ELECTRIC CHARGES AND FIELDS

Electric Charges

Electric charges are the physical property of matter that causes it to experience a force when placed in an electromagnetic field.

Charges → SI Unit = Coulomb

- +ve → Proton
- −ve → Electron

- Like charges repel
  - ⊕ and ⊕ → repel
  - ⊖ and ⊖ → repel

- unlike charges attract
  - ⊕ and ⊖ → attract
  - ⊖ and ⊕ → attract

Conductors

Substances which allows electricity to pass through them

Insulators

Substances which doesn't allow electricity to pass through them

Earthing :

The process where excess of charge from a body goes to ground by touching the charge carrying conducting body to Earth is called Earthing.

Charge by Contact

When a charged object is touched to another object, the other object also gets charged with same polarity due to charge transfer.

Charge by Induction

When a charged object is brought closer to another object (without touching), the original object doesn't lose any charge and other object get charged as well with oppasite polarity. The other extreme end of the newly charged object develops polarity same as that of the charged object.

Basic Properties of Electric Charge

1. Additivity of Charges
2. Conservation of Charges
3. Quantization of Charges

Additivity:

Charges are scalars and can be added algebraically. For, eg, if point charges are q_1, q_2, q_3, q_n. Then,

$$q_{TOTAL} = q_1 + q_2 + q_3 + q_n$$

Conservation :

The net charges on a closed isolated system is always conserved. Thats conservation of charges.

Quantization :

The electric charge is always an integral multiple of e is termed as quantisation of charge. Thus, charge q on a body is always given by $q = ne$. SI unit of q is Coulomb (C).

- The value of e is 1.602×10^{-19} C.
- There are about 6×10^{18} electrons in a charge of $-1C$.

$$\boxed{\text{Charge} = ne}$$

where, n = integer
e = basic unit of charge

* e could be positive as proton $= +e$
and negative as electron $= -e$

6 July

Coulomb's Law

Coulomb's law states that Force exerted between two point charges :

- is inversely proportional to square of distance between them.
- is directly proportional to product of magnitude of the two charges.

$\therefore F = \dfrac{k|q_1 q_2|}{r^2}$, where $k = \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \dfrac{Nm^2}{C^2}$

So, $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{|q_1 q_2|}{r^2}$, Here, $\varepsilon_0 = 8.854 \times 10^{-12} \dfrac{C^2}{Nm^2}$ is called permittivity of free space.

Coulomb's law can be represented in vector form as,

$$F_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$F_{12} = -F_{21}$$

∴ Thus, coulomb's law agrees with newton's third law

Forces between multiple charges - Superposition principle

As per principle of superposition, the force on any charge due to a number of other charges is the vector sum of all the forces on that charge due to other charges, taken one at a time.

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$F_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{13}^2} \hat{r}_{13}$$

$$\therefore F_1 = F_{12} + F_{13} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}\right]$$

Electric Field

Electric field is a force produced by a charge near its surroundings. This Force is exerted on other charges when brought in vicinity of this field.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \quad \text{and} \quad \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r^2} \hat{r}$$

$$\therefore F = qE$$

SI unit of Field = $\frac{N}{C}$

Electric field due to a charge at a point is the force that a unit positive charge would experience if placed at that point.

Electric field due to system of charges

According to the superposition principle, the total electric field at a point in space is equal to the vector sum of individual fields present

~~Electric~~

Electrostatic Force due to continuous charge distribution

① Linear charge distribution

⇒ λ = linear charge density

$\therefore \lambda = \frac{\Delta Q}{\Delta l}$

② Surface charge distribution

⇒ σ = surface charge density

$\therefore \sigma = \frac{\Delta Q}{\Delta S}$

③ Volume charge distribution

⇒ ρ = volume charge density

$\therefore \rho = \frac{\Delta Q}{\Delta V}$

Electric field due to a Ring

$$E_x = \frac{kQx}{(R^2 + x^2)^{\frac{3}{2}}}$$

Electric field due to an arc

$\left(\lambda = \frac{Q}{\cancel{2}\theta R}\right)$

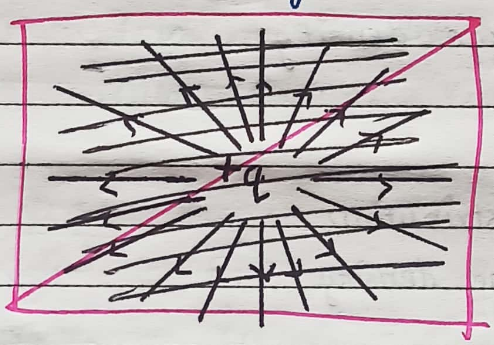
$$E_y = \frac{2k\lambda}{R}\sin\theta$$

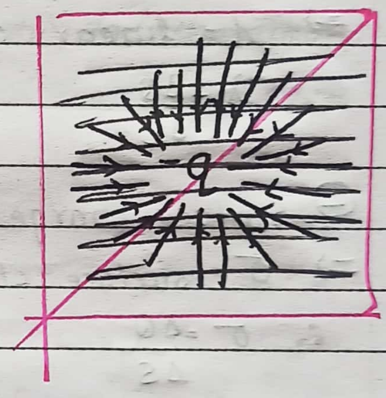
Electric Field Lines

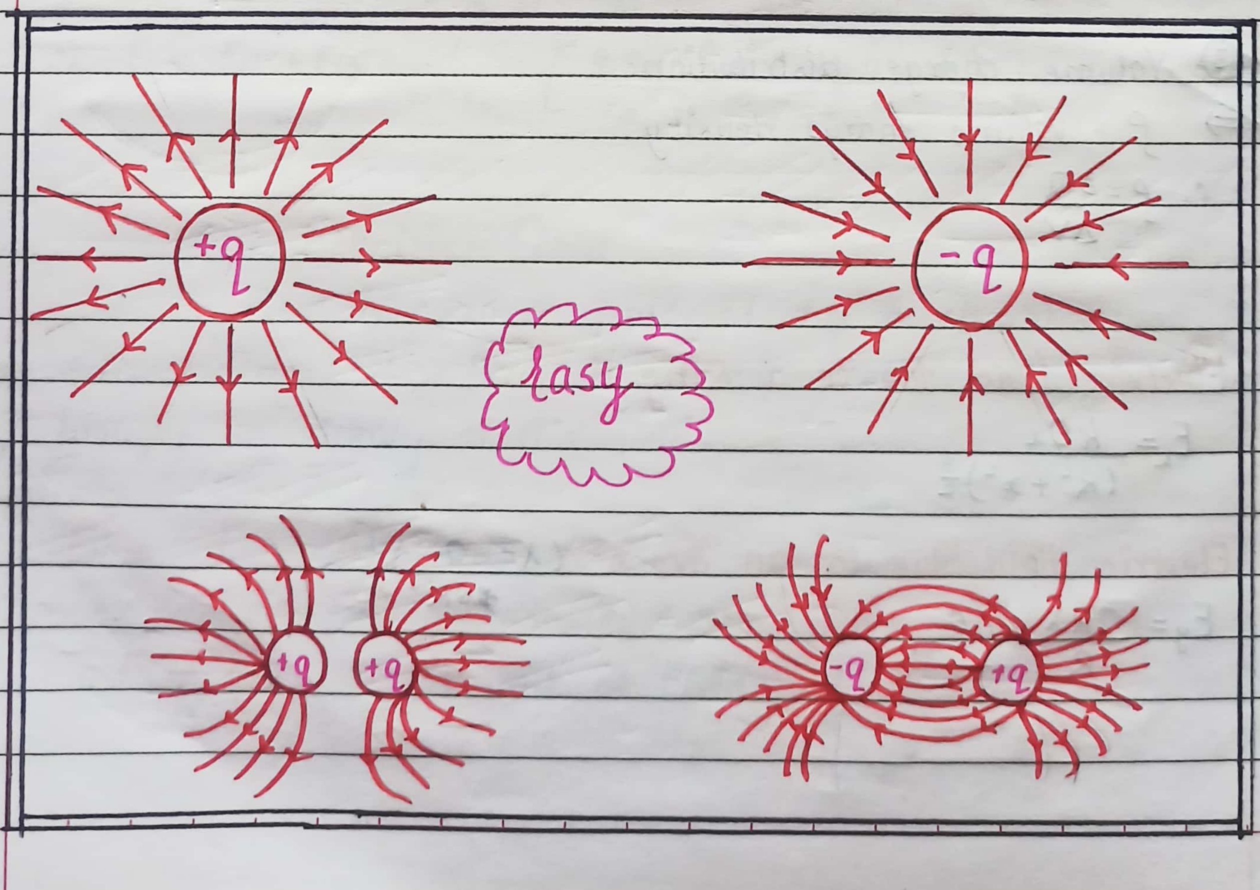
The concept of electric field was introduced by michael faraday. Electric field lines are a way of pictorially mapping the electric field around a configuration of charges.

- This lines can start from positive charge and end on infinite or can start from infinite and end on negative charge.
- The tangent at any point on these lines gives direction of field at that point

Representation of Electric Field Lines due to positive and negative charges:







Some More Properties of Field Lines :

1. Two Fields lines cannot intersect.
2. Field lines cannot have non differentiable points.
3. Field lines should be continuous.
4. Electrostatic field cannot form any kind of loop or circulate.
5. In general, the trajectory of a charged particle need not follow a field line.

Electric Flux

Electric flux is the number of field lines that intersect at a given area. Electric flux can be defined as, ϕ,

$d\phi = \int \vec{E} \cdot \vec{ds}$ $\therefore \phi = E \cdot ds \cos\theta$

- The SI unit of Flux is Nm^2/C.
- The net flux of a closed body is zero.

~~Gauss's Law~~

Flux through a sphere:

$\cancel{d\phi = \vec{E} \cdot \vec{ds}}$

$\cancel{d\phi = Eds}$

$\cancel{\phi = EA_{sphere}}$

$\cancel{\phi = kq4\pi}$ $\phi = \frac{kq}{R^2} 4\pi R^2$

$\cancel{\phi = q4\pi}$

$\cancel{\phi = kq4\pi}$

$\cancel{\phi = \frac{q4\pi}{4\pi\varepsilon_0}}$

$\cancel{\phi = \frac{q}{\varepsilon_0}}$

DERIVATION :) Gauss Law

$d\phi = \int \vec{E} \cdot \vec{dS}$, $\therefore \phi = E S_{urface}$

For a Sphere

$\phi = E A_{sphere}$

$\phi = \frac{kq}{R^2} \times 4\pi R^2$

$\phi = kq4\pi$

$\phi = \frac{q4\pi}{4\pi\varepsilon_0}$ $\left(\because k = \frac{1}{4\pi\varepsilon_0}\right)$

$\phi = \frac{q}{\varepsilon_0}$

$\phi = \frac{q}{\varepsilon_0}$, That's how its derived

Gauss's Law

Gauss Law states that the total electric flux through a closed surface is zero if no charge is enclosed by the surface.

Electric flux $(\phi) = \dfrac{q}{\varepsilon_0}$ (q = Total charge enclosed by surface)

Properties:

- Gauss's law is true for any closed surface, no matter what its shape and size.
- The charge q is the sum of all charges enclosed by the surface. The charge may located anywhere inside surface.
- Gauss law is based on inverse square dependence on distance contained in the Coulomb's Law.

Application of gauss's Law:

For some symmetrical charge configurations, however, it is possible to obtain the electric field in a simple way using the gauss's law. This is best understood by some examples.

~~Field due to a uniformly charged thin spherical shell:~~

~~Case I:~~

~~If~~ $\cancel{r \geq R}$

~~Then,~~ $\cancel{\vec{E} = \dfrac{kQ}{r^2}}$

~~Case II:~~

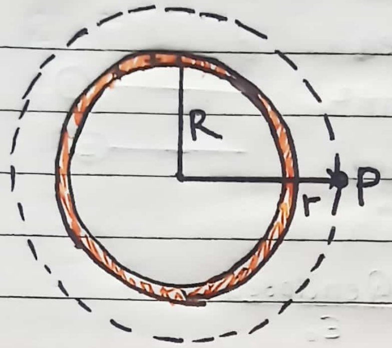
~~If~~ $\cancel{r \leq R}$ $\cancel{r < R}$

~~Then,~~ $\cancel{Q_{enclosed} = zero}$

~~i.e.~~ $\cancel{\vec{E} = 0}$

Field due to a uniformly charged thin spherical ~~sph~~ shell:

Field outside the shell:



Due to spherical symmetry, the electric field at each point of the gaussian ~~s~~ surface has same magnitude.

$\phi = |\vec{E}| \int_{surface} |d\vec{S}|$, $\therefore \phi = |\vec{E}| \cdot \text{Area}$

$\phi = |\vec{E}| \cdot 4\pi r^2$ —— ①

and $\phi = \dfrac{q_{enclosed}}{\varepsilon_0}$ .... (Gauss's Law) —— ②

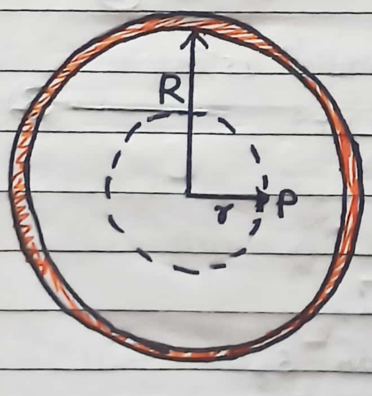
Compare ① and ②,

$$|\vec{E}| \cdot 4\pi r^2 = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\therefore |\vec{E}| = \frac{q}{\varepsilon_0} \cdot \frac{1}{4\pi r^2}$$, It can be written as,

$$\boxed{|\vec{E}| = \frac{q}{4\pi\varepsilon_0 r^2}}$$

Field inside the shell:



Net Flux = Charge enclosed

$|\vec{E}| \cdot 4\pi r^2 = 0$ (since gaussian surface encloses no charge)

$\therefore |\vec{E}| = 0$

Thus, electric field due to uniformly charged thin shell is zero at all points.

$$\boxed{|\vec{E}| = 0}$$

Field due to uniformly charged sphere :

~~Inside~~ Outside the sphere :

$$\phi = \vec{E} \cdot \text{Area}$$

$$\phi = \vec{E} \cdot 4\pi r^2 \quad \text{———— (1)}$$

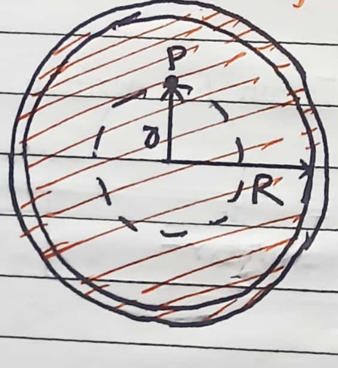
$$\phi = \frac{Q_{enclosed}}{\varepsilon_0} \quad \text{———— (2)}$$

$$\vec{E} \cdot 4\pi r^2 = \frac{Q_{enclosed}}{\varepsilon_0}$$

$$\vec{E} = \frac{1}{4\pi r^2} \cdot \frac{Q}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

~~Outside~~ Inside the sphere :



$$|\vec{E}|_r = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

Now,

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad \text{(For Bigger sphere) ———— (1)}$$

$$\rho = \frac{q}{\frac{4}{3}\pi r^3} \quad \text{(For smaller sphere) ———— (2)}$$

It can be written as,

$$q = \rho \cdot \frac{4}{3}\pi r^3 \quad \text{———— (3)}$$

From (1), (2) and (3),

~~put value of q (eq. 3) in equation 1,~~

put value of ρ (equation 1) in equation 3,

$$q = \frac{Q}{\cancel{\frac{4}{3}}\cancel{\pi} R^3} \cdot \cancel{\frac{4}{3}}\cancel{\pi} r^3 \quad \therefore q = \frac{Q r^3}{R^3} \quad \text{———— (4)}$$

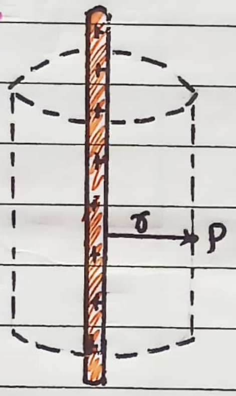
$$|\vec{E}|_r = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

put equation 4 in above equation,

$$|\vec{E}|_r = \frac{1}{4\pi\varepsilon_0} \times \frac{Qr^3}{R^3} \times \frac{1}{\cancel{r^2}}$$

$$\boxed{|\vec{E}|_r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^3} (r)}$$

Field due to infinitely long straight uniformly charged wire :



Flux through the two ends of gaussian surface is 0.
$\vec{E}$ is perpendicular to the surface with constant magnitude throughout the length.

$\phi = |\vec{E}| \int ds$

$\phi = |\vec{E}| \cdot Area$

$\phi = |\vec{E}| \cdot 2\pi rh$ —— ①

$\cancel{\phi} = \cancel{\frac{Q_{enclosed}}{\varepsilon_0}}$

$\phi = \frac{Q_{enclosed}}{\varepsilon_0}$ —— ②

$\lambda = \frac{Q_{enclosed}}{h}$

$\therefore Q_{enclosed} = \lambda h$

~~(Charge enclosed)~~

From ① and ②,

$|\vec{E}| \cdot 2\pi rh = \frac{Q_{enclosed}}{\varepsilon_0}$ —— ③

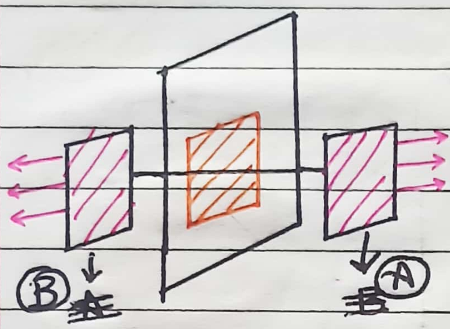
Put Q value in equation ③,

$$|\vec{E}| \cdot 2\pi rh = \frac{\lambda h}{\varepsilon_0}$$

$$|\vec{E}| = \frac{\lambda \cancel{h}}{2\pi r\varepsilon_0 \cancel{h}} = \frac{\lambda}{2\pi r\varepsilon_0}$$

$$\boxed{\therefore |\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Field due to a uniformly charged Infinitely plane sheet:



The total flux would be combination of faces A and B.

$\phi_A = |\vec{E}| . Area$

$\phi_B = |\vec{E}| . Area$

$\phi_{A+B} = 2|\vec{E}| Area$

$= 2EA$

Charge enclosed $= \sigma A$

$$\phi = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$\therefore$ Now, $2|\vec{E}|\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$

$$\boxed{\therefore |\vec{E}| = \frac{\sigma}{2\varepsilon_0}}$$

Some Common Values

PROTON	Mass	1.67×10^{-27} kg
	Charge	$+1.602 \times 10^{-19}$ C
ELECTRON	Mass	9.11×10^{-31} kg
	Charge	-1.602×10^{-19} C
k (coulombs law constant)		9×10^{9} $\frac{Nm^2}{C^2}$
ε_0 (Permittivity of free space)		8.854×10^{-12} $\frac{C^2}{Nm^2}$

Application of Gauss Law

Cases	Electric Field	
Uniformly charged thin spherical shell	Field outside the shell	$\frac{q}{4\pi\varepsilon_0 r^2}$
	Field inside the shell	0
Uniformly charged Sphere	Field outside the sphere	$\frac{Q}{4\pi\varepsilon_0 r^2}$
	Field inside the sphere	$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R^3} (r)$
Infinitely long straight uniformly charged wire	$\frac{\lambda}{2\pi\varepsilon_0 r}$	
Uniformly charged Infinitely Plane sheet	$\frac{\sigma}{2\varepsilon_0}$ $\frac{\sigma}{2\varepsilon_0}$	