength of X-rays is very small in comparison to the wavelength of light. Hence they carry much more energy (This is the only difference between X-rays and light).
- X-Rays are invisible.
- They travel in a straight line with speed of light.
- X-Rays carry no charge so they are not deflected in magnetic field and electric field.
- $\lambda_{\text{gamma rays}} < \lambda_{\text{X–rays}} < \lambda_{\text{UV- rays}}$
- X-rays do not pass through heavy metal and bones.
- Lead is the best absorber of X-rays.
- For X-rays photography of human body parts $BaSO_4$ is the best absorber.
- X-rays are not emitted by hydrogen atom because energy level of hydrogen atom are too close to each other.

Absorption of X-rays :

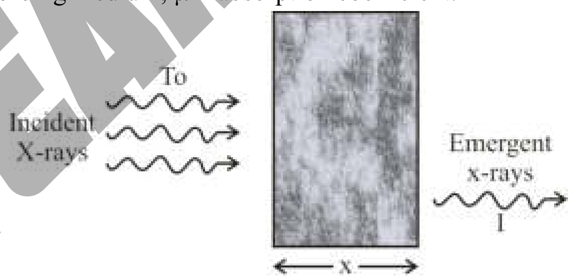
X-rays are absorbed when they are incident on substance intensity of emergent X–rays

$$I = I_o e^{-\mu x}$$

So, intensity of absorbed X-rays

$$I' = I_o - I = I_o (1 - e^{-\mu x})$$

Where, x = thickness of absorbing medium, μ = absorption coefficient.



$\mu \propto \lambda^3$; (λ = wavelength of $\propto$-ray)

$\mu \propto \nu^{-3}$ (ν = Frequency of X-ray)

$\mu \propto Z^4$ (Z = Atomic number of Target)

Classification of X-rays:

In X-ray tube, when high speed electrons strikes the target, they penetrate the target. They loses their kinetic energy and comes to rest inside the metal the electron before finally being stopped makes several collisions with the atoms in the target At each collision one of the following two types of X-rays may get formed.

i. Continuous X-rays

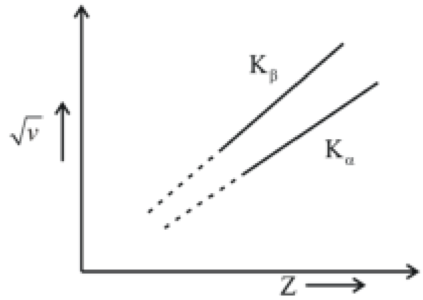
ii. Characteristic X-rays.

■ **Mosley's Law:**

Mosley studied the character of heavy elements and concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequencies.

He also gave the following relation

$$\boxed{\sqrt{\nu} = a(z - b)}$$



Where,

ν = Frequency of emitted line,
Z = Atomic number of target
a = proportionality constant
b = Screening constant or shielding constant
(Z – b) = Effective atomic number, a and b does not depend on the nature of target different values of b are as follows

b = 1	for	K – Series
b = 7.4	for	L – Series
b = 19.2	for	M – Series

- Mosley's law supported Bohr's theory.
- It experimentally determined the atomic number (Z) of elements.
- This Law established the importance of ordering of elements in periodic table by Atomic number and not by atomic weight
- Gaps in Moseley's data for A = 43, 61, 72, 75 suggested existence of new elements which were later discovered.
- The atomic numbers of Cu, Ag and Pt were established to be 29, 47 and 78 respectively.
- Wavelength of characteristics spectrum $\frac{1}{\lambda} = R(Z-b)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ and energy of X-ray radiations.

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc(Z-b)^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

- If transition takes place from $n_2 = 2$ to $n_1 = 1$(k_α – line)

i. $a = \sqrt{\frac{3RC}{4}} = 2.47\times10^{15}\,Hz$

ii. $\nu_{k_\alpha} = RC(Z-1)^2\left(1-\frac{1}{2^2}\right) = \frac{3RC}{4}(Z-1)^2 = 2.47\times10^{15}(Z-1)^2\,Hz$

iii. In general the wavelength of all the K-lines are given by

$$\frac{1}{\lambda_k} = R(Z-1)^2\left(1-\frac{1}{n^2}\right) \quad \text{where } n = 2, 3, 4, \ldots.$$

iv. $E_{k\alpha} = 10.2(Z-1)^2\,eV$

Applications of X-rays:

- In study of crystal structure
- Structure of DNA was also determined using X-ray diffraction.
- In medical science
- In radiography
- In radiotherapy
- In engineering
- In laboratories
- In detective department
- In art the change occurring in old oil paintings can be examined by X-rays.

■ Key Points:-

- Discovery of positive rays helps in discovering isotopes.
- The De-Broglie wavelength of electrons in first Bohr orbit of an atom is equal to circumference of orbit.
- A particle having zero rest mass and non zero energy and momentum must travels with a speed equal to speed of light.
- A photon is not material particle. It is a quanta of energy.
- By coating the metal surface with a layer of barium oxide or strontium oxide it's work function is lowered.

Exemplar Problems
Modern Physics

Atomic Physics

1. A hydrogen atom in is ground state absorbs 10.2 eV of energy. The angular momentum of electron of the hydrogen atom will increase by the value of: (Given, Plank's constant = 6.6 × 10^{-34} Js)

(a) 2.10×10^{-34} Js (b) 1.05×10^{-34} Js
(c) 3.15×10^{-34} Js (d) 4.2×10^{-34} Js

JEE MAIN 2022

2. In Bohr's atomic model of hydrogen, let K. P and E are the kinetic energy, potential energy and total energy of the electron respectively. Choose the correct option when the electron undergoes transitions to a higher level :

(a) All K. P and E increase.
(b) K decreases. P and E increase.
(c) P decreases. K and E increase.
(d) K increases. P and E decrease.

JEE MAIN 2022

3. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths λ_1 / $\lambda 2$ of the photons emitted in this process is

(a) 20/7 (b) 27/5
(c) 7/5 (d) 9/7

JEE MAIN 2019

4. A He+ ion is in its first excited state. Its ionisation energy is

(a) 54.40 eV (b) 13.60 eV
(c) 48.36 eV (d) 6.04 eV

JEE MAIN 2019

5. Let *T*1 and T2 be the energy of an electron in the first and second excited states of hydrogen atoms, respectively. According to the Bohr's model of an atom, the ratio *T*1 : *T*2 is

(a) 9 : 4 (b) 1 : 4
(c) 4 : 1 (d) 4 : 9

NEET 2022

6. For which one of the following, Bohr model is not valid ?

(a) Singly ionised helium atom (He^+)
(b) Deuteron atom
(c) Singly ionised neon atom (Ne^+)
(d) Hydrogen atom

NEET 2020

7. For which one of the following, Bohr model is not valid?

(a) Hydrogen atom
(b) Singly ionised helium atom (He+)
(c) Deuteron atom
(d) Singly ionised neon atom (Ne+)

NEET 2020

8. The total energy of an electron in an atom in an orbit is – 3.4 eV. Its kinetic and potential energies are, respectively

(a) 3.4 eV, 3.4 eV (b) – 3.4 eV, – 3.4 eV
(c) – 3.4 eV, – 6.8 eV (d) 3.4 eV, – 6.8 eV

NEET 2019

9. The ratio of kinetic energy to the total energy of an electron in a Bohr orbit of the hydrogen atom, is

(a) 1 : 1 (b) 1 : –1
(c) 2 : –1 (d) 1 : –2

NEET 2018

10. Who made the observation that there must be very large empty space within the atom?

(a) Einstein (b) Arrhenius
(c) Chadwick (d) Rutherford

UPTGT Science - 2021

11. If the atomic size increase, the non-metallic character will

(a) Increase
(b) Decrease
(c) Sometimes increases and sometimes decreases
(d) No change

UPTGT Science - 2021

12. When hydrogen atom is raised from the ground state to excited state

(a) P.E. increases and K.E. decreases
(b) P.E. decreases and K.E. increases
(c) Both P.E. and K.E. increases
(d) Both P.E. and K.E. decreases

UP TGT Physics 2016

13. Which is the following is incorrect statement?

(a) Bohr's model of atom fails to explain spectra of multi electron atoms
(b) Bohr's atomic model fails to explain Zeeman effect.
(c) Bohr's atomic model fails to explain Stark effect
(d) Bohr's atomic model treats electron as a particle and as a wave

UPTGT Science - 2021

14. **Who postulated that the momentum of an electron in an atom is quantized?**
(a) Erwin Schrodinger
(b) Niels Henrik David Bohr
(c) Loius Victor Pierre Ramond Duc de Broglie
(d) Wolfgang Pauli

UPTGT Science - 2021

15. **If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the I[st] excited state of Li^{++} is :**
(a) 122.4 eV (b) 30.6 eV
(c) 13.6 eV (d) 3.4 eV

UKPSC Lecturer (Mains) 2020

16. **In a Bohr orbit of hydrogen atom, the ratio of Kinetic energy of an electron to its potential energy is–**
(a) $\frac{1}{2}$ (b) 2
(c) $-\frac{1}{2}$ (d) −2

(UPPGT 2011)

17. **Wavelength of 2nd member of Balmer series is:**
(a) $\frac{16}{3R}$ (b) $\frac{8}{5R}$
(c) $\frac{4}{3R}$ (d) $\frac{2}{R}$

UPPCS Pre 2010

18. **Bohr Magneton is the unit of :**
(a) magnetic field
(b) magnetization
(c) magnetic dipole moment
(d) magnetic flux density

UPPCS Pre 2010

19. **The minimum orbital angular momentum of the electron in a hydrogen atom is:**
(a) h (b) $\frac{h}{2\pi}$
(c) $\frac{h}{2}$ (d) $\frac{h}{\lambda}$

UPPCS (Pre) 2003

20. **If the electron in the hydrogen atom jumps from third orbit to second orbit, the wavelength of radiations emitted in terms of Rydberg constant will be:**
(a) $\frac{5R}{36}$ (b) $\frac{5}{36R}$
(c) $\frac{36}{5R}$ (d) $\frac{36R}{5}$

UPPCS (Pre) 2002
UPPGT 2005
UP PGT Physics 2016 (Exam : 02.02.2019)

21. **If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius $_{100}Fm^{257}$ in *n* times the Bohr radius, then n is**
(a) 100 (b) 200
(c) 4 (d) $\frac{1}{4}$

(UPPGT 2010)

22. **Rydberg's constant is equal to**
(a) $R = \frac{me^2}{8\varepsilon_0^2 ch^3}$ (b) $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$
(c) $R = \frac{m^2e^4}{8\varepsilon_0^2 ch^3}$ (d) $R = \frac{m^4e^4}{8\varepsilon_0^2 ch^3}$

(UPPGT 2010)

23. **The ionization energy of Li^{+2} ion is related to H atom as**
(a) 2 : 1 (b) 3 : 1
(c) 9 : 1 (d) 4 : 1

(UPPGT 2013)
UPPCS Pre 2004

24. **If the wavelength of first member of Balmar series is 6563Å. The wavelength of first member of Lyman series shall be**
(a) 1120 Å (b) 1320 Å
(c) 1015 Å (d) 1215 Å

(UPPGT 2013)
UPPCS Pre 2002

25. **In the following nuclear reaction,**
$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\gamma} D_4$$
Mass number of D is 182 and atomic number is 74.
Mass number and atomic number of D_4 respectively will be.
(a) 174 and 71 (b) 174 and 69
(c) 172 and 69 (d) 172 and 71

JEE MAIN 2022

26. **The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci?**
(a) 30 days (b) 35 days
(c) 40 days (d) 25 days

JEE MAIN 2022

27. **The Q-value of a nuclear reaction and kinetic energy of the projectile particle, K_p are related as :**
(a) $Q = K_p$ (b) $(K_p + Q) < O$
(c) $Q < K_p$ (d) $(K_p + Q) > 0$

JEE MAIN 2022

28. **Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be:-**

(a) 13.6 eV (b) 2815.2 eV
(c) 331.2 eV (d) 27.2 eV

JEE MAIN 2021

29. The half -life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, (NA = × 10^{23}/mol)

(a) 240 Ci (b) 357 Ci
(c) 535 Ci (d) 252 Ci

JEE MAIN 2021

30. In the given nuclear reaction, the element X is

$$^{22}_{11}Na \rightarrow X + e^{+} + v$$

(a) $^{22}_{11}Mg$ (b) $^{23}_{11}Na$
(c) $^{23}_{10}Ne$ (d) $^{22}_{10}Ne$

NEET 2022

31. A radioactive nucleus $^{A}_{Z}X$ undergoes spontaneous decay in the sequence $^{A}_{Z}X \rightarrow _{Z-1}B \rightarrow _{Z-3}C \rightarrow _{Z-2}D$, where Z is the atomic number of element X. The possible decay particles in the sequence are

(a) $\beta^{-}, \alpha, \beta^{+}$ (b) $\alpha, \beta^{-}, \beta^{+}$
(c) $\alpha, \beta^{+}, \beta^{-}$ (d) $\beta^{+}, \alpha, \beta^{-}$

NEET 2021

32. A nucleus with mass number 240 breaks into two fragments each of mass number 120, the binding energy per nucleon of unfragmented nuclei is 7.6 MeV while that of fragments is 8.5 MeV. The total gain in the Binding Energy in the process is

(a) 2016 MeV (b) 0.9 MeV
(c) 9.4 MeV (d) 804 MeV

NEET 2021

33. When a uranium isotope $^{235}_{92}U$ is bombarded with a neutron, it generates $^{89}_{36}Kr$, three neutrons and :

(a) $^{91}_{40}Zr$ (b) $^{101}_{36}Kr$
(c) $^{103}_{36}Kr$ (d) $^{144}_{56}Ba$

NEET 2020

34. The energy required to break one bond in DNA is 10^{-20} J. This value in eV is nearly

(a) 6 (b) 0.6
(c) 0.06 (d) 0.006

NEET 2020

35. The binding energy of a nucleus is a measure of its

(a) Stability (b) Mass
(c) Charge (d) Momentum

UPPSC Ashram Paddhati PGT - 2021

36. Which of the following statements is true?

(a) α particles have a well defined range
(b) In β -decay,the neutron-proton ratio increases
(c) Neutrino can pass easily through vast amount of matter
(d) 1Å photon can produce electron-positron pair

UPPSC Ashram Paddhati PGT - 2021

37. Surface energy E_S, of a nucleus having mass number A and atomic number Z is proportional to

(a) $(A-Z)^{\frac{1}{2}}$ (b) $(A-2Z)^{2}$
(c) $A^{\frac{1}{3}}$ (d) $A^{\frac{2}{3}}$

UPPSC Ashram Paddhati PGT - 2021

38. What is the particle X in the following nuclear reaction?

$$_{4}Be^{9} + _{2}He^{4} \rightarrow _{6}C^{12} + X$$

(a) Electron (b) Proton
(c) Neutron (d) Meson

UPPSC Poly. Lect. 2021 Paper - II

39. The particles heavier than nucleons are called -

(a) Hyperons (b) Leptons
(c) Gravitons (d) Mesons

UPPSC Poly. Lect. 2021 Paper - II
(UPPGT 2013)
UPPCS Pre 2003

40. The binding energy per nucleon is maximum for nuclei -

(a) ^{56}Fe (b) ^{208}Pb
(c) ^{4}He (d) ^{101}Mo

UPPSC Poly. Lect. 2021 Paper - II
UKPSC Lecturer (Mains) 2020

41. When γ-rays fall on heavy substance, electron and positron are produced, the phenomenon is known as -

(a) β – decay (b) Meson production
(c) Cosmic ray (d) Pair production

UPPSC Poly. Lect. 2021 Paper - II

42. The average binding energy per nucleon of most nuclei is of the order of

(a) 10^{-12} eV (b) 10^{-12} MeV
(c) 10^{-12} BeV (d) 10^{-12} J

UP PGT - 2021

43. At the peak of the nuclear binding energy curve

(a) Z is even but N is odd
(b) Z is odd but N is even
(c) Both Z and N are odd
(d) Both Z and N are even

UP PGT - 2021
RPSC School Lect. 26.07.2016

44. γ-emission is associated with transition between
(a) Atomic energy levels
(b) Nuclear energy levels
(c) Both (a) and (b)
(d) Neither (a) nor (b)
UPPSC GDC - 2021

45. **The magic numbers are related to which nuclear property?**
(a) Force (b) Decay
(c) Explosion (d) Stability
UPPSC GDC - 2021

46. **$_7N^{13}$ is emitted in the reaction of deuteron and $_6C^{12}$ nucleus, another emitted particle is**
(a) electron (b) positron
(c) proton (d) neutron
RPSC School Lect. 26.07.2016

47. **Nuclear forces are**
(a) Always repulsive
(b) Always attractive
(c) Attractive at large distances and repulsive at short distances
(d) Repulsive at large distances and attractive at short distances
UP PGT Physics 2016 (Exam : 02.02.2019)

48. **Natural Radioactivity was discovered by**
(a) Rutherford (b) Pierre Curie
(c) Becquerel (d) Roentgen
UPPSC GDC - 2021

49. **The fission of a $_{92}U^{235}$ nucleus**
(a) may produce pairs of fission products other than Ba and Kr
(b) always produces Ba and Kr as fission product
(c) produces Ba and any other fission product
(d) produces Kr and any other fission product/Kr
UPPSC Ashram Paddhati PGT - 2021

50. **In nuclear reactions the conversion of -**
(a) Mass only
(b) Energy only
(c) Momentum only
(d) Mass, energy and momentum
UPPSC Poly. Lect. 2021 Paper - II

51. **Radioactive substances do not exit -**
(a) $\alpha-$rays (b) $\beta-$rays
(c) Positrons (d) Protons
UPPSC Poly. Lect. 2021 Paper - II

52. **The half life of a radio-active element depends upon -**
(a) Amount of element present
(b) Temperature
(c) Pressure
(d) None of these
UPPSC Poly. Lect. 2021 Paper - II

53. **If 25% of a radioactive material decays in 5 years, how much of the original material will be left undecayed after 20 years?**
(a) 5% (b) 10%
(c) 25% (d) 32%
UP PGT - 2021

54. **Which one of the following is used as a coolant/moderator in a nuclear reactor?**
(a) Distilled water (b) Fresh water
(c) Heavy water (d) Ice
UP PGT - 2021

55. **Identify the incorrect statement.**
(a) Radioactivity makes it possible to establish the age of many geological and biological specimens.
(b) Radiocarbon is the beta-active carbon isotope $^{14}_{6}C$.
(c) Living plants and animals have different ratio of radiocarbon to ordinary carbon.
(d) Cosmic rays are high energy atomic nuclei chiefly protons.
UP PGT - 2021

56. **Neutrino is emitted in**
(a) α-decay (b) β-decay
(c) γ-decay (d) None of the above
UPPSC GDC - 2021

57. **In β-decay of nucleus**
(a) Mass number changes but atomic number remains unchanged
(b) Both mass number and atomic number changes
(c) Mass number remains unchanged but atomic number changes
(d) None of the above
UPPSC GDC - 2021

58. **Which among the following is a pair of isotones?**
(a) $^{23}_{11}Na, ^{23}_{12}Mg$ (b) $^{23}_{11}Na, ^{24}_{12}Mg$
(c) $^{24}_{11}Na, ^{24}_{12}Mg$ (d) $^{24}_{11}Na, ^{23}_{12}Mg$
UPTGT Science - 2021

59. **What is the cause of nuclear fission?**
(a) Centrifugal force (b) Coulomb force
(c) Surface tension (d) Viscous force
UPTGT Science - 2021

60. **Which of these can be the fuel in a nuclear fission reactor?**
(a) Cadmium (b) Thorium
(c) Helium (d) Deuterium
UPTGT Science - 2021

61. **A radioactive element has half life of 10 seconds. A single nucleus of the element will decay**
(a) in less than 10 seconds
(b) at exactly 10 seconds
(c) only after 10 seconds
(d) cannot be predicted
UPTGT Science - 2021

62. A half life equation of the radioactive substance is

(a) $t_{1/2} = \frac{0.6931}{\lambda}$ (b) $t_{1/2} = \frac{\lambda}{0.6931}$

(c) $t_{1/2} = \frac{0.936}{\lambda}$ (d) $t_{1/2} = \frac{\lambda}{0.936}$

UPTGT Science - 2021

63. Which represents Curie's law for paramagnetic material ?

(a) $\chi \propto (T - T_C)$ (b) $\chi \propto \frac{1}{T - T_C}$

(c) $\chi \propto \frac{1}{T}$ (d) $\chi \propto T$

UKPSC Lecturer (Mains) 2020

64. The half-life of a radioactive element, which has only $\frac{1}{32}$ of its original mass left after a lapse of 60 days is :

(a) 12 days (b) 32 days

(c) 60 days (d) 64 days

UKPSC Lecturer (Mains) 2020

65. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closet approach is:

(a) 5.3×10^{-14} m (b) 5.1×10^{-13} m

(c) 5.9×10^{-14} m (d) 5.9×10^{-13} m

UKPSC Lecturer (Mains) 2020

66. In Einstein's model of specific heat of solids, which of the following assumption is not true ?

(a) A crystal consists of atoms which may be regarded as identical and independent harmonic oscillators

(b) Any number of oscillators may be present in the same quantum state

(c) The oscillators are quantum oscillators and have discrete energy

(d) All the oscillators vibrate with different frequency

RPSC School Lect. 08.01.2020

67. An element $_{84}X^{202}$ emits an α – particle and then a β^- particle. If the final product is $_aY^b$, then a and b are

(a) 82, 198 (b) 83, 198

(c) 82, 199 (d) 83, 199

UP PGT Physics 2016 (Exam : 02.02.2019)

68. The half life of radon is 3.8 days starting with 9.6 mg of radon, how much will be left after 19 days

(a) 0.5mg (b) 0.6 mg

(c) 0.3mg (d) 0.2 mg

UP PGT Physics 2016 (Exam : 02.02.2019)

69. The activity of a radioactive element decreases in 9 years to 1/3 of its initial activity A_0. After next 9 years its activity will be

(a) $\frac{A_0}{3}$ (b) $\frac{A_0}{6}$

(c) $\frac{A_0}{9}$ (d) $\frac{A_0}{27}$

RPSC School Lect. 26.07.2016

70. What is the half-life of $_6C^{14}$, if its disintegration constant (λ), is 2.31×10^{-4} years^{-1}

(a) 0.3×10^4 year (b) 0.3×10^6 year

(c) 3.0×10^3 year (d) 0.3×10^3 year

UP LT Grade Science 2018 (Exam : 29.07.2018)

71. The time taken for a sample of radium D to decrease to 10%, if its half–life is 22 years, will be:

(a) 29 years (b) 89 years

(c) 73.11 years (d) 90 years

(UPPGT 2013)
UPPCS (Pre) 2002

72. In a radioactive decay neither the atomic number nor the mass number changes. The particle emitted in the decay, is:

(a) Proton (b) Neutron

(c) Electron (d) Photon

(UPPGT 2013)
UPPCS (Pre) 2002

73. A proton, a neutron, an electron and an α-particle have same energy. If $\lambda_p, \lambda_n, \lambda_e$ and λ_α are the de Broglie's wavelengths of proton, neutron, electron and α particle respectively, then choose the correct relation from the following :

(a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_n < \lambda_p < \lambda_e$

(c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

JEE MAIN 2022

74. An a particle and a carbon 12 atom has same kinetic energy K. The ratio of their de-Broglie wavelength is : ($\lambda_\alpha : \lambda_{C12}$) is :

(a) $1:\sqrt{3}$ (b) $\sqrt{3}:1$

(c) 3 : 1 (d) $2:\sqrt{3}$

JEE MAIN 2022

75. For a specific wavelength 670 nm of light coming from a galaxy moving with velocity v, the observed wavelength is 670.7 nm. The value of v is :

(a) 3×10^8 ms^{-1} (b) 3×10^{10} ms^{-1}

(c) 3.13×10^5 ms^{-1} (d) 4.48×10^5 ms^{-1}

JEE MAIN 2022

76. The de Brogue wavelengths for an electron and a photon are λ_e and λ_p respectively. For the same kinetic energy of electron and photon. which of the following presents the correct relation between the de Brogue wavelengths of two ?

(a) $\lambda_p \propto \lambda_e^2$ (b) $\lambda_p \propto \lambda_e$

(c) $\lambda_p \propto \sqrt{\lambda_e}$ (d) $\lambda_p \propto \sqrt{\frac{1}{\lambda_e}}$

JEE MAIN 2022

77. Let K_1 and K_2 be the maximum kinetic energies of photo–electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2, respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then:

(a) $K_1 > \frac{K_2}{3}$ (b) $K_1 < \frac{K_2}{3}$

(c) $K_1 = \frac{K_2}{3}$ (d) $K_2 = \frac{K_1}{3}$

JEE MAIN 2022

78. The light or two different frequencies whose photons have energies 3.8 eV and 1.4 eV respectively illuminate a metallic surface whose work function is 0.6 eV successively. The ration of maximum speeds of emitted electrons for the two frequencies respectively will be.

(a) 1 : 1 (b) 2 : 1

(c) 4 : 1 (d) 1 : 4

JEE MAIN 2022

79 A proton, a deuteron and an a-particle with same kinetic energy enter into a uniform magnetic field at right angle to magnetic field. The ratio of the radii of their respective circular paths is :

(a) $1:\sqrt{2}:\sqrt{2}$ (b) $1:1:\sqrt{2}$

(c) $\sqrt{2}:1:1$ (d) $1:\sqrt{2}:1$

JEE MAIN 2022

80. A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of particle is:-

(a) $\frac{1}{16}$ times the mass of e^-

(b) 8 times the mass of e^-

(c) 16 times the mass of e^-

(d) $\frac{1}{8}$ times the mass of e^-

JEE MAIN 2021

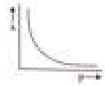
81. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths ? (mP = 1.00727 u, me = 0.00055u)

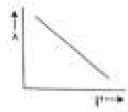
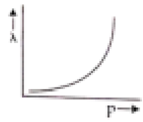
(a) 1860: 1 (b) $(1680)^2 : 1$

(c) 41.4 : 1 (d) 43 : 1

JEE MAIN 2021

82. The graph which shows the variation of the de Broglie wavelength (λ) of a particle and its associated momentum (p) is

(a)  (b) 

(c)  (d) 

NEET 2022

83. When two monochromatic lights of frequency, v and $\frac{v}{2}$ are incident on a photoelectric metal, their stopping potential becomes $\frac{V_s}{2}$ and V_s respectively. The threshold frequency for this metal is

(a) $\frac{3}{2}$v (b) 2v

(c) 3v (d) $\frac{2}{3}$v

NEET 2022

84. The number of photons per second on an average emitted by the source of monochromatic light of wavelength 600 nm, when it delivers the power of 3.3×10^{-3} watt will be ($h = 6.6 \times 10^{-34}$ Js)

(a) 10^{15} (b) 10^{18}

(c) 10^{17} (d) 10^{16}

NEET 2021

85. Light of frequency 1.5 times the threshold frequency is incident on a photosensitive material. What will be the photoelectric current if the frequency is halved and intensity is doubled?

(a) Doubled (b) Four times

(c) One-fourth (d) Zero

NEET 2020

86. The shortest wavelength of X-rays emitted from X-ray tube depends upon

(a) Current in the tube

(b) Voltage applied to the tube

(c) Nature of the gas in the tube

(d) None of these

UPPSC Ashram Paddhati PGT - 2021

87. According to Moseley's law frequency of K_α line will be -

(a) $\frac{1}{2}CRZ^2$ (b) $\frac{2}{3}CRZ^2$

(c) $\frac{3}{4}CRZ^2$ (d) $\frac{4}{3}CRZ^2$

UPPSC Poly. Lect. 2021 Paper - II

88. **If 50kV is applied in an X-ray tube, the minimum wavelength for X-rays will be nearly-**
(a) 3 nm (b) 2 nm
(c) 0.2Å (d) 2Å
UPPSC Poly. Lect. 2021 Paper - II
TGT 2004

89. **In X-ray emission tubes, X-ray is emitted by the acceleration of**
(a) Atoms (b) Protons
(c) Electrons (d) Neutrons
UPPSC Ashram Paddhati PGT - 2021

90. **The short wavelength limit of X-ray depends on-**
(a) Nature of target
(b) Potential applied on X-ray
(c) Nature of filament
(d) None of the above
UPPSC Poly. Lect. 2021 Paper - II

91. **The wavelength of k_α x-ray line of an element of atomic number z = 11 is λ. The wavelength of k_α x-ray line another element of atomic number z is 4λ. Then z is**
(a) 11 (b) 44
(c) 6 (d) 4
UP PGT - 2021

92. **Mark the wrong statement.**
(a) The characteristic X-ray spectrum consists of sharp peaks superimposed on the continuous spectrum.
(b) The characteristic X-rays from substances of higher atomic number are of longer wavelength than those from lower atomic number
(c) X-rays from higher atomic number are more penetrating.
(d) Characteristic X-rays are generated when an electron from a higher inner shell jumps to lower
UP PGT - 2021

93. **The source of solar energy is**
(a) burning of hydrogen
(b) nuclear fission reactions
(c) nuclear fusion reactions
(d) radiant energy received from other solar systems
UP PGT - 2021

94. **At the same temperature, which of the following will exert maximum pressure?**
(a) Gas of classical molecules
(b) Gas of Bosons
(c) Gas of Fermions
(d) None of the above
UPPSC GDC - 2021

95. **The wavelength of the matter waves associated with a fast moving sub-atomic particle depends upon-**
(i) charge **(ii) mass**
(iii) velocity **(iv) spin state and**
(v) momentum
The correct factors are
(a) Only (iii)
(b) Only (i), (ii) and (iii)
(c) Only (ii), (iii) and (iv)
(d) Only (ii), (iii) and (v)
UPTGT Science - 2021

96. **If microwaves, X-rays, infrared rays, gamma rays, ultraviolet rays, radio waves and visible parts of E.M. spectrum are denoted by M, X, I, G, U, R and V respectively. Which of the following represents the arrangement in ascending order of their wavelength?**
(a) R, M, I, V, U, X and G
(b) M, R, V, X, U, G and I
(c) G, X, U, V, I, M and R
(d) I, M, R, U, V, X and G
UKPSC Lecturer (Mains) 2020

97. **Which experiment showed that atoms have discrete energy levels?**
(a) Davisson and Germer (b) Frank and Hertz
(c) Planck (d) Rutherford
UPTGT Science - 2021

98. **The equation which does NOT represent Einstein's photoelectric equation, is-**
(a) $K_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ (b) $K_{max} = \frac{h}{c}(\nu - \nu_0)$
(c) $K_{max} = h\nu - h\nu_0$ (d) $K_{max} = h\nu - \phi_0$
UPPSC Ashram Paddhati PGT - 2021

99. **According to Einstein theory of specific heat of solids, all atoms of a solid vibrate with**
(a) Zero frequency
(b) Only one and the same frequency
(c) Different frequency
(d) Unknown frequency
UPPSC Ashram Paddhati PGT - 2021

100. **In Compton effect change in wavelength depends on which parameter?**
(a) Material (b) Scattering angle
(c) Wavelength (d) Polarization
UPPSC Ashram Paddhati PGT - 2021
UPPCS (Pre) 1997

101. **The mass of photon in motion is**
(a) $\frac{h\nu}{c^2}$ (b) $\frac{h\nu}{c}$
(c) $\frac{p}{\lambda}$ (d) Zero
UPPSC Poly. Lect. 2021 Paper - II
UP LT Grade Science 2018 (Exam : 29.07.2018)

102. The photoelectric threshold wavelength for a metal is 3000Å. What is the kinetic energy of an electron ejected from it by radiation of wavelength 1200Å?

(a) 6.2 eV (b) 3.1 eV
(c) 1.65 eV (d) 1.03 eV

UP PGT - 2021

103. If 1.20 V potential difference must be applied to stop the fastest photoelectrons emitted by a nickel surface under the action of ultraviolet light of wavelength 2000Å, calculate the work function of nickel.

(a) 6.21 eV (b) 5.01 eV
(c) 4.80 eV (d) 4.50 eV

UP PGT - 2021

104. Fermi - dirac Statistics cannot be applied to -

(a) Electrons (b) Protons
(c) Fermions (d) Photons

UPPSC Poly. Lect. 2021 Paper - I

105. The relation between Einstin's B_{21} and B_{12} coefficient is

(a) $\frac{B_{21}}{B_{12}} = 1:2$ (b) $\frac{B_{21}}{B_{12}} = 2:1$
(c) $\frac{B_{12}}{B_{21}} = 2:3$ (d) $\frac{B_{21}}{B_{12}} = 1:1$

UPPSC GDC - 2021

106. Maximum change in wavelength in a Compton scattering is -

(a) 0.0243 Å (b) 0.2430 Å
(c) 0.0486 Å (d) 0.4860 Å

UPPSC Poly. Lect. 2021 Paper - II

107. What is the value of Compton shift?

(a) $(1-\cos\phi)$ (b) $\frac{h}{m_0 c}(1+\cos\phi)$
(c) $\frac{h}{m_0 c}(1-\cos\phi)$ (d) $\frac{m_0 c}{h}(1-\cos\phi)$

UPPSC Poly. Lect. 2021 Paper - II

108. The velocity of individual wave is called :

(a) Group velocity (b) Phase velocity
(c) Particle velocity (d) None of these

UKPSC Lecturer (Mains) 2020

109. Davisson and Germer experiment showed that the electron behaves like a

(a) Particle
(b) Wave
(c) Particle as well as wave
(d) None of these

UPPSC Ashram Paddhati PGT - 2021
(UPPGT 2013)
UPPCS (Pre) 2002

110. The de-Broglie wavelength of a particle of mass m and energy E is

(a) $\sqrt{2hmE}$ (b) $\frac{h}{\sqrt{2mE}}$
(c) $\frac{1}{\sqrt{2hmE}}$ (d) $\frac{E}{\sqrt{2hm}}$

UPPSC Ashram Paddhati PGT - 2021

111. Frequency of light sufficient for creation of electron – hole pair in GaAs (Eg = 1.42eV) is -

(a) 1.42 MHz (b) 3.43 MHz
(c) 1.42×10^{14} MHz (d) 3.43×10^{14} MHz

UPPSC Poly. Lect. 2021 Paper - II

112. The de-Broglie wavelength of an electron accelerated to a potential difference of V volt is-

(a) $\sqrt{\frac{12.5}{V}}$Å (b) $\sqrt{\frac{150}{V}}$Å
(c) $\frac{hv}{\sqrt{2mkT}}$ (d) $\frac{h}{\sqrt{2mV}}$

UPPSC Poly. Lect. 2021 Paper - II

113. For which of the following particles will it be most difficult to experimentally verify the de-Broglie relationship :

(a) An electron (b) A proton
(c) An α-particle (d) A dust particle

UKPSC Lecturer (Mains) 2020

114. The shortest wavelength present in the radiation from an X-ray machine whose accelerating potential is 50,000 V will be

(a) 0.025 nm (b) 0.3426 nm
(c) 0.0248 Å (d) 0.3426 Å

UKPSC Lecturer (Mains) 2020

115. The ratio of de-Broglie wavelengths λ_p and λ_e associated with protons and electrons, respectively having same kinetic energy is :

(a) $\frac{m_e}{m_p}$ (b) $\sqrt{\frac{m_e}{m_p}}$
(c) $\sqrt{\frac{m_p}{m_e}}$ (d) $\sqrt{\frac{m_e}{2m_p}}$

UKPSC Lecturer (Mains) 2020

116. The wavelength of the de-Broglie wave by Davisson and Germer is given by :

(a) $\lambda = \frac{h}{2m_o eV}$ (b) $\lambda = \frac{h}{(2m_o eV)^{1/2}}$
(c) $\lambda = h(2m_o eV)^{1/2}$ (d) $\lambda = h(2m_o eV)$

UKPSC Lecturer (Mains) 2020

117. **What is the ratio of de-Broglie wavelengths of proton, deuteron and α-particle if they are accelerated through same potential difference?**

(a) $1:1:\frac{1}{\sqrt{2}}$ (b) $1:\frac{1}{\sqrt{2}}:\frac{1}{2}$

(c) $1:\frac{1}{\sqrt{2}}:\frac{1}{2\sqrt{2}}$ (d) 1 : 1 : 1

RPSC School Lect. 08.01.2020

118. **If the angular velocity ω and the propagation number K of a wave are related by $\omega^2 = c^2K^2 + \omega_0^2$, where c and ω_0 are constants, then the product of group velocity and phase velocity is**

(a) c^2 (b) $c^2/\sqrt{2}$

(c) c^2/K^2 (d) $c^2/\sqrt{2}K^2$

RPSC School Lect. 26.07.2016

119. **The number of photons emitted per second by a 25 watt source of monochromatic light of wavelength 6000Å is**

(a) 7.6×10^{17} (b) 6.1×10^{17}

(c) 7.6×10^{19} (d) 6.1×10^{19}

UKPSC Lecturer (Mains) 2020

120. **A non-monochromatic light is used in an experiment on photoelectric effect. The stopping potential**

(a) is related to the mean wavelength

(b) is related to the shortest wavelength

(c) is related to the longest wavelength

(d) is not related to the wavelength

RPSC School Lect. 08.01.2020

121. **Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is charged to $\frac{3\lambda}{4}$, the speed of fastest emitted electrons will be**

(a) $v\left(\frac{3}{5}\right)^{1/2}$ (b) $> v\left(\frac{4}{3}\right)^{1/2}$

(c) $< v\left(\frac{4}{3}\right)^{1/2}$ (d) $v\left(\frac{4}{5}\right)^{1/2}$

UKPSC Lecturer (Mains) 2020

122. **Einstein's photoelectric equation is given by :**

(a) $W = h\nu + \frac{1}{2}mv^2$ (b) $h\nu = W + \frac{1}{2}mv^2$

(c) $h\nu + \frac{1}{2}mv^2 + W = 0$ (d) $h\nu + W = \frac{1}{2}mv^2$

UKPSC Lecturer (Mains) 2020

123. **The photoelectric threshold wavelength for tungsten is 2300 Å. If this surface is irradiated by ultra-violet light of wavelength 1800 Å, the kinetic energy of emitted electrons is**

(a) 1.5 eV (b) 1.8 eV

(c) 2.4 eV (d) 2.7 eV

UKPSC Lecturer (Mains) 2020

124. **A small plate of metal (work function = 1.17 eV) is placed at a distance of 2 m from a monochromatic source of light of wavelength 4.8×10^{-7} m and power 1 W . Light falls normally on the plate. The number of photons striking the plate per square meter per second is :**

(a) 4.8×10^{15} (b) 4.8×10^{16}

(c) 6.2×10^{15} (d) 6.2×10^{16}

UKPSC Lecturer (Mains) 2020

125. **The momentum associated with a photon of light of wavelength 500 nm is :**

(a) 1.11×10^{-27} kg m/s (b) 1.21×10^{-27} kg m/s

(c) 1.33×10^{-27} kg m/s (d) 1.41×10^{-27} kg m/s

UKPSC Lecturer (Mains) 2020

126. **A 5 MeV α-particle loses its energy in G.M. counter, one electron-ion pair is produced for each 30 eV of energy lost in the gas of the G.M. counter. The G.M. counter has a multiplication of 10,000 and the total capacitance between the two electrodes is 50 pF. The amplitude of voltage pulse developed will be :**

(a) 1.6 V (b) 3.2 V

(c) 4.6 V (d) 5.3 V

UKPSC Lecturer (Mains) 2020

127. **An X-ray tube operates on 30kV. The minimum wavelength of X-ray emitted is :**

(a) 0.133 Å (b) 0.4 Å

(c) 1.4 Å (d) 6.6 Å

UKPSC Lecturer (Mains) 2020

128. **A particle of rest mass m_0 moves with speed of $\frac{c}{\sqrt{2}}$. Its kinetic energy is :**

(a) $0.41\ m_0c^2$ (b) m_0c^2

(c) $1.41\ m_0c^2$ (d) $2.41\ m_0c^2$

UKPSC Lecturer (Mains) 2020

129. **In a cloud chamber, a short, thick and continuously straight track is obtained on the photographic plate. The incident particle is**

(a) Neutron (b) α-particle

(c) β-particle (d) γ-radiation

RPSC School Lect. 08.01.2020

130. **Which of the following is dependent on the intensity of incident radiation in a photoelectric effect experiment ?**

(a) work function of the surface

(b) amount of photoelectric current

(c) stopping potential

(d) maximum kinetic energy of photoelectrons

RPSC School Lect. 26.07.2016

131. **The experiment which demonstrated the discrete energy states of the atoms was performed by**
(a) Franck and Hertz
(b) Davisson and Germer
(c) Stern and Gerlach
(d) Paschen and Back
RPSC School Lect. 26.07.2016

132. **A proton acquire 500 V in crossing the gap in Dee of cyclotron. If it completes 2000 rotation, then its acquired energy will be**
(a) 1 MeV (b) 0.5 MeV
(c) 2 MeV (d) 4 MeV
RPSC School Lect. 26.07.2016

133. **The work funciton of sodium is 2.3 eV. What will be the maximum kinetic energy of the photoelectrons emitted by the surface exposed to the radiation of 2000 Å?**
(a) 2.3 eV (b) 3.9 eV
(c) 6.2 eV (d) 3.9V
UP TGT Physics 2016

134. **A metal surface is illuminated by a light of given intensity and frequency to cause photoemission. If the intensity of illumination is reduced to one fourth of its original value, then the maximum kinetic energy of the emitted photoelectrons would becomes.**
(a) Unchanged
(b) $1/16^{th}$ of original value
(c) twice the original value
(d) four times the original value
UP TGT Physics 2016

135. **What potential must be applied to stop the fastest photoelectrons emitted when ultraviolet light of 2000 Å wavelength falls on a nickel surface?**
(Work function of nickel = 5.01 eV)
(a) 5.01 V (b) 3.02 V
(c) 2.10 V (d) 1.20 V
UP PGT Physics 2016 (Exam : 02.02.2019)

136. **In photoelectric effect, the rate of emission of photoelectrons depends on**
(a) Intensity of radiation
(b) Frequency of radiation
(c) Both (A) and (B) above
(d) None of the above
UP PGT Physics 2016 (Exam : 02.02.2019)

137. **In photoelectric effect, the most energetic electrons are emitted from**
(a) Inner shells (b) Surface of metal
(c) Nucleus (d) Deep inside the metal
UP PGT Physics 2016 (Exam : 02.02.2019)

138. **A beam of light of frequency υ and of intensity 10 lumen falls on a metal surface of work function ϕ . In which of the following conditions, there shall be no emission of electrons ?**
(a) $\upsilon > \frac{\phi}{h}$ For (b) $\upsilon < \frac{\phi}{h}$ For
(c) $\upsilon \geq \frac{\phi}{h}$ For (d) $\upsilon = \frac{10\phi}{h}$ For
UP LT Grade Science 2018 (Exam : 29.07.2018)

139. **Two photons are moving in opposite direction each with velocity c. Their relative velocity will be**
(a) 2c (b) c
(c) 0 (d) $\sqrt{2}c$
(UPPGT 2013)

140. **According to the law of photoelectric emission, the number of electrons emitted per second depends on–**
(a) the intensity of incident light
(b) the nature of incident light
(c) the frequency of incident light
(d) the speed of incident light
(UPPGT 2011)

Answer Key

1. (b)	2. (b)	3. (a)	4. (b)	5. (a)	6. (c)	7. (d)	8. (d)	9. (b)	10. (d)
11. (b)	12. (a)	13. (d)	14. (b)	15. (b)	16. (c)	17. (a)	18. (c)	19. (b)	20. (c)
21. (d)	22. (b)	23. (c)	24. (d)	25. (a)	26. (b)	27. (d)	28. (b)	29. (b)	30. (d)
31. (d)	32. (a)	33. (d)	34. (c)	35. (a)	36. (c)	37. (d)	38. (c)	39. (a)	40. (a)
41. (d)	42. (d)	43. (d)	44. (b)	45. (d)	46. (d)	47. (c)	48. (c)	49. (a)	50. (d)
51. (d)	52. (d)	53. (d)	54. (c)	55. (c)	56. (b)	57. (c)	58. (b)	59. (b)	60. (b)
61. (d)	62. (a)	63. (c)	64. (a)	65. (a)	66. (d)	67. (b)	68. (c)	69. (c)	70. (a)
71. (c)	72. (d)	73. (b)	74. (b)	75. (c)	76. (a)	77. (b)	78. (b)	79. (d)	80. (d)
81. (d)	82. (a)	83. (a)	84. (d)	85. (d)	86. (b)	87. (c)	88. (c)	89. (c)	90. (b)
91. (c)	92. (b)	93. (c)	94. (c)	95. (d)	96. (c)	97. (b)	98. (b)	99. (b)	100. (b)
101. (a)	102. (a)	103. (b)	104. (d)	105. (d)	106. (c)	107. (c)	108. (b)	109. (b)	110. (b)
111. (d)	112. (b)	113. (d)	114. (a)	115. (b)	116. (b)	117. (c)	118. (a)	119. (c)	120. (b)
121. (b)	122. (b)	123. (a)	124. (b)	125. (c)	126. (d)	127. (b)	128. (a)	129. (b)	130. (b)
131. (a)	132. (c)	133. (b)	134. (a)	135. (d)	136. (a)	137. (b)	138. (b)	139. (b)	140. (a)

08.

Electronics

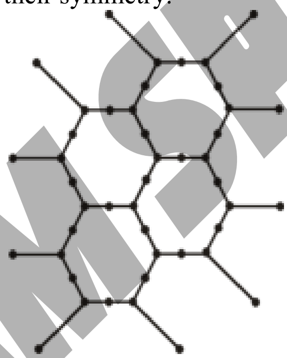
(1) Introduction of Solids

■ **Solids:-** It is a state of matter which has a definite shape and a definite volume. The characteristic properties of the solid depends upon the nature of forces acting between their constituent of particles (i.e., ions atoms or molecules)

Solids are divided into two categories–

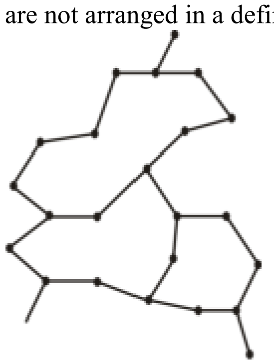
1. Crystalline Solids

(i) These solids have definite external geometrical form.
(ii) Ions, atoms or molecules of these solids are arranged in a definite fashion in all it's three dimensions.
(iii) Examples:- Quartz, Calcite, mica, diamond etc.
(iv) They have well defined facets or faces.
(v) They are ordered at short range as well as at long range.
(vi) They have sharp melting point.
(vii) Bond strengths are identical throughout the solid.
(viii) These are considered as true solids.
(ix) An important property of crystals is their symmetry.



2. Amorphous or glassy solids:-

(i) These solids have no definite external geometrical form.
(ii) Ions, atoms or molecules of these solids are not arranged in a definite fashion.



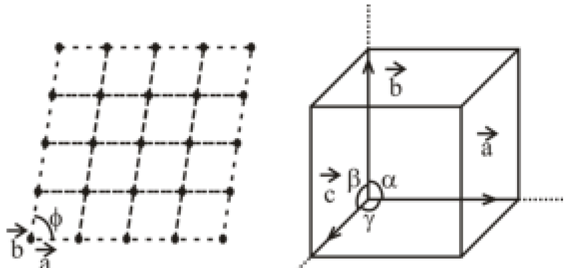
(iii) Example:- Rubber, Plastic, Paraffin wax, Cement etc.
(iv) They do not possess definite facets or faces.
(v) These have short range order
(vi) They are isotropic
(vii) They do not have a sharp melting point
(viii) Bond strengths vary
(ix) These are considered as pseudo-solids or super cooled liquids.
(x) Amorphous solids do not have any symmetry.

■ **Terms Related with Crystal Structure:-**

(i). Crystal lattice:- It is a geometrical arrangement of points in space where if atoms or molecules of a solid are placed, we obtain an actual crystal structure of the solid.

(ii). Basis:- The atoms or molecules attached with every lattice point in a crystal structure is called the basis of crystal structure.

(iii). Unit cell:- It is defined as that volume of the solid form which the entire crystal structure can be constructed by the translational repetition in three dimensions. The length of three sides of a unit cell (3D) are called primitives or lattice constant. They are denoted by a, b, c.



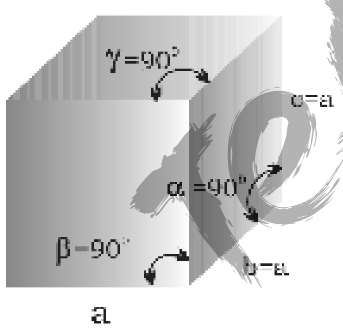
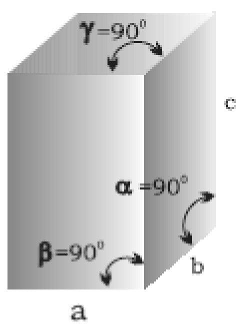
2D Unit Cell 3D Unit Cell

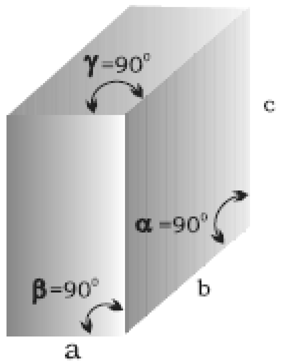
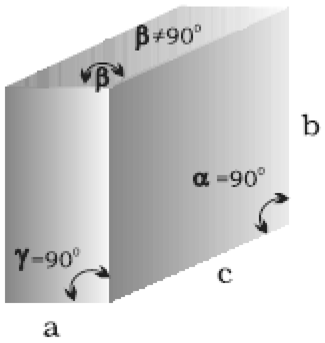
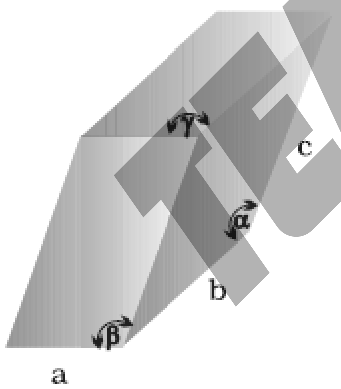
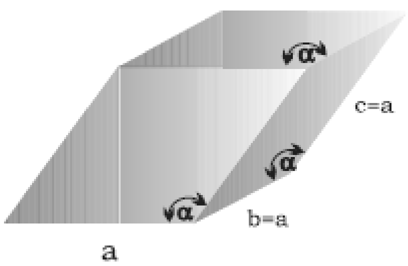
(iv) Primitive cell:- A primitive cell is a minimum volume unit cell or the simple unit cell with particles only at the corners and all other types of unit cells are called non-primitive unit cells.

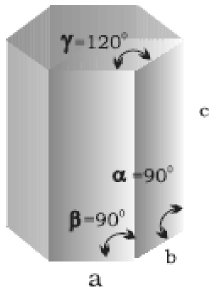
There is only one lattice point per primitive

(v) Crystallographic axis:- The lines drawn parallel to the lines of intersection of the faces of the unit cell are called Crystallographic axis.

All the crystals on the basis of the shape of their unit cells have been divided into seven crystal system as shown in the following table. These system arranged in 14 Bravais lattices.

System	Lattice Constants	Angle between lattice constants	Example
Cubic  Number of lattices = 3	$a = b = c$	$\alpha = \beta = \gamma = 90^o$	Diamond, NaCl, Li, Ag, NH_4Cl, Pb, Zinc blende, Cu etc.
Tetragonal  Number of lattices = 2	$a = b \neq c$	$\alpha = \beta = \gamma = 90^o$	White tin (SnO_2), $NiSO_4$ TiO_2, $CaSO_4$ etc.

Orthorhombic  Number of lattices = 4	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^o$	$HgCl_2$, KNO_3, Gallium Rhombic Sulphure, $BaSO_4$etc.
Monoclinc  Number of lattices = 2	$a \neq b \neq c$	$\alpha = \gamma = 90^o$ and $\beta \neq 90^o$	$KClO_3$, $FeSO_4$, Monoclinc Sulphure Na_2SO_4. 10 H_2O elc.
Triclinic  Number of lattices = 1	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^o$	$K_2Cr_2O_7$, $CuSO_4.5H_2O$, H_3BO_3 etc.
Rhombohedral  Number of lattices = 1	$a = b = c$	$\alpha = \beta = \gamma \neq 90^o$	Calcite($CaCO_3$), As, Sb, Bi, HgS (Cinnabar) etc.

Hexagonal  Number of lattices = 1	$a = b \neq c$	$\alpha = \beta = 90^\circ$ and $\gamma = 120^\circ$	Zn, Cd and Ni, Graphite, ZnO, CdS etc.

Comparison of Cell Properties of Some Crystal Structures

S.No.	Properties	Diamond Cubic (DC)	Simple Cubic (SC)	Body Centered Cubic (BCC)	Face Centered Cubic (FCC)	Hexagonal Close Packed Structure (HCP)
1.	Volume of unit cell	a^3	a^3	a^3	a^3	$\frac{3}{2}\sqrt{3}a^2c$
2.	Number of atoms per unit cell	8	1	2	4	6
3.	Number of atoms per unit volume	$\frac{8}{a^3}$	$\frac{1}{a^3}$	$\frac{2}{a^3}$	$\frac{4}{a^3}$	$\frac{4}{\sqrt{3}a^2c}$
4.	Number of nearest neighbours (coordination number)	4	6	8	12	12
5.	Nearest neighbour distance (2r)	$\frac{a\sqrt{3}}{4}$	a	$\frac{a\sqrt{3}}{2}$	$\frac{a\sqrt{2}}{2}$	a
6.	Atomic radius	$\frac{a\sqrt{3}}{8}$	$\frac{a}{2}$	$\frac{a\sqrt{3}}{4}$	$\frac{a\sqrt{2}}{4}$	$\frac{a}{2}$
7.	Atomic packing factor	0.034	0.52	0.68	0.74	0.74
8.	Examples	Germanium, Silicon and Carbon	Polonium	Sodium, Lithium and Chromium	Aluminium, Copper, Silver and Lead	Magnesium Zinc and Cadmium

■ **Bonding Forces in Crystals:-** The properties of a solid are mainly determined by the type of bonding that exists between the atoms. According to bonding in crystals they are classified into following types:-

1. **Ionic crystal:-** This type of bonding is formed due to transfer of electrons between atoms and consequent attraction between them.
   - (i) In NaCl Crystal, the electron of Na atom is transferred to chlorine atom. In this way Na atom changes into Na^+ ion and Cl atom changes into Cl^- ion.
   - (ii) Cause of bonding is electrostatic force between positive and negative ions.
   - (iii) These crystals are usually hard, brittle and possess high melting and boiling point.
   - (iv) These are bad conductors of electricity.
   - (v) Common examples are NaCl, CsCl, LiF etc.
2. **Covalent Crystal:-** Covalent bonding is formed by sharing of electrons of opposite spins between two atoms.
   - (i) The conductivity of these solids rise with rise in temperature.
   - (ii) These crystals Posses high melting point.
   - (iii) Bonding between H_2, Cl_2 molecules, Ge, Si quartz, diamond etc. are common examples of covalent bonding.
3. **Metallic bonds:-** This type of bonding is formed due to attraction of valence (free) electrons with the positive ion cores.
   - (i) Their conductivity decreases with rise of temperature
   - (ii) When visible light falls on a metallic crystal, the electrons of atom absorb visible light, so they are opaque to visible light.

4. **Vander Waal's crystal:-** These crystals consist of neutral atoms or molecules bonded together in solid phase by weak, short range attractive force & called Vander Waal's forces.
(i) This bonding is weakest and occurs in solid CO_2, methane, paraffin, ice etc.
(ii) They are normally insulator and soft, easily compressible and posses low melting point.

5. **Hydrogen bonding:-** Hydrogen bonding is due to permanent dipole interaction.
(i) This bond is stronger than Vander Waal's bond but much weaker than ionic and covalent bond.
(ii) They posses low melting point
(iii) Common examples are H_2O, HF etc.

(2) Energy Bands of solids

Theory of energy band is based on the Pauli Exclusion Principle. In isolated atom the valence electrons can exist only in one of the allowed orbital's each of a sharply defined energy called energy levels. When two atom are brought nearest to each other, there are alterations in energy levels and they spread in the form of bands.

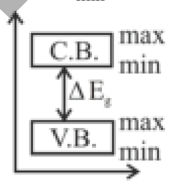
■ Energy bands are of following types:-

1. **Valence band:-** The energy band formed by a series of energy levels containing valence electrons is known as valence band. At '0' K, the electron fill the energy levels in valence band starting from lowest one.
(i) This band is always filled with electrons.
(ii) This is the band of maximum energy.
(iii) Electrons are not capable of gaining energy from external electric field.
(iv) No flow of current due to electrons present in this band.
(v) The highest energy level which can be occupied by an electron in valence band at '0' K is called Fermi level.

2. **Conduction band:-** The higher energy level band is called the conduction band.
(i) It is also called empty band of minimum energy.
(ii) This band is partially filled by the electrons.
(iii) In this band the electrons can gain energy from external electric field.
(iv) The electrons in the conduction band are called the free electrons. They are able to move anywhere within the volume of the solid.
(v) Current flows due to such electrons.

3. **Forbidden energy gap:- (ΔE_g)**
Energy gap between conduction band and valence band is known as forbidden energy gap.

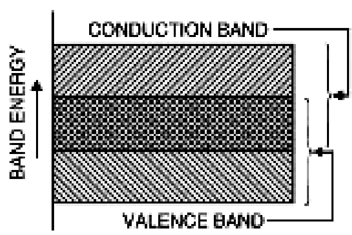
$$\Delta E_g = (C.B.)_{min} - (V.B.)_{max}$$



(i) No Free electron is present in forbidden energy gap.
(ii) Width of forbidden energy gap depends upon the nature of substance.
(iii) As temperature increase (↑), forbidden energy gap decrease (↓) very slightly.

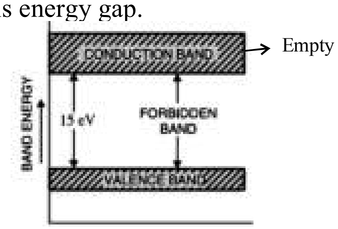
■ Classification of solids on the basis of energy bands- Depending on whether the energy band gap is zero, large or small the solid may be classified into conductors, insulators and semiconductor.

1. **Conductors (metals):-**
In case of metals either the conduction band is partially filled and valence band is partially empty or conduction band and valence band overlap in case of overlapping, electrons from valence band can easily move into conduction band thus large number of electrons are available for conduction.
In case valence band is empty, electrons from its lower level can move to higher level making conduction possible. This is the reason why resistance of metals is low or the conductivity is high.



2. Insulators

In insulators, the valence band is completely filled whereas the conduction band is completely empty. As there is no electron in conduction band so no electrical conduction is possible. The energy gap between conduction band and valence band is so large ($E_g > 3ev$) that electron in valence band can be provided so much energy from any external source that is can jump this energy gap.



3. Semiconductors

The energy band structure of a semiconductor is shown in figure. It is similar to that of an insulator but with a comparatively small energy gap ($E_g < 3ev$)

At absolute zero temperature the conduction band of semi conductor is totally empty and valence band is completely filled.

Therefore, they are insulators at low temperatures

■ Holes in Semi conductors:-

(i) When an electron is removed from a covalent bond it leaves a vacancy behind. An electron from a neighbouring atom can move into this vacancy leaving the neighbour with a vacancy. In this way the vacancy formed is called hole (or crater) and can travel through the material and serve as an additional current carrier.

(ii) A hole is considered as a seat of positive charge, having magnitude of charge equal to that of an electron.

(iii) Holes act as virtual charge, although there is no physical charge on it.

(iv) Effective mass of hole is more than electron.

(v) Mobility of hole is less than electron.

(3) Semiconductors

Semiconductors are the materials which have a conductivity between conductors (generally metals) and non-conductors or insulators (such as ceramics).

example : Germanium. Gallium arsenide and Silicon are the most commonly used semiconductors. Silicon is used in electronic circuit fabrication and gallium arsenide is used in solar cells, laser diodes etc.

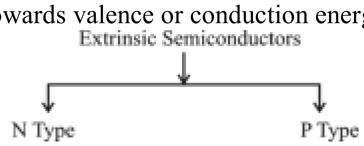
■ Types of semiconductors

(i) Intrinsic Semiconductor

- A semiconductor in its pure state is called intrinsic semiconductor. It has thermally generated current carriers.
- They have four electrons in the outermost orbit of atom and atoms are held together by covalent band.
- Free electrons and holes both are charge carriers and n_e (in C.B.) = n_h (in V.B.)
- The drift velocity of electrons (V_e) is greater than that of holes (V_n)
- Fermi energy level lies at the centre of the C.B. and V.B.
- In pure semiconductor, impurity must be less than 1 in 10^8 part of semiconductor
- In intrinsic semi conductor $n_e^{(\sigma)} = n_n^{(o)} = n$, where $n_e^{(\sigma)}$ = electron density in conduction band, $n_n^{(o)}$ = Hole density in V.B, n_i = density of intrinsic carriers.

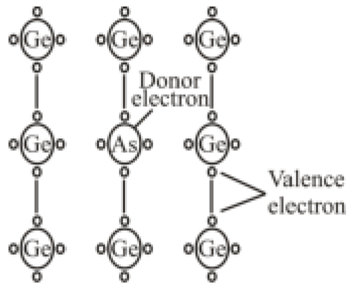
(ii) Extrinsic Semiconductor:-

- An impure semiconductor is called extrinsic semiconductor when pure semiconductor material is mixed with small amounts of certain specific impurities with valency different from that of the parent material the number of mobile electrons/holes drastically changes.
- The process of addition of impurity is called doping.
- In extrinsic semiconductor $n_e \neq n_h$.
- The number of atoms of impurity element is about 1 in 10^8 atoms of the semiconductor. In extrinsic semiconductors fermi level shifts towards valence or conduction energy band.

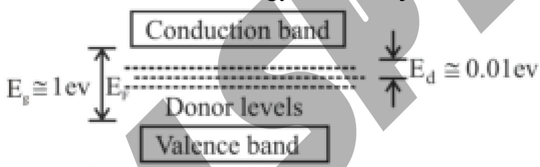


1. N type semiconductor – These are obtained by adding a small amount of pentavalent [Phosphorus (P), Arsenic (As), Antimony (Sb) etc] Impurity to a pure sample of semiconductor (Ge).

$$\begin{matrix}\text{Intrinsic} \\ \text{S.C.}\end{matrix} + \begin{matrix}\text{Pentavalent} \\ \text{impurity}\end{matrix} \longrightarrow \text{N} - \text{type semiconductor}$$



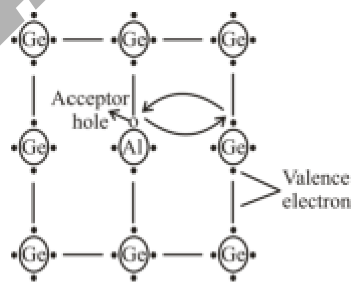
- Majority charge carriers $\rightarrow$ electrons
  Minority charge carriers $\rightarrow$ holes
- $n_e >> n_h$, $i_e >> i_h$
- conductivity $\sigma \approx n e \mu_e e$
- N–type semiconductor is electrically neutral (not negatively) charged. Impurity is called donor impurity because one impurity atom generates one electron. Donor energy level lies just below the conduction band.



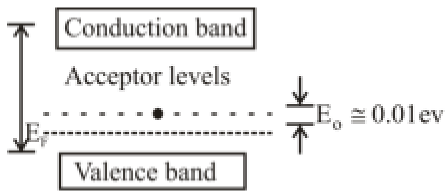
2. P type semiconductor:-

These are obtained by adding a small amount of trivalent [Boron (B), Aluminium (Al) Gallium (Ga) etc.] impurity to a pure sample of semiconductor (Ge).

$$\begin{matrix}\text{Intrinsic} \\ \text{S.C.}\end{matrix} + \begin{matrix}\text{trivalent} \\ \text{impurity}\end{matrix} \longrightarrow \text{P} - \text{type semiconductor}$$



- Majority charge carriers – holes
  Minority charge carriers – electrons
- $n_h >> n_e$; $i_h >> i_e$
- Conductivity $\sigma \cong n_n \mu_n e$
- P type semiconductor is also electrically neutral (not positively charged). Impurity is called acceptor impurity acceptor energy level lies just about the valence band.



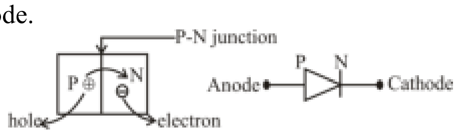
Densities of charge carriers:-

material	Type	Density of conduction electrons (m^{-3})	Density of holes (m^{-3})
Copper	Conductor	9×10^{28}	0
Silicon	intrinsic semiconductor	7×10^{15}	7×10^{15}
Silicon doped with phosphorus (1 part in 10^6)	N-type semiconductor	5×10^{22}	1×10^{9}
Silicon doped with Aluminium (1 part in 10^6)	P-type semiconductor	1×10^{9}	5×10^{22}

(4) P-N junction & Diodes

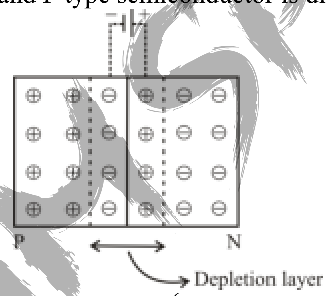
■ P-N junction Diode:-

When a P-type semiconductor is suitably joined to an N-type semiconductor, then resulting arrangement is called P–N junction or P–N junction diode.



1. Depletion region:- Depletion region or depletion layer is a region in a P-N junction diode where no mobile charge carriers are present. Depletion layer acts like a barrier that opposes the flow of electrons from n-side and holes from p-side. On account of different concentration of charge carries in the two sections of P-N junction the electron from N-region diffuse through the junction into P-region and the hole from P-region diffuse into N-region.

Due to diffusion, neutrality of both N and P type semiconductor is disturbed.



- The thickness of depletion layer is 1 micron = 10^{-6} m.
- Width of depletion layer $\propto \dfrac{1}{\text{Dopping}}$
- Depletion is directly proportional to temperature the P-N junction diode is equivalent to capacitor in which the depletion layer acts as a dielectric.

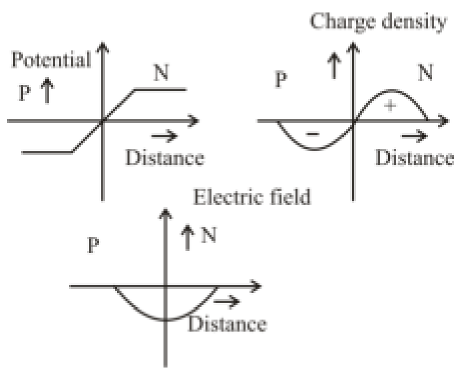
2. Potential barrier:- It is the potential difference created across the P-N junction due to the diffusion of electrons and holes.

For Ge, $\mathbf{V_B = 0.3V}$ and for silicon $\mathbf{V_B = 0.7V}$

On the average the potential barrier in P-N junction is ~ 0.5V and the width of depletion region ~ 10^{-6}m.

So the barrier electric field $E = \dfrac{V}{d} = \dfrac{0.5}{10^{-6}} = 5\times10^{5}$ V/m

■ Some important graphs:-

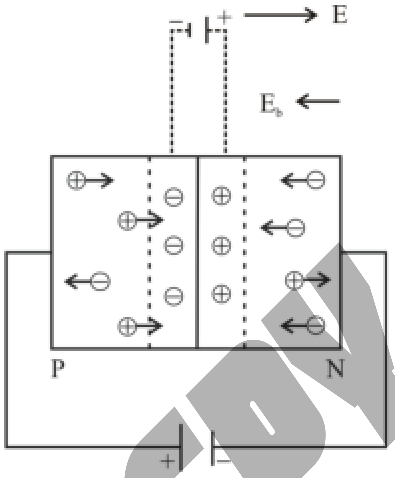


3. **Diffusion and drift current:-**

- Diffusion current is a current in a semiconductor caused by the diffusion of charge carriers (holes and/ or electrons). This current is due to transport of charges occurring because of non-uniform charged particles in a semiconductor.
- Drift current is the current flow in semiconductor due to the applied electric field. It is due to motion of charge carriers due to the force exerted on them by an electric field.

■ **Biasing**:- It means the way of connecting emf sourced to P-N junction diode. It is of following two types.

(1) Forward Biasing: Positive terminal of the battery is connected to the P-crystal and negative terminal as the battery is connected to N-Crystal.



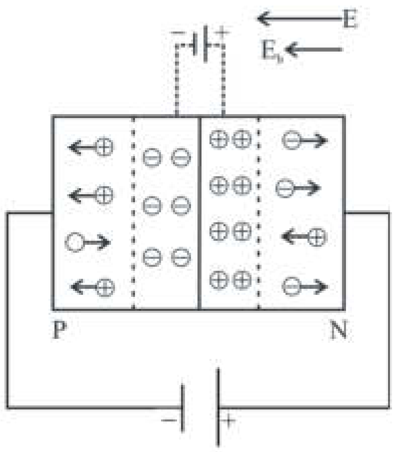
- In forward biasing width of depletion layer decreases.
- In forward biasing resistance offered $R_{Forward} \approx 10\Omega - 25\Omega$
- Forward bias opposes the potential barrier and for V > V_B a forward current is set up across the junction.

The current is given by $i = i_s\left(e^{ev/KT} - 1\right)$

Where–

i_s = saturation current in the exponent.
e = 1.6×10^{-19} C
k = Boltzmann's constant.

2. Reverse Biasing:- Positive terminal of the battery is connected to the N-crystal and negative terminal of the battery is connected to P-crystal.

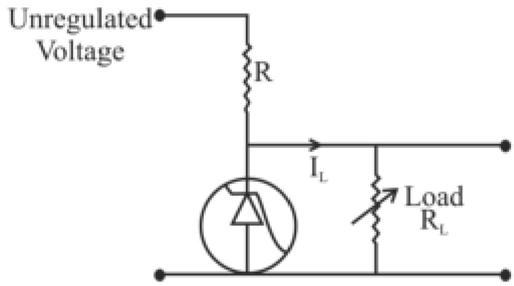


- In reverse biasing width of depletion layer increases.
- In reverse biasing resistance offered reverse $\approx 10^5\,\Omega$
- The net reverse current is given by-

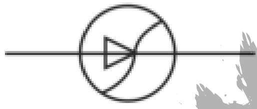
I = Drift current – diffusion current

■ Special Purpose Diodes:-

1. **Zener diode-** It is a highly doped p-n junction which is not damaged by high reverse current. It can operate continuously without being damaged in the region of reverse background voltage. In the forward bias the Zener diode acts as ordinary diode. It can be used as voltage regulator.

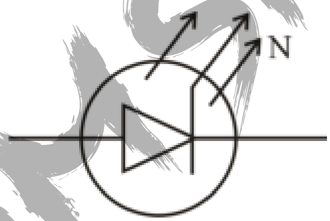


(A) Zener diode as a voltage regulator

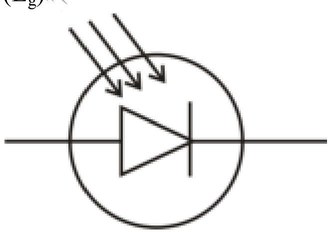


Symbol of zener diode

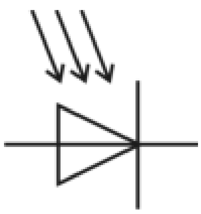
2. **Light Emitting diode (LED):-** These are specially designed diodes which give out light radiations when forward biased. LED's are made of GaAsP, GaP etc.



3. **Photo diode:-** Photo diode is a special type of photo-detector. (i.e. $h\nu > E_g$).
Here, Photon energy ($h\nu$) > Band energy (E_g)



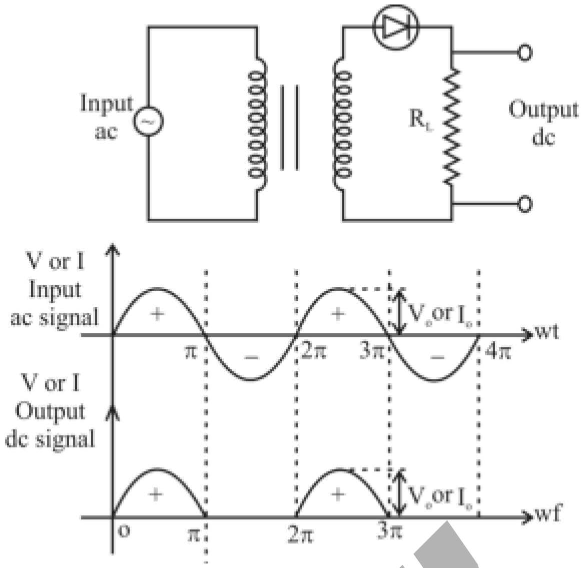
4. **Solar cells:-** It is based on the photovoltaic effect. One of the semiconductor region is made so thin that the light incident on it reaches the P–N Junction and gets absorbed. It converts solar energy into electrical energy.



■ P-N Junction Diode as a Rectifier:-

Rectifier is a circuit which converts ac to unidirectional pulsating output. In other words it converts ac to dc. It is of following types.

1. **Half wave rectifier:-** When the P-N junction diode rectifies half of the ac wave, it is called half wave rectifier.



(i) During positive half cycle

$$\text{Diode} \longrightarrow \text{forward biased}$$

$$\text{Output signal} \longrightarrow \text{obtained}$$

(ii) During negative half cycle

$$\text{Diode} \longrightarrow \text{reverse biased}$$

$$\text{Output signal} \longrightarrow \text{reverse obtained}$$

(iii) Output voltage is obtained across the load resistance R_L. It is not constant but pulsating (mixture of ac and dc) in nature.

(iv) Average output in one cycle

$$I_{dc} = \frac{I_o}{\pi} \text{ and } V_{dc} = \frac{V_o}{\pi}$$

$$I_o = \frac{V_o}{r_f + R_L}$$

$$V_{dc} = \frac{V_o}{\pi\left(1 + \frac{r_f}{R_L}\right)}$$

(r_f = Forward biased resistance)

(v) r.m.s. output: $I_{rms} = \frac{I_o}{2}$

$$V_{rms} = \frac{V_o}{2}$$

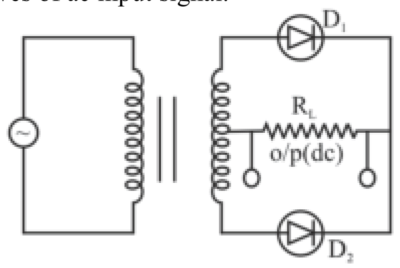
(vi) The ratio of the effective alternating component of the output voltage or current to the dc component is known as ripple factor.

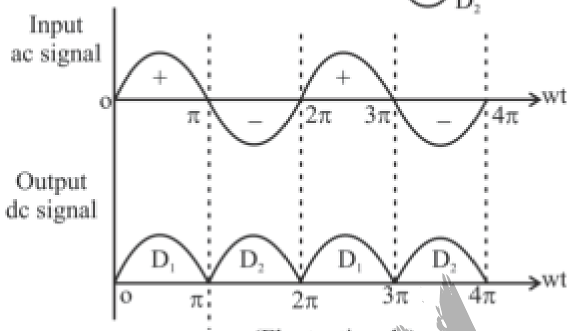
$$r = \frac{I_{ac}}{I_{dc}} = \left[\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1\right]^{1/2} = 1.21$$

(vii) The ripple frequency (ω) for half wave rectifier is same as that of a.c.

(viii) Form factor $= \frac{I_{rms}}{I_{dc}} = \frac{\pi}{2} = 1.57$

2. Full wave rectifier:- It rectifies both halves of ac input signal.





(Fluctuating dc)

Full wave rectifier:-

(i) During positive half cycle

Diode : $D_1 \rightarrow$ forward biased

$D_2 \rightarrow$ reverse biased

Output Signal $\rightarrow$ obtained due to D_1 only

(ii) During negative half cycle

Diode : $D_1 \rightarrow$ reverse biased

$D_2 \rightarrow$ forward biased

Output Signal $\rightarrow$ obtained due to D_2 only

(iii) Fluctuating dc $\rightarrow \boxed{\text{Filter}} \rightarrow$ Constant dc

(iv) Output voltage is obtained across the load resistance R_L. It is not constant but pulsating in nature.

(v) Average output; $V_{av} = \dfrac{2V_0}{\pi}, I_{av} = \dfrac{2I_0}{\pi}$

$$V_{dc} = \frac{2V_0}{\pi\left(1 + \frac{r_f}{R_L}\right)}$$

(vi) r.m.s OUTPUT; $V_{rms} = \dfrac{V_0}{\sqrt{2}}, I_{rms} = \dfrac{I_0}{\sqrt{2}}$

(vii) Ripple factor; $r = 0.48 = 48\%$

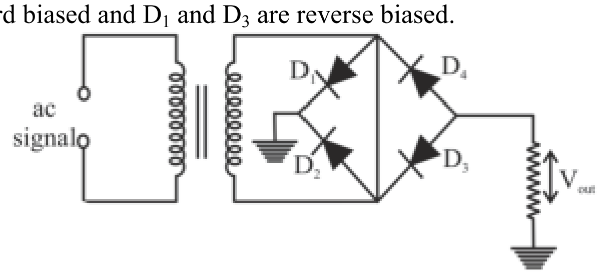
(viii) Ripple frequency ; The ripple of full wave rectifier $= 2\times$ (Frequency of input ac)

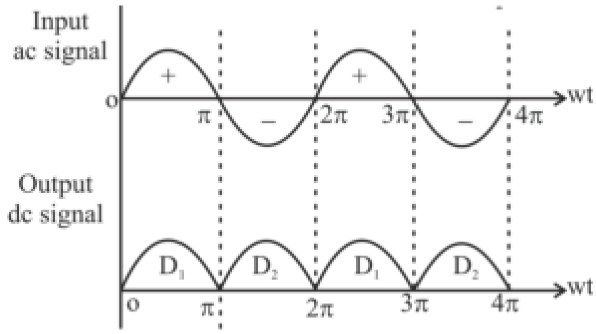
(ix) Peak inverse voltage (piv); it's value, is $2V_0$

(x) Efficiency ; $n = 81.2\%$

3. Full wave bridge rectifier:- Four diodes D_1, D_2, D_3 and D_4 are used in the circuit.

During positive half cycle D_1 and D_3 are forward biased and D_2 and D_4 are reverse biased. During negative half cycle D_2 and D_4 are forward biased and D_1 and D_3 are reverse biased.





(Fluctuating dc)

Parameter	Half Wave (or one pulse)	Full Wave (two Pulse) Center-tap	Bridge
DC output voltage	$\frac{V_m}{\pi}$	$\frac{2V_m}{\pi}$	$\frac{2V_m}{\pi}$
RMS value of output voltage V_{ms}	$\frac{V_m}{2}$	$\frac{V_m}{\sqrt{2}}$	$\frac{V_m}{\sqrt{2}}$
Ripple voltage V_r	0.3856 V_m	0.3077 V_m	0.3077 V_m
Voltage ripple factor (V.R.F.)	1.211	0.482	0.482
Rectification efficiency (η)	40.53%	81.06%	81.06%
TUF	0.2865	0.672	0.8106%
PIV	V_m	2Vm	Vm
C.F.	2	$\sqrt{2}$	$\sqrt{2}$
No. of diodes	1	2	4
Ripple frequency	f	2f	2f

(5) Transistors

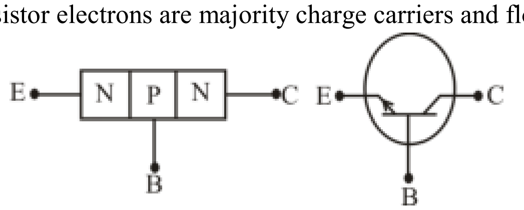
Transistor is a type of a semiconductor device that can be used to both conduct and insulate electric current or voltage. A transistor basically acts as switch and an amplifier. In simple words, we can say that a transistor is a miniature device that is used to control or regulate the flow of electronic signals.

■ **It consists of three main regions:-**

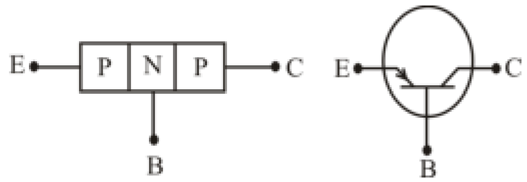
- **Emitter (E):-** It provides majority charge carriers by which current flows in the transistor. Therefore the emitter semiconductor is heavily doped.
- **Base (B):-** The based region is lightly doped and thin.
- **Collector (C):-** The size of collector region is larger than the two other regions.

■ **Junction transistors are of two types-**

1. **NPN transistor –** It is formed by sandwiching a thin layer of P-type semiconductor between two N-type semiconductors. In NPN transistor electrons are majority charge carriers and flow from emitter to Base.



2. **PNP transistor:-** It is formed by sandwiching a thin layer of N-type semiconductor between two P-type semiconductor. In PNP transistor holes are majority charge carriers and flow from emitter to base. In the symbols of both NPN and PNP transistor arrow indicates the direction of conventional current.



■ Working of transistors:-

- There are four possible ways of biasing the two P-N junctions (emitter junction and collector junction of transistor.)
  (a) Active mode:- Also known as linear mode operation.
  (b) Saturation mode:- Maximum collector current flows and transistor acts as a closed switch from collector to emitter terminals.
  (c) Cut off mode:- Denotes operations like an open switch where only leakage current flows.
  (d) Inverse mode:- The emitter and collector are interchanged.
- A transistor is mostly used in the active region of operation i.e. emitter base junction is forward biased and collector base junction is reverse biased.
- From the operation of junction transistor it is found that when the current in emitter circuit changes is corresponding change in collector current.
- In each state of the transistor there is an input port and an output port. In general each electrical quantity (V or I) obtained at the output is controlled by the input.

■ Transistor configurations:-

A transistor can be connected in a circuit in the following three different configurations. Common base (CB), Common emitter (CE) and Common Collector (CC) configuration

(i) CB Configurations- Base is common to both emitter and collector.

(ii) CE Configurations- Emitter is common to both base and collector.

(iii) CC Configurations- Collector is common to both base and emitter.

An n-channel FET (Field-Effect Transistor) consists of a block of N-type material with contacts at each end together with a strip of P-type material on one side that is called the gate.

■ Transistor as an Amplifier:-

A device which increases the amplitude of the input signal is called amplifier.

The transistor can be used as an amplifier in the following three configurations.

(i) CB amplifier (ii) CE amplifier (iii) CC amplifier

Different gains in CE/CB Amplifiers-

(1) Transistor as CB amplifier-

(i) $\text{ac current gain } \alpha_{ac} = \dfrac{\text{Small change in collector current } (\Delta i_c)}{\text{Small change in emitter current } (\Delta i_e)}$

(ii) $\text{dc current gain } \alpha_{dc}\,(\text{or } \alpha) = \dfrac{\text{collector current } (i_c)}{\text{emitter current } (i_e)}$

value of α_{dc} lies between 0.95 to 0.99

(iii) $\text{Voltage gain } A_v = \dfrac{\text{Change in output voltage } (\Delta V_0)}{\text{Change in input voltage } (\Delta V_i)}$

$\Rightarrow A_v = \alpha_{ac} \times \text{Resistance gain}$

$\text{Power gain} = \dfrac{\text{Change in output power } (\Delta P_0)}{\text{Change in input power } (\Delta P_c)}$

$\text{Power gain} = \alpha_{ac}^2 \times \text{Resistance gain}$

(2) Transistor as CE amplifier-

(i) $\text{ac current gain } \beta_{ac} = \left(\dfrac{\Delta_{ie}}{\Delta_{ib}}\right) V_{CE} = \text{constant}$

(ii) $\text{dc current gain } \beta_{dc} = \dfrac{i_c}{i_b}$

(iii) Voltage gain; $A_v = \frac{\Delta v_0}{\Delta v_1} = \beta_{ac} \times \text{Resistance gain}$

(iv) Power gain = $A_v = \frac{\Delta p_0}{\Delta p_1} = \beta_{ac}^2 \times \text{Resistance gain}$

(v) Trans conductance (g_m), The ratio of the change in collector current to the change in emitter base voltage is called conductance i.e.,

$$g_m = \frac{\Delta I_C}{\Delta V_{EB}} = \frac{A_V}{R_L}; R_L = \text{Load Resistance}$$

(3) Relation between α and β

$$\beta = \frac{\alpha}{1-\alpha} \quad \text{or} \quad \alpha = \frac{\beta}{1+\beta}$$

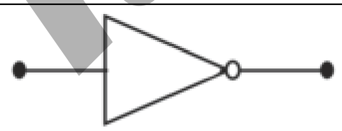
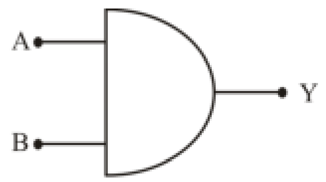
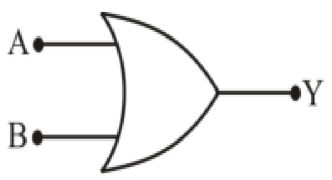
■ Comparison of Amplifier Configurations

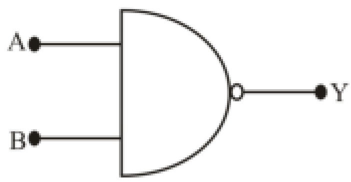
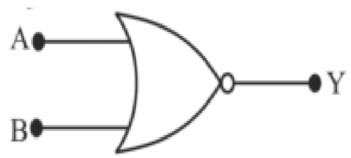
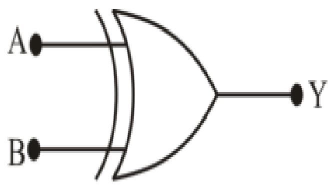
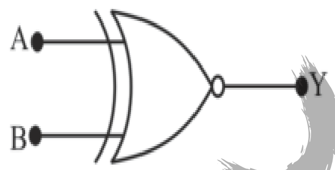
S. No.	Particulars	Common Base	Common Emitter	Common Collector
1.	Current Gain	Nearly Unity	High	Very High
2.	Voltage Gain	High	Very High	Nearly Unity
3.	Power Gain	High	High	Low
4.	Input Impedance	Low	High	Very High
5.	Output Impedance	Very High	High	Low
6.	Phase Reversal	No	Yes	No

(6) Logic Gates and Truth Tables

■ **Logic Gates:-** The digital circuit that can be analysed with the help of Boolean algebra is called logic gate or logic circuit. A logic gate has two or more inputs but only one output.
There are primarily three logic gates namely the OR gate, the AND gate and the NOT gate.

■ **Truth table:-** The operation of a logic gate or circuit can be represented in a table which contains all possible inputs and their corresponding outputs is called the truth table. To write the truth table we use binary digit 1 and 0.

Name	Logical symbol	Boolean Expression	Truth table
Not (Inverter)		$Y = \bar{A}$	A | $Y = \bar{A}$ 0 | 1 1 | 0
AND		Y = A. B	A | B | Y = A.B 0 | 0 | 0 0 | 1 | 0 1 | 0 | 0 1 | 1 | 1
OR		Y= A + B	A | B | Y = A + B 0 | 0 | 0 0 | 1 | 1 1 | 0 | 1 1 | 1 | 1

NAND (Universal)		$Y = \overline{A \cdot B}$	(see table below)
NOR (Universal)		$Y = \overline{A + B}$	(see table below)
XOR		$Y = A \oplus B$ $= \overline{A}B + A\overline{B}$	(see table below)
X-NOR		$Y = \overline{A \oplus B}$	(see table below)

A	B	A.B	$Y = \overline{A.B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

A	B	$A + B$	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	$Y = A \oplus B = \overline{A}B + A\overline{B}$
0	0	0
0	1	1
1	0	1
1	1	0

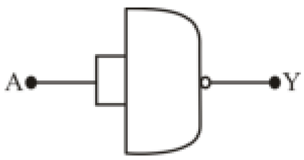
A	B	$A \oplus B$	$Y = \overline{A \oplus B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	0	1

■ Logic gates using 'NAND' gate

The NAND gate is the building block of the digital electronics all the logic gates like the OR the AND and the NOT can be constructed from the NAND gates.

1. Construction of the 'NOT' gate from the 'NAND' gate

When both the inputs (A and B) of the NAND gate are joined together then it works as the NOT gate.

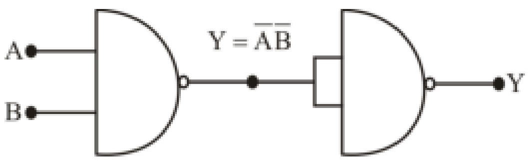


Truth Table and logic symbol

Input	Output
A = B	Y
0	1
1	0

2. Construction of the 'AND' gate from the 'NAND' gate

When the output of the NAND gate is given to the input of the NOT gate (made from the NAND gate) then the resultant logic gate works as the AND gate.

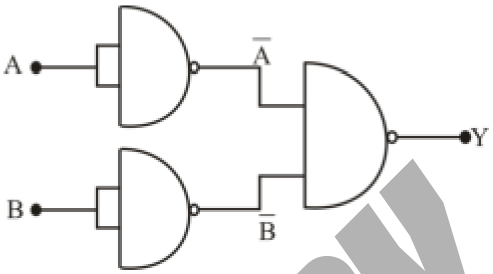


Truth Table and logic symbol

A	B	Y'	Y
0	0	1	0
0	1	1	0
1	0	1	0
1	1	0	1

3. Construction of the 'OR' gate by the 'NAND' gate

When the output of two NOT gates (obtained from the NAND gate) is given to the inputs of the NAND gates the resultant logic gate works as the OR gate.



A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

■ Key Points:-

(i) The most efficient packing of atoms in cubic lattice structure occurs for fcc.
(ii) The lattice for NaCl crystal is fcc
(iii) The space lattice of diamond is fcc.
(iv) Carbon, Silicon, germanium, tin can crystallize in the diamond structure.
(v) At room temperature $\sigma_{Ge} > \sigma_{si}$
(vi) Semiconductor device are current control devices
(vii) The Semiconductor device are temperature sensitive devices.
(viii) The electric field setup across the potential barrier is of the order of 3×10^5 V/m for Ge and 7×10^5 V/m for Si
(ix) If Eg $\approx$ 0 eV, the material is good conductor or metal and if Eg $\approx$ 1 eV, the material is a simeconductor. If Eg $\approx$ 6 eV then the material is an insulator.
(x) Voltage obtained from a diode rectifier is a mixture of alternating and direct voltage.
(xi) The base is much thinner than the emitter while collector is wider than both emitter & base
(xii) CC (Common Collector) amplifier is called power amplifier or current booster or emitter follower.
(xiii) When a P N junction is forward biased, it can emit light hence can serve as a light emitting diode (LED). The wavelength of the emitted light is.

$$\lambda = \frac{c}{f} = \frac{hc}{Eg}$$

(xiv) The Fermi energy of a given material is the energy of a quantum state that the probability 0.5 of being occupied by an electron.
(xv) A good emitter should have low work function, high melting point, high working temperature, high electrical and mechanical strength.
(xvi) When triode amplifier are in series then the total voltage gain-
$A = A_1A_2A_3 =$ ----------
(xvii) NOR and NAND gates is a universal gates because they can be used to perform the basic logic function AND, OR and NOT.
(xviii) Output in Ex- OR gate is '1' only when inputs are different.
(xix) If both inputs of NAND gate are shorted then it will become 'NOT' gate.

Exemplar Problems
Electronics

Semiconductors and Diodes

1. For using a multimeter to identify diode from electrical components. choose the correct statement out of the following about the diode :

(a) It is two terminal device which conducts current in both directions.

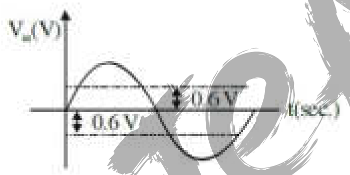
(b) It is two terminal device which conducts current in one direction only

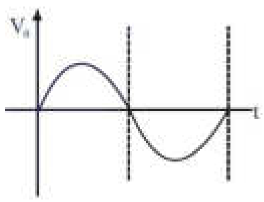
(c) It does not conduct current gives an initial deflection which decays to zero.

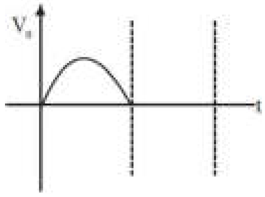
(d) It is three terminal device which conducts current in one direction only between central terminal and either of the remaining two terminals

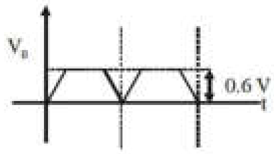
JEE MAIN 2022

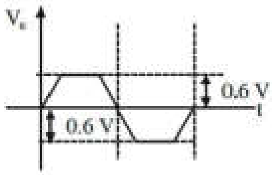
2. In the given circuit the input voltage V in is shown in figure. The cut–in voltage of p–n junction diode (D_1 or D_2) is 0.6 V. Which of the following output voltage (V_0) waveform across the diode is correct?



(a) 

(b) 

(c) 

(d) 

JEE MAIN 2022

3. The electron concentration in an n-type semiconductor is the same as hole concentration in a p-type semiconductor. An external field (electric) is applied across each of them. Compare the currents in them.

(a) No current will flow in p-type, current will only flow in n-type

(b) Current in n-type = current in p-type

(c) Current in p-type > current in n-type

(d) Current in n-type > current in p-type.

NEET 2021

4. For a *p*-type semiconductor, which of the following statements is true?

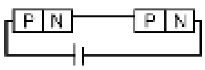
(a) Electrons are the majority carriers and pentavalent atoms are the dopants

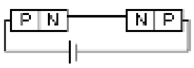
(b) Electrons are the majority carriers and trivalent atoms are the dopants.

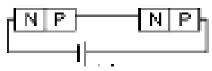
(c) Holes are the majority carriers and trivalent atoms are the dopants.

(d) Holes are the majority carriers and pentavalent atoms are the dopants.

NEET 2019

5. (a) 

(b) 

(c) 

In the given circuits (a), (b) and (c), the potential drop across the two *p-n* junctions are equal in

(a) Both circuits (a) and (c)

(b) Circuit (a) only

(c) Circuit (b) only

(d) Circuit (c) only

NEET 2022

6. **Consider the following statements (A) and (B) and identify the correct answer.**

(A) A zener diode is connected in reverse bias, when used as a voltage regulator.

(B) The potential barrier of p-n junction lies between 0.1 V to 0.3 V.

(a) (A) is incorrect but (B) is correct.
(b) (A) and (B) both are correct.
(c) (A) and (B) both are incorrect
(d) (A) is correct and (B) is incorrect.

NEET 2021

7. **The increase in the width of the depletion region in a p-n junction diode is due to :**

(a) reverse bias only
(b) both forward bias and reverse bias
(c) increase in forward current
(d) forward bias only

NEET 2020

8. **An LED is constructed from a *p-n* junction diode using GaAsP. The energy gap is 1.9 eV. The wavelength of the light emitted will be equal to**

(a) 10.4×10^{-26} m (b) 654 nm
(c) 654 Å (d) 654×10^{-11} m

Odisha NEET 2019

9. **For a transistor to act as a switch, it must be operated in**

(a) Active region
(b) Saturation state only
(c) Cut-off state only
(d) Saturation and cut-off state

JEE MAIN 2022

10. **The positive feedback is required by an amplifier to act an oscillator. The feedback here means :**

(a) External input is necessary to sustain ac signal in output.
(b) A portion of the output power is returned back to the input.
(c) Feedback can be achieved by LR network.
(d) The base-collector junction must be forward

JEE MAIN 2022

11. **In a pure semiconductor**

(a) holes and electrons have equal mobility
(b) holes have higher mobility
(c) electrons have higher mobility
(d) only electrons are mobile

UPTGT Science - 2021

12. **When a p-n junction is reverse biased, its depletion region**

(a) become wider
(b) becomes narrower
(c) disappears
(d) does not change

UPTGT Science - 2021

13. **In n type semi-conductor, if doping concentration is high then fermi level at 0 K lies**

(a) at the centre of band gap
(b) very near to valence band
(c) between the donor level and the bottom of conduction band
(d) between the donor level and centre of band gap

RPSC School Lect. 26.07.2016

14. **In the case of p – n junction, diode at high value of reverse bias, the current rises sharply. The value of reverse voltage is known as**

(a) Cut in voltage
(b) Break-down voltage
(c) Cut off voltage
(d) Inverse voltage

UP TGT Physics 2016

15. **A p–type of semiconductor is**

(a) Uncharged
(b) + Vely charged
(c) –Vely charged
(d) Uncharged only at 0^0C

UP TGT Physics 2016

16. **A P-n junction has a thickness of the order of**

(a) 1 cm (b) 10^{-3}cm
(c) 10^{-6}m (d) 10^{-9}cm

UPPSC Ashram Paddhati PGT - 2021

17. **The forbidden band is 1.1eV in silicon. For which of the following wavelengths, it will be transparent?**

(a) 100 nm (b) 200nm
(c) 400nm (d) 1200nm

UPPSC Ashram Paddhati PGT - 2021

18. **Solar cell is a**

(a) Transistor (b) Diode
(c) p-n junction (d) Voltmeter

UPPSC Ashram Paddhati PGT - 2021

19. **Avalanche effect occurs in a pn junction, usually when -**

(a) depletion layer is thick
(b) depletion layer is thin
(c) with highly doped p & n sides
(d) None of these

UPPSC Poly. Lect. 2021 Paper - II

20. **n-type semiconductor is obtained by doping of Germanium with impurity of -**

(a) Aluminum (b) Indium
(c) Arsenic (d) Gallium

UPPSC Poly. Lect. 2021 Paper - II

21. **For detection of light intensity, we use -**

(a) LED in forward bias
(b) LED in reverse bias
(c) Photodiode in forward bias
(d) Photodiode in reverse bias

UPPSC Poly. Lect. 2021 Paper - II
UPPCS (Pre) 2000

22. **A Zener diode is suitable for-**
(a) Voltage regulation (b) Current regulation
(c) Both (a) and (b) (d) Rectifier

UPPSC Poly. Lect. 2021 Paper - II

23. **As reverse bias across the collector-base junction increases, the width of the collector-base depletion layer increases, and the effective width of base decreases. This is called**
(a) Tunnel effect (b) Early effect
(c) Zeeman effect (d) Schottky barrier

UPPSC GDC - 2021

24. **A Zener diode can be put to break down by applying**
(a) Forward bias (b) Large reverse bias
(c) No bias (d) None of the above

UPPSC GDC - 2021

25. **Which of the following is correct about depletion region capacitance?**
(a) It is always zero
(b) It is constant
(c) It is independent of reverse voltage
(d) it depends upon the magnitude of reverse voltage

UPPSC GDC - 2021

26. **The colour of light emitted by a LED depends on :**
(a) its forward bias.
(b) its reverse bias.
(c) the amount of forward current.
(d) the type of semiconductor material used.

UKPSC Lecturer (Mains) 2020

27. **Semiconductor Lasers are formed by using:**
(a) combination of semiconductor and insulator
(b) amorphous semiconductors
(c) non–degenerate semiconductors
(d) degenerate semiconductors

UPPCS (Pre) 2004

28. **Zener Breakdown occurs**
(a) mostly in silicon diodes
(b) mostly in Ge-diodes
(c) because of rupture of covalent bonds
(d) due to thermally generated carriers

(UPPGT 2013)
UPPCS Pre 2007, 2002

29. **The depletion layer width of a p-n junction has maximum value in**
(a) forward bias (b) reverse bias
(c) zero bias (d) a.c. bias

UPPCS Pre 2006, 2009

30. **A zener diode is used for**
(a) rectification (b) modulation
(c) detection (d) voltage regulation

UPPCS Pre 2009

31. **In p-n junction holes diffuse from the p-region to n-region because of :**
(a) the potential difference across the p-n junction
(b) the attraction of the free electrons of n-region
(c) the higher hole concentration in p-region is higher than in n-region
(d) All of the above

UPPCS Pre 2010

32. **The diodes which are used for voltage regulation, are:**
(a) Photodiodes (b) PIN diodes
(c) Varacter diodes (d) Zener diodes

(UPPGT 2013)
UPPCS (Pre) 2004

Transistors

33. **In half wave rectification, if the input frequency is 60 Hz, then the output frequency would be**
(a) 120 Hz (b) Zero
(c) 30 Hz (d) 60 Hz

NEET 2022

34. **Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.**

Assertion A : n-p-n transistor permits more current than a p-n-p transistor.

Reason R : Electrons have greater mobility as a charge carrier.

Choose the correct answer from the options given below :
(a) Both A and R true. and R is correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

JEE MAIN 2022

35. **For transistor action, which of the following statements is correct?**
(a) Base, emitter and collector regions should have same size.
(b) Both emitter junction as well as the collector junction are forward biased.
(c) The base region must be very thin and lightly doped.
(d) Base, emitter and collector regions should have same doping concentrations.

NEET 2020

36. **For transistor action, which of the following statements is correct?**
(a) Base, emitter and collector regions should have same doping concentrations.
(b) Base, emitter and collector regions should have same size.
(c) Both emitter junction as well as the collector junction are forward biased.
(d) The base region must be very thin and lightly doped.

NEET 2020

37. **In a common emitter transistor amplifier the audio signal voltage across the collector is 3 V. The resistance of collector is 3 kΩ. If current gain is 100 and the base resistance is 2 kΩ, the voltage and power gain of the amplifier is**
(a) 15 and 200 (b) 150 and 15000
(c) 20 and 2000 (d) 200 and 1000

NEET 2017

38. **In a common base connection, the current amplification factor is 0.95 . If the emitter current is 0.9 mA, the value of base current will be.**
(a) $50\,\mu A$ (b) $55\,\mu A$
(c) $45\,\mu A$ (d) $51\,\mu A$

UPPSC Ashram Paddhati PGT - 2021

39. **A transistor having $\alpha = 0.975$ and negligible reverse saturation current is operated in Common Emitter configuration (CE) mode. If $I_B = 250\mu A, I_C$ will be -**
(a) 10.45 mA (b) 10.15 mA
(c) 9.35 mA (d) 9.75 mA

UPPSC Poly. Lect. 2021 Paper - II

40. **Which of the following circuit is being used impedance matching?**
(a) transistor in common base configuration
(b) transistor in common emitter configuration
(c) transistor in common collector configuration
(d) zener diode in reverse bias

UP PGT - 2021

41. **Which of the following is correct relation between common-emitter current gain β and common base current gain α?**
(a) $\beta = \frac{1}{1-\alpha}$ (b) $\beta = \frac{\alpha}{1+\alpha}$
(c) $\beta = \frac{\alpha}{1-\alpha}$ (d) $\beta = \frac{1-\alpha}{1}$

UPPSC GDC - 2021

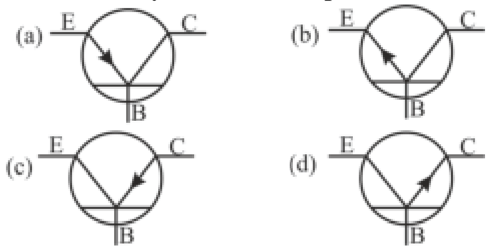
42. **In an n-p-n transistor circuit collector current is 10 mA. If (only) 90% of electrons emitted reach collector, emitter current will be :**
(a) 11 mA (b) 17 mA
(c) 9 mA (d) 15 mA

UKPSC Lecturer (Mains) 2020

43. **Voltage gain of a transistor amplifier with a load resistance 1 kΩ and input resistance 200Ω is 450. The value of β is :**
(a) 60 (b) 80
(c) 70 (d) 90

UKPSC Lecturer (Mains) 2020

44. **Which is a symbol of the n-p-n transistor?**



UP PGT Physics 2016 (Exam : 02.02.2019)

45. **In a PNP transistor in the common base configuration the emitter current changes from 0.6 mA to 0.4 mA when the emitter to base voltage is changed from 0.68 V to 0.64 V. The dynamic input resistance is**
(a) 5×10^{-3} Ohm (b) 1.133 Ohm
(c) 1.60 Ohm (d) 200 Ohm

UPPCS Pre 2007

46. **For a transistor, 'β' is the base transport factor an 'γ' is the emitter injection efficiency, then current transport ratio 'α' is given by**
(a) $\alpha = \beta\gamma$ (b) $\alpha = \beta/\gamma$
(c) $\alpha = \gamma/\beta$ (d) $\alpha = \frac{\beta-\gamma}{\beta+\gamma}$

UPPCS Pre 2007

47. **In which configuration of a transistor, the voltage gain is highest?**
(a) Common base
(b) Common emitter
(c) Common collector
(d) Same in all the above three

UPPCS (Pre) 1999
UPPCS Pre 2007

48. **A common emitter amplifier has**
(a) large voltage gain but small current and power gains
(b) large current gain but small voltage and power gains
(c) large voltage current and power gains
(d) none of the above

UPPCS Pre 2005

49. **Three amplifier stages each with a gain of 10 are cascaded. The net gain will be**
(a) 30 (b) 1000
(c) 3.33 (d) 10

UPPCS Pre 2005, 2007

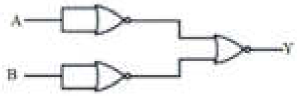
50. The ripple voltage present in the output of a half wave rectifier is

(a) more than
(b) less than
(c) equal to
(d) independent of the d.c. voltage present in its output

UPPCS Pre 2005

Logic Gates & Truth Tables

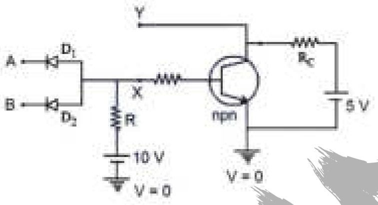
51. Identify the logic operation performed by the given circuit :



(a) AND gate (b) OR gate
(c) NOR gate (d) NAND gate

JEE MAIN 2022

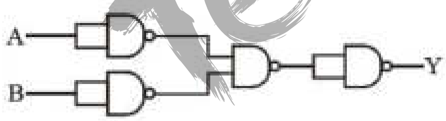
52. In the following circuit, the correct relation between output (Y) and inputs A and B will be:



(a) $Y = AB$ (b) $Y = A + B$
(c) $Y = \overline{AB}$ (d) $Y = \overline{A + B}$

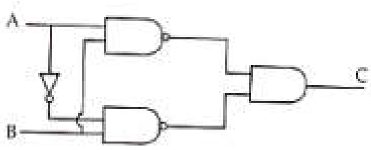
JEE MAIN 2022

53. The following logic gate is equivalent to:



(a) NOR Gate (b) OR Gate
(c) AND Gate (d) NAND Gate

JEE MAIN 2021

54. 

The truth table for the given logic circuit is

(a)

A	B	C
0	0	0
0	1	1
1	0	0
1	1	1

(b)

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

(c)

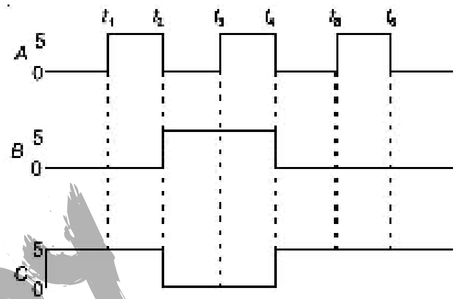
A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

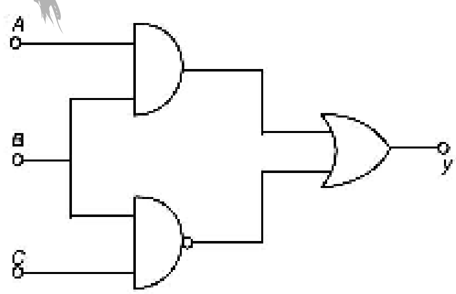
(d)

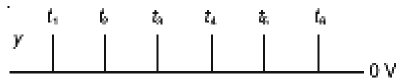
A	B	C
0	0	1
0	1	0
1	0	1
1	1	0

NEET 2022

55. For the given circuit, the input digital signals are applied at the terminals A, B and C. What would be the output at the terminal y?





(a) 

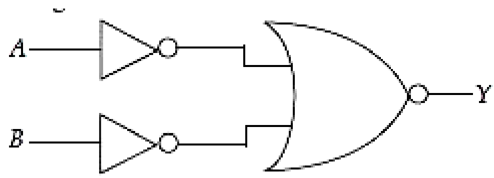
(b) 

(c) 

(d) 

NEET 2021

56. For the logic circuit shown, the truth table is



(a)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(b)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(c)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

NEET 2019

57. A digital gate having two inputs A and B and output C has the following truth table

A	B	C
1	1	0
1	0	1
0	1	1
0	0	1

Given truth table corresponds to

(a) OR gate (b) AND gate

(c) NOR gate (d) NAND gate

UPPSC Ashram Paddhati PGT - 2021
UPPCS Pre 2009
UPPCS (Pre) 2000

58. The output of NOR gate can be represented as-

(a) $Y = \overline{A} + \overline{B}$ (b) $Y = \overline{A}.\overline{B}$

(c) $Y = A + B$ (d) $Y = \overline{A}.\overline{B}$

UPPSC Poly. Lect. 2021 Paper - II

59. Which of the following gates is universal gate?

(a) XOR (b) NOR

(c) NOT (d) AND

UPPSC Poly. Lect. 2021 Paper - II

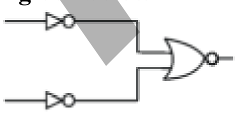
60. Which of the following is not the basic logic gate?

(a) OR (b) NOT

(c) XOR (d) AND

UPPSC Poly. Lect. 2021 Paper - II

61. The circuit given below



represents

(a) OR Gate (b) AND Gate

(c) NOR Gate (d) NAND Gate

UPPCS Pre 2006

62. The given truth table

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

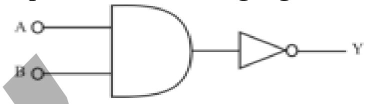
represents a

(a) NAND gate (b) NOR gate

(c) NOT gate (d) XOR gate

UPPCS Pre 2008, 2006, 2004, 1999

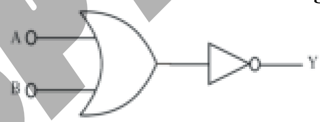
63. The output of the following logic circuit is:



(a) $y = \overline{A + B}$ (b) $y = \overline{A.B}$

(c) $y = \overline{A} + \overline{B}$ (d) $y = \overline{A}.\overline{B}$

64. The combination of following gates represent:

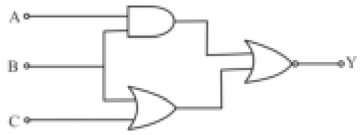


(a) NAND gate (b) NOR gate

(c) NOT gate (d) XOR gate

UPPCS Pre 2008

65. The Boolean expression and the output Y of the following logic circuit with inputs A = 1, B = 1 and C = 0 will be



(a) $Y = A.B + (B + C), Y = 1$

(b) $Y = \overline{A.B + (B + C)}, Y = 0$

(c) $Y = A.B + B + C, Y = 0$

(d) $Y = A.B + \overline{B + C}, Y = 1$

UPPCS Pre 2008

Answer Key

1. (b)	2. (d)	3. (d)	4. (c)	5. (a)	6. (d)	7. (a)	8. (b)	9. (d)	10. (b)
11. (c)	12. (a)	13. (c)	14. (b)	15. (a)	16. (c)	17. (d)	18. (c)	19. (a)	20. (c)
21. (d)	22. (a)	23. (b)	24. (b)	25. (d)	26. (d)	27. (d)	28. (c)	29. (b)	30. (d)
31. (c)	32. (d)	33. (d)	34. (a)	35. (c)	36. (d)	37. (b)	38. (c)	39. (d)	40. (c)
41. (c)	42. (a)	43. (d)	44. (b)	45. (d)	46. (b)	47. (b)	48. (c)	49. (b)	50. (a)
51. (a)	52. (c)	53. (a)	54. (d)	55. (c)	56. (a)	57. (d)	58. (b)	59. (b)	60. (c)
61. (b)	62. (b)	63. (b)	64. (b)	65. (b)					

09.

Communications and Communication system

The term communication refers to the transmitting, receiving and processing of information by electronic means.

■ Basic **Communication System:-**

A basic communication system consists of an information source, a transmitter, a link and a receiver.

(1) Source of Information:- The idea/message that is to be conveyed is information. The message may be individual one or a set of messages. The message may be a symbol, code, group of words or any pre decided unit.

(2) Transmitter: - In radio transmission, the transmitter consists of a transducer, modulator, amplifier and transmitting antenna.

Note:-

Transducer;- Converts sound signal into electric signal.

Modulator:- Mixing of audio electric signal with high frequency radio waves.

Amplifier:- Boosting the power of modulated signal

Antenna:- Signal is radiated in the space with the aid of an antenna.

3. Communication Channel:- The function of communication channel is to carry the modulated signal from transmitter to receiver. The communication channel is also called transmission medium or link. The term channel refers to the frequency range allocated to a particular service or transmission.

4. Receiver:- The receiver consists of

Pick up antenna	-	To pick the signal
Demodulator	-	To separate out the audio signal from the modulated signal
Amplifier	-	To boost up the weak audio signal
Transducer	-	To convert back audio signal in the form of electrical pulses into sound waves.

■ Types of Communication System:-

Communication systems can be classified or according to the nature of information or mode of transmission or types of transmission channel.

1. According to the nature of information source.

(i). Speech transmission

(ii). Picture transmission

(iii). Facsimile transmission (FAX):- This involves exact reproduction of a document or picture which is static.

2. According to the mode of transmission.

(i). **Analog Communication:**- The communication system which makes use of analog signals is called analog communication system.

Example of analog Communication Systems are telegraphy, Television broadcast, RADAR, Teleprinting

(ii). Digital communication:- In this communication system digital signals are used.

Examples:-Facsimile, Mobile Phone, E-mail, Communication Satellite,Global Positioning System (GPS), Tele Conferencing.

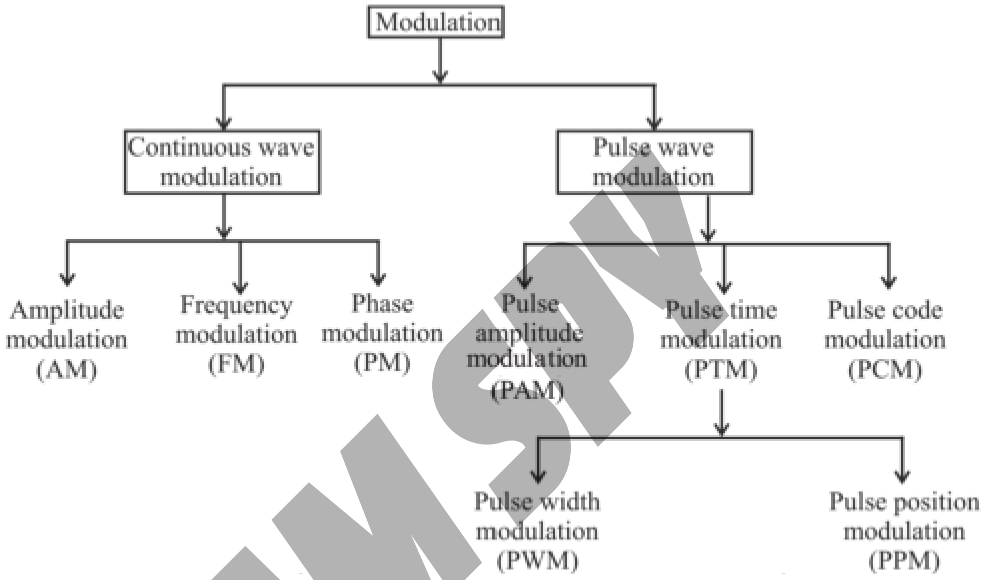
(iii). According to the transmission Channel:-

(i) Line Communication (ii) Space Communication

■ Modulation:-

The process of superimposing the audio signal over a high frequency carrier wave is called Modulation. In the process of Modulation any one characteristic of carrier wave is varied in accordance with the instantaneous value of audio signal.

- Digital and analog signals to be transmitted are usually of low frequency and hence cannot be transmitted.
- These signals require some carrier to be transported. These carriers are known as carrier waves or high frequency signals.
- The process of placement of a low frequency (LF) signal over the high frequency (HF) signal is known as modulation.



Need of Modulation :-

(i) Energy carried by low frequency audio waves (20 Hz to 20000 Hz) is very small.

(ii) For efficient radiation and reception of signal the transmitting and receiving antenna's should be very high approximately 5000 m.

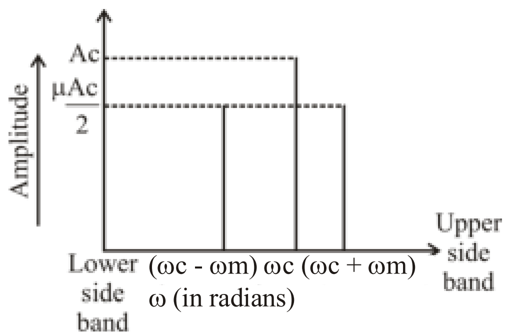
(iii) The frequency range of audio signal is so small that overlapping of signals creates a confusion.

Types of Modulation:-

1. Amplitude Modulation:- In this type of modulation the amplitude of high frequency carrier wave is varies in accordance to instantaneous amplitude of modulating signal.

Band width required for amplitude modulation = Twice the frequency of the modulation signal.

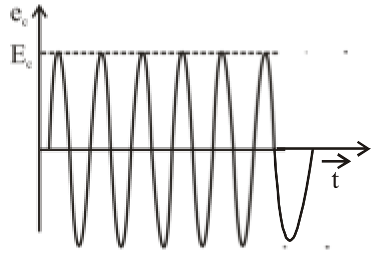
$$\text{Modulation index}\,(\mu) = \frac{\text{Change in amplitude of carrier wave}}{\text{Amplitude of carrier wave}}$$



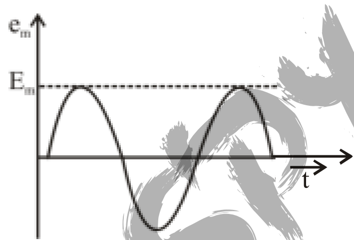
2. Frequency Modulation:- (FM)

The process of changing the frequency of a carrier wave in accordance with the audio frequency signal is known as frequency modulation.

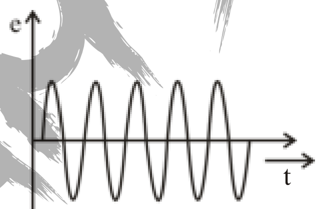
- Audio quality of AM transmission is poor. There is a need to eliminate amplitude sensitive noise. This is possible if we eliminate amplitude variation.
- In FM the overall amplitude of FM wave remains constant at all times.
- In FM, the total transmitted power remains constant.



HF Carrier wave



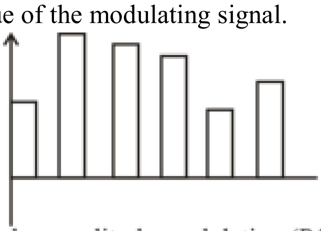
LF modulation wave



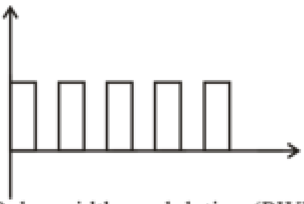
FM wave

3. Pulse modulation:- Here the carrier is in the form of pulses.

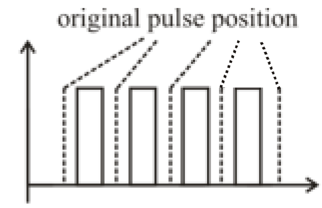
- Pulse amplitude modulation (PAM):- The amplitude of the pulse varies in accordance with the modulating signal.
- Pulse width modulation (PWM):- The pulse duration varies in accordance with the modulating signal.
- Pulse position modulation (PPM):- In PPM, the position of the pulses of the carrier wave train is varied in accordance with the instantaneous value of the modulating signal.



Pulse amplitude modulation (PAM)



Pulse width modulation (PWM)



Pulse position modulation (PPM)

■ **Demodulation:-** The process of extracting the audio signal from the modulated wave is known as demodulation or detection.The wireless signals consist of radio frequency (high frequency) carrier wave modulated by audio frequency (low frequency). The diaphragm of a telephone receiver or a loud speaker cannot vibrate with high frequency. So it is necessary to separate the audio frequencies from the radio frequency carrier wave.

■ **Band Width:-** The type of communication system needed for a given signal depends on the band of frequencies. Band width is given by difference of highest and lowest frequency within the band.

Various transmission media used for communication with their range is listed below.

Services	**Frequency bands**	**Remarks**
Standard AM broadcast	540 -1600 KHz	Radio broadcast
FM broadcast	88 - 108 MHz	Music channel
Television	54-72 MHz 76-88 MHz 174-216 MHz 420-890 MHz	VHF TV UHF TV
Cellular Mobile radio	896 - 901 MHz 840 - 935 MHz	Mobile to base station Base station to mobile
Satellite Communication	5.925 - 6.425 GHz 3.7 - 4.2 GHz	Uplink Downlink

■ **Antenna:-** An antenna converts electrical energy into electromagnetic waves at transmitting end and pick up transmitted signal at receiving end and converts electromagnetic waves into electrical signal.

(i) Hertz Antenna:- It is a straight conductor of length equal to half the wavelength of radio signal to be transmitted or received.

(ii) Marconi Antenna:- It is a straight conductor of length equal to quarter of a wave length of radio signal to be transmitted or received.

■ **Earth's Atmosphere:-** The gaseous envelop surrounding the Earth is called Earth's atmosphere. It contains the following layers,

(i) Troposphere- This region extends up to a height of 12 km from earth surface.

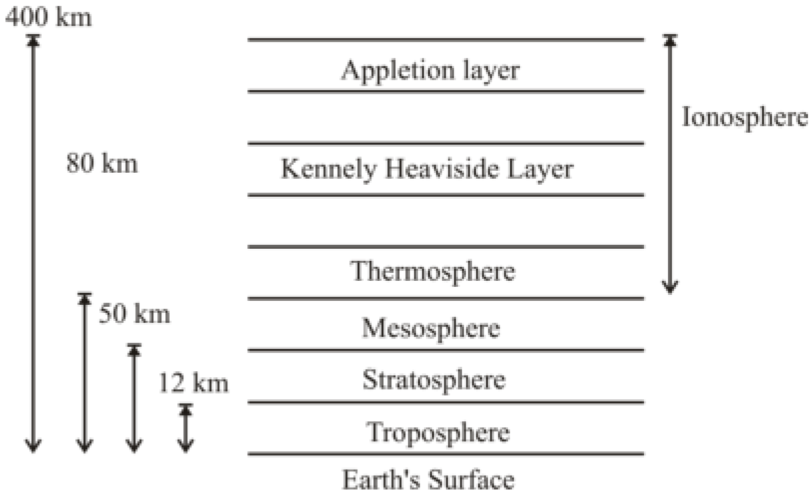
(ii) Stratosphere- This region extends from 12 km to 50 km. In this region, most of the atmospheric ozone is concentrated from 30 to 50 km. This layer is called ozone layer.

(iii) Mesosphere- The region extends from 50 km to 80 km.

(iv) Ionosphere- This region extends from 80 km to 400 km. This layer helps in radio Transmission.

- In ionosphere the electron density is very large in a region beyond 110 km from Earth's surface which extends vertically for a few kilometers. This layer is called **Kennelly Heaviside layer**.
- In ionosphere a layer having large electron density is found at height 250 km from Earth's surface is called Appleton layer.

• Atmosphere of Earth in respect of communication is shown below-



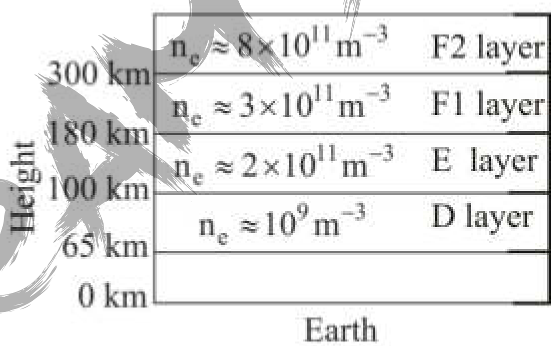
• There are four main layers in Earth's atmosphere in Ionosphere having high density of electrons and positive ions. These layers are D, E, F_1 and F_2,

(i) D layer - It is at a virtual height of 65 km from surface of Earth and having electron density ≈ 10^9 m^{-3}.

(ii) E layer - It is at a virtual height of 100 km from the surface of Earth having electron density ≈ 2×10^{11} m^{-3}.

(iii) F_1 layer- It is at a virtual height of 180 km from the surface of Earth having electron density ≈ 3×10^{11} m^{-3}.

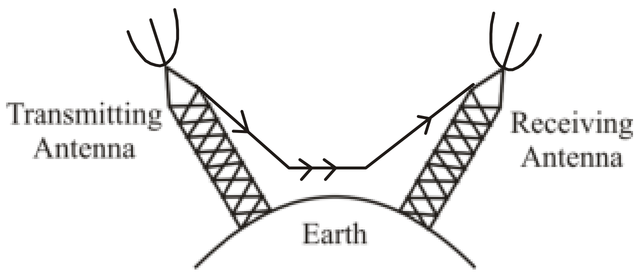
(iv) F_2 layer - It is at a vertical height of about 300 km in height time and about 250 to 400 km in day time. The electron density of this layer is ≈ 8×10^{11} m^{-3}.



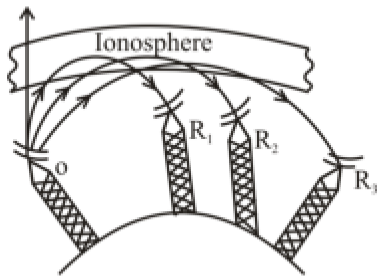
■ Propagation of Radio waves:-

The three modes are discussed below:-

(i) Ground wave or surface wave propagation:- It is suitable for low and medium frequency upto 2 MHz. It is used for local broad casting.



(ii) Sky wave propagation:- It is suitable for radio waves of frequency between 2 MHz to 30 MHz. It is used for long distance radio communication.



Critical Frequency- The highest frequency of radio wave that can be reflected back by the ionosphere is called critical frequency.

Critical frequency, $V_c = 9(N_{max})^{1/2}$

Where,

N_{max} = Density of electrons/ metre^3.

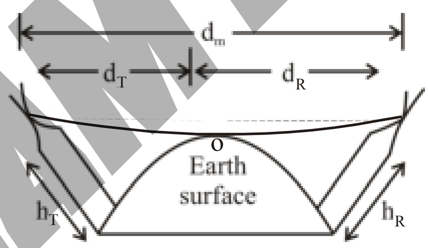
Skip distance:- The minimum distance from the transmitter at which a sky wave of a frequency but not more than critical frequency, is sent back to the Earth.

$$\text{Skip distance } (D_{skip}) = 2h\sqrt{\left(\frac{V_{max}}{V_c}\right)^2 - 1}$$

Where, h = height of reflecting layer of atmosphere.

V_{max} = maximum frequency of electromagnetic waves, V_c = critical frequency.

(iii) Space wave propagation- It is suitable for 30 MHz to 300 MHz. It is used in television communication and radar communication. It is also called line of sight communication.



Range is limited due to curvature of Earth. If h be the height of the transmitting antenna then signal can be received upto a maximum distance.

$$d = \sqrt{2Rh}$$

If height of transmitting and receiving antennas be h_T and h_R respectively. The effective range will

$$\boxed{d = \sqrt{2Rh_T} + \sqrt{2Rh_R}}$$

Note:- The variation in the strength of a signal at receiver due to interference of waves is called fading.

- **Microwave propagation-** Microwave communication is used in radar to locate the flying objects in space. These waves can be transmitted as beam signals in a particular direction, much better than radio wave. There is no diffraction of micro wave around corners of an obstacle which happens to lies along its passage.

- **Satellite Communication:-**
  - It is carried out between a transmitter and a receiver through a satellite. A geostationary satellite is utilised for this purpose, whose time period is 24 hours.
  - A communication satellite is a space craft, provided with microwave receiver and transmitter. It is placed in an orbit around the Earth.

- The India remote sensing satellite are IRS-IA, IRS-IB and IRS-IC.
- The line of sight microwave communication through satellite is possible. The communication satellite is always at a fixed location with respect to the Earth e.g. the satellite which is acting as a repeater must be at rest with respect to the Earth. The satellite communication can be used for establishing mobile communication with great use. The communication satellites are now being used in Global positioning system (GPS).
- The ordinary users are finding their positions within accuracy of 100 m. There are two types of satellites used for long distance transmission.

(i) Passive satellite:- It acts as reflector only for the signals transmitted from earth, moon the natural satellite of earth is a passive satellite.

(ii) Active satellite:- It carries all the equipment used for receiving signals sent from the Earth, processing them and then re-transmitting them to the Earth. Now-a-days active satellite are in use.

Merits of Satellites Communications:-

(i) Covers wide area for broadcasting as compared to other communication systems i.e. it has wide coverage range.

(ii) Found to be much economical as compared to other communication systems on Earth. In fact the cost involved in satellite communication is independent of the distance.

(iii) Used effectively in mobile communication.

(iv) Most cost effective in remote and hilly areas such as Ladakh, Himachal Pradesh etc.

(v) Permits transmission of data at high rate.

(vi) Very accurate for search, rescue and navigation purposes.

Demerits of satellite communication-

(i) If a system on the satellite goes out of order due to environmental stresses it is at most impossible to repair it.

(ii) In satellite communication, there is a time delay between transmission and reception due to extremely large communication path length greater than (2×36000 Km) this delay causes a time gap while communicating.

■ Optical Communication:-

(i) The use of optical carrier waves for transmission of information from one place to another is called optical communication.

(ii) The useful optical frequency range is **10^{12}Hz to 10^{16}Hz** which is very high as compared to radio and microwave frequencies (10^6Hz to 10^{11}Hz).

(iii) The information carrying capacity $\propto$ bandwidth $\propto$ frequency of carrier wave. So optical communication is better than others because of high frequency.

iv) Light emitting diodes (LED) and diode lasers are preferred for optical source LED's are used for small distance transmission while diode laser is used for very large distance transmission.

(v) In order to transmit information signal via an optical communication system, it is necessary to modulate light with the information signal.

■ Optical Fibre:-

The optical fibre are used to transmit light signals from one place to another without any practical loss in the intensity of light signal.

Types of optical Fibre-

(i) Monomode Optical Fibre

(ii) Multimode Optical Fibre

(iii) Graded Index Multimode Fibre

Applications of Optical Fibres:-

(i) A bundle of optical fibres is called light pipe this pipe can transmit an image since the pipe is flexible. It can be twisted in any desired manner. Hence it is used medical and optical examination of even the inaccessible parts of human body e.g in endoscopy.

(ii) Optical fibres are used in transmission and reception of electrical signals by converting them first into light signals.

(iii) Optical fibers are used in telephone and other transmitting cables each fiber can carry upto 2000 telephone messages without much loss of intensity.

LED and Diode laser in communication-

Light emitting Diode (LED) and diode laser are preferred sources for optical communication links to the following features-

(i) Each produces light of suitable power required in optical communication diode laser provides light which is monochromatic and coherent. This light is obtained as a parallel beam it is used in very long distance transmission.

(ii) LED provides almost monochromatic light. This is suitable for small distance transmission. It is in fact, a low cost device as compared to diode lasers.

■ Internet Telephony:-

Modem- The term modem is contraction of the term modulator and demodulator. Modem is a device which can modulate as well as demodulate the signal. It connects one computer to another through ordinary telephone lines.

The internet:-

Internet permits communication and sharing of all types of information between any two or more computers connected through a large and complex network. The exchange of information on internet is very fast as electronic signals of computers are communicated through electromagnetic waves.

Application of internet include internet surfing, E-mails, File transfer, E-banking, E- commerce, E-booking and social networking.

FAX (Facsimile Telegraphy)

Mobile Phone can wirelessly send and receive radio frequency signals. The central concept of this system is to divide the service area into a suitable number of cells centered an office called MTSO (Mobile Telephone Switching Office). Mobile phones operate typically in UHF of frequencies (about 800 - 950 MHz).

■ Global positioning System (GPS)

- GPS is a space based satellite navigation system that provides accurate information about time and location anywhere on or near the earth.
- GPS uses 24 satellites which are continuously orbiting, monitoring and mapping the earth surface.
- Every such satellite orbits around the earth twice a day at a distance of about 20,000 Km from it.
- The orbits of these satellites are aligned in such a way that at least four of them always keep looking any given point on the earth surface.

■ Key Points:-

(i) MASER is a Microwave Amplification by Stimulated Emission of Radiation. It is used as a microwave amplifier to oscillator. The principle of MASER is identical to that of LASER only frequency range is $\leq 10^{11}$ Hz in MASERS.

(ii) In frequency modulation M_f (frequency modulation index) is inversely proportional to modulating frequency F_m while in PM it does not vary with modulating frequency more over. FM is more noise immune.

(iii) Parallel wire lines are never used for transmission of microwaves. This is because at the frequency of microwaves separation between the two wires approaches half a wavelength $\left(\frac{\lambda}{2}\right)$. Therefore radiation loss of energy becomes maximum.

$$\text{Number of channel accommodated for Transmission} = \frac{\text{Total band width of channel}}{\text{band width needed perchannel}}$$

Bit rate = Sampling rate × no. of bits per sample.

(iv) Modulation factor determines the strength and quality of the transmitted signal.

(v) In Hertz antenna conductor length, $\ell = \frac{\lambda}{2}$

and in Macroni Antenna conductor length, $\ell = \frac{\lambda}{4}$

(vi) In a digital signal, information is carried by the pattern of pulses and not by the shape of pulses.

(vii) Sampling converts an analog signal into digital for example when an analog signal is sampled at interval of 125μ. The number of samples taken per second $= \frac{1}{125 \times 10^{-6}} = 8000$

(viii) AGC stands for Automatic Gain Control.

(ix) Sputnik-I launched by Russia in 1957 was the first active satellite.

Exemplar Problems

Communications & Communications System

1. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal ?
(a) 42.8 m (b) 42.8 mm
(c) 21.4 mm (d) 21.4 m
JEE MAIN 2022

2. Amplitude modulated wave is represented by .
$$V_{AM} = 10\left[1+0.4\cos\left(2\pi\times10^4 t\right)\right]\cos\left(2\pi\times10^7 t\right).$$
The total bandwidth of the amplitude modulated wave is :
(a) 10 kHz (b) 20 MHz
(c) 20 kHz (d) 10 MHz
JEE MAIN 2022

3. A sinusoidal wave y(t) = 40sin(10 x 10^6 πt) is amplitude modulated by another sinusoidal wave x(t) = 20sin (1000πt). The amplitude of minimum frequency component of modulated signal is :
(a) 0.5 (b) 0.25
(c) 20 (d) 10
JEE MAIN 2022

4. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission :
(a) 375×10^7 (b) 75×10^7
(c) 375×10^8 (d) 75×10^9
JEE MAIN 2022

5. The TV transmission tower at a particular station has a height of 125 m. For doubling the coverage of its range, the height of the tower should be increased by :
(a) 125 m (b) 250 m
(c) 375 (d) 500 m
JEE MAIN 2022

6. We do not transmit low frequency signal to long distances because
(a) The size of the antenna should be comparable to signal wavelength which is unreal solution for a signal of longer wavelength.
(b) Effective power radiated by a long wavelength baseband signal would be high.
(c) We want to avoid mixing up signals transmitted by different transmitter simultaneously.
(d) Low frequency signal can be sent to long distances by superimposing with a high frequency wave as well.
Therefore, the most suitable options will be :
(a) All statements are true
(b) (a), (b) and (c) are true only
(c) (a), (c) and (d) are true only
(d) (b), (c) and (d) are true only
JEE MAIN 2022

7. Match List I with List II

	List -I		List-II
A	Facsimile	I.	Static Document Image
B.	Guided media Channel	II.	Local Broadcast Radio
C.	Frequency Modulation	III.	Rectangular wave
D.	Digital Signal	IV.	Optical Fiber

Choose the correct answer from the following options :
(a) A –IV, B-III, C-II, D-I
(b) A-I, B-IV, C-II, D-III
(c) A –IV, B-II, C-III, D-I
(d) A-I, B-II, C-III, D-IV
JEE MAIN 2022

8. Match List–I with List–II.

	List-i		List-ii
(a)	10 km height over earth's surface	(i)	Thermosphere
(b)	70 km height over earth's surface	(ii)	Mesosphere
(c)	180 km height over earth's surface	(iii)	Stratosphere
(d)	270 km height over earth's surface	(iv)	Troposphere

(a) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
(b) (a)–(i), (b)–(iv), (c)–(iii), (d)–(ii)
(c) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)
(d) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)
JEE MAIN 2021

9. **Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight ?**
(Assume radius of earth is 6400 km)
(a) 19.77 m
(b) 39.45 m
(c) 79.1 m
(d) 158.2 m

JEE MAIN 2021

10. **The process of changing a weak signal without altering its shape into a strong one is known as**
(a) Biasing
(b) Rectification
(c) Modulation
(d) Amplification

UP PGT - 2021

11. **The function of a detector is to demodulate the modulated carrier wave and the steps for the process are**
(a) Demodulation and filtering
(b) Demodulation and rectification
(c) Rectification and filtering
(d) Generation and filtering

UP PGT - 2021

12. **The total power content of an Amplitude Modulated (AM) wave is 1500 W. For 100% modulation, the power transmitted by carrier wave will be :**
(a) 250 W
(b) 500 W
(c) 750 W
(d) 1000 W

UKPSC Lecturer (Mains) 2020

13. **If the highest modulating frequency of the wave is 6 kHz, the number of stations that can be accommodated in a 180 kHz, bandwidth is:**
(a) 5
(b) 10
(c) 12
(d) 15

UKPSC Lecturer (Mains) 2020

14. **A modulated carrier wave has maximum and minimum amplitudes of 750 mV and 250 mV respectively. What will be the value of percentage modulation ?**
(a) 100%
(b) 75%
(c) 50%
(d) 25%

UKPSC Lecturer (Mains) 2020

15. **Which device provide logical regeneration of a LAN ?**
(a) Router (b) Bridge
(c) Switch (d) Hub

UKPSC Lecturer (Mains) 2020

16. **The internet is a system of :**
(a) Software bundles
(b) Web page
(c) Website
(d) Interconnected networks

UKPSC Lecturer (Mains) 2020

17. **Which of the following is used to produce radio waves of constant amplitude?**
(a) Rectifier
(b) Amplifier
(c) Oscillator
(d) FET

UKPSC Lecturer (Mains) 2020

18. **For an amplitude modulated wave, the maximum amplitude in found to be 8 V while the minimum amplitude is found to be 2 V. The modulation index is**
(a) 0.1
(b) 0.4
(c) 0.6
(d) 0.707

RPSC School Lect. 26.07.2016

19. **The thermionic current in vacuum tubes is given by :**
(a) Child equation
(b) Richardson-Dushman equation
(c) Max well's equations
(d) Photoelectric equation

(UPPGT 2005)

20. **Semiconductor Lasers are formed by using:**
(a) combination of semiconductor and insulator
(b) amorphous semiconductors
(c) non–degenerate semiconductors
(d) degenerate semiconductors

UPPCS (Pre) 2004

Answer Key

1. (c)	2. (c)	3. (d)	4. (b)	5. (c)	6. (c)	7. (b)	8. (a)	9. (b)	10. (d)
11. (c)	12. (d)	13. (d)	14. (c)	15. (a)	16. (d)	17. (c)	18. (c)	19. (b)	20. (d)