Parisharam GOAT (2026)

Matrices

Notes and Practice Sheet

TOPIC TO BE COVERED AS PER CBSE LATEST CURRICULUM (2024-25)

- Concept, notaion, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices.
- Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. On commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2).
- Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

Definition 1 : A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called the elements or the entries of the matrix.

We denote matrices by capital letters. The following are some examples of matrices:

$$A=\begin{bmatrix}-2 & 5\\ 0 & \sqrt{5}\\ 3 & 6\end{bmatrix}, B=\begin{bmatrix}2+i & 3 & -\frac{1}{2}\\ 3.5 & -1 & 2\\ \sqrt{3} & 5 & \frac{5}{7}\end{bmatrix},$$

$$C=\begin{bmatrix}1+x & x^3 & 3\\ \cos x & \sin x+2 & \tan x\end{bmatrix}$$

ORDER OF MATRIX

The order of matrix is a relationship with the number of elements present in a matrix.

The order of a matrix is denoted by $m \times n$, where m and n are the number of rows and columns respectively and the number of elements in a matrix will be equal to the product of m and n.

Types of Matrices

In this section, we shall discuss different types of matrices.

(i) Column matrix

A matrix is said to be a column matrix, if it has only one column.

For example, $A=\begin{bmatrix}0\\ \sqrt{3}\\ -1\\ 1/2\end{bmatrix}$ is a column matrix of order 4×1.

In general, $A=\left[a_{ij}\right]_{m\times 1}$ is a column matrix of order $m \times 1$.

(ii) Row matrix

A matrix is said to be a row matrix, if it has only one row.

For example, $B = \begin{bmatrix}-\frac{1}{2} & \sqrt{5} & 2 & 3\end{bmatrix}_{1\times 4}$ is a row matrix.

In general, $B = [b_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$

(iii) Square matrix

A matrix in which the number of rows are equal to the number of columns, is said to be a square matrix. Thus an $m\times n$ matrix is said to be a square matrix if $m=n$ and is known as a square matrix of order 'n'.

For example

$A = \begin{bmatrix}3 & -1 & 0\\ \frac{3}{2} & 3\sqrt{2} & 1\\ 4 & 3 & -1\end{bmatrix}$ is a square matrix of order 3.

In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m.

(iv) Diagonal matrix

A square matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if all its non-diagonal elements are zero, that is a matrix $B = [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if $b_{ij}=0$, when $i \neq j$.

For example

$A=[4], B=\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}, C=\begin{bmatrix}-1.1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$, are

diagonal matrices of order 1, 2, 3, respectively.

(v) Scalar matrix

A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal, that is, a square matrix $B=[b_{ij}]_{n\times n}$ is said to be a scalar matrix, if

$b_{ij} = 0$, when $i \neq j$

$b_{ij} = k$, when $i = j$, for some constant k.

For example

$A=[3],\ B=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix},\ C=\begin{bmatrix}\sqrt{3} & 0 & 0\\ 0 & \sqrt{3} & 0\\ 0 & 0 & \sqrt{3}\end{bmatrix}$

are scalar matrices of order 1, 2 and 3, respectively.

(vi) Identity matrix

A square matrix in which elements in the diagonal are all 1 and rest are all zero is called an identity matrix. In other words, the square matrix $A=[a_{ij}]_{n\times n}$ is an identity matrix,

if $a_{ij}=\begin{cases}1 & \text{if } i=j\\ 0 & \text{if } i\neq j\end{cases}$.

We denote the identity matrix of order n by I_n. When order is clear from the context, we simply write it as I.

For example

$[1], \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ are identity matrices of order 1, 2 and 3,

(vii) Zero matrix

A matrix is said to be zero matrix or null matrix if all its elements are zero.

For example

$[0], \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, [0, 0]$ are all zero matrices.

We denote zero matrix by O. Its order will be clear from the context.

Equality of matrices

Definition 2 : Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) They are of the same order

(ii) Each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

For example

$\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ and $\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ are equal matrices but $\begin{bmatrix}3 & 2\\ 0 & 1\end{bmatrix}$ and $\begin{bmatrix}2 & 3\\ 0 & 1\end{bmatrix}$ are not equal matrices.

Symbolically, if two matrices A and B are equal, we write $A = B$.

Operations on Matrices

Addition

In general, if $A=[a_{ij}]$ and $B=[b_{ij}]$ are two matrices of the same order, say $m\times n$. Then, the sum of the two matrices A and B is defined as a matrix $C=[c_{ij}]_{m\times n}$, where $c_{ij}=a_{ij}+b_{ij}$, for all possible values of i and j.

Note :

1. We emphasise that if A and B are not of the same order, then $A + B$ is not defined.

For example

If $A=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 3\\ 1 & 0 & 1\end{bmatrix}$, then $A + B$ is not defined.

Multiplication of a matrix by a scalar

$A=\left[a_{ij}\right]_{m\times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k.

For example

If $A=\begin{bmatrix} 3 & 1 & 1.5 \\ \sqrt{5} & 7 & -3 \\ 2 & 0 & 5 \end{bmatrix}$, then

$$3A=3\begin{bmatrix} 3 & 1 & 1.5 \\ \sqrt{5} & 7 & -3 \\ 2 & 0 & 5 \end{bmatrix}=\begin{bmatrix} 9 & 3 & 4.5 \\ 3\sqrt{5} & 21 & -9 \\ 6 & 0 & 15 \end{bmatrix}$$

Negative of a matrix. The negative of a matrix is denoted by $-A$. We define $-A = (-1)A$.

For example

Let $A = \begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}$, then $-A$ is given by

$$-A = (-1)A = (-1)\begin{bmatrix} 3 & 1 \\ -5 & x \end{bmatrix}=\begin{bmatrix} -3 & -1 \\ 5 & -x \end{bmatrix}$$

Difference of matrices

If $A = [a_{ij}]$, $B = [b_{ij}]$ are two matrices of the same order, say $m\times n$, then difference $A - B$ is defined as a matrix $D = [d_{ij}]$, where $d_{ij}=a_{ij}-b_{ij}$, for all value of i and j. In other words, $D = A - B = A + (-1)B$, that is sum of the matrix A and the matrix $-B$.

Properties of matrix addition

(i) Commutative law : $A + B = B + A$

(ii) Associative law : $(A + B) + C = A + (B + C)$

(iii) Existence of additive identity : $A + O = O + A$

(iv) The existence of additive inverse :
$A + (-A) = (-A) + A = 0$

Properties of scalar multiplication of a matrix

(i) $k\left(A+B\right)=kA+kB$,

(ii) $\left(k+l\right)A=kA+lA$

Multiplication of matrices

The product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let $A = [a_{ij}]$ be an $m\times n$ matrix and $B = [b_{jk}]$ be an $n\times p$ matrix. Then the product of the matrices A and B is the matrix C of order $m\times p$.

Remark :

If AB is defined, then BA need not be defined.

Properties of multiplication of matrices

(i) The associative law : $(AB)C = A(BC)$

(ii) The distributive law.

For three matrices A, B and C.

(i) $A(B + C) = AB + AC$

(ii) $(A + B)C = AC + BC$, whenever both sides of equality are defined.

(iii) The existence of multiplicative identity :
$IA=AI=A$.

Transpose of a Matrix

Definition-3 : If $A=\left[a_{ij}\right]$ be an $m\times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A.

Transpose of the matrix A is denoted by A' or (A^T).

In other words, if $A=\left[a_{ij}\right]_{m\times n}$, then $A'=\left[a_{ji}\right]_{n\times m}$.

For example

If $A=\begin{bmatrix} 3 & 5 \\ \sqrt{3} & 1 \\ 0 & \frac{-1}{5} \end{bmatrix}_{3\times 2}$, then $A'=\begin{bmatrix} 3 & \sqrt{3} & 0 \\ 5 & 1 & \frac{-1}{5} \end{bmatrix}_{2\times 3}$

Properties of transpose of the matrices

For any matrices A and B of suitable orders, we have

(i) $(A')' = A$,

(ii) $(kA)' = kA'$ (where k is any constant)

(iii) $(A + B') = A' + B'$

(iv) $(AB)' = B'A'$

Symmetric and Skew Symmetric Matrices

Definition 4*A* square matrix $A = [a_{ij}]$ is said to be symmetric if $A' = A$, that is, $[a_{ij}] = [a_{ji}]$ for all possible values of i and j.

For example

$$A=\begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -1.5 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$ is a symmetric matrix as

$A' = A$.

Definition-5 :

A square matrix $A = [a_{ij}]$ is said to be skew symmetric matrix, if $A' = A$

For example :

The matrix $B=\begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}$ is a skew symmetric matrix as $B' = -B$

Theorem-1 :

For any square matrix A with real number entries, $A + A'$ is a symmetric matrix and $A - A'$ is a skew symmetric matrix.

Theorem-2 :

Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

$$A=\frac{1}{2}\left(A+A'\right)+\frac{1}{2}\left(A-A'\right)$$

Definition-6

If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that $AB = BA = I$, then B is called the inverse matrix of A and it is denoted by A^{-1}. In that case A is said to be invertible.

Note :

1. A rectangular matrix does not possess inverse matrix, since for products BA and AB to be defined and to be equal, it is necessary that matrices A and B should be square matrices of the same order.

2. If B is the inverse of A, then A is also the inverse of B.

Theorem-3 :

(Uniqueness of inverse) Inverse of a square matrix, if it exists, is unique.

Theorem-4 :

If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}A^{-1}$.

Exercise

MCQ's

1. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of $x + y$ is

(A) 4 (B) 5
(C) 6 (D) 9

2. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

(A) Square matrix (B) Diagonal matrix
(C) Unit matrix (D) Scalar matrix

3. If the sum of all elements of a 3×3 scalar is 9, then the product of its all-diagonal elements is

(A) 0 (B) 9
(C) 27 (D) 729

4. If $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{Max}(i, j) - \text{Min}(i, j)$. Then, A^2 is equal to

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

5. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is

(A) 0 (B) 5
(C) 10 (D) 25

6. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(A) A (B) $I - A$
(C) $I + A$ (D) $3A$

7. If the matrix $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then $a + b + c =$

(A) –5 (B) 0
(C) 5 (D) None of these

8. If $A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$ is a symmetric matrix, then $x + y + z$ is equal to

(A) 10 (B) 6
(C) 8 (D) 0

9. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is

(A) 0 (B) 1
(C) –10 (D) 10

10. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is

(A) 1 (B) 2
(C) 0 (D) –2

11. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

(A) A null matrix (B) A unit matrix
(C) $-A$ (D) A

12. For what value of $x \in \left[0, \frac{\pi}{2}\right]$, is $A + A^T = \sqrt{3}I$, where $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$?

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) 0 (D) $\frac{\pi}{2}$

13. If A is a square matrix $A^2 = A$, then $(I + A)^2 - 3A$ is equal to

(A) I (B) A

(C) $2A$ (D) $3I$

14. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2I$, then the value(s) of x is / are

(A) $\pm\sqrt{7}$ (B) 0

(C) ±5 (D) 25

15. If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to

(A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$

16. Statement-1 : Every scalar matrix is a diagonal matrix.

Statement-2 : In a diagonal matrix, all diagonal elements are 0.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

17. Statement-1 : For any symmetric matrix A, $B^T AB$ is a skew-symmetric matrix

Statement-2 : A square matrix P is skew-symmetric if $P^T = -P$.

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

(C) Statement-1 is true, Statement-2 is false.

(D) Statement-1 is false, Statement-2 is true.

18. If A and B are square matrices of the same order, then $(A + B)(A - B)$ is equal to

(A) $A^2 - B^2$

(B) $A^2 - BA - AB - B^2$

(C) $A^2 - B^2 + BA - AB$

(D) $A^2 - BA + B^2 + AB$

19. If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $x + y$ equals

(A) 0 (B) –1

(C) 2 (D) None of these

20. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then values of a and b are

(A) $a = 4, b = 1$ (B) $a = 1, b = 4$

(C) $a = 0, b = 4$ (D) $a = 2, b = 4$

21. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

22. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A (B) $I - A$

(C) 1 (D) $3A$

23. The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is

(A) A is a diagonal matrix

(B) A is a zero matrix

(D) A is a square matrix

(C) A is a scalar matrix

24. If A and B are symmetric matrices, then ABA is

(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Diagonal matrix

(D) Scalar matrix

25. If $A=\begin{bmatrix}5 & x\\ y & 0\end{bmatrix}$ and A^T, then

(A) $x=0, y=5$ (B) $x+y=5$
(C) $x=y$ (D) None of these

26. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then $A^T+A=I_2$, if

(A) $\theta=n\pi, n\in Z$

(B) $\theta=(2n+1)\frac{\pi}{2}, n\in Z$

(C) $\theta=2n\pi+\frac{\pi}{3}, n\in Z$

(D) None of these

27. The number of all possible matrices of order 3×3 with each entry 0 or 1 or –1 is

(A) 27 (B) 3^6
(C) 81 (D) 3^9

28. The trace of the matrix $A=\begin{bmatrix}1 & -5 & 7\\ 0 & 7 & 9\\ 11 & 8 & 9\end{bmatrix}$ is

(A) 17 (B) 25
(C) 3 (D) 12

29. If $A=\begin{bmatrix}2 & -1 & 3\\ -4 & 5 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 3\\ 4 & -2\\ 1 & 5\end{bmatrix}$, then

(A) Only *AB* is defined
(B) Only *BA* is defined
(C) *AB* and *BA* both are defined
(D) *AB* and *BA* both are not defined

30. If $A=\begin{bmatrix}4 & x+2\\ 2x-3 & x+2\end{bmatrix}$ is a symmetric matrix, then x =

(A) 3 (B) 5
(C) 2 (D) 4

31. If α and β are the roots of the equation $1+x+x^2=0$, then $\begin{bmatrix}1 & \beta\\ \alpha & \alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ 1 & \beta\end{bmatrix}=$

(A) $\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$ (B) $\begin{bmatrix}-1 & -1\\ -1 & 2\end{bmatrix}$

(C) $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$ (D) $\begin{bmatrix}-1 & -1\\ -1 & -2\end{bmatrix}$

32. If $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, then A^3-2A^2 is

(A) A null matrix (B) An identity matrix
(C) A (D) $-A$

33. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A=B^2$, then x equals

(A) ±1 (B) –1
(C) 1 (D) 2

34. If $A=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$, then AA^T is equal to

(A) 14 (B) $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$

(C) $\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{bmatrix}$ (D) [14]

35. If matrix $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ and $A^2=kA$, then the value of k is

(A) 1 (B) –2
(C) 2 (D) –1

36. For any 2×2 matrix P, which of the following matrices can be Q such that $PQ=OP$?

(A) [1]

(B) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

(C) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

(D) No such matrix exists as matrix multiplication is not commutative.

37. P and Q are matrices such that both $P + Q$ and PQ are defined. Which of the following is true about P and Q ?

(A) P and Q can be any matrices but of the same order

(B) P and Q must be square matrices of the same order

(C) P and Q must be square matrics or same order

(D) P and Q must be square matrices not necessarily of the same order order of P of m and n

38. If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then

(A) $AB = 0$ (B) $AB = -BA$

(C) $BA = O$ (D) $AB = BA$

39. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is

(A) 7 (B) 6

(C) 8 (D) 18

40. If a matrix has 36 elements, the number of possible order it can have, is

(A) 13 (B) 3

(C) 5 (D) 9

41. If matrices A and B are of order 1×3 and 3×1 respectively, then the order of $A^T B^T$ is

(A) 1×1 (B) 3×1

(C) 1×3 (D) 3×3

42. If $[x, \quad 2 \quad 0]\begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \quad 1]\begin{bmatrix} -2 \\ x \end{bmatrix}$, then the value of x is

(A) -1 (B) 0

(C) 1 (D) 2

43. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true?

(A) $a_{ij} = \begin{cases} 0, \text{ if } i = j \\ 1, \text{ if } i \neq j \end{cases}$ (B) $a_{ij} = 1 \text{ for all } i, j$

(C) $a_{ij} = 0 \text{ for all } i, j$ (D) $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ 1, \text{ if } i = j \end{cases}$

44. If A and B are two skew-symmetric matrices, then $(AB + BA)$ is

(A) A skew symmetric matrix

(B) A symmetric matrix

(C) A null matrix

(D) An identity matrix

45. For any square matrix A, $(A - A^T)$ is always

(A) An identity matrix

(B) A null matrix

(C) A skew-symmetric matrix

(D) A symmetric matrix

46. If X, Y and XY are matrices of orders 2×3, $m \times n$ and 2×5 respectively, then the number of elements in Y is

(A) 6 (B) 10

(C) 15 (D) 35

47. If A is matrix of order 2023×2024 and B is a matrix such that AB' and $B'A$ both are defined, then the order of matrix B is

(A) 2023×2024 (B) 2023×2023

(C) 2024×2024 (D) 2024×2023

48. If A is matrix of order 2023×2024 and B is a matrix such that AB and BA both are defined, then the order of matrix B is

(A) 2023×2024 (B) 2023×2023

(C) 2024×2024 (D) 2024×2023

49. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then value of x is / are

(A) ± 3 (B) $\pm\sqrt{7}$

(C) ± 5 (D) 0

50. If all entries of a square matrix of order 3 are either 5 or 0, then how many Identity matrices are possible?

(A) 1 (B) 8
(C) 2 (D) 0

51. If all entries of a square matrix of order 3 are either 3 or 0, then how many Scalar matrices possible?

(A) 1 (B) 8
(C) 6 (D) 2

52. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 (B) 18
(C) 81 (D) 512

Subjective Type Questions

53. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

54. If A is a square matrix such that $A^2 = A$, show that $(I + A)^3 = 7A + I$.

55. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ and $f(x) = x^2 - 4x + 7$. Show that $f(A) = O$. use this result to find A^5.

56. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find A.

57. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

(i) The order of the matrix,
(ii) The number of elements,
(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

58. Construct a 3×4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i + j|$

59. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ (ii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

60. If $A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$, then compute $3A - 5B$.

61. Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

62. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x, y, z and w.

63. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

64. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = f(x + y)$.

65. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

66. If $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

67. Express the matrix $B=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

68. If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then prove that $A^n=\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n\in\mathrm{N}$.

69. Let $A=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B=\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}, C=\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$.

Find a matrix D such that $CD - AB = O$.

70. Find the values of x, y, z if the matrix $A=\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.

71. Construct a 3×3 matrix $A=\left[a_{ij}\right]$ such that

(a) $a_{ij}=\begin{cases} i+j; & i>j \\ \frac{i}{j}; & i=j \\ i-j; & i<j \end{cases}$ (b) $a_{ij}=\begin{cases} 2^i; & i>j \\ i.j; & i=j \\ 3^j; & i<j \end{cases}$

SOLUTIONS

1. (B)
2. (A)
3. (C)
4. (C)
5. (D)
6. (A)
7. (A)
8. (D)
9. (A)
10. (B)
11. (B)
12. (B)
13. (A)
14. (C)
15. (C)
16. (C)
17. (D)
18. (C)
19. (A)
20. (B)
21. (C)
22. (C)
23. (A)
24. (A)
25. (C)
26. (C)
27. (D)
28. (A)
29. (C)
30. (B)
31. (B)
32. (A)
33. (C)
34. (D)
35. (C)
36. (B)
37. (B)
38. (B)
39. (D)
40. (D)
41. (D)
42. (A)
43. (D)
44. (B)

45. **(C)**

46. **(C)**

47. **(A)**

48. **(D)**

49. **(C)**

50. **(D)**

51. **(D)**

52. **(D)**

53. $(A-I)^3+(A+I)^3-7A=A$

54. **(Prove)**

55. $A^5=\begin{bmatrix}-118 & -93\\ 31 & -118\end{bmatrix}$

56. $A=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$

57. (i) 3×4

(ii) 12

(iii) $a_{13}=19$, $a_{21}=35$, $a_{33}=-5$, $a_{24}=12$, $a_{23}=5/2$

58. $A=\begin{bmatrix}1 & \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{5}{2} & 2 & \frac{3}{2} & 1\\ 4 & \frac{7}{3} & 3 & \frac{5}{2}\end{bmatrix}$

59. (i) $x=4, y=2, z=0$ or $x=2, y=4, z=0$

(ii) $x=2, y=4, z=3$

60. $3A-5B=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

61. (i) $x=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}, y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$

(ii) $x=\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}, y=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$

62. $x=2, y=4, w=3, z=1$

63. Prove

64. Prove

65. $K=1$

66. Prove

67. Prove

68. Prove

69. $D=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$

70. $x=\pm\frac{1}{\sqrt{2}}, y=\pm\frac{1}{\sqrt{6}}, z=\pm\frac{1}{\sqrt{3}}$

71. (a) $\begin{bmatrix}1 & -1 & -2\\ 3 & 1 & -1\\ 4 & 5 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 9 & 27\\ 4 & 4 & 27\\ 8 & 8 & 9\end{bmatrix}$

Must Do Questions From Matrices

Example : 16, 17, 20, 22, 23, 24, 25

Exercise 3.1 : 1, 3, 4-(iii), 5-(i), 6-(ii and iii), 8, 10

Exercise 3.2 : 5, 6, 7, 8, 12, 13, 15, 16, 17, 18, 20, 21, 22

Exercise 3.3 : 4, 6, 10, 11, 12

Miscellaneous : (1 – 11) complete