PROBABILITY

- Probability is simply how likely something is to happen.
- Whenever we're sure/unsure about the outcome of an event we can talk about the Probabilities of certain outcomes - how likely they are.
- The Analysis of events governed by Probability is called statistics.

Probability : [P(E)]

$$P(E) = \frac{\text{Favourable outcome}}{\text{Total no. of outcome}}$$

$$0 \leq P(E) \leq 1$$

Impossible Event — Sure / certain event.

Q. Dice → Even no
Probability ??

Sample Space = (1,2,3,4,5,6) ⇒ $n(S) = 6$

Favourable outcome = (2,4,6) ⇒ $n(F) = 3$

Probability = $\frac{F.O}{S.S}$

$P(E) + P(\bar{E}) = 1$

Happen — Not happen.

$$P(E) = \frac{n(S)}{n(F)}$$

$$P(E) = \frac{3}{6} = \frac{1}{2} = 0.5 \rightarrow 0 \leq P(E) \leq 1$$

Experimental Probability Vs Estimated Probability:

- Experimental (or Empirical) Probability: Probability calculated based on actual experiments or trials.

Example:
If you flip a coin 100 times and get heads 55 times.

Estimated Probability –
→ Probability based on Predictions and theoretical assumptions.

Example:
For a coin toss, the Probability of getting heads is :

Probability Basics:

- Impossible Events
- Certain (or sure) Event

The Probability of any event is between 0 and 1.

$$0 \leq P(A) \leq 1$$

Terms in Probability:

SAMPLE SPACE:

- Set of all possible outcomes in a probability experiments.
- For instance, in a coin toss, it's "head" and "Taill" (H,T).

SAMPLE POINTS:

- One of these possible results in an experiments.
- For example, in rolling a fair six sided cube (dice), sample Points are 1 to 6.

EXPERIMENTS:

- A Process or trial with uncertain results.
- Examples include coin tossing, card selection, or rolling of a dice.

EVENT:

- A Specific result or set of

results we are intrested in.

- For example, getting an event number when rolling the dice

FAVORABLE OUTCOME:

- The outcomes that satisfy the Event.
- For the event of rolling an even number, the favorable outcomes are (2,4,6).....

Probability of Complementary Events:

$P(E) = 1 - P(E)$

Sample Space:

COINS ↪ (H,T) ⟿ ② times = 'n'

⊛ When 1 coin is tossed $\Rightarrow 2^1 = 2$

Total outcome = (H,T) ⇒ n(T) = 2

Total outcome $= 2^n$

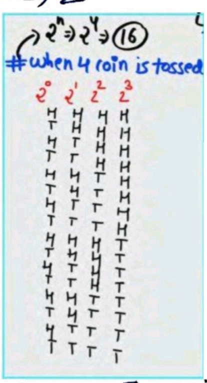
⊛ When 2 coin is tossed $\Rightarrow 2^2 = 4$

Total outcome (HH, HT, TH, TT) ⇒ n(T) = 4

⊛ When 3 coin is tossed $\Rightarrow 2^3 = 8$

⊛ When 4 coin is tossed

↪ $2^n = 2^4 = 16$



$2^2\ 2^1\ 2^0$

H	H	H
T	H	H
H	T	H
T	T	H
H	H	T
T	H	T
H	T	T
T	T	T

⊛ When 5 coin is tossed

↪ Total outcome = 5 $\Rightarrow 2^5 = 32$

DICE (1,2,3,4,5,6)

↪ 6^n = Total outcomes

⊕ When 1 dice is flipped.

↪ $6^n \Rightarrow 6^1 = 6$

Total outcomes = 6 (1,2,3,4,5,6)

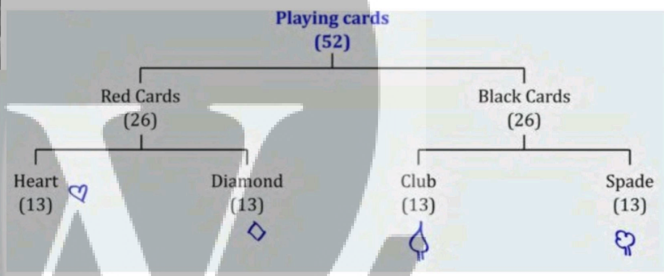
⊕ When 2 dice is flipped.

↪ $6^n \Rightarrow 6^2 = 36$

Total outcomes = 36

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) - - - - - -
(4,1) - - - - - -
(5,1) - - - - - -
(6,1) - - - - - -

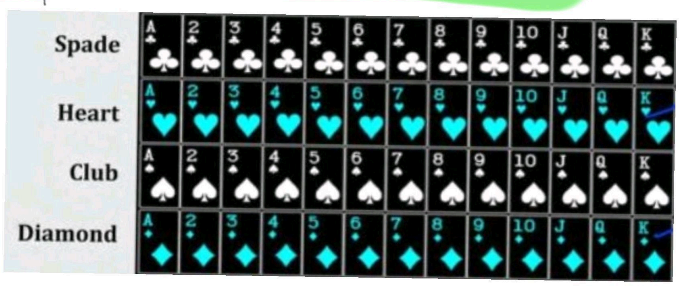
⊕ Playing Cards:



Cards (52)

Spade ③	Club ③	Diamond ③	Heart ③
1 King	1 King	1 King	1 King
1 Queen	1 Queen	1 Queen	1 Queen
1 Jack	1 Jack	1 Jack	1 Jack
1 Ace	1 Ace	1 Ace	1 Ace
2-10 Cards	2-10 Cards	2-10 Cards	2-10 Cards
Total = 13	Total = 13	Total = 13	Total = 13

Spade, Club, Heart, Diamond:



Event:
i.e. A subset of the sample space.
Example:
Drawing an Ace from a deck →
A = (Ace of Heart, Ace of Spades, Ace of clubs, Ace of diamonds)

Types of Events:

(I) Impossible Event:
Cannot happen ; ϕ (empty set)

(II) Sure Event:
Always happens ; Sample Space (S)

(III) Simple / Elementary Event:
only 1 outcome
Example: Drawing the "Queen of Hearts"

(IV) Compound Event:
More than one outcome.
Example: Getting HH or HT when tossing 2 coins.

(V) Mutually Exclusive:
cannot happen together
$P(A \cap B) = 0$
Example: Even and odd number on dice.

(VI) Exhaustive Events:
Together cover all outcomes
$A \cup B \cup C = S$

(VII) Mutually Exclusive or Exhaustive:
No overlap + cover everything
$A \cap B = \phi$, $A \cup B = S$.

(VIII) Complementary Event:
Not $A = A' = S - A$
$P(\text{not } A) = 1 - P(A)$

A or B (union):
Event $A \cup B$ = A or B or Both
$P(A \cup B)$

A and B (intersection):
Event $A \cap B$ = both A and B happen
$P(A \cap B)$

A but not B (A−B):
In A, not in B
$A - B = A \cap B'$

1. Random Experiment and Sample Space:

When we perform an experiment where all outcomes are known but which one occurs is uncertain it's called a random experiment.
The set of all possible outcomes is called the sample space (S).
Example:
Tossing two coin → S = (HH, HT, TH, TT)

2. Events:

Definitions: Any subset of a sample space is called an Event.
Example:
"Exactly one head" → $E = \{HT, TH\}$

3. Occurence of an Event:

If the outcome of an experiment belongs to the event set, then event occurs.
Example:
Throw a dice, E = (1, 2, 3) → if 2 appears, event E occurs.

Types Of Events:

(i) Impossible Event:

Definition: An event that cannot happen at all.

Set form: $E = \phi$

Example-1: Getting a multiple of 7 on a die → { } (No outcome possible)

Example-2: Getting a red king when you draw from a box of only black cards → Impossible.

(ii) Sure Events:

Definition: An event that always happens or includes all possible outcomes.

Set form: $E = SE = SE = S$

Example-1: Getting an odd or even number on a die → (1, 2, 3, 4, 5, 6)

Example-2: When you see toss a coin, getting head or tail → (H, T)

(iii) Simple Event:

Definition: An event that has only one outcome.

Example-1: Tossing two coins. Each of (HH), (HT), (TH), (TT) is a simple Event.

Example-2: Rolling a die and getting a 4 → {4}.

(iv) Compound Event:

Definition: An event that has more than one outcome.

Example-1: Tossing three coins

→ Event "At least one head"

E = {HTT, THT, TTH, HHT, HTH, THH, HHH}

Example-2: Rolling a die → Event "Even number appears"

E = (2, 4, 6).

Type of Event	Meaning	Example (Die / Coin)	Set Notation
Impossible	Cannot occur	Multiple of 7 on a die	(~~E = ∅~~ $E = \phi$
Sure	Must occur	Odd or even number on a die	(E = S)
Simple	One outcome only	Getting 4 on a die	(E = {4})
Compound	More than one outcome	Even number on a die	(E = {2,4,6})

2 Coin Toss:

e.g. Total possible outcomes = (HH, TT, HT, TH) → sample Space

(HH) (TT) | (HH, TT) | (HH, TT, TH) …

Events

* E_1 = (HH)
* E_2 = (TT)

E_3 = (HT)

→ simple Event (1 event)

E_4 = (HH, TT)

E_5 = (HT, TT)

E_6 = (HT, TT, HH).

→ compound event ↓ 2 or 2 more (event)

Algebra Of Events:

M. Imp

Probability

OR e.g. (+) | AND e.g. (×)

⇒ A or B — A ∪ B

A and B — A ∩ B

* (A − B)
* (A but not B)

→ $A \cap B'$

Type	Meaning	Notation / Example
Complementary Event	Event 'not A'	(A' = S - A)
A or B	Either A or B or both	(A ∪ B)
A and B	Both A and B occur	(A ∩ B)
A but not B	A occurs, B doesn't	(A - B = A ∩ B')

Example :

Rolling a die → $S = \{1,2,3,4,5,6\}$

A: Prime → $\{2,3,5\}$

B: Odd → $\{1,3,5\}$

* $A \cup B = \{1,2,3,5\}$
* $A \cap B = \{3,5\}$
* $A - B = \{2\}$
* $A' = \{1,4,6\}$

Mutually Exclusive Events:

Two or more events are mutually exclusive if they cannot happen at the same time that is

$A \cap B = \phi$ (No common outcomes)

Easy Example:

Rolling a die

A: Getting an odd number (1,3,5)

B: Getting an even number (2,4,6)

→ $A \cap B = \phi$

So A and B are mutually exclusive.

Eg: 1 coin flip

→ S.S = (H,T)

A = H] Dono ek saath nhi ho sakte
B = T

cannot happen at the same time ?

Mutually Exclusive.

Drawing one card from a deck:

A = Getting a king
B = Getting a Queen) $A \cap B = \phi$

You can't get both at once

→ Mutually exclusive.

Exhaustive Events:

Definitions:

Events are exhaustive if together they include all possible outcomes of the experiment.

That is,

$A \cup B \cup C = S$ (Their union = entire sample space)

Easy Example:

Rolling a die {sample space (S) = 1,2,3,4,5,6

A = Getting an even number (2,4,6)

B = Getting an odd number (1,3,5)

→ $A \cup B = \{1,2,3,4,5,6\} = S$

So A & B are Exhaustive Events.

* Tossing a coin

A: Head B: Tail → $A \cup B = \{H,T\} = S$

Exhaustive, since one must occur.

* Tossing 3 coins.

A: No Head = {TTT}

B: Exactly one Head = {HTT, THT, TTH}

C: At least two heads = {HHT, HTH, THH, HHH}.

→ $A \cap B = A \cap C = B \cap C = \phi$ and

$A \cup B \cup C = S$

Hence, A, B and C are mutually exclusive and exhaustive.

Q. Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

A : Only Heads on 3 faces (HHH)

B : Only Tails on 3 faces (TTT)

$A \cap B = \phi$

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

(ii) Three events which are mutually exclusive and exhaustive.

A : (TTT)

B : (HHT, HTH, THH, HHH)

C : (HTT, THT, TTH)

A ⇒ No heads

C ⇒ Exactly one head

B ⇒ Atleast two head

Mutually Exclusive

$A \cap B = \phi$

$B \cap C = \phi$

$C \cap A = \phi$

Exhaustive

$A \cup B \cup C = S$

(iii) Two events which are not mutually exclusive.

A : (HHH) → 3 heads on all faces.

B : HHT, HTH, THH, HHH

Atleast two heads

$A \cap B \neq \phi$ → Not exclusive.

Q. Two dice are thrown. The events A, B and C are as follows:

A : Getting an even number on the first die.

B : Getting an odd number on the first die.

C : Getting the sum of the numbers on the dice ≤ 5.

Describe the events :

i) A'

ii) not B

iii) A or B

iv) A and B

v) A but not C

vi) B or C

vii) B and C

viii) $A \cap B' \cap C'$

A = (2,1) (2,2) (2,3) (2,4) (2,5)
(4,1) - - - - - - -
(6,1) - - - - - - -

B = (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(3,1) - - - - - - -
(5,1) - - - - - - -

C = (11) (12) (21) (13) (31) (22) (14) (41) (23) (32)

C'(not C) =

Ans) A'(not A)
→ (11) (12) (13) (14) (15) (16)
(3,1) - - - - - - - -
(5,1) - - - - - - - -

(vii) B and C

$B \cap C$ ⇒ (1,1) (1,2) (1,3) (3,1) (1,4) (3,2)

$A \cap B'$ = (2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(4,1) - - - - - -
(6,1) - - - - - -

Q. State true or false: (give reason for your answer).

(i) A and B are mutually exclusive. $A \cap B = \phi$

(ii) A and B are mutually exclusive and exhaustive. $A \cup B = S$

(iii) $A = B'$ ✓

(iv) A and C are mutually exclusive $A \cap C \neq \phi$

(v) A and B' are mutually exclusive. $A \cap B = \phi$

(vi) A', B', C are mutually exclusive and exhaustive. F

A = Even no. 1 dice

B = odd no. 1 dice

C = ≤ 5.

Q. A fair coin is tossed four times, and a person win Re. 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amount.

① H, H, H, H
₹1, ₹1, ₹1, ₹1
→ ₹4 $\frac{4}{16}$

② H, H, H, T
₹1, ₹1, ₹1 − ₹1.50
→ ₹1.50 $\frac{3}{16}$

③ H, H, T, T
₹1 ₹1 −₹1.50 −₹1.50
→ −₹1 $\frac{2}{16}$

④ H, T, T, T
₹1 −₹1.50 −₹1.50 −₹1.50
−₹3.5 $\frac{1}{16}$

⑤ T, T, T, T
−₹1.5 −₹1.5 −₹1.5 −₹1.5
−₹6

Total जीत
₹4
₹4
₹4
+ ₹4
₹16

Q. Check whether the following probabilities P(A) and P(B) are consistently defined

(i) $P(A)=0.5$
$P(B)=0.7$
$P(A\cap B)=0.6$

$P(A)+P(B)=1.2$ ✗✗✗

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$=0.5+0.7-0.6$
$=1.2-0.6$
$=0.6$

Q. Check whether the following probabilities P(A) and P(B) are consistently defined. $0\le P(E)\le 1$

(ii) $P(A)=0.5$, $P(B)=0.4$
$P(A\cup B)=0.8$

$P(A)+P(B)=0.9$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$0.8=0.5+0.4-P(A\cap B)$
$P(A\cap B)=0.9-0.8$
$P(A\cap B)=0.1$ ✓

Q. Given $P(A)=\frac{3}{5}$ and $P(B)=\frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

$P(A \text{ or } B)=P(A)+P(B)-P(A\cap B)$
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$=3/5+1/5-0$ $\quad P(A\cap B)=0$
$P(A \text{ or } B)=4/5$ Ans.

Q. If E and F are events such that $P(E)=\frac{1}{4}$, $P(F)=\frac{1}{2}$ and $P(E \text{ and } F)=\frac{1}{8}$, Find (i) P(E or F)
(ii) P(not E and not F).
→ De-morgan Law

$P(\text{not } E)=1-\frac{1}{4}$
$P(\bar{E})=\frac{3}{4}$

$P(\text{not } F)=1-\frac{1}{2}$
$P(\bar{F})=\frac{1}{2}$

$P(E \text{ or } F)=P(E)+P(F)-P(E \text{ and } F)$
$=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}$
$=\frac{2+4-1}{8}$
$P(E \text{ or } F)=\frac{5}{8}$

$P(E\cup F)=\frac{5}{8}$

$P(E\cup F)'=P(E'\cap F')$
$P(E'\cap F')=P(E\cup F)'$
$=1-P(E\cup F)$
$=1-\frac{5}{8}$
$=\frac{3}{8}$ Ans.

Q. Events E and F are such that $P(\text{not } E \text{ or not } F) = 0.25$. State whether E and F are mutually exclusive.

$P(E \cap F) = 0.25$

$P(E \cap F) \neq 0$ so $\rightarrow$ not mutually exclusive.

$P(E' \cup F') = 0.25$

$P(E' \cup F') = P(E \cap F)'$

$0.25 = 1 - P(E \cap F)$

$P(E \cap F) = 1 - 0.25$

$P(E \cap F) = 0.75$.

De-Morgan Law.

* $(E \cup F)' = E' \cap F'$
* $(E \cap F)' = E' \cup F'$