DETERMINANTS

No : of Row = No. of Column → $[a_{ij}]_{m\times m}$

Row1 → Row2 →
$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$
→ Determinant of order 2

Column 1 Column 2

* $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Expand Along R_1

Minor & Cofactor

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

→ Minor of $a_{12} = M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

→ Cofactor of $a_{12} = C_{12} = (-1)^{1+2}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$

$C_{ij} = (-1)^{i+j} M_{ij}$

NOTE:-
A determinant of order 3 will have 9 minors each minor will be determinant of order 2 and a determinant of order 4 will have 16 minors each minors will be determinant of order 3

NOTE:

Value of determinant = Sum of Product of Element of any Row or Column with their corresponding Cofactors

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} \text{ (along } R_1)$$
$$= a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33} \text{ (along } C_3)$$

OR

$$\left\{\sum_{j=1}^{3} a_{ij}C_{ij} \ \forall\ i = 1,2,3\right\} \quad \left\{\sum_{i=1}^{3} a_{ij}C_{ij} \ \forall\ j = 1,2,3\right\}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$

$i < j$

{No: of Elements $= n^2$}

$i > j$

$\boxed{i = j}$

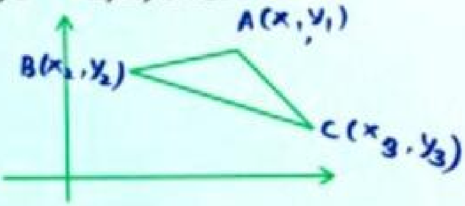
$\left\{ \begin{array}{l} i = \text{Row number} \\ j = \text{Column number} \end{array} \right\}$

Number of elements above Diagonal = Number of elements below diagonal = $(n^2 - n)/2$

Area of Triangle

(i) Area of a triangle whose vertices are (x_r, y_r); $r = 1, 2, 3$ is:

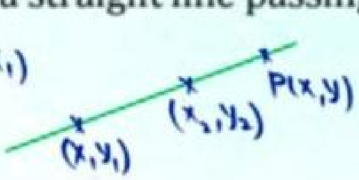
$$D = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right| = ar(\Delta ABC)$$



If $D = 0$, then the three points are collinear.

(ii) Equation of a straight line passing through (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$L: y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$



$$\Delta = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Golden Points

(1)If we multiply the elements of a row (or column) to their respective cofactors and add then we always get the value of determinant

(2) If we multiply the elements of a row(or column) to the respective cofactors of any other row (or column) and add then we get the result as 0

(3) The value of cofactor of an element does not depend on the elements of the row and the column in which it is present

★ Sum of (Elements of a row × cofactors of same row) = determinant value.

★ Sum of (Elements of a column × cofactors of same column) = determinant value.

★ Sum of (Elements of a row × cofactors of some other Row) = 0

★ Sum of (Elements of a column × cofactors of some other column) = 0

★ Sum of (Elements of a row × cofactors of some column) = kaddu.

★ Sum of (Elements of a column × cofactors of some row) = kaddu

Properties of Determinants

P-1 The value of determinant remains unchanged if the rows and columns are interchanged.

P-2 If any two rows or columns of a determinant are interchanged then the value of determinant is changed in sign only.

A Golden Point

If any row or column of a determinant be passed over n rows or columns, the resulting determinant will be $(-1)^n$ times the original determinant.

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = (-1)^2 \begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \end{vmatrix}$$

$$\downarrow R_1 \leftrightarrow R_2$$

$$-\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} \xrightarrow{R_2 \leftrightarrow R_3} (-1)(-1)\begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_1 & b_1 & c_1 \end{vmatrix}$$

P-3 If any two rows or (any two columns)of a determinant are identical then the value of determinant is always zero

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} \qquad R_1 = R_3 \Rightarrow \Delta_1 = 0$$

Proof: $\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$

$$\downarrow R_1 \leftrightarrow R_3$$

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix} = -\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \end{vmatrix}$$

$$\Delta_1 = -\Delta_1$$

$$2\Delta_1 = 0$$

$$\Delta_1 = 0$$

$$\Delta_2 = \begin{vmatrix} a_1 & b_1 & a_1 \\ a_2 & b_2 & a_2 \\ a_3 & b_3 & a_3 \end{vmatrix} \qquad C_1 = C_3 \Rightarrow \Delta_2 = 0$$

P-4 if all elements of any row or column are zero then the value of determinant is also zero

P-5 We can take a factor common from any row or column of a determinant and similarly, we can put back a factor into the determinant in any row or column

Ex. $$\begin{vmatrix} 6 & 12 & 18 \\ 1 & 2 & 1 \\ 4 & 5 & 3 \end{vmatrix} = 6\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 4 & 5 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 12 & 6 \\ 4 & 5 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 24 & 30 & 18 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 2 & 3 \\ 6 & 2 & 1 \\ 24 & 5 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 12 & 3 \\ 1 & 12 & 1 \\ 4 & 30 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 18 \\ 1 & 2 & 6 \\ 4 & 5 & 18 \end{vmatrix}$$

P-6 If each element of any row or column can be expressed sum of two terms then the value of the determinant can be expressed as sum of two determinants

e.g. $$\begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Fusion

fission

Dhyan Rahe

we split determinant along Rows one at a time and similarly along columns also one at a time

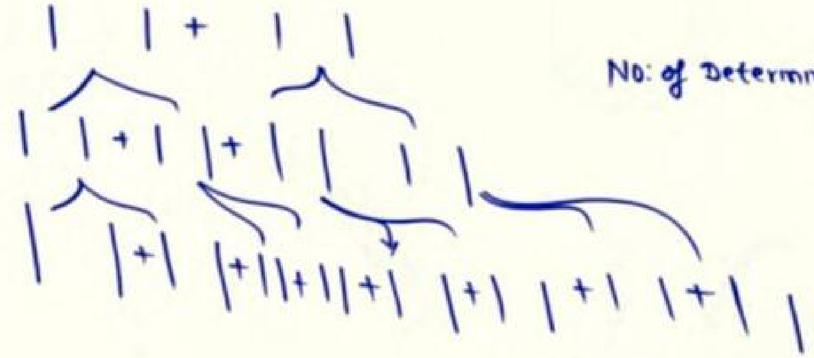
Lallu $$\begin{vmatrix} a_1+l & b_1+m & c_1+n \\ a_2+p & b_2+q & c_2+r \\ a_3+\alpha & b_3+\beta & c_3+\gamma \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} l & m & n \\ p & q & r \\ \alpha & \beta & \gamma \end{vmatrix}$$

"Lallu mat Bano"

Along R_1

Along R_2

Along R_3



No: of Determinants = 8.

Order	No: of Det formed by splitting along each Row/column
If order 3 →	2^3
If order 4 →	2^4
⋮	
If order n →	2^n.

Note:

(1) We can add determinants of same order only. And they can be added only when all corresponding rows (or column) except one are identical in the two determinants and addition takes place along the non identical row(or column)

(2)

$$\sum_{r=1}^{n} \begin{vmatrix} f(r) & a & b \\ g(r) & c & d \\ h(r) & e & f \end{vmatrix} = \begin{vmatrix} \sum_{r=1}^{n} f(r) & a & b \\ \sum_{r=1}^{n} g(r) & c & d \\ \sum_{r=1}^{n} h(r) & e & f \end{vmatrix}$$

C_2 & C_3 do not depend on r

King Property

P-7 The value of determinant is not changed by adding to the elements of any row or column the same multiples of the corresponding elements of any other row or column respectively.

★ Kisi Bhi row mai Kisi dusri row ko kisi no: se multiply karke add yaa subtract kar saktay hai

★ Kisi Bhi column mai kisi dusray column ko kisi no: se multiply karke add yaa subtract kar saktay hai

Which of the following is a valid row or column operation:

(i) $R_1 \to R_1 - \sqrt{3}R_2 + \pi \cdot R_3$ ✓✓

(ii) $C_1 \to C_1 + \sin\theta \cdot C_2 - \sqrt{5}C_3$ ✓✓

(iii) $R_2 \to R_2 + R_3^2$ ✗

(iv) $R_3 \to R_3 - R_1 \cdot R_2$ ✗

(v) $C_1 \to C_1 - \frac{C_3}{C_2}$ ✗

(vi) $C_1 \to C_1 + C_2 + C_3$ ✓

Special Determinants

Yaad Rakhe!!

(a) $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

Proof: $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix}$

$= (y-x)(z-x)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+x \end{vmatrix}$

$= (y-x)(z-x)(z+x-y-x)$

$= -(x-y)(z-x)(z-y)$

$= -(x-y)\cdot(z-x)\cdot -(y-z)$

$= (x-y)(y-z)(z-x)$

(b) $\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$

(c) $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

(d) $\begin{vmatrix} 1 & x & x^4 \\ 1 & y & y^4 \\ 1 & z & z^4 \end{vmatrix} = (x-y)(y-z)(z-x)(x^2+y^2+z^2+xy+yz+zx)$

(e) $\begin{vmatrix} 1 & x^2 & x^4 \\ 1 & y^2 & y^4 \\ 1 & z^2 & z^4 \end{vmatrix} = (x^2-y^2)(y^2-z^2)(z^2-x^2)$

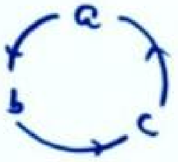
$x \to x^2$
$y \to y^2$ in part (a)
$z \to z^2$

Cyclic Determinant / Circulant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$



$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$$= (a+b+c)\big((c-b)(b-c)-(a-b)(a-c)\big) = (a+b+c)(bc-c^2-b^2+\cancel{bc}-a^2+ac+ab-\cancel{bc})$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)(bc+ca+ab-a^2-b^2-c^2)$$

$$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

Kaam Ki Baat

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)\cdot 2(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)\cdot(2a^2+2b^2+2c^2-2ab-2bc-2ca)$$

$$= -\frac{1}{2}(a+b+c)(a^2+b^2-2ab+a^2+c^2-2ac+b^2+c^2-2ca)$$

$$= -\frac{1}{2}(a+b+c)\left((a-b)^2+(b-c)^2+(c-a)^2\right)$$

B_1: if $a, b, c \in R^+$ & are not all equal then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} < 0$

B_2: $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \iff a+b+c=0$ or $a=b=c$

B_3: If $a, b, c \in R^-$ & are all diff then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} > 0$

Two Golden Points

G_1: The value of determinant corresponding to a triangular determinant is equal to product of its principal diagonal elements.

jiske Diagonal ke uper yaa Neechay waloy saaray elements zero ho

G_2: If in a determinant of odd order, $a_{ij} = -a_{ji}$ & all diagonal elements are zero, then the value of determinant is zero.

$$\Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 3 & 5 \end{vmatrix} = 1\cdot 2\cdot 5 = 10$$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{vmatrix} = 1\cdot 4\cdot 6 = 24$$

ASNC:

yu Daalo $\longrightarrow$ $\downarrow$ yu Nikaalo

- ☐ It is generally used when the given determinant has several terms in power two and we want to convert other terms also into power two

Multiplication of Two Determinants (Det. of same order only can be multiplied)

[R by C]

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1 l_1 + a_1 l_2 & a_1 m_1 + b_2 m_2 \\ a_2 l_1 + b_2 l_2 & a_2 m_1 + b_2 m_2 \end{vmatrix}$$

Det. can be multiplied [R by R], [R by C], [C by C]

[R by R]

$$\begin{vmatrix} a_1 l_1 + b_1 m_1 & a_1 l_2 + m_2 b_1 \\ a_2 l_1 + b_2 m_1 & a_2 l_2 + b_2 m_2 \end{vmatrix} \xrightarrow{\text{Row}}$$

[C by C]

$$\begin{vmatrix} a_1 l_1 + a_2 l_2 & b_1 l + b_2 l_2 \\ a_1 m_1 + a_2 m_2 & b_1 m_1 + b_2 m_2 \end{vmatrix}$$

$\downarrow$ Column

An Important Property 5

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, then $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = D^2$

where A_1, B_i, C_i are cofactors of corresponding elements of D.

Proof: let $D' = \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$

$$DD' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} \xrightarrow{R \leftrightarrow C} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$DD' = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \times \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$= \begin{vmatrix} \overbrace{a_1 A_1 + b_1 B_1 + c_1 C_1}^{D} & \overbrace{a_1 A_2 + b_1 B_2 + c_1 C_2}^{0} & \overbrace{a_1 A_3 + b_1 B_3 + c_1 C_3}^{0} \\ a_2 A_1 + b_2 B_1 + c_2 C_1 & a_2 A_2 + b_2 B_2 + c_2 C_2 & a_2 A_3 + b_2 B_3 + c_2 C_3 \\ a_3 A_1 + b_3 B_1 + c_3 C_1 & a_3 A_2 + b_3 B_2 + c_3 C_2 & a_3 A_3 + b_3 B_3 + c_3 C_3 \end{vmatrix}$$

$$= \begin{vmatrix} D & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{vmatrix} = D^3$$

$D \cdot D' = D^3$

$D' = D^2$

Kaam Ki Baat

① If in Determinant D of order 3 each element is replaced by its respective cofactor then value of New Determinant D' is square of initial Determinant. i.e $D' = D^2 = D^{3-1}$

② If D is a Determinant order n & each element of D is replaced by its respective cofactor then value of New Determinant D' is $(n-1)^{th}$ power of D i.e $D' = D^{n-1}$

③ If D is a Determinant of order n & has value 0 then its cofactor Determinant also has value 0.

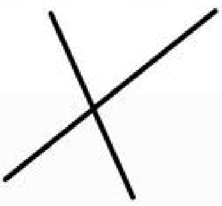
Linear Equations involving Two Variables

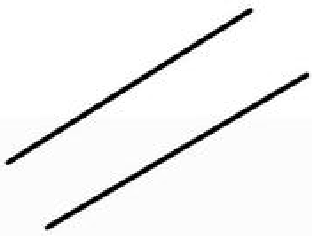
$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Linear Equations

- Consistent
  - Intersecting lines
    $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
    Unique Solution
  - Coincident Lines
    $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
    Infinite Solution
- Inconsistent
  - Parallel Lines
    $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
    No Solution





Linear Equations in Three Variables

CRAMER's RULE

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$$\boxed{x = \frac{\Delta_x}{\Delta},\ y = \frac{\Delta_y}{\Delta},\ z = \frac{\Delta_z}{\Delta}}$$

Consider these Determinant

Proof: $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$

$\Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

$$x\Delta = \begin{vmatrix} a_1x & b_1 & c_1 \\ a_2x & b_2 & c_2 \\ a_3x & b_3 & c_3 \end{vmatrix}$$

$$C_1 \to C_1 + yC_2 + zC_3$$

$$x\Delta = \begin{vmatrix} a_1x+b_1y+c_1z & b_1 & c_1 \\ a_2x+b_2y+c_2z & b_2 & c_2 \\ a_3x+b_3y+c_3z & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} = \Delta_x$$

$x = \dfrac{\Delta_x}{\Delta}$, lly $\quad y\Delta = \begin{vmatrix} a_1 & b_1y & c_1 \\ a_2 & b_2y & c_2 \\ a_3 & b_3y & c_3 \end{vmatrix}$

$$C_2 \to C_2 + xC_1 + zC_3$$

$$y\Delta = \begin{vmatrix} a_1 & a_1x+b_1y+c_1z & c_1 \\ a_2 & a_2x+b_2y+c_2z & c_2 \\ a_3 & a_3x+b_3y+c_3z & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} = \Delta_y$$

$y = \dfrac{\Delta_y}{\Delta}$ lly $z = \dfrac{\Delta_z}{\Delta}$.

Nature of soln

we know: $x \cdot \Delta = \Delta_x$, $y \cdot \Delta = \Delta_y$, $z \cdot \Delta = \Delta_z$

* if $\Delta \neq 0$

$x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}$ $\Rightarrow$ unique soln
& system is Consistent

* if $\Delta = 0$ & atleast one of Δ_x, Δ_y or Δ_z is non zero $\Rightarrow$ No soln system is Inconsistent.

Ex: $\Delta = 0$, $\Delta_x = 0$, $\Delta_y = 1$, $\Delta_z = -1$

$\downarrow$

$y \cdot \Delta = \Delta_y$

$y \cdot 0 = 1$

$0 = 1$ (N.D)

* If $\Delta_x = \Delta_y = \Delta_z = \Delta = 0$

$x \cdot \Delta = \Delta_x$, $y \cdot \Delta = \Delta_y$, $z \cdot \Delta = \Delta_z$

$x \cdot 0 = 0$, $0 \cdot y = 0$, $0 \cdot z = 0$

$\downarrow$, $\downarrow$, $\downarrow$

$x \in R$, $y \in R$, $z \in R$

$\Downarrow$

∞ many soln

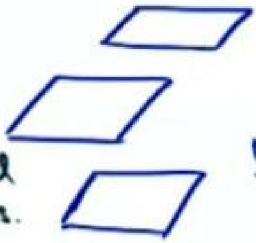
& System is consistent

Nichod!!

★ $\Delta \neq 0 \Rightarrow$ Unique Soln & Consistent System

★ $\Delta = 0$ but atleast one of Δ_x, Δ_y or Δ_z is non zero $\Rightarrow$ No Solution
Inconsistent system.

★ $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ then ∞ many soln, Consistent System



parallel planes.

No soln

$$x+y+2z=3$$
$$x+y+2z=4$$
$$x+y+2z=7$$

$$\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix} = 0, \quad \Delta_x = \begin{vmatrix} 3 & 1 & 2 \\ 4 & 1 & 2 \\ 7 & 1 & 2 \end{vmatrix} = 0$$

$$\Delta_y = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 4 & 2 \\ 1 & 7 & 2 \end{vmatrix} = 0, \quad \Delta_z = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 4 \\ 1 & 1 & 7 \end{vmatrix} = 0$$

Homogeneous system

$d_1, d_2, d_3 = 0$

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

Homogenous System is always consistent
b'coz $x=0$ & $y=0$ & $z=0$ is always a soln

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_x = \begin{vmatrix} 0 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & b_3 & c_3 \end{vmatrix} = 0$$

lly $\Delta_y = 0$, $\Delta_z = 0$

System of linear Eqn

- unique soln: $\Delta \neq 0$ $\Downarrow$ $\left.\begin{matrix} x=0 \\ y=0 \\ z=0 \end{matrix}\right\}$ Trivial soln OR zero soln only
- ~~No soln~~
- ∞ many soln, $\Delta = 0$ } non Trivial solns OR non zero solns.

$$a_1x+b_1y+c_1z=0$$
$$a_1x+b_1y+c_1z=0$$
$$a_1x+b_1y+c_1z=0$$

yaad Rahe:

$\alpha \cdot$ variable $= \beta$
- for unique soln $\alpha \neq 0$
- for ∞ many soln $\alpha = \beta = 0$
- for no solution $\alpha = 0, \beta \neq 0$