Cantor's Diagonal Argument

Cantor's Diagonal Argument is a technique to show that the integers and reals cannot be put into a one to one correspondance is larger than the countable infinite set of integers.

It can be applied to any set. Let A be any set, consider the power set P(A) consisting of all subsets of S. Cantor's Diagonal method can be used to show that P(A) is larger than A i.e there exists an injection but no bijection from A to P(A).

Theorem: For any set A, we have $|A| < |P(A)|$ where P(A) is a power set of A.

Proof:- Let the f^n $g: A \to P(A)$ defined by $g(a) = a$; is injective. Hence $|A| <= P(A)$

We only need to show that $|A| \neq P(A)$
Let $a \in A$ be called a bad element if 'a' does not belong to the set, which is its image if $a \notin f(a)$

Let B be the set of bad elements then
$$B = \{ x : x \in A, x \notin f(x) \}$$

Now B is a subset of A
Since $f: A \to P(A)$ is onto. $B \in A$ such that $f(b) = B$

Page :
Date :

4

If 'b' is a good or bad element. If $b \notin B$, then $b \in F(b) = B$ which is also contradiction.

If $b \notin B$, then $b \in F(b) = B$ which is also a contradiction, Thus, the original assumption that $|A| = |P(A)|$ has lead the contradiction.

Thus $|A| \neq |P(A)|$ which proves the theorem.

Sets :- A set is a collection of well defined distinct objects. Sets are denoted by Capital letter. Elements of sets are denoted by small letter.

1. Finite and infinite sets : A finite sets is a set that has a specific number of elements, whereas an infinite set has an unlimited number of elements.

2. Null or empty sets : A set has no element is called the null set or empty set. It is denoted by the symbol ϕ or $\{\}$.

3. Singleton set : A set that contains only one element is called a singleton set. For example $\{1\}$ is a singleton set.

4. Equal set : Two sets are said to be equal if they have the same elements. For example $\{1, 2, 3\}$ and $\{3, 2, 1\}$

5. Universal set : The collection of all elements under consideration is called the universal set, denoted by the symbol U.

Relation :- Relation refers to the connection or association b/w two or more objects. A relation is a set of ordered pair where the first element is related to the second element

Types of relations

(i) **Reflexive relations :** A relation called reflexive if every element in the set is related to itself. $(a,a) \in R \ \forall a \in A$

$A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\}$

(ii) **Symmetric relations :**

If whenever (a,b) is in the relation, then (b,a) is also in the relation.

$(a,b) \in R, \ (b,a) \in R \ \forall \ a, b \in A$

$A = \{1, 2, 3\}$

$R = \{(1,2)\ (2,1)\ (2,3)\ (3,2)\ (3,1)\ (1,3)\}$

(iii) **Transitive relations :**

If whenever (a,b) and (b,c) are in the relations then also (a,c) in the relations.

$(a,b) \in R, \ (b,c) \in R, \ (c,a) \in R \ \forall a,b,c \in A$

$A = \{1, 2, 3\}$

$R = \{(1,2)\ (2,3), \ (3,1)\}$

(iv) **Asymmetric relations :**

If whenever (a,b) is in the relation (b,a) cannot be in the relation

Eg → If A is a parent of B, then B cannot be parent of A.

Functions :-

A rule that assigns a unique output value to every input value. It is a relation b/w a set of inputs and a set of possible outputs with the property that each input is related to exactly one output.

Types of functions →

1. One to one function : If each element in the range corresponds to exactly one element in the domain.

2. Onto function :
If every element in the range is the image of at least one element in the domain.

3. Bijective function:
A function is called bijective if it both one to one and onto.

4. Composite function:
Function formed by chaining two or more functions together.

5. Identity function:
Function that returns the input value itself

6. Polynomial function:
Function that is defined by a polynomial expression.

Demorgan's law is a set of two logical rules that deal with the negation of logical expression in propositional logic.

First Demorgan's Law :-

The negation of a conjunction is the disjunction of the negation of the individual propositions.

- $\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

We start by assuming the left-hand side of the equivalence is true.

By the defination of negation, this means that $(P \wedge Q)$ is false.
Therefore, atleast one of P and Q must be false.
$\sim P \vee \sim Q$

By the definition of disjunction, this means that at least one of $\sim P$ and $\sim Q$ must be true.
Therefore the right-hand side of the equivalence is also true.
$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$

Second Demorgan's law :-

- $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$

We start by assuming the left hand side of the equivalence is true

By the defination of negation, this means that $(P \vee Q)$ is false.
Therefore, both P and Q must be false.

$$\sim P \wedge \sim Q$$

By the definition of ~~disjunction~~, conjunction, this mean that both $\sim P$ and $\sim Q$ must be true. Therefore, the right-hand side of the equivalence is also true.

$$\sim (P \vee Q) \equiv \sim P \wedge \sim Q$$

Therefore, both Demorgan's Laws are proven.

$$f(x) = x^3 - 2$$

- One - One

If $F(x_1) = F(x_2)$

$$x_1^3 - 2 = x_2^3 - 2$$

$$x_1^3 = x_2^3$$

$$x_1 = \sqrt[3]{x_2^3}$$

- Onto

If $f^{-1}(y) = x$

$$x^3 - 2 = y$$

$$x^3 = y + 2$$

$$x = \sqrt[3]{y+2}$$

$$\boxed{f^{-1}(y) = \sqrt[3]{y+2}}$$

Q State and prove Division Algorithm law

Statement :

For a given integer a and $b>0$ there exist unique integers q and r such that

$$a = bq + r$$
$$0 \leq r < b$$

The integers q and r are called the quotient and the remainder respectively.

Proof ;

We consider the infinite sequence of multiple of b given below :

$$\ldots -b, 0, b \ldots bq \ldots$$

Then obviously either a must be equal to one of the multiples of b say bq in this sequence or it must lie b/w two consecutive multiples say $bq + b$ i.e $b(q+1)$

we have

$$bq \leq a < b(q+1) \quad \text{for some } q$$
$$0 \leq \underline{a - bq} < b \qquad [\,bq \text{ se minus}\,]$$

let $a - bq = r$

The we have

$$a = bq + (a - bq)$$
$$= bq + r \,; \qquad 0 \leq r < b$$

This completes, the existence part of the theorem

Page:

Date:

For uniqueness we assume the possible of two different representation of a

$$a = bq + r \quad , \quad 0 \le r < b \quad \text{---} (1)$$
$$a = bq_1 + r_1 \quad , \quad 0 \le r_1 < b \quad \text{---} (2)$$

for some integers q, q_1, r and r_1

From equation (1) and (2)

$$bq + r = bq_1 + r_1$$
$$b(q - q_1) = r_1 - r$$

This shows that b divides $r_1 - r$.

But this is not possible because both r and r_1 are positive integers less than b.

Hence, q and r must be unique.

This Theorem known as Division Algorithm

Page :

Date :

6 Show that $1^2 + 2^2 + 3^2 ---- + n^2 = \frac{n(n+1)(2n+1)}{6}$

$P(1) = 1^2 = 1$

$P(1)$ is true

$P(k) = 1^2 + 2^2 ------ k^2 = \frac{k(k+1)(2k+1)}{6}$

Now, we shall have to prove that $P(k+1)$ is also true

$\therefore P(k+1) = 1^2 + 2^2 ----- k^2 + (k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= (k+1)\left[\frac{k(2k+1)}{6} + \frac{(k+1)}{1}\right]$

$= (k+1)\left[\frac{2k^2 + k + 6k + 6}{6}\right]$

$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$

$= \frac{(k+1)(2k^2 + 4k + 3k + 6)}{6}$

$= \frac{(k+1)[2k(k+2) + 3(k+2)]}{6}$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

$= \frac{(k+1)(k+1)+1\,[2(k+1)+1]}{6}$

$P(k+1)$ is also True

Page :

Date :

7 Implication is tautology

(a) $\underbrace{(p \to q) \vee r}_{A} \to \underbrace{[(p \vee r) \to (q \vee r)]}_{B}$

p	q	r	Ⓐ $(p\to q)\vee r$	$(p \vee r)$	$q \vee r$	Ⓑ $p\vee r \to q \vee r$	$p \to q$	$A \leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	T	T	T	T	F	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	F	F	T	T	T

all are true

It is tautology

(b) $(p \wedge q \to r) \vee (p \to r) \vee q \to r$

p	q	r	$p \wedge q$	Ⓐ $p \wedge q \to r$	$p \to r$	$q \to r$	Ⓑ $(p \to r) \vee (q \to r)$	$A \leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

all are true, It is tautology

Page :
Date :

8. Find the validity of the following argument :

"If my friend does not come to meet me, then I will go to office"
"If I go to office, then I will complete my work"
Using rule of inference, can we complete, "If I am not happy. Then I will complete my work"?

We have two condition

If A, then B
If B, then C

We can infer: If A, then C

- If my friend does not come to meet me, then I will go to office ($A \rightarrow B$)
- If I go to office, then I will complete my work ($B \rightarrow C$)
- If I am not happy, then I will complete my work ($\sim H \rightarrow C$)

The conclusion is not directly supported by the given premises and does not match the structure we used in the rule of inference.

Q Implication is tautology, contradiction or contingency

(a) $(p \to q) \vee r \leftrightarrow [(p \vee r) \to (q \vee r)]$

p	q	r	$p \to q$	$p \to q \vee r$ (A)	$p \vee r$	$q \vee r$	$(p \vee r) \to (q \vee r)$ (B)	$A \leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	T

It is contingency

(b) $(p \wedge q \to r) \leftrightarrow (p \to r) \vee (q \to r)$

p	q	r	$p \wedge q$	$p \wedge q \to r$ (A)	$p \to r$	$q \to r$	$(p \to r) \vee (q \to r)$ (B)	$A \leftrightarrow B$
T	T	T						
T	T	F						
T	F	T						
T	F	F						
F	T	T						
F	T	F						
F	F	T						
F	F	F						

10. If I study then I will not fail in mathematics, If I do not play football then I will study, But I failed in mathematics. Therefore, I must have played football.

Let p: I study , $\sim r$ = played football
q: I will not fail in mathematics
r: I will fail in mathematics

logical statement

$$\underbrace{p \to q}_{T} \wedge \underbrace{r \to p}_{T} \wedge \underbrace{\sim q}_{T} \to \sim r$$

$p \to q$		$r \to p$	
T	F	T	T
F		T	

F → F True

It is valid

11 What are quantifiers? Give examples.

Quantifiers are logical operators that express the quantity or scope of a statement with a logical formula. They allow us to generalize statements over a range of elements in a set.

There are two quantifiers :

→ The Universal quantifier ($\forall$)

→ Existential quantifier ($\exists$)

1. The Universal quantifiers ($\forall$) :-

Express that a statement is true in a given set [all element]. Denoted : "for all"

Page :
Date :

Example :- ~~$\forall x(P(x))$~~ $\forall n (n>0)$ can be read as 'for all n, n is greater than 0'.
This statement states that every number n in the set of numbers is greater than 0.

2. Existential quantifiers ($\exists$) :
Express that there exists at least one element in a given set for which a statement is true.
Denoted : "there exists"

Example :- $\exists n (n>5)$ can be read as " There exists an n such that n is greater than 5
This statement states that there is at least one number n in the set of number that is greater than 5.

12. Prove using contraposition : if $(3n+7)$ is an odd integer, then 'n' is an even integer.

Suppose that $(3n+7)$ is an odd integer
$3n+7 = 2k+1$ for some integer
Then
$$3n+7 = 2k+1$$
$$3n = 2k+1-7$$
$$n = \frac{2k-6}{3}$$
$$n = 2\left[\frac{(k-3)}{6}\right]$$

This show n is an even integers
$\therefore$ n is a multiple of 2.

Page:
Date:

13. Translate these statements into English where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consist of all people.

(i) $\forall x\,(C(x) \rightarrow F(x))$
All people are comedian if they funny

(ii) $\exists x\,(C(x) \land F(x))$
There exist some people which are comedian and funny

Negate the above statements and express in predicate logic as well as English
(i) Not all people are comedian if they are funny.
(ii) There exist some people with are not comedian or not funny.

14. Explain Disjunctive normal form (D.N.F) and Conjunctive normal form (CNF)

Disjunctive Normal form :-
A statement form which consists of disjunctive b/w conjunctive is called DNF.

Ex ⇒ $(p \land q)$
$(p \land \sim q) \lor (\sim p \land r) \lor (r \land \sim q)$

Conjunctive Normal form :-
A Statement form which consists of conjuctive b/w disjuction is called Conjuction Normal Form

Ex ⇒ $(p \lor q)$
$(\sim p \lor q) \land (\sim p \lor r) \land (r \lor \sim q)$

Page :

Date :

15. Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. This is the distributive law of disjunction over conjunction.

Given $p \vee (q \wedge r)$

By Distributive law

$(p \vee q) \wedge (p \vee r)$

Hence $p \vee (q \wedge r)$ are logically equivalent with $(p \vee q) \wedge (p \vee r)$

16. Explain and properties of Rule of Inference for propositional logic.

1. Modus Ponens

If we have a premise of the form "A implies B" and another premise that asserts A, we can validly conclude B

$(A \rightarrow B), A \vdash B$

2. Modus Tollens

If we have a premise of the form "A implies B" and another premise that asserts "not B" $(\neg B)$, we can validly conclude "not A" $(\neg A)$

$(A \rightarrow B), \neg B \vdash \neg A$

3. Disjunctive Syllogism

If we have a premise stating "A or B" and another premise asserting "not A" $(\neg A)$, we can validly conclude "B".

$(A \vee B), \neg A \vdash B$

Page :
Date :

4. Addition

If we have a premise asserting "A", we can validly conclude "A or B", where B can be any proposition.

$A \vdash A \vee B$

5. Simplification

If we have a premise stating "A and B", we can validly conclude "A".

$A \wedge B \vdash A$

6. Conjunction

If we have premises asserting both "A" and "B" we can validly conclude "A and B"

$A, B \vdash A \wedge B$

Page :
Date :

2022

1.

(a) If $A = \{1, 2, 3\}$ $B = \{3, 4\}$ $C = \{4, 5, 6\}$ determines the $A \times (B \cup C)$

$B \cup C = \{3, 4, 5, 6\}$

$A \times B \cup C = \{(1,3), (1,4), (1,5)(1,6), (2,3), (2,4)$
$(2,5), (2,6), (3,3)(3,4), (3,5), (3,6)\}$

(b) Find the number of ways that a party of seven persons can arrange themselves around a circular table.

No. of ways $= {}^7P_7$

$= \frac{7!}{(7-7)!} = 7! = 5040$

(c) How many ways can a committee consisting of 3 man and 2 woman be chosen from seven men and five women?

${}^7C_3 * {}^5C_2$

$\frac{7!}{3!(7-3)!} * \frac{5!}{2!(5-2)!} = \frac{7!}{3!4!} * \frac{5!}{2!3!}$

$\frac{7.\cancel{6}.5.\cancel{4!}}{\cancel{3!}\cancel{4!}} * \frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{2!}\cancel{3!}}$

$35 * 10$

350

Page :
Date :

d) Describe the proof of contradiction.

Contradiction occurs when we get a statement p, such that p is true and its negation ~p is also true.

Proof :-

Consider two statement p and q

p : $x = a/b$, where a and b are co-primes

q : 2 divides both a and b

Here in this case, we have to assume that statement "p is true" and also manage to show that statement q is true.

Hence we have arrived at a contradiction because statement q implies the negation of statement p is true.

e) Define normal subgroup

A normal subgroup H of a group, G is a subgroup of G which satisfies the similarity transformation with any fixed arbitrary element in G.

If G is an abelian group and x is an arbitrary element of G, then HX is a right coset of H in G and XH is a left coset of G.

$$XH = HX.$$

Page :
Date :

f) graph	Tree

graph	Tree
• It is non-linear data structure	• It is also non-linear data structure
• It is a set of nodes and edges	• It is a set of nodes and edges
• It has no unique node	• It has a unique node called Root.
• Graph can form cycle.	• Trees cannot form cycle

g) State the well-ordering principle.
The well-ordering principle state that the positive integers are well-ordered. If each and every non-empty subset has a smallest or least element is said to an ordered set.
Every non-empty subset S of the positive integers has a least element.

h) Prove that sum of degree of all vertices in a graph is equal to twice the number of edges in graph.

To prove :— $\sum_{i=1}^{n} deg(V_i) = 2e$

Proof :
Let there be a ~~graph~~ graph G with $V = \{V_1, V_2 ... V_3\}$ vertices and 'e' edges
Each edge in a graph contribute 2-degree

Page :

Date :

1) loop with edge 'e' contribute 2-degree
2) Edge with two vertices contribute

∴ a single edge contribute two degree
single edge 1 = 2
$e = 2e$ —— (i)

Total degree of graph G

$\deg(V_1) + \deg(V_2) \; --- \; + \deg(V_n)$ —— (2)

From (1) & (2)

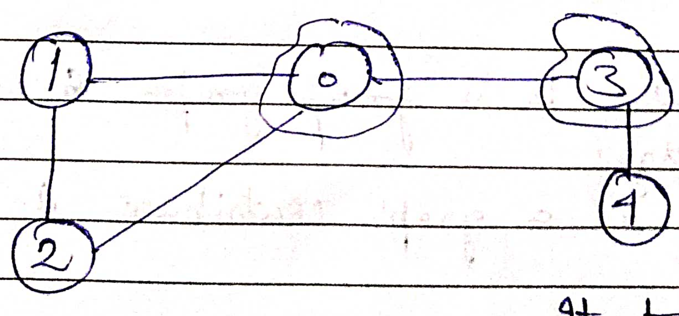
$\deg(V_1) + \deg(V_2) \; ---- \; \deg(V_n) = 2e$

$$\boxed{\sum_{i=1}^{n} \deg(V_i) = 2e}$$

(i) Describe the articulation points by giving suitable example.

A vertex is an undirected connected graph is an articulation point if removing it disconnects the graph.
Articulation point represent vulnerabilities in a connected network. They are useful for designing reliable networks.

Input :



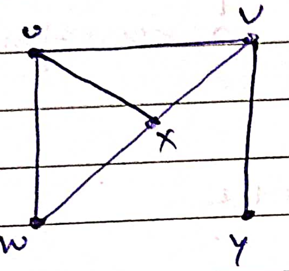
output : 0, 3

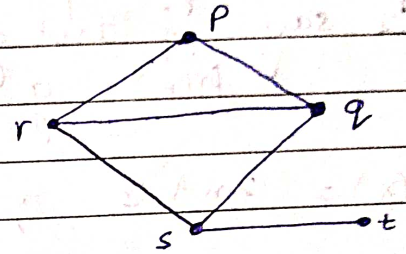
- Iterate over all the vertics
- Remove v from graph
- See if the graph remains connected
- Add v back to the graph

Page :
Date :

j) What is isomorphism of graph ?
Two graph G_1 & G_2 are isomorphic if
i) Number of vertices are same
ii) Number of edges are same
iii) An equal number of vertices with given degree.
iv) Vertex correspondance & edge correspondance valid

Ex -

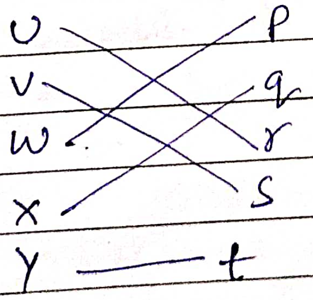




(iii) degree :-

u = 3		p = 2	
v = 3		q = 3	
w = 2		r = 3	
x = 3		s = 3	
y = 1		t = 1	

(iv) Vertex Correspondance & edge Correspondance



Page :

Date :

2 (b) Define Composition of function. If f and g be the function from the set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$

(i) $f \circ g$

(ii) $g \circ f$

The composition of a function is an operation where two function's say f and g, generate a new functions

$$h(x) = g\,f(x)$$

$$f(x) = 2x+3 \quad ; \quad g(x) = 3x+2$$

i) $f \circ g = f[g(x)]$

$= 2[(3x+2)]+3$

$= 6x+4+3$

$= 6x+7$

ii) $g \circ f = g[f(x)]$

$= 3(2x+3)+2$

$= 6x+9+2$

$= 6x+11$

Page :

Date :

3(a) State the pigeon hole principle. Find the minimum number of students in a class to be sure that three of them are born in the same month.

If n pigeonhole are occupied by n+1 or more pigeons, then atleast one pigeonhole occupied by more than one pigeon.

Let take a example

pigenhole = 6

pigeon = 7 , n+1 = 6+1=7

atleast

1	7, 2	3
4	5	6

Minimum number of Students

$n = 12$ [months] $m = ?$

$Kn+1 = 3$

According to definati - $K+1 = 3$

$\boxed{K = 2}$

$m = Kn+1$

$= 2 \times 12 + 1$

$\boxed{m = 25}$

Page :

Date :

(B) State and prove the Schroeder-Bernstein theorem

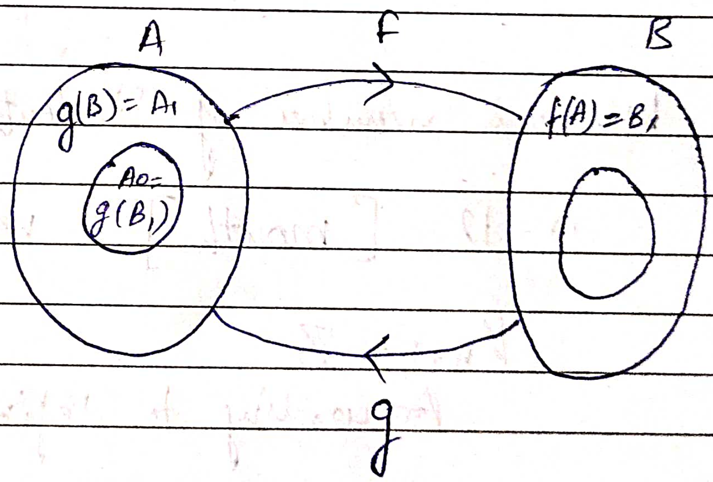
Statement - If A & B are two sets such that A is equivalent to subset of B, B is equivalent to subset of A, then A is equivalent to B.

Proof :-

let A and B are two non-empty disjoint sets.

And let $F : A \to B$ and $g : B \to A$

let $F(A) = B_1$ — (1)



the clearly $B_1 \leq B$

Given that -

A is equivalent to subset of B

$A \sim B_1$

F is one-one & onto

Similarly $g(B) = A_1$ — (1)

$g(B) \leq A$ where $A_1 \leq A$

$\Rightarrow$ $\boxed{A_1 \subseteq A}$

Page :
Date :

Given that -
B is equivalent to subset of A
$A \sim B$

g is one - one & onto

let $g(B_1) = A_2$ where $A_2 \leq A_1 \leq A$ — (2)
$g[f(A)] = A_2$

$g f(A) = A_2$
$g f$ is one - one & onto

∴ Product of two one - one & onto map is also one-one & onto

Hence $A \sim A_2$
Card A = card A_2 ← (3)

from eqn (2)
$A_2 \leq A_1 \leq A$ [by equivalent theorem]

we get Card A_1 = card A — (4)
again from eqⁿ (1) $g(B) = A_1$ & g is one-one & onto
$[B \sim A_1]$
Card B = Card A_1 ← (5)

from eqⁿ (4) and (5)
Card A = Card B
$[A \sim B]$

Hence A is equivalent to B

Page :
Date :

4 (a) What is conjunctive normal form? Obtain the conjunctive normal form of the given boolean function:

$$f(x, y, z) = (x+z)(x+y')$$

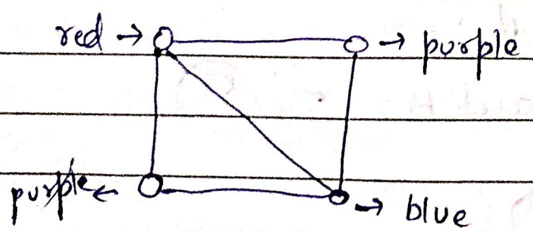
A statement form which consists of conjunction b/w disjunction is called CNF.

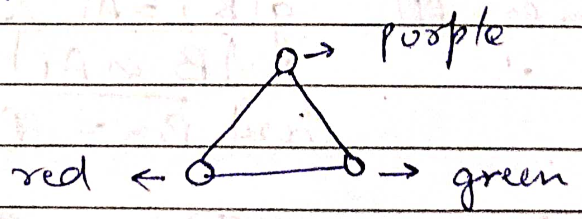
Ex = • $p \wedge q$
• $(\neg p \vee q) \wedge (\neg p \vee r) \wedge (r \vee \neg q)$

$$f(x, y, z) = (x+z)(x+y')$$
$$= (x \vee z) \wedge (x \vee y')$$

(b) What do you mean by graph coloring and chromatic numbers of graph?
Determine the chromatic number of following graph:-

⇒ Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called coloring of graph.

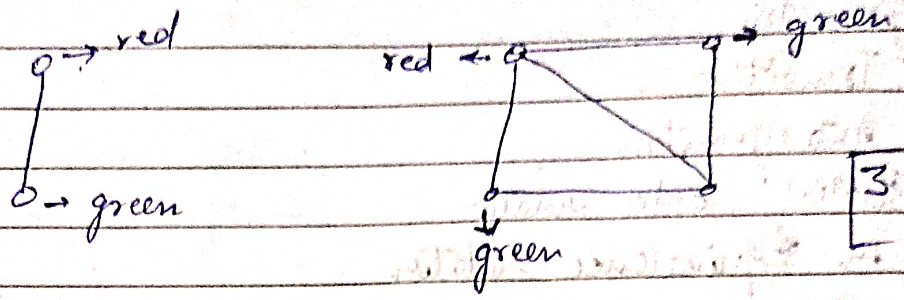




[2 purple because they are not connected]

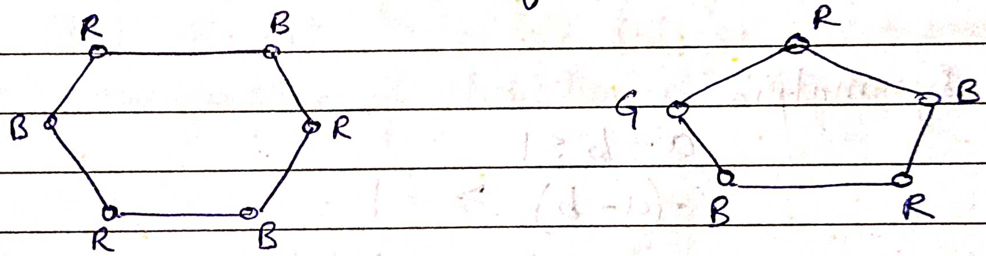
Chromatic number :- The least number of colors required for coloring of a graph G is called its Chromatic number

Ex -

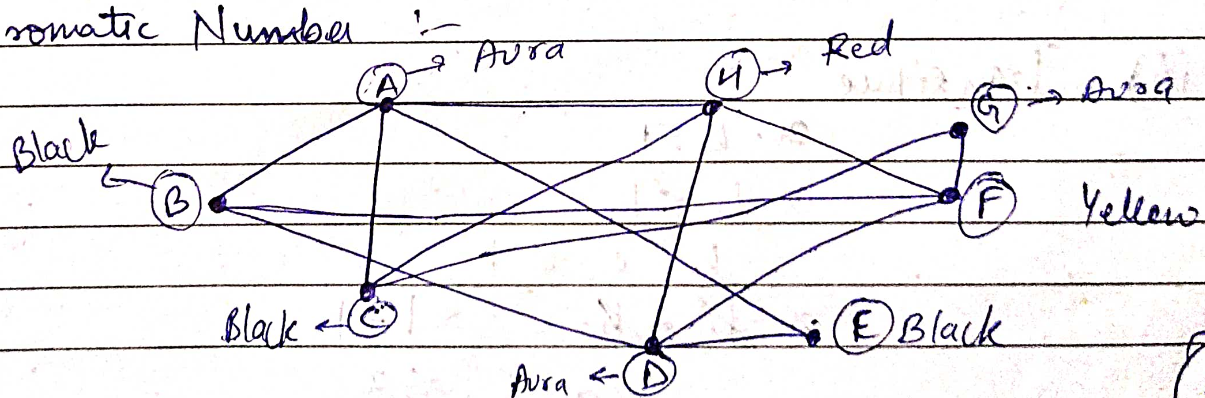


[3 minimum colors]

- The chromatic number of graph G is denoted by $\chi(G)$
- If $\chi(G) = K$, then the graph is called K-Chromatic
- Chromatic number of null graph is 1
- If a graph is circuit with n vertices then
  (i) It is 2-chromatic if n is even
  (ii) It is 3-chromatic if n is odd



Chromatic Number :-



4 No.

A, F, D → Black X   A → Aura
C, E, H → Aura X
C → Black ✓ , H, G → Black X , H → Aura X H → Red
D → Red X , Black X Aura ✓
F → Black X Red X , Aura ✓   G → Yellow X Black X

Page :

Date :

5. (a) If R is a relation on the set of positive integer I such that (a, b) ∈ R if only

i) Reflexive
ii) Symmetric
iii) Transitive
iv) Anti symmetric
v) Partial order relation
vi) An Equivalence relation

i) Reflexive

$$a - b \leq 1$$

$$a = 1 \;,\; b = 1$$

$$1 - 1 \leq 1$$

$$0 \leq 1$$

True

ii) Symmetric

$$a - b \leq 1$$

$$-(a - b) \geq -1$$

$$b - a \geq -1$$

False

iii) Transitive

$$a - b \leq 1$$

$$a - b \leq 1$$

$$b - c \leq 1$$

$$a - \cancel{b} + \cancel{b} - c \leq 1 + 1$$

$$a - c \leq 2$$

$$(a, c) \in R$$

False

Page :

Date :

iv) Anti Symmetric

$(a, b) \in R$

but $(b, a) \notin R$

R is anti-symmetric

v) Partial order relation

R is reflexive & anti symmetric

True

vi) An Equivalence relation

$\therefore$ R is reflexive, but not symmetric & Transitive

False

(b) Write the properties of group in algebric structure, prove that the set I of all integers is a group with respect to the operation of addition of integers.

A non empty set S is called Algebric structure w.r.t to binary operation $*$ if $(a * b) \in S$ $\forall (a, b) \in S$

Properties of group

- Closure : $(a * b)$ belongs to G for all $a, b \in G$
- Associativity : $a * (b * c) = (a * b) * c$ $\forall a, b, c \in G$
- Identity element : There exists $e \in G$ such that $a * e = e * a = a$ $\forall a \in G$
- Inverse : $\forall a \in G$, there exists $a^{-1} \in G$

tient Structures, ...
nal Subgroups, Algebraic Structures
oolean Algebra and Boolean Ring,
n Function, Disjunctive and

... introduction to parallel processors, Concurrent access to memory and cache coherency.

MODULE-4: Memory organization: Memory interleaving, concept of hierarchical memory organization, cache memory, cache size Vs block size, ... write policies.

Page :

Date :

Proof :

We have to satisfy 4 axioms

There exists an identity element in your group that fixes every element under given binary operation

$0 + k = k + 0 = k$

Closure :- We need to show $(a+b)$ stays in the set

Yes, after sum up $a+b$ = integer

Associativity : Yes, addition in Z is associative

$(a+b)+c = a+(b+c)$

Inverse : We have to find a and a^{-1} commute and their operation together give the identity element

$a+(-a) = (-a)+a = 0$