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IIT JEE TOP STUDENT HANDWRITTEN NOTES - PHYSICS

(VECTORS)

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VECTORS

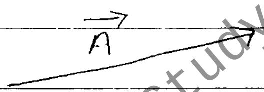
Types of physical quantity –

1) Scalar quantities – They have only magnitude and they do not follow vector law of addition. Ex: speed, time, current, etc

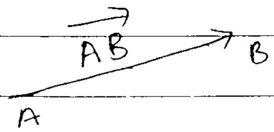
2) Vector quantities – They have both direction as well as magnitude and they follow vector law of addition. Ex. displacement, force, etc.

Notations of vectors

1) One point form



2) Two point form



- Graphically vectors can be represented with the help of an arrow

- The length of the arrow is proportional to magnitude of vector.

- The head of arrow tells direction of vector.

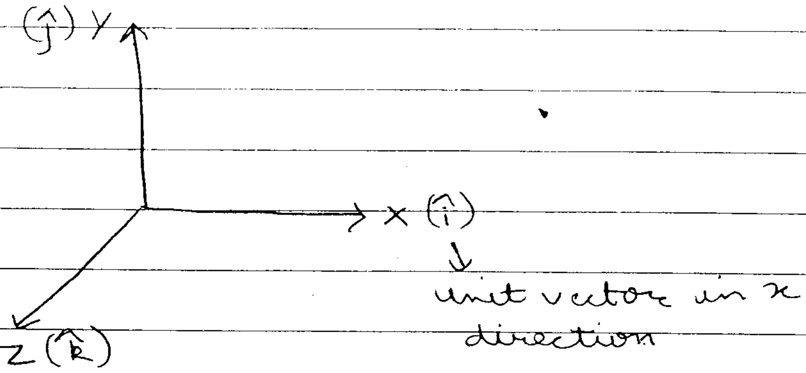
- A vector can be shifted anywhere in space if its magnitude and direction remain unchanged.

- Mathematically vector is shown as

$$\vec{AB} = |\vec{AB}|\,\hat{AB}$$

$|\vec{AB}|$ → Tells value

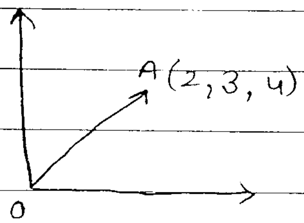
$\hat{AB}$ → Tells direction



Types of Vectors

1) Position vector

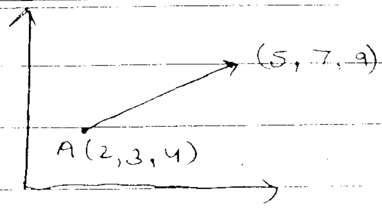
It tells the position of a point w.r.t origin.



$$\vec{OA} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

2) Displacement vector

It tells the position of a point w.r.t some other point except origin.



$$\vec{AB} = \text{P.V. of } \vec{B} - \text{P.V. of } \vec{A}$$
$$= (5\hat{i} + 7\hat{j} + 9\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k})$$
$$= 3\hat{i} + 4\hat{j} + 5\hat{k}$$

3) Unit vector

It is a vector whose magnitude is 1. It can tell us the direction of any vector.

$$\vec{AB} = |\vec{AB}|\ \hat{AB}$$
$$\hat{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$\vec{AB} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$|\vec{AB}| = \sqrt{x^2 + y^2 + z^2}$$

Q Find unit vector of $\vec{AB} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$$|\vec{AB}| = \sqrt{2^2 + 3^2 + 4^2}$$
$$= \sqrt{29}$$

$$\hat{AB} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

4) Parallel vectors

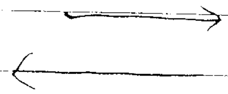
Two vectors having same direction

$\longrightarrow$

$\longrightarrow$

5) Anti Parallel vectors

Vectors having opposite direction



6) Equal vectors

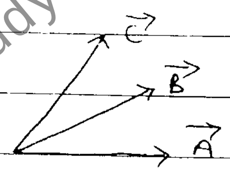
If two vectors have equal magnitude and same direction and also they represent same physical quantity then they are called equal vector.

7) Null vector or zero vector

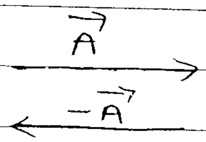
Its magnitude is 0 and its direction is unknown

8) Coinitial vectors

If vectors start from same point then they are called coinitial vector.



Note:

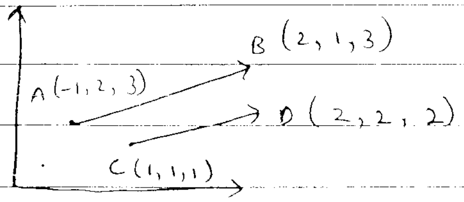


Addition of vectors

a) If vectors are in $\hat{i}, \hat{j}, \hat{k}$ form

$$\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$
$$\vec{A} + \vec{B} = (a_1+a_2)\hat{i} + (b_1+b_2)\hat{j} + (c_1+c_2)\hat{k}$$

Q



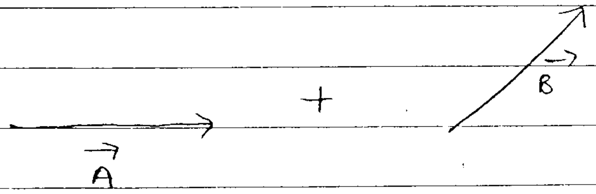
$$\vec{AB} = (2\hat{i} + 1\hat{j} + 3\hat{k}) - (-1\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 3\hat{i} - \hat{j} + 0\hat{k}$$

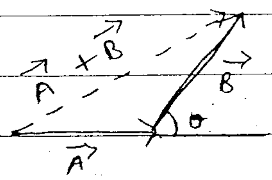
$$\vec{CD} = (2\hat{i} + 2\hat{j} + 2\hat{k}) - (1\hat{i} + 1\hat{j} + 1\hat{k})$$
$$= \hat{i} + \hat{j} + \hat{k}$$

$$\vec{AB} + \vec{CD} = (3\hat{i} - \hat{j}) + (\hat{i} + \hat{j} + \hat{k})$$
$$= 4\hat{i} + \hat{k}$$

b) If vectors are given in arrow form

1) Triangle law of vector addition
If two vectors represent two consecutive sides of a triangle in same order ~~than~~ then the resultant is given by third side taken in opposite order.

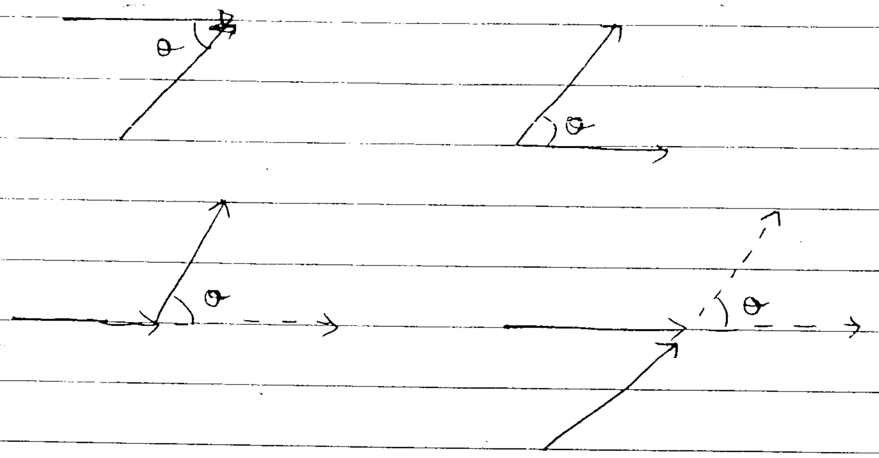




$$|\vec{A} + \vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$$

Mathematical analysis of Δ law of vector addition

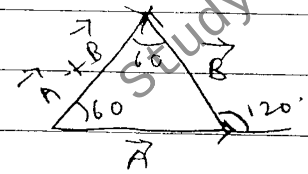
To find angle between 2 vectors tails of both or head of both should coincide



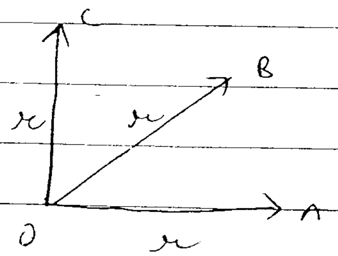
Angle made by resultant with $\vec{A}$ is 'α' then

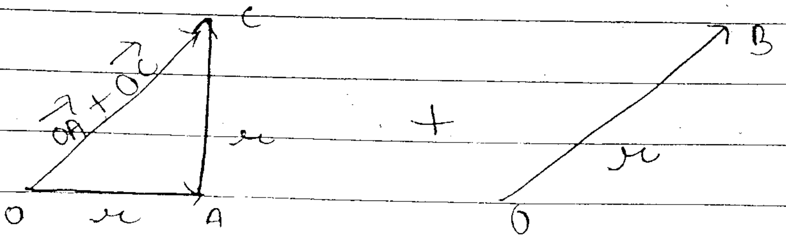
$$\tan\alpha = \frac{Q\sin\theta}{|\vec{A}| + Q\cos\theta}$$

Q If $|\vec{A}| = |\vec{B}| = |\vec{A} + \vec{B}|$



Q $\vec{OA} + \vec{OB} + \vec{OC}$





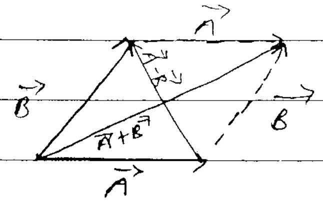
$\vec{OA} + \vec{OC} = \sqrt{2}r$

$\vec{OA} + \vec{OC} + \vec{OB} = \sqrt{2}r + r$

$= r(\sqrt{2}+1)$

Note: If we add 2 vectors $\vec{A}$ & $\vec{B}$ then ~~their~~ they will always lie between $|\vec{A}| + |\vec{B}|$ & $|\vec{A}| - |\vec{B}|$

2) Law of Parallelogram

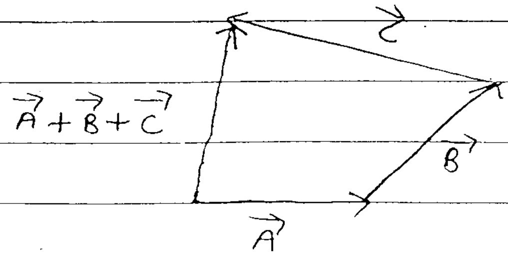


If two vectors represent two adjacent sides of a //gm then their diagonal represent addition and subtraction of vectors.

Note: $\sin 37 \rightarrow 3/5$
$\cos 37 \rightarrow 4/5$
$\tan 37 \rightarrow 3/4$
$\sin 53 \rightarrow 4/5$
$\cos 53 \rightarrow 3/5$
$\tan 53 \rightarrow 4/3$

3) Polygon law of vector addition

It is also an extension of Δ law and it is used to add more than two vectors.



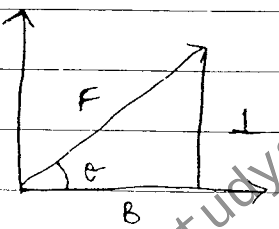
If 3 or more vectors automatically closes the figure in same order then ~~these~~ the resultant of all those vector is zero

$$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$$

Components of Vector

- Any vector can be resolved in two or more directions
- The resolved or splitted parts are called components and the process of ~~spd~~ splitting is called resolution of vector.

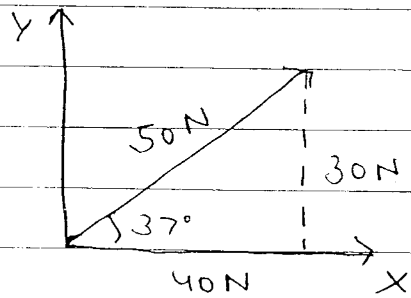


$\cos\theta = B/F$

$B = F\cos\theta$

$\perp = F\sin\theta$

Ex.



~~Fx~~

$F_x = \cancel{B}\, F\cos\theta$

$= 50 \times 4/5 = 40N$

$F_y = F\sin\theta$

$= 50 \times 3/5 = 30N$

* Resultant vector is independent of coordinates
- If sum of three vectors is 0 then all of them should be in same plane

Q. Find a vector whose direction is same as $\vec{A} = \hat{i} + 3\hat{j} + \hat{k}$ and magnitude same as $\vec{B} = 5\hat{i} + \hat{j} + \hat{k}$.

Magnitude of $\vec{B} = \sqrt{5^2+1^2+1^2} = \sqrt{27}$

$$\hat{A} = \frac{\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{11}}$$

$$= \frac{\sqrt{27}}{\sqrt{11}} (\hat{i} + 3\hat{j} + \hat{k})$$

Note: $\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

① If $\vec{A}$ is $\parallel$ to $\vec{B}$
then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

② If $\vec{A}$ is $\perp$ to $\vec{B}$
then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Dot Product of Vectors / Scalar Product

It is mathematically expressed as

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$
$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

If $\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$\vec{A} \cdot \vec{B} = a_1a_2 + b_1b_2 + c_1c_2$

$\hat{i}\hat{i} = \hat{j}\hat{j} = \hat{k}\hat{k} = 1$

$\hat{i}\hat{j} = \hat{j}\hat{k} = \hat{k}\hat{i} = 0$

If $\vec{A}$ is $\perp$ $\vec{B}$ then

$\vec{A} \cdot \vec{B} = |A||B| \cos 90^\circ = 0$

Ex. $\vec{A} = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{B} = 3\hat{i} - 5\hat{j} + x\hat{k}$

Find x if $\vec{A}$ is $\perp$ to $\vec{B}$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$1(3) + (1)(-5) + (-3)(x) = 0$

$3 - 5 - 3x = 0$

$x = -\frac{2}{3}$

Ex. Find angle b/w

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = \hat{i} - \hat{j} + 3\hat{k}$

$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$

$\vec{A}\,\vec{B} = 1(1) + (1)(-1) + 1(3)$

$= 1 - 1 + 3$

$= 3$

$|\vec{A}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$|\vec{B}| = \sqrt{1^2+(-1)^2+3^2} = \sqrt{11}$

$\theta = \cos^{-1} \frac{3}{\sqrt{3}\sqrt{11}}$

$= \cos^{-1} \frac{3}{\sqrt{33}}$

Use Dot product to find projection or component of 1 vector on other

a) Component of $\vec{A}$ along $\vec{B} = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$

b) Component of $\vec{B}$ along $\vec{A} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|}$

Calculation of work

$W = \vec{F} \cdot (\vec{r_f} - \vec{r_i})$

Ex. If a block is acted by a force $\vec{F} = \hat{i} + \hat{j} + 3\hat{k}$ and block moves from (1, 2, 3) to (8, 9, 7)

Find work done by force

$\vec{r_f} - \hat{r_i} = (8-1) + (9-2) + (7-3)$

$= 7 + 7 + 4$

$W = 1(7) + 1(7) + 3(4)$

$= 7 + 7 + 12$

$= 26$

Calculation of Power

$P = \vec{F} \cdot \vec{V}$

Ex. If force acting on a particle is $\vec{F} = 2\hat{i} + 3\hat{j} + \hat{k}$ and it moves with velocity of $\vec{V} = 2\hat{i} - \hat{j} - \hat{k}$ Find power.

$$P = \vec{F} \cdot \vec{V}$$
$$= 2(2) + 3(-1) + (1)(-1)$$
$$= 4 - 3 - 1$$
$$= 0$$

Cross Product / Vector Product

It is defined as

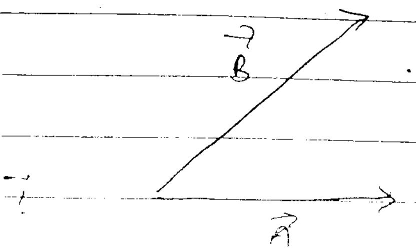
$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \; \hat{n}$$

Tells direction of result of cross product

Direction of cross product is calculated by using -

1) Right hand thumb rule
2) Right hand screw rule

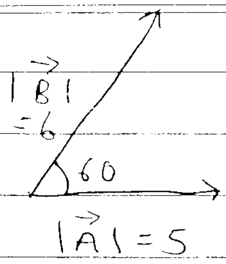
Right hand thumb rule



$\vec{A} \times \vec{B} \Rightarrow$ outside the page
$\vec{B} \times \vec{A} \Rightarrow$ inside the page

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$
$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Ex.



$$\vec{A} \times \vec{B} = \cancel{6}\, 5 \times 6 \times \frac{\sqrt{3}}{2}\ \hat{k}$$

$$= 15\sqrt{3}\ \hat{k}$$

If vectors are given as

$$\vec{A} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$$
$$\vec{B} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{array}{c} \\ A \\ B \end{array}\begin{vmatrix} +\hat{i} & -\hat{j} & +\hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= +\hat{i}\,(b_1c_2 - b_2c_1) - \hat{j}\,(a_1c_2 - a_2c_1) + \hat{k}\,(a_1b_2 - a_2b_1)$$

Ex. $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{B} = \hat{i} + 3\hat{j} + 4\hat{k}$

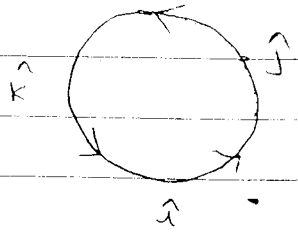
$$\vec{A} \times \vec{B} = \begin{vmatrix} +\hat{i} & -\hat{j} & +\hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 4 \end{vmatrix}$$

$$= +\hat{i}\,(3\times4 - 3\times1) - \hat{i}\,(2\times4 - 1\times1) + \hat{k}\,(2\times3 - 1\times3)$$
$$= 9\hat{i} - 7\hat{j} + 3\hat{k}$$

$$\vec{B} \times \vec{A} = -9\hat{i} + 7\hat{j} - 3\hat{k}$$

Properties of cross product

1)



$\hat{i} \times \hat{j} = \hat{k}$ $\qquad$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\qquad$ $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\qquad$ $\hat{i} \times \hat{k} = -\hat{j}$

2) If two vectors are $\parallel$ then cross product is 0.
i.e. $\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = 0$

3) Using cross product we can find a vector $\perp$ to both $\vec{A}$ & $\vec{B}$ also $\perp$ to plane containing $\vec{A}$ & $\vec{B}$. The direction that is $\perp$ is given by

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

Ex. $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$
$\vec{B} = -\hat{i} + 3\hat{j} + 7\hat{k}$

Find unit vector in direction $\perp$ to $\vec{A}$ & $\vec{B}$

$\vec{A} \times \vec{B} = 18\hat{i} - 15\hat{j} + 9\hat{k}$ (using determinant)

$|\vec{A} \times \vec{B}| = \sqrt{18^2 + (-15)^2 + 9^2} = \sqrt{630}$

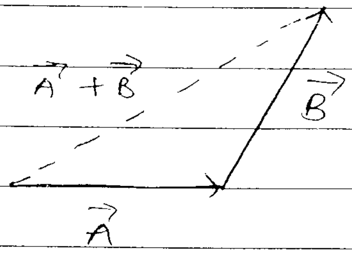
$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$= \frac{18}{\sqrt{630}}\hat{i} - \frac{15}{\sqrt{630}}\hat{j} + \frac{9}{\sqrt{630}}\hat{k}$$

Applications of cross product

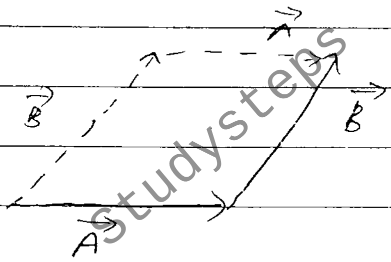
1) To prove that 2 vectors are ∥.

2) Area of triangle



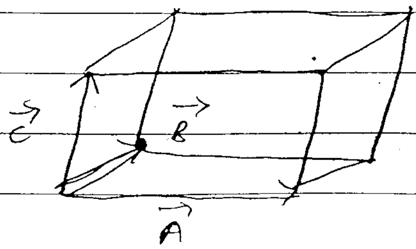
$$\text{Area} = \frac{1}{2}\left|(\vec{A}\times\vec{B})\right|$$

3) Area of parallelogram



$$\text{Area} = |\vec{A}\times\vec{B}|$$

4) Volume of parallelopipe



$$\text{Volume} = \left|(\vec{A}\times\vec{B})\cdot\vec{C}\right|$$

5) Calculation of torque

$$\vec{T} = \vec{\Delta r} \times \vec{F}$$

$$\downarrow$$

$$\vec{r_P} - \vec{r_Q}$$