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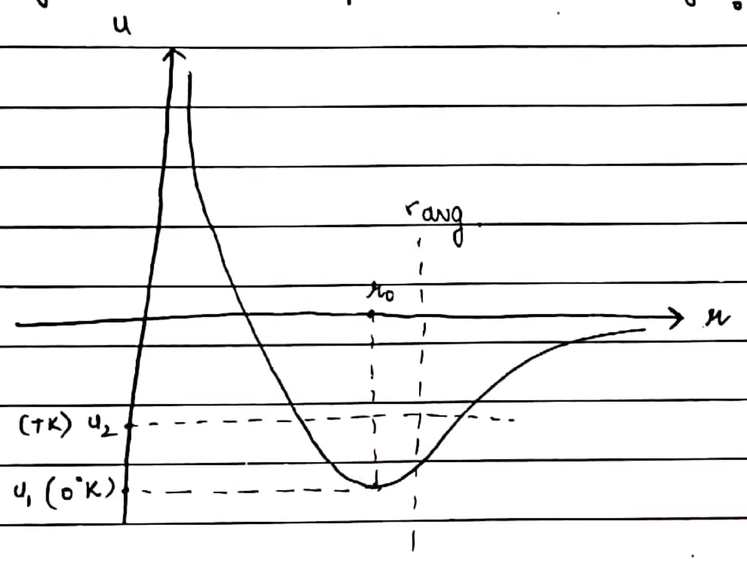
Thermal Expansion

- Expansion of matter on increasing its temperature is called thermal expansion.

→ Why materials expands on heating ?



→ The above graph represent a potential energy of the particles of solid with separation 'r'.

→ At T = 0 K, they remain in equilibrium at separation r_0.

→ On increasing the temp., they start oscillating and the average separation increases due to asymmetric nature of graph.

This is the reason behind thermal expansion.

→ Linear Expansion -

Expansion in length is linear expansion.

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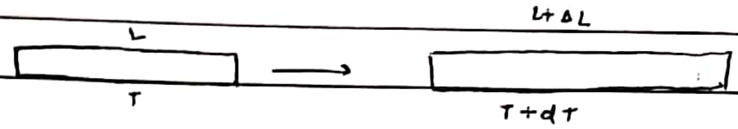


$\rightarrow \quad dL \propto L\,dt$

$dL = \alpha L\,dt$

$$\int_{L}^{L+\Delta L} \frac{dL}{L} = \alpha \int_{T}^{T+\Delta T} dT$$

$$\ln\left(\frac{L+\Delta L}{L}\right) = \alpha \Delta T$$

$$1 + \frac{\Delta L}{L} = e^{\alpha \Delta T}$$

$$1 + \frac{\Delta L}{L} = 1 + \alpha \Delta T$$

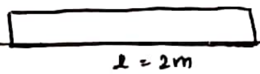
$$\boxed{\Delta L = L\alpha\Delta T}$$

$\Delta L = L\alpha\Delta T$

$\frac{\Delta L}{L} = \alpha\Delta T \longrightarrow$ Fractional change in length.

$L_f = L + L\alpha\Delta T$

$$\boxed{L_f = (1+\alpha\Delta T)\,L}$$

Q. 

, $\alpha = 10^{-5}/^\circ C$, $\Delta T = 200^\circ C$, Find L_f

$L_f = (1 + 10^{-5} \times 200) \times 2$

$L_f = (1 + 2\times10^{-3}) \times 2$

$L_f = \frac{\cancel{1002}\,501}{\cancel{1000}\,\cancel{500}\,250} \times \cancel{2} = \frac{501}{250} = 2.004$

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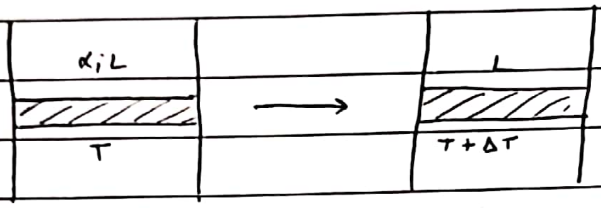
NOTE :-

At temp. $T+\Delta T$, length of rod is $L+\Delta L$, but in this situation, strain of rod is zero because at this new temperature, $L+\Delta L$ is its original length.

→ **Thermal stress**

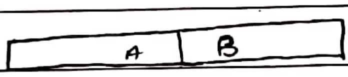


└ Now rod has compressive stress.

strain → $\frac{\Delta L}{L} = \alpha \Delta T$

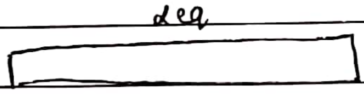
stress → $Y \alpha \Delta T$

Q.



Find α equivalent of this composite rod?

α_{eq}



(L_1+L_2)

$\Delta L = L_1\alpha_1 \Delta t + L_2 \alpha_2 \Delta T$

$\Delta L = (L_1+L_2)\, \alpha_{eq} \cdot \Delta T$

$L_1\alpha_1 \Delta T + L_2 \alpha_2 \Delta T = (\alpha_1 + L_2) \cdot \alpha_{eq} \cdot \Delta T$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{L_1 + L_2}$$

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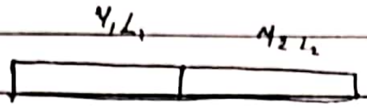
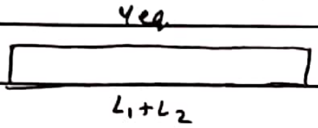
Q.


Find Y_{eq}.

On applying force

$$\Delta L = \frac{F L_1}{A Y_1} + \frac{F L_2}{A Y_2}$$



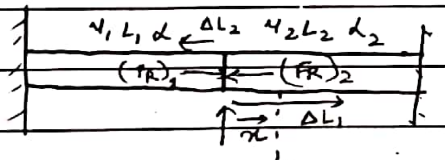
$$\rightarrow \Delta L = \frac{F_1 (L_1 + L_2)}{A Y_{eq}}.$$

$$\frac{F L_1}{A Y_1} + \frac{F L_2}{A Y_2} = \frac{F(L_1 + L_2)}{A Y_{eq}} = 0$$

$$\frac{(L_1 + L_2)}{Y_{eq}} = \frac{L_1}{Y_1} + \frac{L_2}{Y_2}$$

$$\downarrow$$

$$\frac{\Sigma L}{Y_{eq}} = \Sigma \frac{1}{Y}$$

Q.


Junction

Find the disp. of junction if temperature of whole assembly is increased by ΔT.

$$\frac{\Delta L_1}{L_1} = \alpha_1 \Delta T \quad , \quad \frac{\Delta L_2}{L_2} = \alpha_2 \Delta T$$

$$(F_R)_1 = Y_1 A \alpha_1 \Delta T \quad , \quad (F_R)_2 = Y_2 A \alpha_2 \Delta T$$

Let junction shifts by x

At equi. $(F_R)_1 = (F_R)_2$

$$Y_1 A \left[\frac{\Delta L_1 - x}{L_1} \right] = Y_2 A \left[\frac{\Delta L_2 + x}{L_2} \right]$$

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$$\frac{Y_1}{L_1}\left[L_1\alpha_1\Delta T - x\right] = \frac{Y_2}{L_2}\left[L_2\alpha_2\Delta T + x\right]$$

$$\boxed{x = \frac{(Y_1\alpha_1 - Y_2\alpha_2)\,\Delta T}{\frac{Y_1}{L_1} + \frac{Y_2}{L_2}}}$$

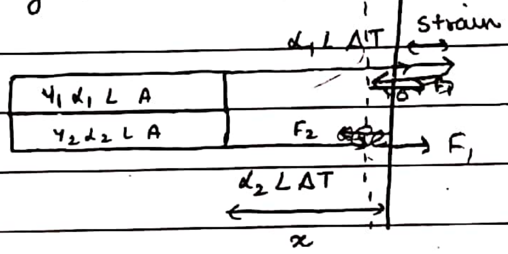
Q.

Y_1	α_1	L	A
Y_2	α_2	L	A

[Assume no Bending]

$\alpha_1 > \alpha_2$

Find final length of composite rod if temp is increased by ΔT.



At steady state,

$F_1 = F_2$

$$Y_1 \cancel{A}\left[\frac{L\alpha_1\Delta T - x}{\cancel{L}}\right] = Y_2 \cancel{A}\left[\frac{x - L\alpha_2\Delta T}{\cancel{L}}\right]$$

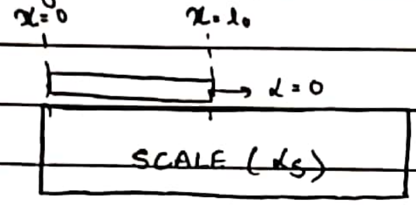
$$Y_1\alpha_1 L\Delta T - Y_1 x = Y_2 x - Y_2 L\alpha_2\Delta T$$

$$x = \frac{(Y_1\alpha_1 + Y_2\alpha_2)\,L\Delta T}{Y_1 + Y_2}$$

$$x = L\Delta T\,\frac{\Sigma Y\alpha}{\Sigma Y}$$

Measuring scale.

Q. The reading of scale at temp T is l_0. Find the reading at temp $T+\Delta T$.



Temp $\rightarrow T$ scale ka koi purana no, abi l_0 ke brabr hoga.

Let new reading be l_{new}.

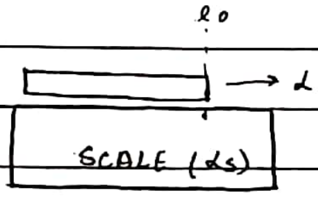
We can write,

$$l_{new}(1+\alpha_S\Delta T) = l_0$$

$$l_{new} = l_0(1+\alpha_S\Delta T)^{-1}$$

$$= l_0(1-\alpha_S\Delta T)$$

Q.



Repeat previous problem.

Let new reading be l_{new}.

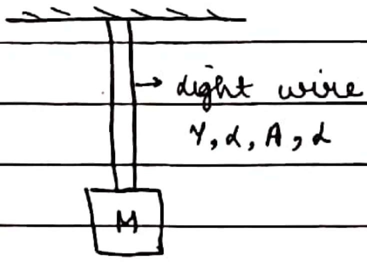
We can write,

$$l_{new}(1+\alpha_S\Delta T) = l_0(1+\alpha\Delta T)$$

$$l_{new} = l_0(1+(\alpha-\alpha_S)\Delta T)$$

$\left[\text{neglecting } -\underbrace{\alpha\cdot\alpha_S(\Delta T)^2}_{\text{very small}}\right]$

Q.



Find the change of temperature of wire ~~by~~ so that it can regain its original length.

Due to elastic elongation,

$$\frac{\Delta L}{L} = \frac{F}{AY}$$

Due to thermal expansion,

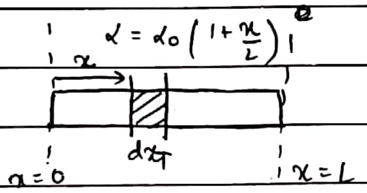
$$\frac{\Delta L}{L} = \alpha \Delta T$$

$$\therefore \frac{F}{AY} + \alpha \Delta T = 0$$

$$\frac{F}{AY} = -\alpha \Delta T$$

$$\Delta T = \frac{F}{-\alpha AY} \quad \text{or} \quad \frac{Mg}{-\alpha AY}$$

Q.



Find elongation in length of rod if temp is increased by ΔT

$$\frac{\Delta L}{L} = \alpha \Delta T$$

$$\frac{ds}{dx} = \alpha \Delta T$$

$$\int ds = \int_0^L \alpha_0 \left(1 + \frac{x}{L}\right) \Delta T \, dx$$

$$\Delta L = \alpha_0 \Delta T \left[L + \frac{L}{2} \right]$$

$$\Delta L = \frac{3 \alpha_0 \Delta T L}{2}$$

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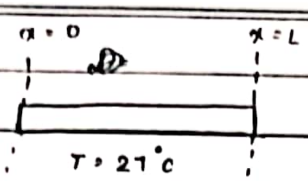

$\alpha = (10^{-6})T$

$l = 2m$

Find the length of at 127°c.

$\int ds = \int L\alpha\, dT$

$\int ds = \int L \times \frac{T}{10^6} \times dT$

$\Delta L = \frac{L}{10^6} \times \int_{300}^{400} T\,dT$

$\Delta L = \frac{2}{10^6} \times \frac{400^2 - 300^2}{2}$

$= \frac{160000 - 90000}{10^6} = \frac{7 \times 10^4}{10^6} = 0.07$

→ But l is not constant, Actual method -

$dl = l\alpha\, dT$

$\int_{l_i}^{l_f} \frac{dl}{l} = 10^{-6} \int_{300}^{400} T\,dT$

$\ln\left(\frac{l_f}{l_i}\right) = 10^{-6}\left[\frac{(16-9)\times 10^4}{2}\right]$

$\ln\left(\frac{l_f}{l_i}\right) = 3.5 \times 10^{-2}$

$l_f = 2e^{0.035}$

$l_f = 2(1 + 0.035)$

$l_f = 2.07$ m.

P.T.O.

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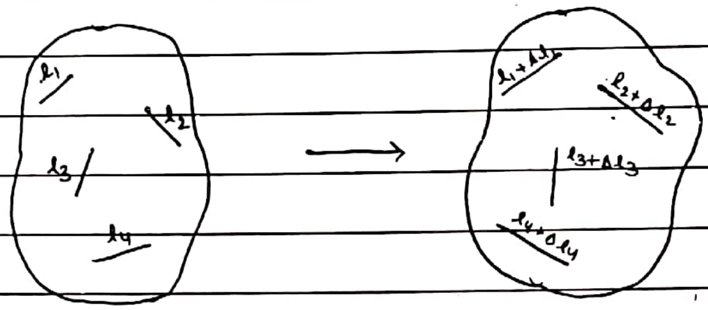
Isotropic Expansion -

- If fractional change in length due to temperature is same in all directions, then expansion is called Isotropic.
- It is analogous to photographic enlargement.



T T+ΔT

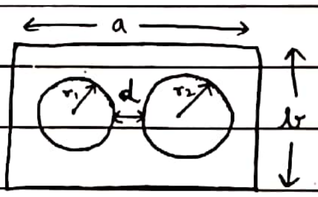
If expansion is Isotropic then,

$$\frac{\Delta l_1}{l_1} = \frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = \frac{\Delta l_4}{l_4}$$

NOTE :-

If not mentioned we have to consider isotropic expansion.

Q.



If temperature of sheet is increased, then choose the quantities which will increase -

A) a b) b c) r_1 d) r_2 e) d

[Method - Zooming in will increase everything]

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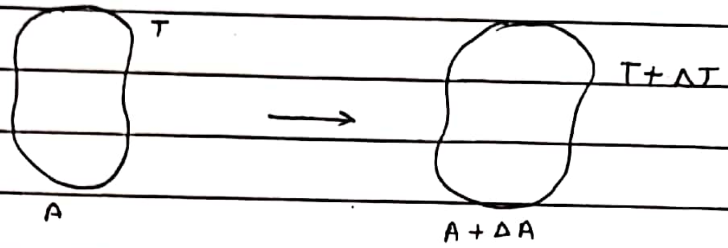
Aerial Expansion / Superficial Expansion -



$$\Delta A \propto A \Delta T$$

$$\boxed{\Delta A = A \beta \Delta T}$$

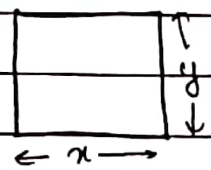
$\beta \rightarrow$ co-efficient of aerial expansion

Unit $\rightarrow \frac{1}{°C}$ or $\frac{1}{K}$

$$\frac{\Delta A}{A} = \beta \Delta T$$

$$A_f = A(1+\beta \Delta T)$$

$\rightarrow$ Relation between α & β.



$$A = xy$$

$$\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

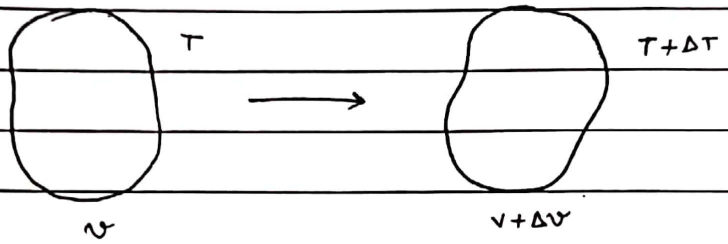
$$\beta \Delta T = \alpha_x \Delta T + \alpha_y \Delta T$$

$$\beta = \alpha_x + \alpha_y$$

If expansion is isotropic -

$$\beta = 2\alpha$$

Volume Expansion -



$$\Delta V \propto V \Delta T$$

$$\boxed{\Delta V = \gamma V \Delta T}$$

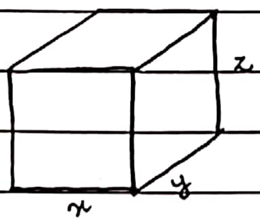
$\gamma \rightarrow$ co-efficient of volume expansion

Unit $\rightarrow \frac{1}{°C}$ or $\frac{1}{K}$

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$V_f = (1 + \gamma \Delta T) v$$

$\rightarrow$ Relation b/w α & γ



$$V = xyz$$

$$\frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\boxed{\gamma = \alpha_x + \alpha_y + \alpha_z}$$

If expansion is isotropic ; $\gamma = 3\alpha$

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

Q. When a cylindrical rod is heated by 80°C, the % change in its length is 0.1 %. Find

a) % change in its radius

b) % change in its area of cross section

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c) % change in its volume
d) % change in its density
e) value of α.

a) $\frac{\Delta L}{L} \times 100 = 0.1$

$\alpha \Delta T \times 100 = 0.1$

$\frac{\Delta R}{R} \times 100 = \frac{\Delta L}{L} \times 100 = 0.1\%$

b) $A = \pi R^3$

$\frac{\Delta A}{A} = \frac{2\Delta R}{R} \times 100 = 2 \times 0.1\% = 0.2\%$

c) $V = \pi R^2 L$

$\frac{\Delta V}{V} = \frac{3\Delta R}{R} \times 100 = 3 \times 0.1 = 0.3\%$

d) $\frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V} = -0.3\%$

e) $\alpha \Delta T \times 100 = 0.1$

$\alpha \times 80 \times 100 = 0.1$

$\alpha = \frac{1}{8 \times 10^4}$

→ **Expansion in liquids** -

- In expansion of liquids, there is no meaning of α and β.

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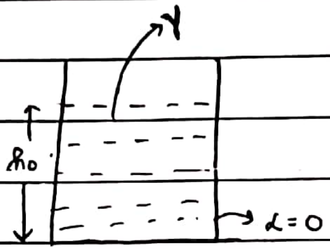
- For liquids, only γ is defined.

Q. Find the height of liquid in beaker at temp. $T+\Delta T$



$$V = A \times h$$

$$\frac{\Delta V}{V} = \cancel{\frac{\Delta A}{A}}^{\,0} + \frac{\Delta H}{H}$$

$$\gamma \Delta T = \frac{\Delta H}{H_0}$$

$$h_0 \gamma \Delta T = \Delta h$$

Final height $\rightarrow$ $h_0(1+\gamma\Delta T)$

OR

$$V_0 = A_0 h_0$$

$$V_f = V_0(1+\gamma\Delta T)$$

$$A_0 = A_0$$

$$A_0 h = V_0(1+\gamma\Delta T)$$

$$h = h_0(1+\gamma\Delta T)$$

final height $\rightarrow$ $h_0(1+\gamma\Delta T)$

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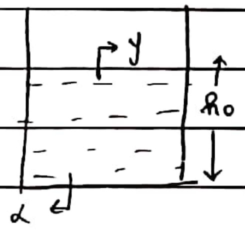
b) Find fractional change in pressure at bottom of the container.

$$\frac{\Delta P}{P} = \frac{\Delta \rho}{\rho} + \frac{\Delta h}{h} \qquad P = \rho g h$$

$$\frac{\Delta P}{P} = \gamma \Delta T + (-\gamma \Delta T)$$

$$\frac{\Delta P}{P} = 0$$

Q. Repeat previous problem



$$V = Ah$$

$$\frac{\Delta V}{V} = \frac{\Delta A}{A} + \frac{\Delta H}{H_0}$$

$$\gamma \Delta T = 2\alpha \Delta T + \frac{\Delta H}{H_0}$$

$$(\gamma - 2\alpha)\Delta T = \frac{\Delta H}{H_0}$$

$$H_0 \Delta T (\gamma - 2\alpha) = \Delta H$$

$$h = h_0 (1 + (\gamma - 2\alpha)\Delta T)$$

change in pressure -

$$\frac{\Delta P}{P} = \rho g h = \frac{\Delta \rho}{\rho} + \frac{\Delta H}{h}$$

$$= -\gamma \Delta T + (\gamma - 2\alpha)\Delta T$$

$$\frac{\Delta P}{P} = -2\alpha \Delta T$$

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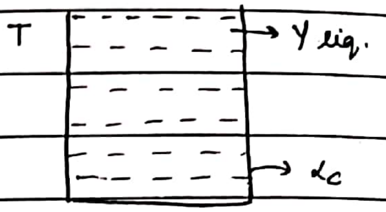
→ Apparent co-efficient of volume expansion.



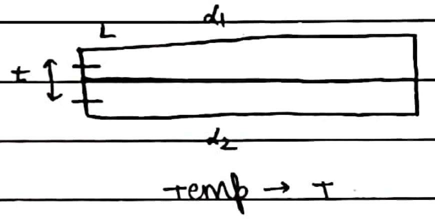
At $T+\Delta T$

$V_{liq} = V_0(1+\gamma \Delta T)$

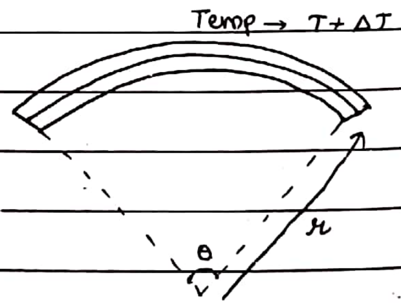
$V_{cont} = V_0(1+3\alpha_c \Delta T)$

$$V_{overflow} = V_0 \underbrace{(\gamma_{liq.} - 3\alpha_c)}_{\gamma_{apparent}} \Delta T$$

→ Bi-metallic strip



$(\alpha_1 > \alpha_2)$



$(r+t)\theta = l_1 = l(1+\alpha_1 \Delta T)$ — ①

$r\theta = l_2 = l(1+\alpha_2 \Delta T)$ — ②

① ÷ ②

$$\frac{r+t}{r} = \frac{1+\alpha_1 \Delta T}{1+\alpha_2 \Delta T}$$

① − ②

$t\theta = l(\alpha_1 - \alpha_2)\Delta T$

$$\theta = \frac{l \Delta T}{t}(\alpha_1 - \alpha_2).$$